Chapter 5

REMEDIATION OF THE LEARNING DIFFICULTIES AND DATA ANALYSIS

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5.1 INTRODUCTION

The diagnosis of the learning difficulties is an incomplete task. In order to complete the process it has to be followed by the remediation of the identified learning difficulties. Diagnosis and remediation goes hand in hand. This chapter gives the description about the entire remedial programme. It includes the detailed schedule of the remedial programme that was carried out by the investigator.

Further, the implementation of the parallel test and the comparative analysis of the performance of the students on achievement test and the parallel test are included in this chapter.

5.2 REMEDIAL PROGRAMME

Remedial programme was conducted with the purpose of reducing the learning difficulties of the students in geometry. It was planned and implemented by the investigator on the group of the students considered for the identification of the learning difficulties. There were thirty-five students initially but thirty-three students were present regularly during the entire remedial programme.

5.2.1 Remedial Programme Schedule

Remdial programme was carried out for forty hours, spread over seven weeks. It was carried out for twenty days, two hours daily and three days per week. The detailed schedule is displayed in the table format as below.

Table – 18

Remedial Programme Schedule

Day	Date	Content
1	14-02-2008	Knowing the students and general understanding about their learning in geometry
2	15-02-2008	Concept of Point and Line

3	16-02-2008	Relationship between point and Line Collinear and Non-collinear points
4	21-02-2008	Betweenness relation of points
5	22-02-2008	Concept of line-segment Mid-point and Bisector of the line-segment
6	23-02-2008	Distance, length of the line-segment and Congruent line-segments
7	28-02-2008	Concept of ray Opposite Rays
8	29-02-2008	Relationship between Point, Line, Line-segment, and Ray
9	01-03-2008	Intersection of Lines, Line-segments, and Rays in any combination
10	07-03-2008	Concept of Plane and Intersection of Planes
11	08-03-2008	Relationship of Point, Line and Plane Coplanar and non-coplanar points Coplanar and non-coplanar lines
12	13-03-2008	Parallel lines, Parallel Planes, Skew lines
13	14-03-2008	Partitions of plane by a line Half-planes and closed half-planes
14	15-03-2008	Worksheet on Plane
15	17-03-2008	Concept of an Angle, Measure of an Angle, Arms and Vertex of an Angle
16	18-03-2008	Partitions of the plane by an angle, Interior and Exterior of an angle
17	19-03-2008	Types of angles and types of pair of angles
18	24-03-2008	Worksheet on angle
19	25-03-2008	Symbols and geometric Figures
20	26-03-2008	Open Session with the students

5.2.2 Day-wise Implementation of the Remedial Programme

Day 1:

Knowing the students and general understanding about their learning in geometry

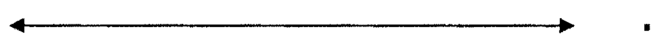
All the students introduced themselves. Investigator tried to become familiar with all of them by having some informal talk with them about mathematics and their feeling about mathematics as a subject. A rapport was build up with the students by making them feel comfortable and natural. It was informed to the students that few classes in geometry will be conducted by the investigator. The purpose of the programme was not disclosed to the students.

A simple game was played with the students. They were asked to take a piece of paper where they have to write as many geometrical terms as they know in five minutes. It was observed that almost all the students are aware about fifteen to eighteen of the geometrical terms viz. point, line, line-segment, parallel lines, intersecting lines, collinear points, non-collinear points, plane, angle, line-segment, ray, half-plane, end-point, mid-point, coplanar lines, non-coplanar lines, coplanar points, non-coplanar points, skew lines, bisector, arms. These terms were written on the black-board and voluntarily students were asked to talk (define or explain or comment) about the geometrical terms written on the black-board. The observation was that only eight students participated voluntarily out of that only two students talked correctly about the four geometrical terms i.e. (i) Parallel lines are not intersecting lines, (ii) Points which are not co-planar are non-coplanar points, (iii) An angle has two arms, (iv) line-segment has two end-points. Investigator could understand that the actual learning of students in geometry was not satisfactory and needs some remedial inputs.

Day 2:

Concept of Point and Line

It was explained to the students that point is a term which can be explained and has got a universal meaning but it is not a defined term. It is represented by a dot. A dot was put on the blackboard and it was explained that it's not a point but it's the representation of the point which is named by a capital alphabet. It was also made clear to the students that point is an undefined term.

Also, the concept of line was explained to the students. A student was called and was asked to draw a line. A point was kept outside the line as shown in the figure.  It was asked to the student does this point lie on the line drawn. Students said no the point does not lie on the line. This was clarified by extending the line and it was explained to the students that line is extended on both the sides. Then the students were asked how far a line can be extended by asking them to demonstrate. The student extended further and stopped at the edge of the blackboard. Investigator probed into and explained that line is extended on both the sides infinitely and line is an undefined term. Also line as a set of points extended infinitely was illustrated i.e. for two points A and B on the line ℓ it is a set of points A, B, all the points between A and B, and the points beyond the points A and B. Similarly with the help of the figure the symbolic representation of line was explained to the students i.e. ℓ or ℓ_1. For points A, B, C and D on the line ℓ the line can be represented as $\overleftrightarrow{AB}$ also i.e. $\ell = \overleftrightarrow{AB}$ and $\overleftrightarrow{AB} = \overleftrightarrow{CD}$, equality of line and distinct lines were explained to the students.

Day 3:

Relationship between point and Line, Collinear and Non-collinear points

Here the teaching aid was used which was prepared with the help of a flannel board on which different objects made from velvet paper can be stuck. Points and lines were prepared with the velvet paper which can easily stay on the flannel board. Different situations were given to the students and they were asked to come forward and display different situations between point and line as per their understanding with the help of this teaching aid. Simultaneously, it was discussed with all the students whether the situation is correct or not. All the postulates of line were explained to the students with the help of figures and one & only one line passes through two distinct points as line is determined by two distinct points was illustrated.

Also, it was clarified to the students that for the points on the line it is symbolically represented as a point belongs to a line and not as a subset of a line i.e. " $A \in \ell$, where A is a point on the line ℓ and $A \notin \ell$, where B is not a point on the line ℓ". Further, Collinear and non-collinear points were illustrated with the help of a teaching aid. An understanding about any two points are collinear was developed among the students and it was explained to them that in case of three or more points the question of collinearity arise. Also the misconception that if a point is given but no line is drawn passing through that point in a figure means that no line passes through that point was clarified. In fact, each point lies on some line and infinitely many lines passes through a given point was explained to the students through a group discussion on their own views about a point and no line drawn on the black-board.

Day 4:

Betweenness relation of points

A figure was drawn on the blackboard having a line and three points on it and the understanding about the betweenness relation of points was revised i.e for A-B-C, points A, B, and C are collinear and point C is in between A and C. Later seven groups were formed in the class having four to five students in each group. Students of each group were asked to draw a figure in their note-book based on the instructions given by the investigator as follows:

1. Draw a line
2. Name the line as ℓ
3. Put four points on the line
4. Name the points as A, B, C, D

5. Put two points outside the line ℓ such that they are collinear with the point B
6. Name these points as E and F
7. List all the betweeness relation of these points

Students' group work was monitored and their doubts were clarified in small groups by taking a round in the class. It was observed that they faced a difficulty in demonstrating the instruction number (5) i.e. putting points collinear outside the line. They were helped by the investigator. Afterwards the discussion was carried out where group-wise they presented their list of betweenness relation among the points. This was followed by a probing by the investigator on different possibilities based on the collinear points and making them understand that there can be betweeness relation for four collinear points too and also for the collinear points not shown on a particular line in the figure. Finally all the groups listed the betweenness relations viz. A-B-C, B-C-D, A-B-D, A-C-D, A-B-C-D. At the end few of the betweenness relation which were incorrect and B-C-D is same as D-C-B were clarified.

Day 5:

Concept of line-segment, Mid-point and Bisector of the line-segment

It started with their understanding about the line-segment. A figure of line-segment was drawn on the black-board as:

Questions were asked to the students regarding the end-points of the line-segment viz. How many end-points are there? Which are they? One point was kept on the line-segment 'C' and again few more questions were asked to the students about how many line-segments are there in the figure? Which are they? and so on. During this question-answer session their doubts regarding the end-points, number of end-points, equal line-segments were clarified. This was followed by the relation of a line-segment and a line by drawing a figure where ℓ was a line and A, B were two points on the line. It was explained to the students that $\overline{AB}$ is a subset of line ℓ and symbolically it is represented as $\overline{AB} \subset \ell$. Further, the set representation of $\overline{AB}$ was explained to the students and all the points in between A and B lying on the $\overrightarrow{AB}$ including points A and B is a $\overline{AB}$, so end-points of the line-segment belongs to $\overline{AB}$ was clarified to the students i.e. $\overline{AB}$ = {A,B} $\cup$ {P/ A-P-B} ; A $\in$ $\overline{AB}$, B $\in$ $\overline{AB}$.

Later, the concept of mid-point and bisector of the line-segment was explained to the students i.e. mid-point is the point in between the end-points of the line-segment which divided the line-segment in two congruent parts (equal lengths). Similarly bisector is a line or a line-segment or a ray which passes through the mid-point of the line-segment (or intersects the line-segment at the mid-point) so that it divides the line-segment in two equal parts. This was illustrated with the help of the figures.

Finally it was instructed that how to find-out the mid-point will be discussed in the next class.

Day 6:

Distance, length of the line-segment and Congruent line-segments

Initially with the help of the figure having a line and the points lying on the line the concept of distance was revised. This was followed by the explanation considering the following points:

Distance between any two points is always positive.

Length of the line-segment is the distance between the two end-points of the line-segment and is always positive.

The teaching aid of the number-line made from the card-board was displayed on the wall and the pushpins were used to represent the points on the number-line. With the help of the teaching –aid the origin of the line, positive and negative direction was explained to the students. Further, Students were involved in finding out the distance between different points represented on the number-line. Afterwards it was explained to the students that the distance between two points can be found by finding the modulus of the difference between the values associated to the points i.e. For points A and B distance between A, B is the length of the $\overline{AB}$ and it is represented as AB which is equal to |a-b| where a and b are the numerical value associated with the points A and B respectively on the number-line. During the calculation work the algebraic rules that a-(-b)=a+b; a,b > 0 and (-a)-(-b)=-a+b (& not (-a)-b; a,b > 0) were clarified.

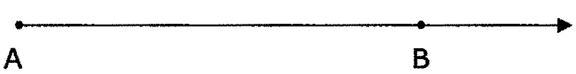
Apart from this it was clarified to the students that the two line-segments having same length are called congruent line-segments. The difference between the same line-segments and congruent line-segments was clarified. Also, students demonstrated their understanding by answering the questions asked by the investigator based on the different situations represented on the teaching-aid of number-line. This understanding of the students was extended to the situations drawn on the black-board which is just a line and points and not the number-line having markings per unit distance. They were given the practice of finding the distance based on the values associated to the points.

At the end of the session students were explained about finding the mid-point of the line-segment with the help of the length of the line-segment and dividing it by two and then locating it on the line. It was also clarified that the line cannot have a bisector or a mid-point by making students play a small game where seven students have to stand in a horizontal line each student is representing a point in the line they have made. One student is asked to locate the student which is a mid-point. Others were asked to join at the two sides continuously as line is extended infinitely. So it was observed by the students that the mid-point (student at the middle of the line) keeps on changing and hence a line does not have a mid-point or a bisector.

Day 7:

Concept of ray, Opposite Rays

The concept of ray in terms of it's a defined term, it has an initial point or one-end-point, it is extended infinitely on one side was revised with the help of the geometrical figure of ray as below.



The geometrical representation of the ray was clarified i.e. $\overrightarrow{AB}$, where A and B are the points on the ray, A is an initial point and A$\in \overrightarrow{AB}$. Based on the figure it was explained to the students how $\overline{AB}$ is a subset of $\overrightarrow{AB}$. Further, the set representation of ray was explained to the students i.e. $\overline{AB}$ $\cup$ {P/ A-B-P}. After this the following points were clarified to the students:

Equality of rays i.e. For B-A-Y, $\overrightarrow{BA} = \overrightarrow{BY}$ and $\overrightarrow{BY} \neq \overrightarrow{AY}$

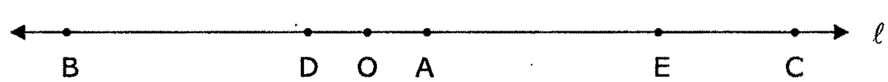
$\overrightarrow{AB} \neq \overrightarrow{BA}$ and the difference between $\overrightarrow{AB}$ & $\overrightarrow{BA}$

In case of opposite rays, the significance of all the three conditions was explained to the students viz. (i) both the rays should lie on the same line, (ii) both the rays should have same initial point and (iii) both the rays should be in opposite direction, by showing the possibility of the situation if one of these condition is not satisfied. So, the definition of the opposite rays was explained to the students i.e. For A-O-B, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are opposite rays. Finally the difference between the two distinct rays and opposite rays was illustrated. Also, it was explained to the students that opposite rays are intersecting as they have the common initial point so its intersection will be a singleton set of initial point but intersecting rays are not always opposite rays.

Day 8:

Relationship between Point, Line, Line-segment, and Ray

This session was devoted to the holistic understanding about the interrelation among the point, line, line-segment and ray. Here the simple statements were asked to the students and they have to say whether its correct or incorrect based on the figure below:



1. D $\in \overleftrightarrow{AE}$
2. $\overrightarrow{DA} \subset \overleftrightarrow{EC}$
3. C$\in \overrightarrow{DA}$

6. $\overrightarrow{AE} \subset \overrightarrow{OA}$
7. $\overline{DE} \subset \ell$
8. $\overrightarrow{AO} \subset \ell$

4. $\overline{AO} \subset \overline{AD}$

5. A $\in \overline{DO}$

9. $\overline{EC} \subset \overleftrightarrow{BO}$

10. $\overrightarrow{BO} = \overleftrightarrow{EC}$

Discussion was carried out based on the responses of the students in the class. The following points were clarified to the students by correcting their responses:

For the points lying on line,

- Each point belongs to the line
- A line-segment formed by any two points is a subset of the line
- A ray formed by any two points is a subset of the line
- A line formed by any two points is the same line
- A point lying in between the end-points of the line-segment always belongs to the line-segment
- A point after the initial point of the ray and towards the ray always belongs to the ray
- Two rays with the same initial point and in same direction are equal
- Two line-segments having same end-points are equal

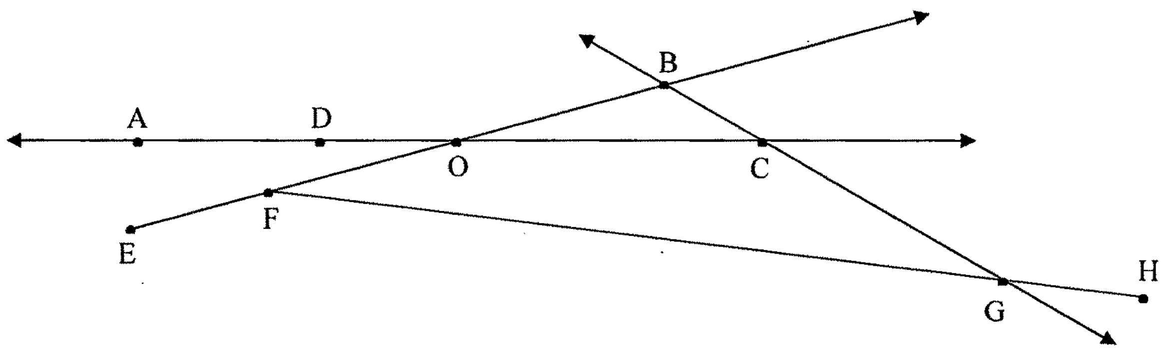
At the end the difference between the geometrical concepts viz. point, line, line-segment, and ray was described

Day 9:

Intersection of Lines, Line-segments, and Rays in any combination

The session started with the simple understanding of the students about the term intersection with respect to the 'Set Theory'. Investigator with the help of students understanding revised that intersection of two sets is the set of common elements to both the sets. Further, it was brought to students' notice that all the geometrical terms are set of points. So, the intersection of any two geometrical terms is set of common points to both of it. This was followed by some examples of intersection of lines, line-segments and rays. Afterwards students were divided in the group of fours and a worksheet on the intersection was distributed wherein they have to fill it up with their responses and later have to share their answers. The worksheet given was as follows:

Look at the figure below and answer the following questions.



$\overleftrightarrow{AD} \cap \overleftrightarrow{DC} =$ ________	$\overleftrightarrow{AD} \cap \overleftrightarrow{CG} =$ ________	$\overrightarrow{EO} \cap \overrightarrow{GB} =$ ________
$\overrightarrow{FB} \cap \overrightarrow{CG} =$ ________	$\overline{AD} \cap \overline{AC} =$ ________	$\overline{DO} \cap \overrightarrow{EF} =$ ________
$\overline{FH} \cap \overline{EO} =$ ________	$\overrightarrow{EB} \cap \overrightarrow{DA} =$ ________	$\overline{OC} \cap \overline{FG} =$ ________
$\overline{FG} \cap \overline{FH} =$ ________	$\overline{OC} \cap \overrightarrow{CG} =$ ________	$\overrightarrow{EO} \cap \overline{OB} =$ ________

Based on the students responses on the above worksheet questions the discussion was carried out where the following points were clarified to the students:

- Intersection of two lines is either a point or an empty set was not understood
- If the intersection of two lines is a line then it is a same line.
- Intersection can never be a line except both are the same line.
- Intersection of two lines can never be a line-segment or plane
- Intersection of a line and a line-segment is a point or an empty set or a line-segment. If its line-segment then it has to be a subset of a line.
- The point of intersection is common to both the lines / line-segments / rays
- Intersection of two line-segments is a line-segment or a point or an empty set
- Intersection of two rays is a point or a line-segment or a ray or an empty set.

- Intersection of two intersecting rays in the same direction is a ray. Intersection of two intersecting rays in opposite direction is a line-segment or a point.
- Intersection of line-segment & ray or two lines or two line-segments can never be a ray

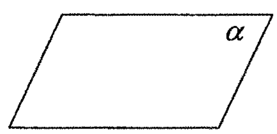
Simultaneously the misconceptions held by the students regarding the intersection were clarified. Especially, the misconception that a line/ray passing through both the lines/line-segments/rays is the intersection was removed from the students.

Towards the end students were asked to illustrate not intersecting lines, line-segments and rays on the black-board. It was explained that in case of lines not intersecting lines are called parallel lines but not intersecting rays and line-segments are not called parallel line-segments and rays respectively.

Day 10:

Concept of Plane and Intersection of Planes

The session was started with the figure drawn on the black-board as below:



It was asked to the students showing the figure on the black-board as "what is this?" There were different answers viz. Rectangle, Parallelogram and Plane. Then it was asked to the students "what is a plane?". One student replied that it is an undefined term. It was explained to the students that if you just consider it as a figure then it's a parallelogram as opposite sides are parallel but in geometry it is the way you represent the plane and name it as α, β, X, Y,...Further it was explained that plane is an undefined term and is extended infinitely on all the four sides of the figure (drawn on the black-board) i.e. If you extend this a bit more it becomes the black-board and still if you extend it becomes the surface of the wall and further also it can be extended infinitely. There is no end to it.

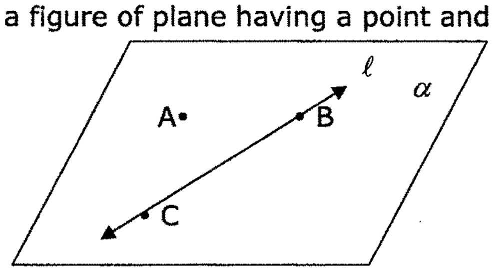
Here the teaching aid was used. It was made up of a wooden stand where one can fit in the thin vertical rods of same size and the tip of the rod had small balls of different colour which represented the points. There were thin cards which represented the plane passing through the points (balls). With the help of this teaching aid it was demonstrated in front of the students that two points cannot determine a plane and three collinear points can also not determine a plane as infinitely many planes can pass through these points. Similarly, the postulates of plane i.e. three non-collinear points determine a plane, each plane contains atleast three non-collinear points was explained to the students. Further, two distinct points determine a line and two distinct points cannot determine a plane was linked to deduce that a line cannot determine a plane and infinitely many lines can pass through a line with the help of the teaching aid. So, the theorems were explained and demonstrated to the students i.e. A line and a point outside it determine a plane, two intersecting distinct lines determine a plane.

Two cards were used to clarify to the students that the intersection of two planes is either a line or an empty set. It was explained to the students with the help of the figure that the intersection is an empty set then they are called parallel planes.

Day 11:

Relationship of Point, Line and Plane, Coplanar and non-coplanar points, Coplanar and non-coplanar lines

During this session the relation of point and line with a plane was explained. Firstly it started with a figure of plane having a point and line as below:



Based on the figure it was clarified to the students that point A $\in \alpha$ and $\overleftrightarrow{CB} \subset \alpha$ or $\ell \subset \alpha$. i.e. point belongs to the plane and line is a subset of plane.

Further, it was explained by relating to their understanding about the collinear and non-collinear points. As in case of a line the points lying on the same line are called collinear and if they are not then they are non-collinear points similarly in case of plane those points lying in the same plane are called co-planar points and if they are not lying in the same plane then they are called non-coplanar points. Further it was explained that if the lines lie in the same plane then are called coplanar lines and if they do not lie in the same plane then are called non-coplanar lines. While explaining the discussion was carried out on the following points:

- Two distinct points determine a line and three non-collinear points determine a plane
- Differences between the collinear points and coplanar points, non-collinear and non-coplanar points
- Difference between coplanar points and co-planar lines

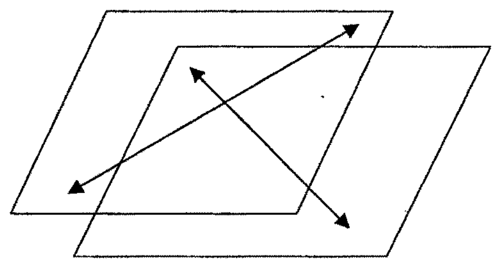
In all the above points the terminology changes as the reference changes with respect to relation between point and line or between point and plane or between line and plane, this was clarified to the students.

Day 12:

Parallel lines, Parallel Planes, Skew lines

This session brought into picture the difference between parallel lines and parallel planes, and parallel lines and skew lines. It's not that simple for the students to get this difference as it requires the visualization an imagination. Here the effort was made to provide concrete experiences to the students by using the teaching aid made from the flannel board and the lines prepared from the velvet paper. Students were involved in holding the cardboards which represents the planes. The concept of parallel lines was done by revising their understanding by asking questions. For the understanding of the parallel planes two big cardboards were used which were held parallel and were not meeting i.e. not intersecting.

For skew lines one line was stuck on the flannel board displayed on the wall another line was drawn on the transparent sheet which was held parallel to the flannel board. Students can see the location of both the lines through the transparency (both the lines seems like crossing each other but they cannot actually as they were in two parallel planes). See the figure below:



It was asked to the students that "are these two lines coplanar?" they said "No". It was followed by next questions i.e. "are these two lines intersecting?" they said "No"; "are they parallel?" students said "No". So it was explained to the students that these two lines are called skew lines as they are lying in two different planes and no plane can pass through both the lines simultaneously i.e. non-coplanar lines are skew lines which is neither intersecting nor parallel lines. Differences between parallel lines, parallel planes, and skew lines were brought to students' understanding.

Day 13:

Partitions of plane by a line, Half-planes and closed half-planes

For this session the teaching aid prepared from flannel board and the lines points and letters prepared from the velvet paper were used. Firstly the figure was drawn on the black-board and the concept of half-planes was explained to the students. Flannel board was displayed on the wall in the class and students were involved in placing the line and points on it based on the instructions given.

Name the plane as α

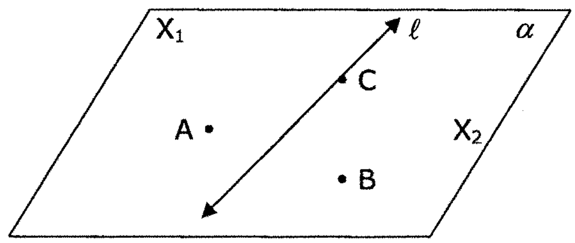
Place the line ℓ

Put point A on one side of the line in the plane α

Put point B on the other side of the line in the plane α

Put point C on the line ℓ

The following figure was generated on the flannel board.



Based on this task it was explained to the students that the plane α is divided into two halves by a line ℓ and they are called the half-planes. These half-planes are named as X_1, X_2 or Y_1,Y_2 or so on. Further it was clarified to the students that point C does not lie in any of the half-planes and hence line divides the plane in three parts i.e. the line itself, part X_1 of the plane and part X_2 of the plane. It was also explained to that these three are disjoint sets. Also, it was clarified that if the line is included with the half-planes then they are called closed half-planes and their intersection is not empty set but it is the line as its common to both the closed half-planes. Towards the end of the session the discussion was carried out with the students on the following points:

If two points are in different half-planes made by a line, then the line-segment joining them always intersects the line.

If two points are in the same half plane made by the then the line-segment joining them never intersects the line (but line may intersect).

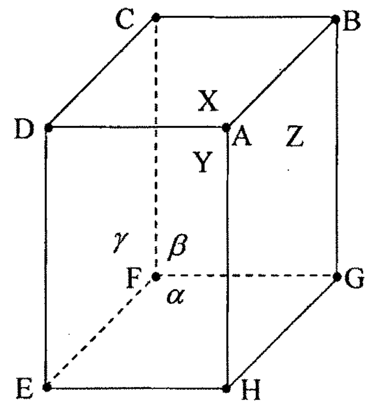
Also, the above points were demonstrated with the help of the teaching aid displayed in the class.

Day 14:

Worksheet on Plane

This session was devoted to the practice of student's understanding about the plane as its more abstract which requires the visualization ability. Students were divided in group of fours. It was prepared as below and was distributed to the groups.

Look at the figure and do as directed. In the given figure $\alpha, \beta, \gamma, X, Y, Z$ are six planes and A,B,C,D,E,F,G,H are the points.



1. List three pairs of coplanar points.
2. List three pairs of non-coplanar points.
3. Name two intersecting lines and write its intersection.
4. Name three intersecting lines and write its intersection.
5. Name two parallel lines
6. List three coplanar-lines
7. Name two skew lines
8. Name two parallel planes
9. Name two intersecting planes. Write its intersection.
10. Name three intersecting planes. Write its intersection.

The responses given by the students were shared group-wise and for each item the list of all the answers given by different groups was written on the black-board. The discussion was carried out for finding the correctness of the answers and the doubts of the students were clarified.

Based on the discussion the following points were summarized:

- Any three points are always coplanar
- For more than three points the question of whether they are coplanar or not arise
- Intersection of two or more lines is a point
- Difference between coplanar lines and skew lines
- Intersection of two distinct planes is a line

- Intersection of three distinct planes is a point

Day 15:

Concept of an Angle, Measure of an Angle, Arms and Vertex of an Angle, Bisector of an angle

The figure of an angle was drawn on the black-board and a model of an angle was used to explain the definition of an angle i.e. The union of two rays having the same initial point and not lying in one line is called an angle. Angles were explained by illustrating that for three non-collinear points A, B and C in a plane three angles are formed i.e. $\angle$ABC, $\angle$BCA and $\angle$CAB. The symbolic representation of angle was illustrated. With the help of this example the concept of arms of an angle are rays with the common initial point and the common initial point is called the vertex of an angle were explained. Also, it was clarified that $\angle$ABC, $\angle$BCA and $\angle$CAB are different angles as their arms and vertices are different. The equality of angles was explained to the students that for $\angle$ABC and the point D lying on the arm $\overrightarrow{BA}$, $\angle$ABC = $\angle$DBA.

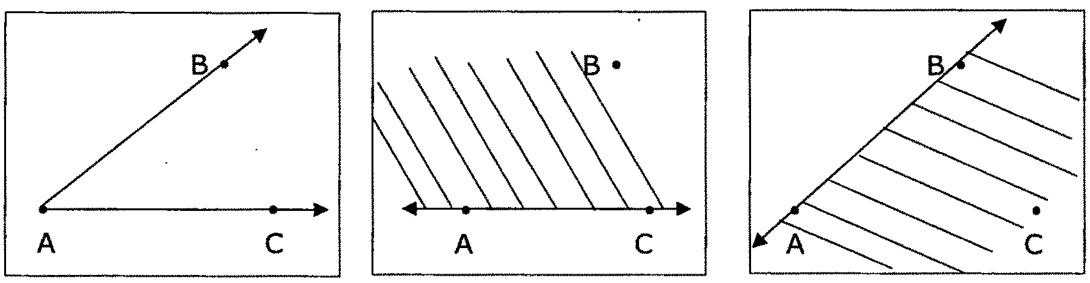
It was explained to the students that the measure of an angle is between 0 and 180 and it is denoted by $m\angle$ABC i.e. measure of an angle ABC. Further the measure of an angle was illustrated with the help of different examples which were drawn on the black-board and were measured with the help of a protractor. The result about sum of the measures of angles was also demonstrated with the help of a figure i.e. for $\angle$BAC and point D such that B-D-C, $m\angle$BAD + $m\angle$DAC = $m\angle$BAC.

Also, it was discussed with the students for $\angle$BAC and point D in the interior of an $\angle$BAC if $m\angle$BAD = $m\angle$DAC then $\overrightarrow{AD}$ is called a bisector of an angle. Every angle has one and only one bisector and bisector of an angle is a ray having the vertex of an angle as the initial point were clarified to the students. Also, it was explained to the students that the two angles formed by the bisector of an angle are congruent.

Day 16:

Partitions of the plane by an angle, Interior and Exterior of an angle and Bisector of an angle

Here the two transparencies were used to demonstrate the partitions of plane by an angle. This was explained by comparing it with the partitions of plane made by a line. Students were instructed to observe the transparencies projected. One transparency was having an angle drawn on it $\angle$BAC, another transparency had the half plane (shaded) made by $\overleftrightarrow{AB}$ and the third transparency had the half plane (shaded) made by $\overleftrightarrow{AC}$.



The above three transparencies were overlapped to explain that the interior of an angle is the overlapped portion of both the transparencies i.e. The interior of $\angle$BAC = the half plane of $\overleftrightarrow{AC}$ on the side of B $\cap$ the half plane of $\overleftrightarrow{AB}$ on the side of C. Simultaneously the exterior of an angle is the portion which is not overlapping portion or the portion of the plane which is left after excluding the interior of an angle and the points of an angle. Hence, it was made clear to the students that an angle divides the plane in three parts viz. $\angle$BAC, Interior of $\angle$BAC, and exterior of $\angle$BAC. Students were called to put the points in the interior or exterior of an angle on the black-board as instructed by the investigator.

This was followed by the understanding about the cross-bar theorem. It was asked to the students by drawing a figure that for $\angle$BAC, if B-D-C then D lies in the exterior or interior of an angle. Further it was asked to the students if D is a point in the interior of an angle then what can you say about the intersection of $\overrightarrow{AD}$ and $\overline{BC}$? At the end it was clarified that this is a cross-bar theorem which states that if D is in the interior of an angle $\angle$BAC, then $\overrightarrow{AD}$ intersects $\overline{BC}$.

Day 17:

Types of angles and types of pair of angles

The model of an angle having two arms rotating was used for explaining types of angles to the students. There are three types of angles based on the measure of an angle was revised with the students viz. Acute angle (measure is less than 90), Right angle (measure is 90), and Obtuse angle (measure is more than 90).

A model of four arms having same initial point was used for explaining the types of pairs of angles based on the measures and arms both viz. Complementary angles, supplementary angles, congruent angles, Adjacent angles, Linear pair of angles, Vertically Opposite angles. The characteristics all the pairs of angles were demonstrated with the help of the model.

The following properties related to the pairs of angles and their interrelations were demonstrated and clarified to the students:

- Vertically opposite angles are always congruent
- All linear pair of angles is adjacent but adjacent angles do not always form a linear pair of angles
- Complementary angles are always adjacent
- Linear pair of angles are not always congruent
- Vertically Opposite angles can never be Adjacent and Linear
- Linear pair of angles is always Supplementary
- Linear pair of angles is always adjacent
- Linear pair of angles can never be complementary
- Supplementary angles are not always Linear
- Complementary angles & Supplementary angles cannot be possible together

Day 18:

Worksheet on Angle

During this session students were divided in group of fours. The worksheet on angle was distributed among the groups. It was observed that students were not able to distinguish between different types of angles and faced problems in identifying the types of pairs of angles and their interrelation so this entire session was devoted to provide practice to the students and develop clear understanding about the same. The worksheet prepared was as follows.

Draw the following angles:

i) Acute Angle
ii) Right Angle
iii) Obtuse Angle

Draw the following:

a) Two Congruent angles
b) Pair of Complementary angles
c) Pair of supplementary angles
d) Adjacent angles
e) Linear pair of angles
f) Vertically opposite angles

After students completed their worksheets they were asked to verify the following statements:

Complementary angles are congruent

Vertically opposite angles are congruent

Supplementary angles are congruent

Linear pair of angles are supplementary

Linear pair of angles are adjacent

Vertically opposite angles are supplementary

The reverse of the above statements were also verified at times. This activity was to help students to clarify their confusion between different types of pairs of angles.

Day 19:

Symbols and Geometrical figures

In this session the focus was on understanding of the meaning of each symbol and its appropriate use. Also, the points to be kept in mind while drawing the geometric figures were explained to the students as follows:

- The meaning and difference between the symbols $\in$, $\notin$, $=$, $\neq$, $\subset$
- 'ϕ' cannot be the point of intersection but it represents empty set
- The difference between ϕ & $\{\phi\}$
- Naming the lines was not considered significant in the geometrical representation & was not clear
- Naming of points on the line / line-segment / ray was not understood
- There is only one point represented by 'A', 'B' , 'C' in a plane or on a line was not clear
- In "$\ell_2 \cap \ell_1 = \{X\}$", X is point of intersection
- 'X' represents point and not line with reference to the given figure
- 'Y' is a plane & not a point with reference to the given figure
- "ℓ_1" represents line & not ray or line-segment
- AB represents distance between points A and B or length of the line segment $\overline{AB}$ & is always a positive number
- On a number-line, O is called the origin point & is not in any direction
- The point of intersection should be represented as a set
- The points should be represented in set form
- The appropriate use of symbols to express the relation of point and line, line and line-segment, line and plane, ray and line, ray and line-segment
- Naming the lines in the geometrical figures
- Naming of points on the line / line-segment / ray
- There is only one point represented by 'A', 'B', 'C',.. in a plane or on a line
- $\overleftrightarrow{XY}$ represents the same line having points X and Y both

The above points with respect to the use of symbols in geometry and the geometric figures were addressed and clarified to the students with the help of illustrations and examples of different geometrical situations on the black-board.

Day 20:

Open Session with the students

This session was organized with the purpose of allowing students to ask questions, clarify their doubts and queries they have in their mind. Students asked some questions related to the geometry which were answered by the investigator. Slowly students were deviated from content questions to their curiosity about what is the use of geometry, where these line, plane and angle is used. All the questions were addressed by the investigator through the informal talk and making them realize about the importance of geometry in day to day life and for different professions.

The rapport was developed between the investigator and the students during the entire programme so they felt like talking to the teacher and expressing their views and ideas freely and naturally. So later they talked about their experience about the programme conducted. They were happy and enjoyed all the sessions. They shared that their doubts and misunderstandings related to the basic concepts of geometry were cleared and it was helpful to them for learning better in geometry.

During the entire remedial programme different inputs were given to the students, they were involved in group discussions, problem solving and group activities conducted by the investigator in the class. Also, different teaching-aids and mathematical manipulatives were used to reach out to the students. It was observed that students participated during the remedial programme and interacted in the class by sharing their ideas and queries. All the identified learning difficulties were addressed during the remedial programme.

The record of the attendance of the students of the experimental group was maintained by the investigator. Two students were not regularly present during the entire remedial programme. So, finally thirty-three out of thirty-five students attended the remedial programme completely.

5.3 ADMINISTRATION OF THE PARALLEL TEST FOR ACHIEVEMENT

The parallel test to the achievement test was constructed by the investigator and the details are included in chapter III. In brief it consisted of six major questions and 100 items in total. It was of hundred marks and each item was of one mark. The parallel test was administered by the investigator to Experimental and control group.

The parallel test was administered to thirty-three students of the experimental group as four students were absent and fifty-two students of the control group as two students were absent. The time given to the students for the test was two hours.

The responses of the students on the parallel test were scored by the investigator. These scores were considered as the scores on the post-test for both the experimental and control group. Finally in case of experimental group the scores of thirty-two students were considered on the parallel test as out of thirty-three students who appeared for the parallel test only thirty-two students were regularly present and attended the entire remedial programme.

5.4 IMPACT OF THE REMEDIAL PROGRAMME

The impact of remedial programme was studied by comparing the pre-test scores and poet-test scores of the students of experimental group. Also, the performance of the students of experimental group on each item in achievement test and parallel test was compared. Finally, the ANCOVA was applied on the scores of control group and experimental group to study the impact of the remedial programme.

5.4.1 Comparative Analysis of Scores

The scores of the thirty-two students of experimental group on both the achievement test and parallel test were considered and compared. The table below displays the scores of the students on both the achievement test and the parallel test.

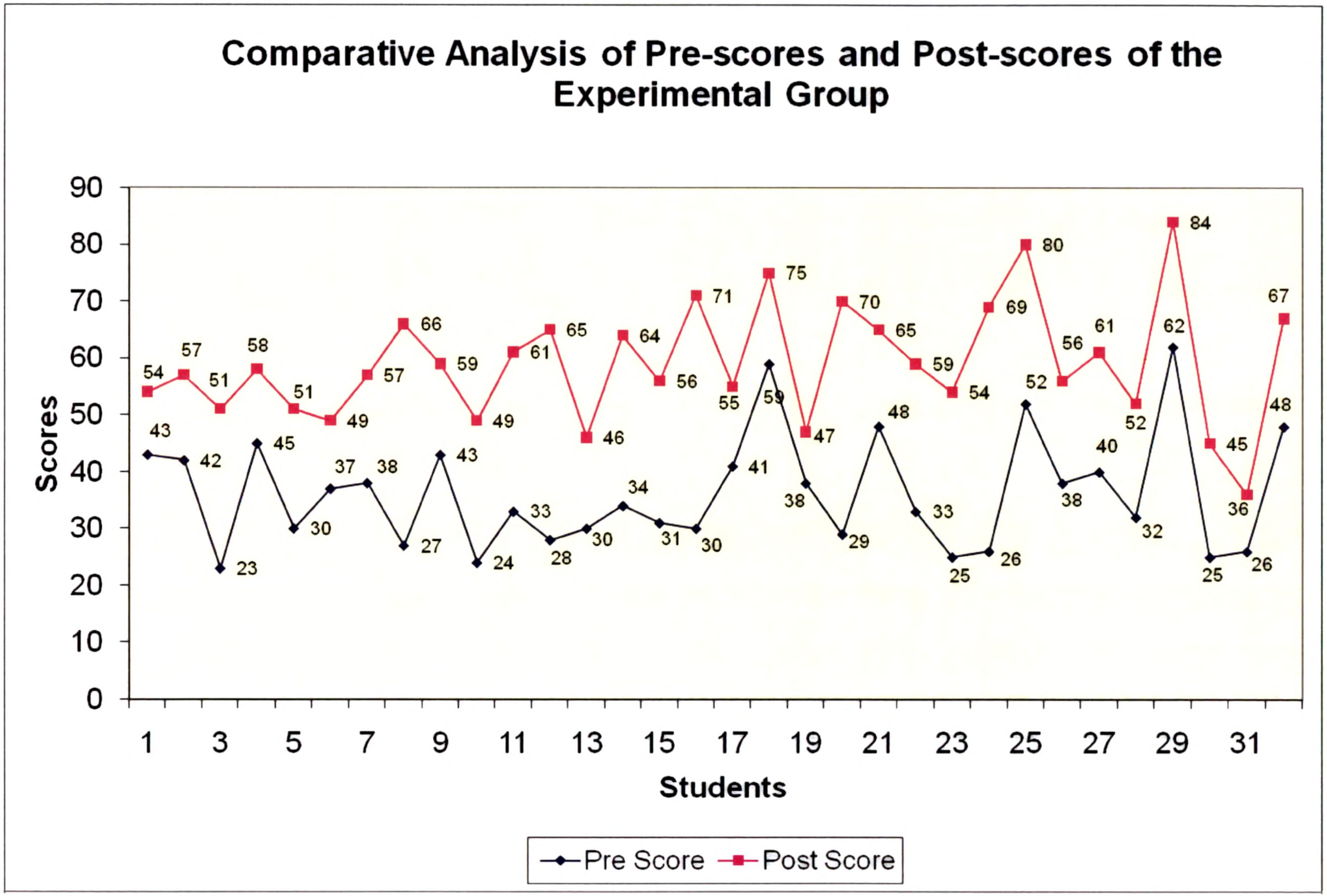
Table – 19
Comparison of Pre-test and Post-test scores

Roll No.	Scores on Achievement test (Pre-scores)	Scores on Parallel test (Post-scores)
1	Absent during diagnosis	Absent
2	43	54
3	42	57
4	23	51
5	45	58
6	30	51
7	37	49
8	38	57
9	27	66
10	43	59
11	24	49
12	33	61
13	28	65
14	Absent during remedial	Absent
15	30	46
16	34	64
17	----	Absent
18	31	56
19	Absent during diagnosis	Absent
20	30	71
21	41	55
22	59	75
23	38	47
24	29	70
25	Absent during remedial	----
26	48	65
27	33	59
28	25	54
29	26	69
30	52	80
31	38	56
32	40	61
33	32	52
34	62	84
35	25	45
36	26	36
37	48	67
Total	**1160**	**1889**
Average	**36.25**	**59.03**

From the above table it is clear that the mean score of the students of experimental group on the pre-test was 36.25 and the mean score of the students on post-test was 59.03 which showed the improvement in the performance of the students. i.e. the performance of the students has increased from mean of 36.25 in achievement test to mean of 59.03 in parallel test for achievement. Thus, there is observed difference of 22.78. Also, the minimum gain in terms of individual student was of nine percent and the maximum gain was of forty-three percent.

Graphically the comparative analysis of the scores of each student on pre-test and post-test is represented as below:

Figure - 2



Comparative Analysis of Pre-scores and Post-scores of the Experimental Group
Scores
0
10
20
30
40
50
60
70
80
90
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
Students
Pre Score
Post Score

Further, the comparative analysis of frequency distribution of pre-scores and post scores in case of the experimental group is presented in a tabular form as below.

Table – 20

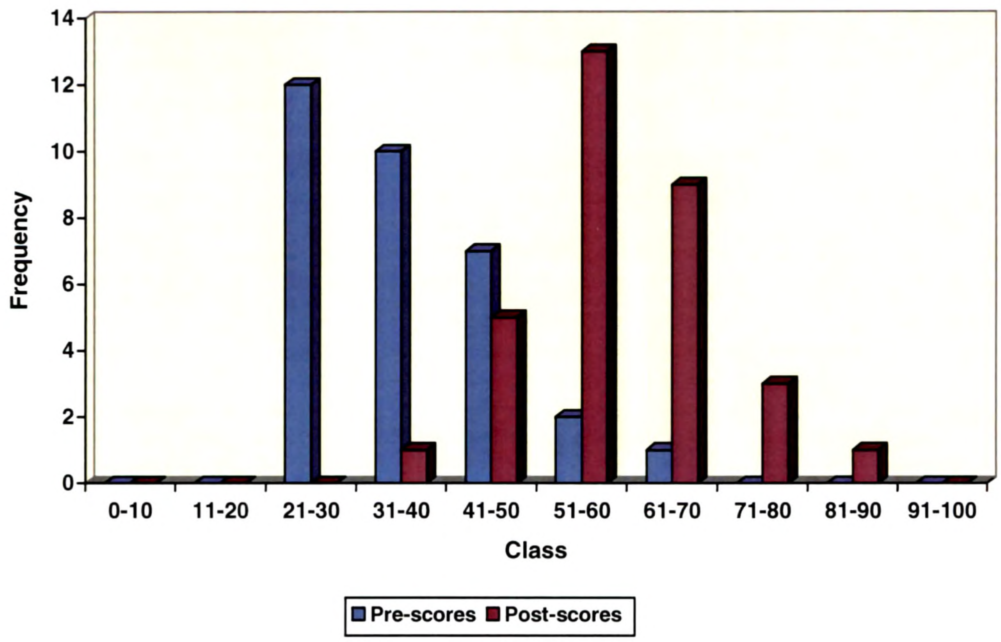
Comparative analysis of the frequency distribution of pre-scores and post scores

Class (Marks)	Frequency of pre-scores (Students)	Frequency of post-scores (Students)
1 – 10	-	-
11 – 20	-	-
21 – 30	12	-
31 – 40	10	01
41 –50	07	05
51 – 60	02	13
61 –70	01	09
71 – 80	-	03
81 – 90	-	01
91 –100	-	-
Total	32	32

Graphically the data of the above table is presented as below.

Figure - 3

Comparative Analysis of Frequency Distribution of Pre-scores and Post-scores



From the graph it is clear that in the pre-test maximum students scored between twenty-one and thirty where as in case of post-test maximum students scored between fifty-one and sixty. It is also observed from the graph that in the pre-test no student scored more than sixty percent except one where as in the post-test there were thirteen students who scored more than sixty percent, three students scored more than seventy percent and two students scored eighty percent and above.

5.4.2 Comparative Analysis of Item-wise Performance

The correct responses for each item in the pre-test and post-test were compared. The table followed by the figure below presents the comparison of the number of correct responses and the percentages of correct responses on each item during the pre-test and post-test.

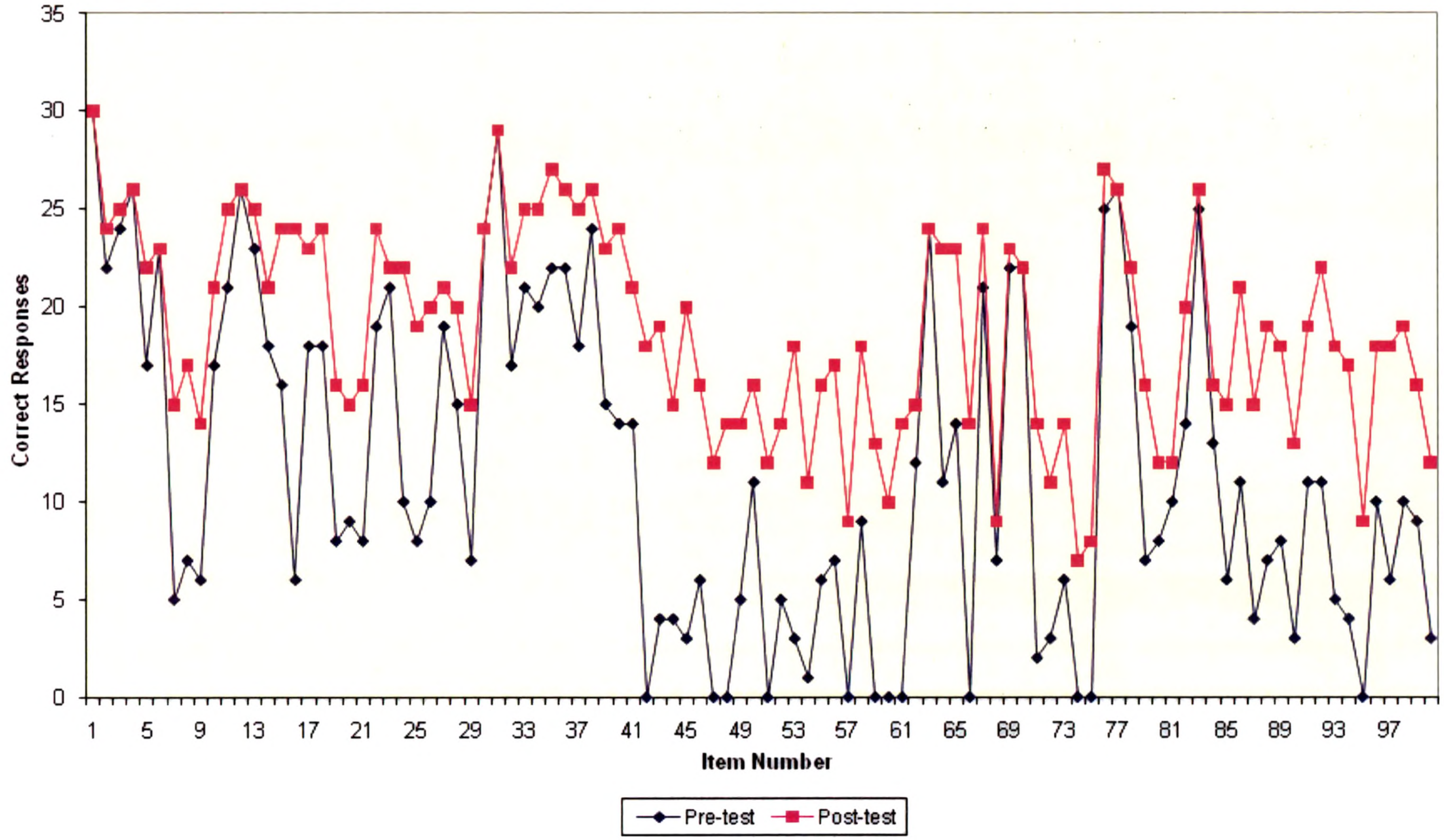
Table – 21
Comparative Analysis of item-wise performance

Question No.	Item No.	Number of Correct Responses in Pre-test	Number of Correct Responses in Post-test	Percentages of Correct Responses in Pre-test	Percentages of Correct Responses in Post-test
Q.I	**1**	30	30	93.8	93.8
	2	22	24	68.8	75
	3	24	25	75	78.1
	4	26	26	81.3	81.3
	5	17	22	53.1	68.8
	6	23	23	71.9	71.9
	7	5	15	15.6	46.9
	8	7	17	21.9	53.1
	9	6	14	18.8	43.8
	10	17	21	53.1	65.6
	11	21	25	65.6	78.1
	12	26	26	81.3	81.3
	13	23	25	71.9	78.1
	14	18	21	56.3	65.6
	15	16	24	50	75
	16	6	24	18.8	75
	17	18	23	56.3	71.9
	18	18	24	56.3	75
	19	8	16	25	50
	20	9	15	28.1	46.9
	21	8	16	25	50
	22	19	24	59.4	75
	23	21	22	65.6	68.8
	24	10	22	31.3	68.8
	25	8	19	25	59.4
	26	10	20	31.3	62.5
	27	19	21	59.4	65.6
	28	15	20	46.9	62.5
	29	7	15	21.9	46.9
	30	24	24	75	75
	31	29	29	90.6	90.6
	32	17	22	53.1	68.8
	33	21	25	65.6	78.1
	34	20	25	62.5	78.1
	35	22	27	68.8	84.4
	36	22	26	68.8	81.3
	37	18	25	56.3	78.1
	38	24	26	75	81.3
	39	15	23	46.9	71.9
	40	14	24	43.8	75
Q.II A	**1**	14	21	43.8	65.6
	2	0	18	0	56.3
	3	4	19	12.5	59.4
	4	4	15	12.5	46.9
	5	3	20	9.38	62.5
Q.II B	**1**	6	16	18.8	50
	2	0	12	0	37.5

	3	0	14	0	43.8
	4	5	14	15.6	43.8
	5	11	16	34.4	50
	6	0	12	0	37.5
	7	5	14	15.6	43.8
	8	3	18	9.38	56.3
	9	1	11	3.13	34.4
	10	6	16	18.8	50
	11	7	17	21.9	53.1
	12	0	9	0	28.1
	13	9	18	28.1	56.3
	14	0	13	0	40.6
	15	0	10	0	31.3
Q.II C	**1**	0	14	0	43.8
	2	12	15	37.5	46.9
	3	24	24	75	75
	4	11	23	34.4	71.9
	5	14	23	43.8	71.9
	6	0	14	0	43.8
	7	21	24	65.6	75
	8	7	9	21.9	28.1
	9	22	23	68.8	71.9
	10	22	22	68.8	68.8
Q.II D	**1**	2	14	6.25	43.8
	2	3	11	9.38	34.4
	3	6	14	18.8	43.8
	4	0	7	0	21.9
	5	0	8	0	25
	6	25	27	78.1	84.4
	7	26	26	81.3	81.3
	8	19	22	59.4	68.8
	9	7	16	21.9	50
	10	8	12	25	37.5
	11	10	12	31.3	37.5
	12	14	20	43.8	62.5
	13	25	26	78.1	81.3
	14	13	16	40.6	50
	15	6	15	18.8	46.9
Q.III	**1**	11	21	34.4	65.6
	2	4	15	12.5	46.9
	3	7	19	21.9	59.4
	4	8	18	25	56.3
	5	3	13	9.38	40.6
	6	11	19	34.4	59.4
	7	11	22	34.4	68.8
	8	5	18	15.6	56.3
	9	4	17	12.5	53.1
	10	0	9	0	28.1
	11	10	18	31.3	56.3
	12	6	18	18.8	56.3
	13	10	19	31.3	59.4
	14	9	16	28.1	50
	15	3	12	9.38	37.5

Figure - 4

Comparative Analysis of Item-wise Correct Responses in Pre-test and Post-test



It is clearly observed form the table and the figure above that for any of the item the number of correct responses have not decreased it has either increased or at the most remained the same. So, it can be concluded that overall there was a gain in the item-wise performance.

Also each item in the pre-test and the post-test were associated with the specific level of instructional objectives (Appendix B). So, from the above data the comprehensive picture related to the average performance of the students at different levels of instructional objectives is compared and presented the table below.

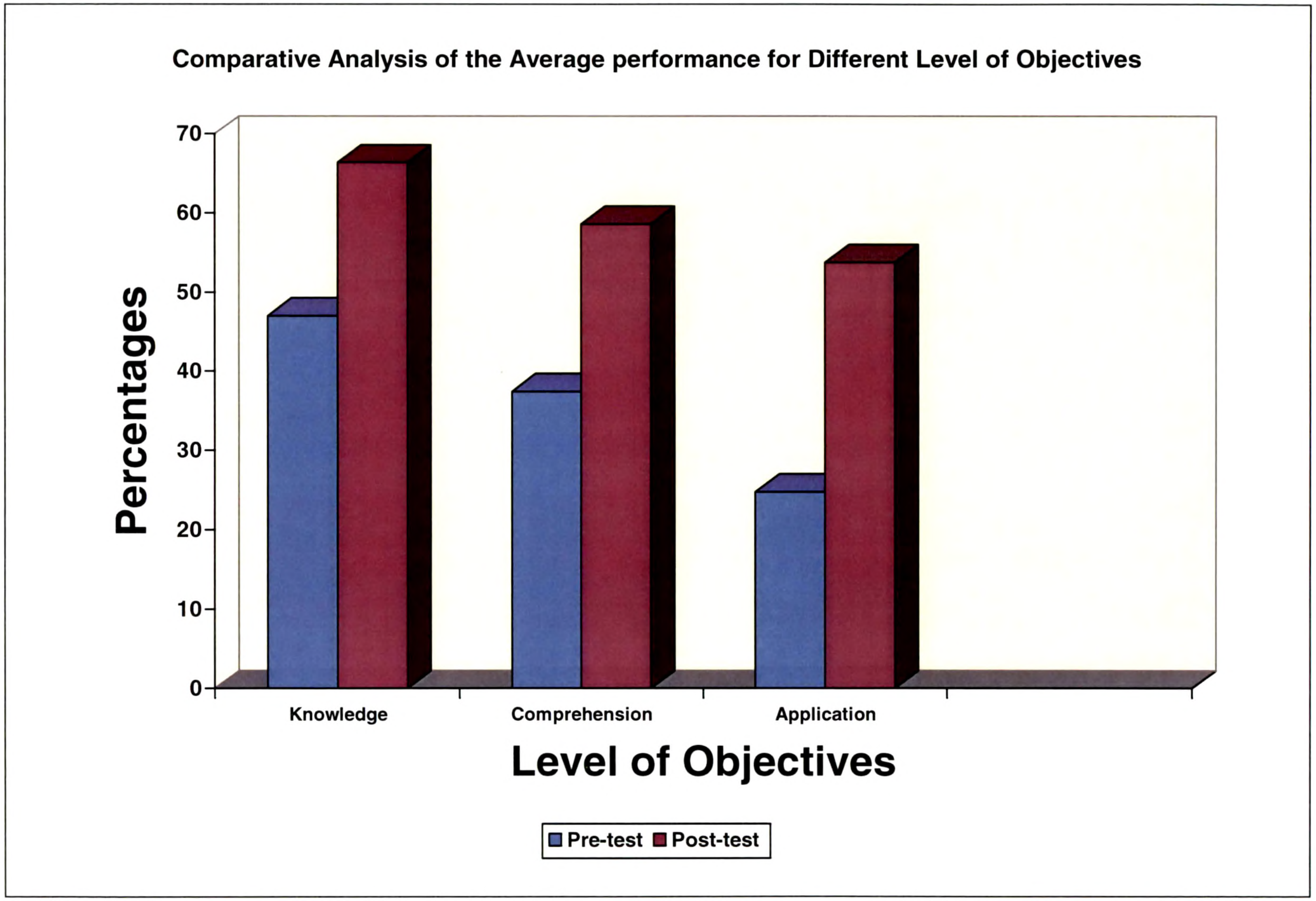
Table – 22

Comparative Analysis of Average Performance at Different Levels of Instructional Objectives

Level of Objectives	Average performance in Pre-test (%)	Average Performance in Post-test (%)
Knowledge	47.02	66.34
Comprehension	37.44	58.59
Application	24.76	53.73

From the table above it is observed that there was increase in the performance at each level of instructional objectives. The gain in case of knowledge, Comprehension and Application level of objectives was 19.3%, 21.2% and 29% respectively. At the same time it is also observed that the performance at application level has not exceeded the performance at comprehension level and the performance at comprehension level has not exceeded the performance at knowledge level. The maximum gain was at application level which is the result of the gain at knowledge and comprehension level. It is observed that if the performance at knowledge and comprehension level is increased then it has cumulative impact on the increase in the performance at application level. This is also presented in form of a graph as below.

Figure - 5


Comparative Analysis of the Average performance for Different Level of Objectives
Percentages
70
60
50
40
30
20
10
0
Knowledge
Comprehension
Application
Level of Objectives
Pre-test
Post-test

5.4.3 Comparative Analysis Between the Mean of Control and Experimental group

Finally, the data collected in terms of the pre-scores and post-scores of the students of control group and experimental group were analyzed using ANCOVA and pre-test score was used as covariate. The output of the analyzed data is displayed in the table below:

Table - 23
Descriptive Statistics

Group	N	Pre-test Mean	Post-test Mean	Std. Deviation
Experimental Group	32	36.25	59.03	10.493
Control Group	52	46.02	46.88	16.783
Total	84	41.14	51.51	15.793

Table - 24
ANCOVA Output

F	df1	df2	Sig.
10.582	1	82	.002

From the above table it is clear that the value of 'F' was 10.58 significant at .002 level i.e. even at .01 level. So, it was found using ANCOVA that there was a significant difference between the mean scores of the control group and the experimental group and it is towards the mean score of the experimental group. Hence, the diagnosis and remediation has a positive impact on the achievement of the students of experimental group.