Chapter 3

FOUR QUADRANT POWER MEASUREMENTS

3.0 Introduction

The increasing application of electronic equipment results in non sinusoidal waveforms, which can cause current and voltage harmonics. The distortion of supply waveforms may cause errors in electrical energy measurement affecting distribution businesses revenues. With today's emphasis on conserving energy many test measurement equipments are required to determine consumption before deciding what can be done to reduce it. Power measurement in AC circuit is more difficult as compare to that of DC circuit. Since the flow of current in AC circuits constantly changes direction (alternating current) due to alternating polarity of the voltage.

Electric energy measured by modern electronic meters depends on the measurement principles and power definitions employed to calculate the quantities that appear in a customer bill. Modern hardware allows the applications of any formulae but which method is to be used for measurement of active and reactive power under various conditions is an issue of debate.

The definitions of active power, reactive power, distortion power, apparent power that appear in various literatures [13], [14] are discussed in details. There have been many attempts to formulate the most appropriate definition of electric powers in non-sinusoidal circuits [15], [16].

In modern power systems, more and more energy is transmitted by non-sinusoidal voltages and currents as a result of widely used nonlinear loads. Such non-sinusoidal voltages and currents create many problems concerning the measurement, determination and calculations of their harmonic contents. The tariffs of the electric companies for billing energy usually depend on methods of measurement and calculation of active power under non-sinusoidal conditions. So it is necessary to determine correctly, power consumed in sinusoidal and non-sinusoidal circuits. The main goal of this work is to develop a model in order to perform experimental measurements of this important power component and to compare the measured results with the calculated ones. In this thesis more focus is given for the measurement technique for bidirectional flow of active and reactive power in an interconnected network under sinusoidal and non-sinusoidal conditions. The work is concentrated towards looking into architecture of an interconnected power system when the active and reactive power can flow in either direction either from source to load or from load to source. In such case direction of flow of active and reactive power is calculated and based on those values of frequency and voltage a new tariff structure is proposed for interconnected system.

3.1 Active and Reactive Power Measurement

Any electric circuit comprising of resistance capacitance and inductance consumes energy in two ways. One dissipated as heat in resistance and the other as stored energy in magnetic field in case of inductance($1/2\,LI^2$), and electric field in case of capacitance ($1/2\,CV^2$).The energy stored in field is called reactive energy. Rate of change of energy is power. Therefore, in case of DC circuits where voltage and current are steady and stable state the rate of change of energy stored in the fields is zero and therefore power input to DC circuit will be power consumed in resistance. In case of AC circuit where voltage and current variables change with time even in steady state the total power input to AC circuit will be the sum of power dissipated in resistance and rate of change of energy stored in magnetic $LI\frac{dI}{dt}$ and electric field $CV\frac{dV}{dt}$ .This change is both positive and negative; meaning energy flow into the field is bilateral. The energy rate of dissipation in resistance is known as active power and rate of change in energy stored in fields is called reactive power. Since there is no rate of change in stored energy in case of DC circuits, the power input in DC circuits corresponds to power dissipated only in resistance. For AC circuits in addition to active power dissipation there will be a rate of change in stored energy in the field, which corresponds to reactive power. Thus, in AC circuits the power input consists of two components namely the active power and reactive power.

Consider a series RLC circuit (Fig.3.1-1) connected to an alternating voltage $e = E_m sin\omega t$

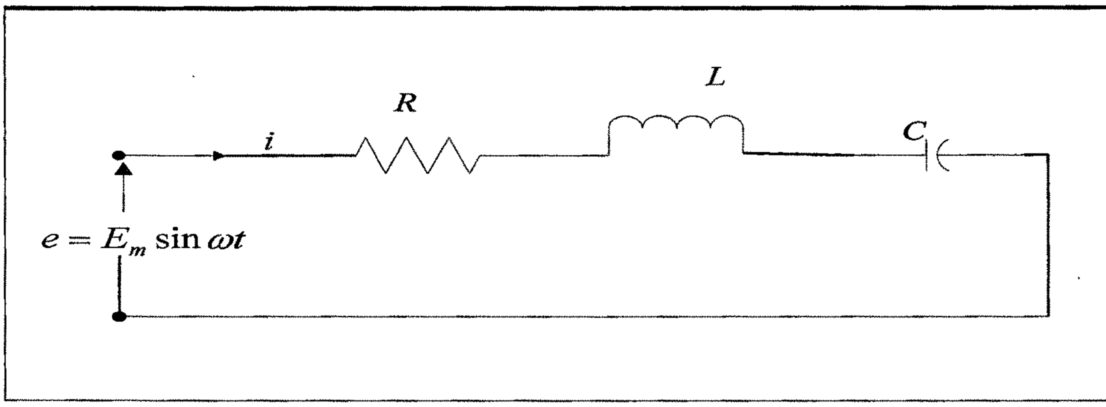

Fig. 3.1-1 Series RLC Circuit

Where

E_m Is the peak magnitude

ω Is the angular frequency given by $\omega = 2\pi f$. Where f is the fundamental frequency.

$$e = iR + L\frac{di}{dt} + v_c \qquad where \quad v_c = \frac{1}{C}\int_0^t i\,dt$$
$$= iR + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt \qquad (3.1)$$

Instantaneous power p is given by Eq. 3.2

$$p = ei = i^2 R + Li\frac{di}{dt} + \frac{i}{C}\int_0^t i\,dt$$
$$= i^2 R + Li\frac{di}{dt} + iv_c \qquad (3.2)$$

Average power over a cycle is given by Eq. 3.3

$$P = \frac{1}{T}\int_0^T p\,dt \qquad Where\ \ T = \frac{2\pi}{\omega}$$
$$= \frac{R}{T}\int_0^T i^2\,dt + \frac{L}{T}\int_0^T i\,di + \frac{1}{T}\int i\,v_c\,dt \tag{3.3}$$

The first term on the RHS of Eq. 3.3 corresponds to I^2_{RMS} R, which is the active power. The second and third terms correspond to reactive power because they are associated with rate of change of stored energy. The integrated instantaneous reactive powers over a cycle will be zero. Thus, the average power over a cycle consumed in an AC circuit is equal to the active power even though the instantaneous power has reactive power component.

If the circuit of Fig.3.1-2 is energized by DC voltage E, then in steady state the voltage is given by Eq. 3.4.

$$E = iR$$
$$since\ \ \frac{dE}{dt} = \frac{dI}{dt} = 0 \tag{3.4}$$

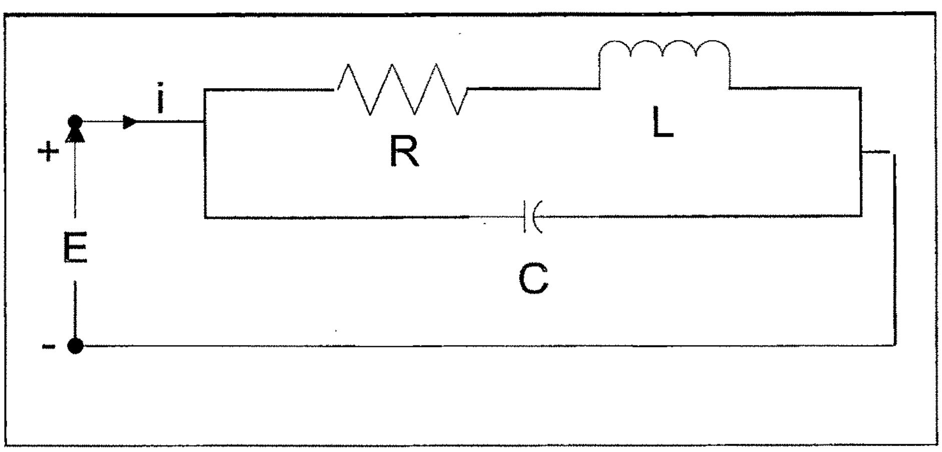

Fig. 3.1-2 RLC Circuit with DC source

So the average active power $P = I^2R$ and there is no reactive power component. Thus active and reactive components of power arise in AC circuits due to time variation of energy stored in the electric and magnetic fields. The average power over a cycle is active power input and average of reactive power components is zero because the reactive power flow is bilateral between source and magnetic/electric fields.

3.2 Measurement of Active and Reactive Power for Different Cases:

The average input power over a complete cycle gives active power dissipated in the circuit and if there is energy storing elements like inductors and capacitors the average reactive power is zero even though there is instantaneous reactive power, which flows bilaterally between source and reactive elements. The calculations of this instantaneous power fed to different elements like resistance, inductance and capacitance is illustrated below:

(A) Resistive circuit

Consider a resistive circuit as shown in Fig. 3.2-1. The average power is given by Eq. 3.5

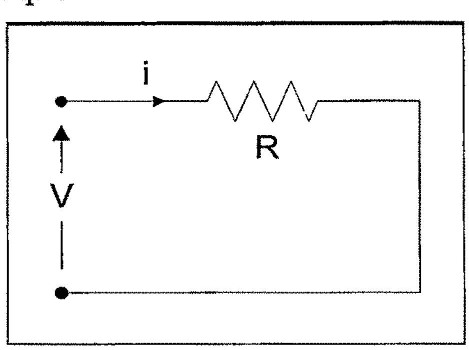

Fig. 3.2-1 Resistive Circuit

Where,

$v = V_m \sin(\omega t)$; $i = \frac{V_m}{R}\sin(\omega t)$ $\quad \omega = 2\pi f$

Instantaneous power $p = vi = \frac{V_m^2}{R}\sin^2(\omega t)$

Average power over a cycle $P = \frac{1}{T}\int_0^T p\,dt$; $T = \frac{2\pi}{\omega}$ (3.5)

Where T is one cycle time period.

$$= \frac{\omega}{2\pi}\frac{V_m^2}{R}\int_0^T \sin^2 \omega t\,dt$$

$$= \frac{\omega}{2\pi}\frac{V_m^2}{2R}\int_0^T (1 - \cos\ 2\omega t)dt$$

$$= \frac{\omega}{2\pi}\frac{V_m^2}{2R}\left[\int_0^T dt - \int_0^T \cos 2\omega t\,dt\right]$$

$$= \frac{\omega}{2\pi}\frac{V_m^2}{2R}\left[[t]_0^T - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right]$$

$$= \frac{\omega}{2\pi}\frac{V_m^2}{2R}[T - 0]$$

$$= \frac{V_m^2}{2R}$$

$$= \frac{V_{RMS}^2}{R} \tag{3.6}$$

Fig. 3.2-2 shows the plot of instantaneous power and average power.

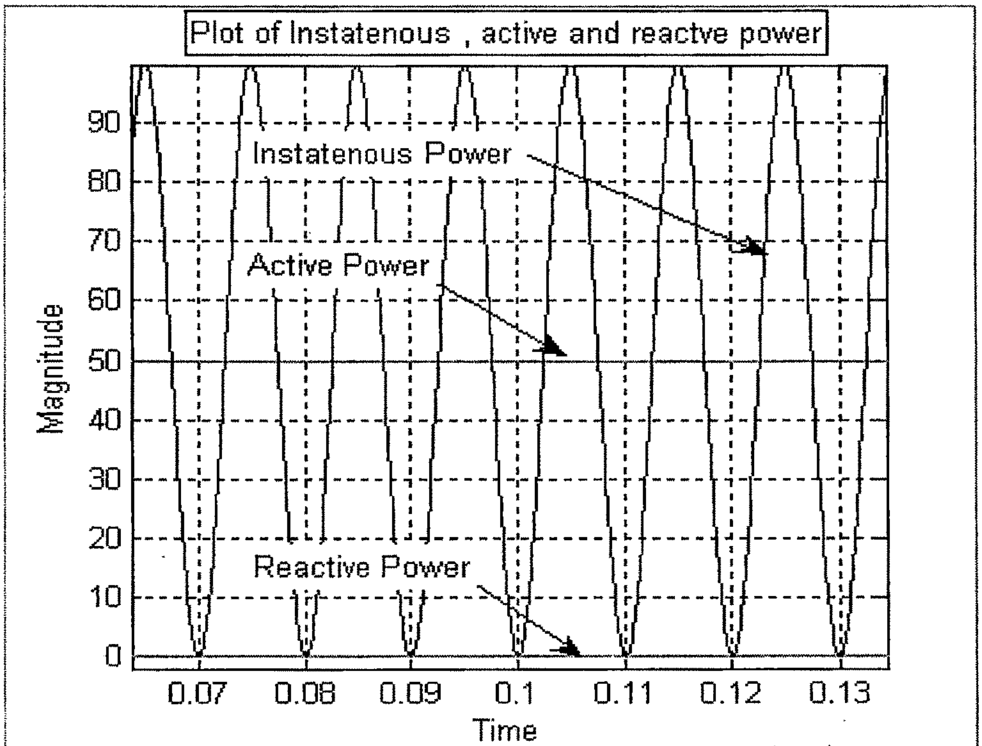

Fig. 3.2-2 Plot of Instantaneous active and reactive power

(B) Inductor circuit

Consider an inductive circuit as shown in Fig. 3.2-3. The average power is given by Eq. 3.7

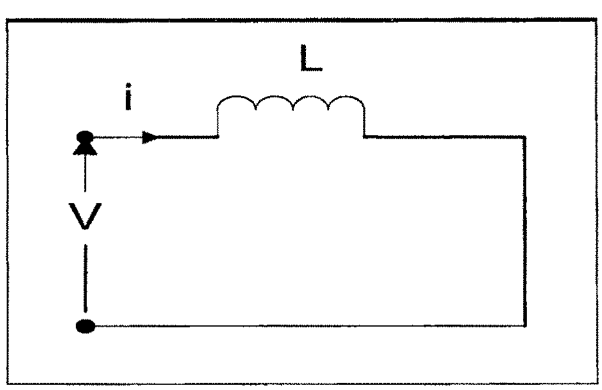

Fig. 3.2-3 Inductive Circuit

$$v = V_m \sin \omega t = L \frac{di}{dt}$$

$$i = \frac{V_m}{L} \int_0^t \sin \omega t \, dt$$

$$= -\frac{V_m}{L} [\cos \omega t]_0^t$$

$$= -\frac{V_m}{\omega L} [\cos \omega t - 1]$$

(3.7)

Fig. 3.2-4 shows the plot of υ and i

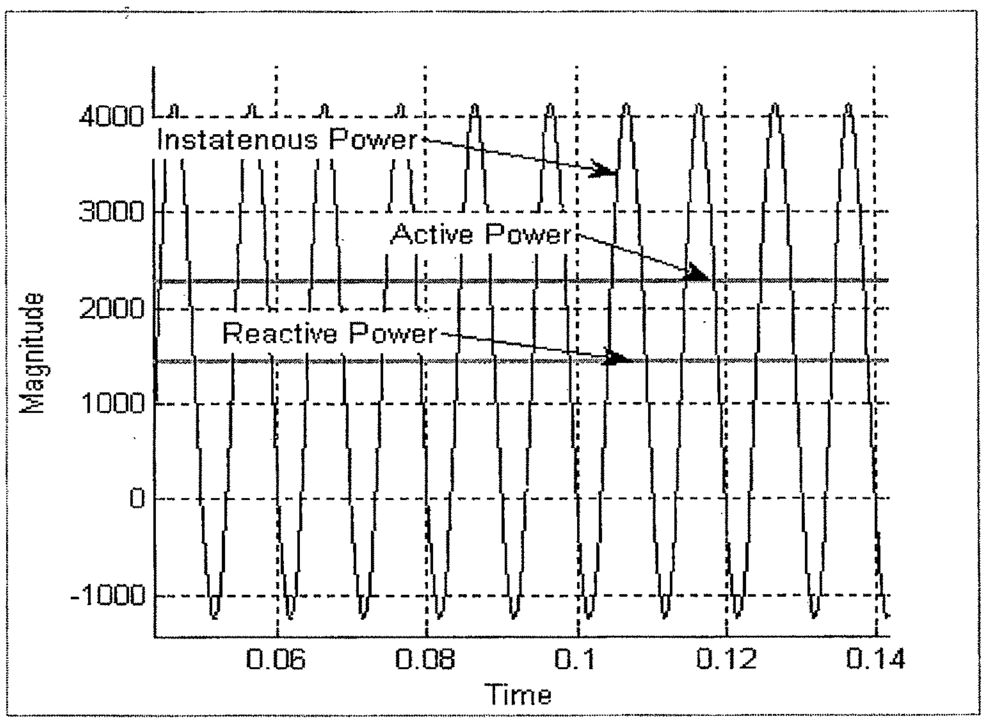

Fig. 3.2-4 Plot of Instantaneous active and reactive power

Due to resistance present in practical circuits, the DC component slowly attenuates and the AC component of current remain as steady state current given by Eq. 3.8

$i = i_{steady\_state} = i_L$

$$i_L = -\frac{Vm}{\omega L}\cos\omega t \qquad (3.8)$$

Instantaneous power input is given by Eq. 3.9 and is derived as follows

$$p = ei$$

$$p = V_m \sin\omega t \cdot \frac{V_m}{\omega L}(1-\cos\omega t) \qquad e = V_m \sin\omega t.$$

$$= \frac{V_m^2}{\omega L}\{\sin\omega t - \sin\omega t\cos\omega t\}$$

$$= \frac{V_m^2}{2\omega L}\{2\sin\omega t - \sin 2\omega t\}$$

(3.9)

Average power P over one cycle is given by Eq. 3.10

$$P = \frac{1}{T}\int_0^T ei \quad = \frac{V_m^2}{2\omega LT}\left[\int_0^T 2\sin\omega t - \int_0^T \sin 2\omega t\right]$$

$$= \frac{V_m^2}{2\omega LT}\left[\left\{-\frac{2\cos\omega t}{\omega}\right\}_0^T + \left\{\frac{\cos 2\omega t}{2\omega}\right\}_0^T\right]$$

$$= 0 \quad \text{since } \omega T = 2\pi$$

(3.10)

The instantaneous power, which is fed into the inductor circuit, give rise to change in stored magnetic energy whose average value is zero. Hence, this power is referred to as reactive power and is represented by Q to differentiate from active power P.

In steady state where the dc component in current has died, and only AC steady state current exists, then the instantaneous reactive power is given by Eq. 3-11

$$p = -V_m \sin\omega t \cdot \frac{V_m}{\omega L}\cos\omega t$$
$$= -\frac{V_m^2}{2\omega L}\sin 2\omega t$$
$$= -\frac{V_{RMS}^2}{\omega L}\sin 2\omega t$$

(3.11)

The average power P is zero and it can be seem that the instantaneous reactive power oscillates between the source and field at twice the frequency. Fig. 3.2-5 shows the plot of instantaneous reactive power.

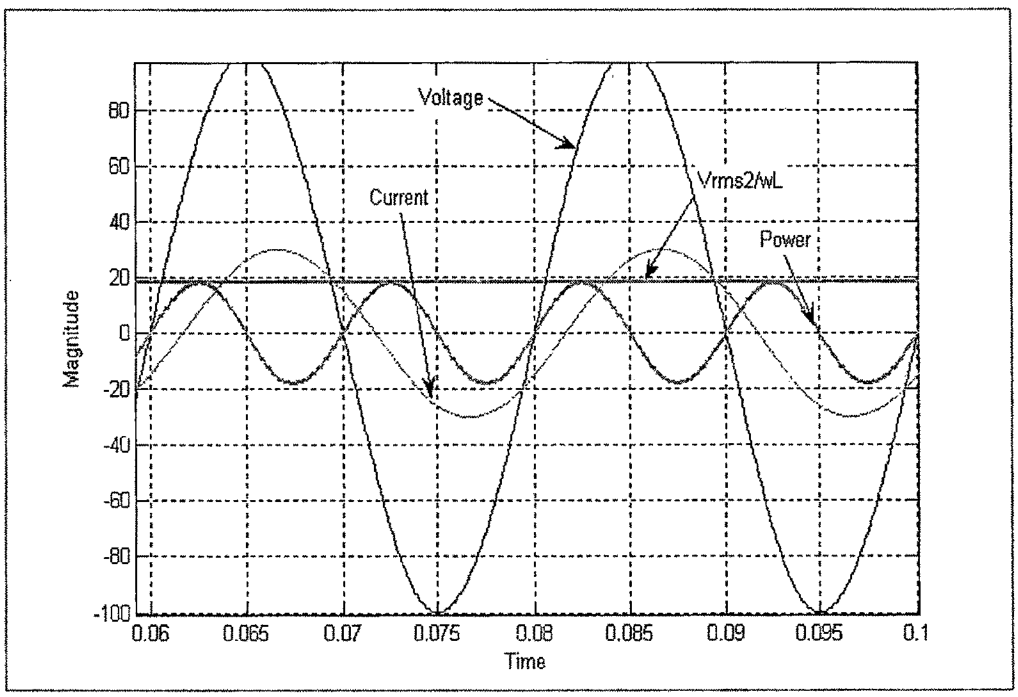

Fig. 3.2-5 Plot for inductive load

The peak value of the instantaneous oscillatory reactive power is $V_{Rms}^2/\omega L$. By definition this peak value is taken as the reactive power of inductor circuit. In spite of this power flow being bilateral, it is considered to flow from source in to the field. It should be noted that the active power in a resistive circuit is given by V_{Rms}^2/R , which is the average of instantaneous power input to the circuit where as for the inductor circuit the reactive power input is given by $V_{Rms}^2/\omega L$, which is the peak of the oscillatory instantaneous power input to the inductor circuit. Similar definition holds good for capacitor circuit also.

(C) Capacitive Circuit:

Consider a capacitive circuit as shown in Fig. 3.2-6. The average power is given by Eq. 3.12

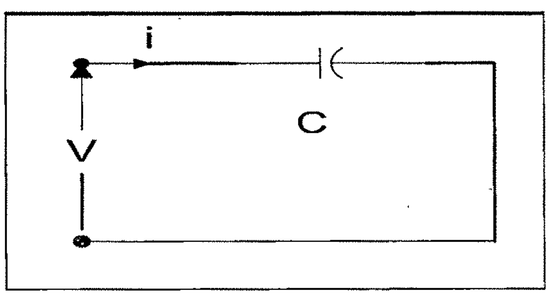

Fig. 3.2-6 Capacitive Circuit

$$v = V_m \sin \omega t \cdot$$
$$i = c\frac{dv}{dt} = V_m \omega C \cos \omega t \qquad (3.12)$$

Instantaneous power p is given by Eq. 3.13

$$p = vi = V_m^2 \omega C \sin \omega t \cos \omega t \cdot$$
$$= \frac{V_m^2}{2/\omega C} \sin 2\omega t$$

(3.13)

Average of p over a cycle is zero.

The instantaneous power input gives rise to rate of change in electro static field set up by capacitor. Hence, this power is called as reactive power Q whose average is zero. This power oscillates between the source and capacitor at twice the frequency. Fig. 3.2-7 shows the plot of this instantaneous reactive power.

The peak value of this oscillatory instantaneous reactive power is given by

$$Q\cdot = \frac{V_m^2}{2/\omega C} = V_{RMS}^2 \omega C$$

(3.14)

This peak value is defined as the reactive power input to the capacitor circuit.

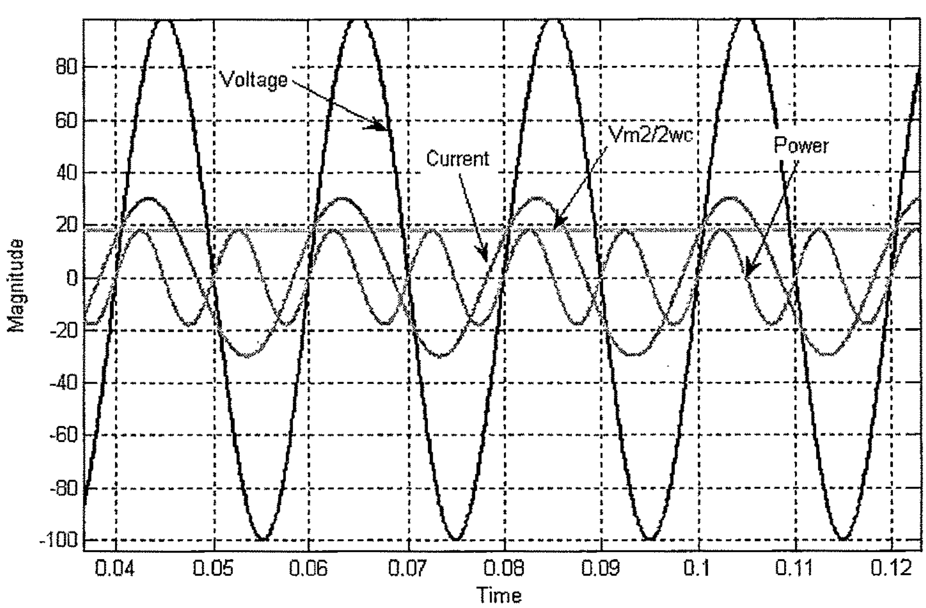

Fig. 3.2-7 Plot for instantaneous active and reactive power

3.3 Various Power Theories

3.3.1 Power Components in Sinusoidal Conditions

Power equation for a system was defined a long time ago. It mutually related the active, reactive and apparent power (P, Q and S) as given by Eq. 3.15

$$S^2 = P^2 + Q^2 \tag{3.15}$$

In addition, the instantaneous power $P(t)$ has been defined by Eq. 3.16

$$p(t) = v(t).i(t) \tag{3.16}$$

Where the voltage and current is given by Eq. 3.17 and Eq. 3.18

$$v(t) = \sqrt{2}\, V_{rms} \sin(\omega t) \tag{3.17}$$

$$i(t) = \sqrt{2}\, I_{rms} \sin(\omega t - \theta) \tag{3.18}$$

Where $\omega = 2\pi f$
f = fundamental frequency

Thus the instantaneous power is given by Eq. 3.19 and as shown in Fig. 3.3.1-1

$$p(t) = P[1 + \cos(2\omega t)] + Q\sin(2\omega t) \tag{3.19}$$

Where

P average power defined as VIcos(θ)
Q reactive power defined as VIsin(θ)
θ Power factor angle

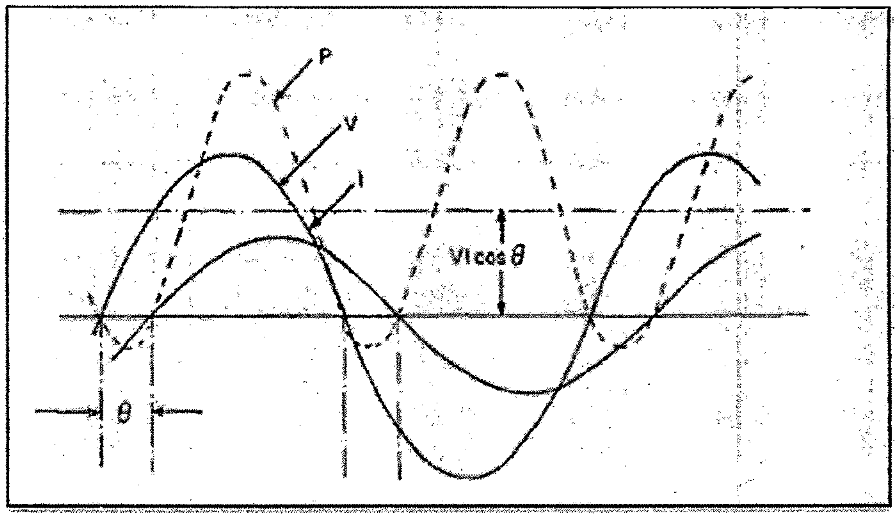

Fig. 3.3.1-1 Instantaneous power plot

3.3.2 Fryze Theory

Fryze introduced his idea of non-active power in 1931 [15] by using a time domain approach when he decomposed the source current into an active component which had the same waveform as the source voltage , and a non-active component by Eq. 3.20

(3.20)

The reactive current is the "useless" current component which does no contribute to the net transfer of enrgy but which has to be delivering to the load together with the active current. This "useless" component causes transmissions losses and voltage drops. The active, reactive and apparent powers are expressed in terms of the rms values of voltages V and these two currents .

$$Q_F = VI_r \quad (3.21)$$

$$Q_F = \sqrt{(VI)^2 - (VI_a)^2} \quad (3.22)$$

$$Q_F = \sqrt{S^2 - P^2} \tag{3.23}$$

Q_F Can be calculated directly from the apparent and active powers, there is no need to have a separate reactive power meter. The drawback of this approach is a loss of accuracy due to the calculations using the differences of squares. The active and reactive powers add geometrically to produce the apparent power. Q_F Is always positive, leading and lagging power factors cannot be distinguished.

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3.3.3 Kusters And Moore Theory

Kusters and Moore [18] extended Fryze decomposition theory for periodic non-sinusoidal waveforms in 1979 based on whether the non active current could be compensated by means of passive elements or not. They defined the active current I_a the same as Fryze in [15] but subdivided the instantaneous non-active current I_r into two components – an inductive and capacitive reactive current I_{ql} and I_{qc}. Their theory reasoned that the inductive (capacitive) reactive current could be completely compensated by means of adding or capacitor (inductor) in parallel with the load. They defined as the instantaneous value of the alternating component of $\int udt$ or in other words that portion of the voltage that did not contribute to the active power transfer from the source to the load. They then define the inductive (capacitive) reactive current as given by Eq. 3.25 and Eq. 3.26.

$$I_{ql} = \bar{u}\left(\frac{1}{T}\int_0^T \bar{u}\, idt\right)/\|\bar{u}\|^2 \tag{3.25}$$

$$I_{qc} = \dot{u}\left(\frac{1}{T}\int_0^T \dot{u}\, idt\right)/\|\bar{u}\|^2 \tag{3.26}$$

Because compensation by means of passive elements did not eliminate all of the non-active current, they defined that portion of the current which could not be compensated as the residual current I_{qlr} or I_{qcr}:

$$I_{qlr} = I - I_a - I_{ql} \qquad I_{qcr} = I - I_a - I_{qc} \tag{3.27}$$

Kusters and Moore definition is valid for single phase where compensation by only passive elements is considered [18]. Because their definition, like Fryze depends on RMS or average values over some time period, it does not lead itself real time compensation.

3.3.4 Enslin And Van Wyk Theory.

In 1998 Enslin and Van Wyk [24-25] generalized Fryze time-domain principles to single phase non-period waveforms by using correlation techniques and also further decomposed the non-active power into a reactive component Q and a de-active component F. To find the effective value of current or voltage, they used the time domain autocorrelation function as given by Eq. 3.28

$$R_{uu}(\tau) = \frac{1}{dT}\int_0^{dT} u(t)u(t-\tau)dt \tag{3.28}$$

$$U = [R_{uu}(0)]^{1/2} \qquad I = [R_{ii}(0)]^{1/2} \tag{3.29}$$

To calculate active power P and an equivalent conductance G, they used the time-domain cross correlation function.

$$R_{ui}(\tau) = \frac{1}{dT}\int_0^{dT} u(t)u(t-\tau)dt \quad P = R_{ui}(0) \qquad G = \frac{P}{U^2} \tag{3.30}$$

They then defined the instantaneous active and non-active current same as Fryze.

$$I_a(t) = Gu(t) \quad \text{And} \quad I_n = I - I_a \tag{3.31}$$

In order to decompose the non-active current into reactive I_r and deactive I_d components, they calculated the reactive power Q and equivalent susceptance B as Eq. 3.32

$$Q = \left[\hat{R}_{ui}^2(\tau) - R_{ui}^2(0)\right]^{1/2} \qquad B = \frac{Q}{U^2} \tag{3.32}$$

Where $\hat{R}_{ui}$ was the maximum value of the cross correlation between voltage and current waveforms, which occurred at the point in the time interval dT where maximum similarity between voltage and current waveforms was present. With the Hilbert transform of the voltage. Enslin and Van Wyk then defined the instantaneous reactive current as Eq. 3.33

$$I_r(t) = BH\{u\}\text{ , where } H\{u\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{u(\tau)d\tau}{\tau - t} \tag{3.33}$$

They then defined the instantaneous deactive current as given by Eq. 3.34

$$I_d = I - I_a - I_r \tag{3.34}$$

The mutual orthogonality of the current components leads to the following apparent power relation given by Eq. 3.35

$$S^2 = P^2 + F^2 = P^2 + Q^2 + D^2 \tag{3.35}$$

Although Enslin and Van Wyk extended Fryze theory to non-periodic single phase systems, their values of the autocorrelation and cross relation functions in Eq. 3.28 and Eq. 3.30 depends on the length of the interval dT and the interval start time

3.3.5 Distortion Power

There is another type of power calculated in non-sinusoidal situations, the so-called distortion power. The IEEE dictionary [17] defines distortion power D as a scalar quantity having amplitude equal to the square-root of the difference of the squares of apparent power U and amplitude of the phasor power S (S is a vector quantity having P and Q rectangular components). Mathematically as given in Eq. 3.36

$$S = \sqrt{(\sum_n P_n)^2 + (\sum_n Q_n)^2} \quad (3.36)$$

$$D = \sqrt{U^2} - S^2 \quad (3.37)$$

The sign of distortion power D from Eq. 3.37 is not definitely determined; it may be given either sign. The dictionary adds that in the absence of other definite information, the sign of D is to be taken the same as the active power. The dictionary does not state what type of definite information is necessary because the statement is essentially meaningless. Distortion power does not exist in the physical sense as do active power and instantaneous power. Both C. Budeanu [13] and many of his followers spent lots of time proving that P, S, and D are "conservative". It has been shown that distortion power

(i) Is not a measure of current waveform relative to voltage waveform,
(ii) Is not equal to zero when the current and voltage waveforms are identical but shifted in time.

Can be equal to zero even when the voltage and current waveforms are not identical [15]. This quantity does not express any distinct energy phenomenon. The only reason it was introduced is because the square of the reactive power defined in (15) is less than $U^2 - P^2$.

3.3.6 IEEE Power Definitions

The concept presented in [17] bases on the main assumption that the object of transmission is to deliver as much of the power as possible through 50Hz positive sequence component to the consumer. Therefore it makes sense to separate the fundamental and the harmonic components from each other respectively.

$$U^2 = U_I^2 + U_H^2 \quad (3.38)$$

$$I^2 = I_I^2 + I_H^2 \quad (3.39)$$

Where U_I and I_I represent fundamental voltage and current signal and U_H and I_H represent the harmonic signal present in the voltage and current signal.

With:

$$U_H^2 = \sum_{m=1} U_n^2 \text{ And } I_H^2 = \sum_{m=1} I_n^2 \quad (3.40)$$

The apparent power S is described by

$$S^2 = (U.I)^2 = S_1^2 + S_N^2 \quad (3.41)$$

The apparent power S given in Eq. 3.41 is divided into two main components, where S_1 is the fundamental apparent power which is turn resolved into the fundamental active power P_1 and fundamental reactive power Q_1 according to well-known equations used under pure 50Hz sinusoidal conditions. The second component in Eq. 3.41 is non-fundamental apparent power S_N and consist of three components as given by Eq. 3.42.

$$S_N^2 = (U_1.I_H)^2 + (U_H.I_1)^2 + (U_H.I_H)^2 \quad (3.42)$$

Because of the fact that the first components is the product of fundamental rms voltage by harmonic current it is named current distortion power and usually this is a dominant term in Eq. 3.42. The second term thinking in similar way is named voltage distortion power and it is a reflection of the voltage distortion power and its reflection of the voltage distortion at the bus. The third term of Eq. 3.42 is called harmonic apparent power S_H

3.3.7 Time Domain Czarnecki Theory

The concept of the momentary current decomposition into two orthogonal components active and reactive current was introduced by Fryze and has found its extension in the work of Czarnecki. His theory [20] is on the decomposition of currents into many orthogonal components. Non active powers are then defined as the product of the rms value of voltage by the rms value of current components. This current decomposition bases on the portioning of the set of N harmonic orders into two subsets N_A and N_B namely:

$$\text{If } P_n \geq 0 \text{ } then\, n \in N_A \qquad (3.43)$$
$$\text{If } P_n < 0 \text{ } then\, n \in N_B$$

Where P_n is the active power transmitted from the source to the load at the n-order harmonic frequency amounts to:

$$P_n = Re\{S_n\} = Re\{U_n^T . I_n^*\} \qquad (3.44)$$

In which the vectors of voltages and currents are calculated as the complex root mean square values for each harmonic frequency. In three-phase circuits these quantities can be obtained for line-to-ground voltages and line currents:

$$U_n = [U_a, U_b, U_c,]^T, I_n = [I_a, I_b, I_c,]^T \qquad (3.45)$$

An explanation for this decomposition lies in the direction of the energy flow. When the load is passive, linear and time-invariant then each of the harmonic active power P_n can not be negative. However, if any of these conditions is not fulfilled, the harmonic currents may be generated in the load so that the energy at that frequency may be transmitted back to the source (P_n < 0). This decomposition also enables to decompose the current observed at the bus into two mutually orthogonal components [21] and their rms values fulfill the relationship: given by Eq. 3.46

Similar relationship for the voltages yields to the statement describing the apparent power S which was first defined by Buchholz and then extended by Czarnecki given by Eq. 3.46

$$\|i\|^2 = \|i_A\|^2 + \|i_B\|^2 \tag{3.46}$$

Similar relationship for the voltages yields to the statement describing the apparent power S:

$$S^2 = \|u\|^2 . \|i\|^2 \tag{3.47}$$
$$S^2 = S_A^2 + S_B^2 + S_F^2$$

The last quantity in Eq. 3.47 occurs also in single-phase circuits under distorted conditions and is called forced apparent power.

3.3.8 Akagi P-Q Theory

In 1983 Akagi and coauthors [29, 30] introduced a novel concept, the *p-q* theory, by applying Park transformation to a three phase three wire system *(a-b-c)* to change it to a two phase plane $(\alpha - \beta)$ an orthogonal reactive axis as shown in Fig. 3.3.4-1.

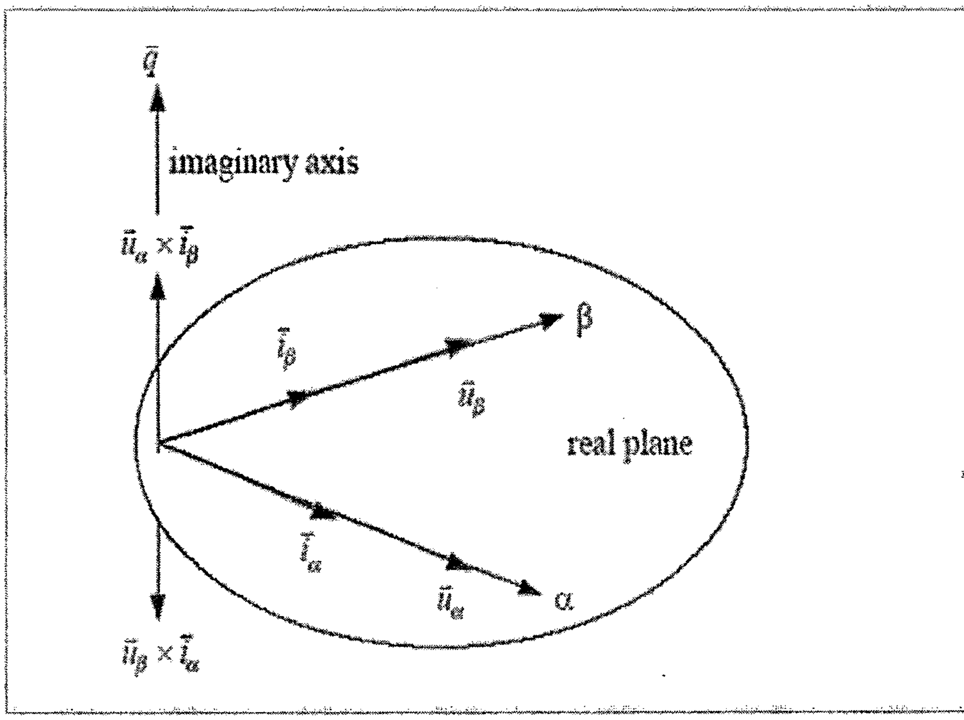


Fig. 3.3.4-1 Two phase plane

They also showed how this theory could be used for three phase four wire systems by introducing the zero phase sequence components. The *p-q* theory transforms instantaneous voltage and current measurement by Eq. 3.48 and Eq. 3.49.

$$\begin{bmatrix} v_0 \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/\sqrt{2} & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (3.48)$$

$$\begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (3.49)$$

Where are phase voltages. Identical relations hold for line currents .

The instantaneous three-phase active power $P_{3\emptyset}$ is given by

$$\begin{aligned} P_{3\emptyset} &= u_a i_a + u_b i_b + u_c i_c \quad (3.50) \\ &= u_\alpha i_\alpha + u_\beta i_\beta + u_0 i_0 \\ &= p_a(t) + p_b(t) + p_c(t) \\ &= p_\alpha(t) + p_\beta(t) + p_0(t) \\ &= p(t) + p_0(t) \end{aligned}$$

Where:

$p = p_\propto(t) + p_\beta(t)$ Is the instantaneous real power and

$p_0 = v_0 i_0$ Is the instantaneous zero sequence power. One advantage of using this theory is to separate zero-sequence components of the system.

Akagi [29] suggested the definition of the reactive power as

$$q(t) = v_\propto i_\beta - v_\beta i_\alpha \quad (3.51)$$

In fact this variable can be re-written in terms as *a-b-c* components.

$$q = -[(u_a - u_b)i_c + (u_b - u_c)i_a + (u_c - u_a)i_b]/\sqrt{3}\,.$$

This Eq. 3.51 is used further for measurement of conventional three-phase reactive power,. The variable q in the new concept consider all the frequency components in current and voltages. This is the reason that made Akagi call it as "instantaneous imaginary power". From Eq. 3.51 it can be seen that q is not influenced by zero sequence components of the system, since it depends only on $\propto$ and β components.

It is important to observe that these quantities are all instantaneous and valid for transient or steady state, and harmonic may be present in voltage and/or current. The instantaneous real power p ,gives the net energy per second being transported from source to load and vice versa at any time. Hence this theory of akagi is used for four quadrant power measurement in three phase four wire system. An another advantage of using this theory is that the active and reactive power are computed on line and real time measurement of both the parameter is possible. It is also possible to determine the direction of power flow both in the case of active and reactive power.

3.4 Three phase sinusoidal voltage applied to linear Load

To establish a connection with the well known conventional active (P) and reactive (Q) power in this section a balances linear load supplied by a three-phase balanced sinusoidal voltage will be analyzed. Let the voltages and the current by given by Eq. 3.52

$$\begin{aligned} v_a &= \sqrt{2}Vsin(\omega t) & i_a &= \sqrt{2}Isin(\omega t - \emptyset) \\ v_b &= \sqrt{2}Vsin(\omega t + 120^\circ), & i_b &= \sqrt{2}Isin(\omega t + 120^\circ - \emptyset), \\ v_c &= \sqrt{2}Vsin(\omega t - 120^\circ) & i_c &= \sqrt{2}Isin(\omega t - 120^\circ - \emptyset) \end{aligned} \quad (3.52)$$

Then using Eq. 3.48 and Eq. 3.49 the $v_\alpha, v_\beta, v_0, i_\alpha, i_\beta$ *and* i_0 are computed

$$\begin{aligned} v_\alpha &= \sqrt{3}Vsin(\omega t) & i_\alpha &= \sqrt{3}Isin(\omega t - \emptyset) \\ v_\beta &= \sqrt{3}Vcos(\omega t) & i_\beta &= \sqrt{3}Icos(\omega t - \emptyset) \\ v_0 &= 0 & i_0 &= 0 \end{aligned} \quad (3.53)$$

Therefore the active and reactive power can be defined as

$$P = v_\alpha i_\alpha + v_\beta i_\beta = 3VIcos\emptyset = P_{3\emptyset} \quad (3.54)$$

$$Q = v_\alpha i_\beta - v_\beta i_\alpha = 3VIsin\emptyset = Q_{3\emptyset} \quad (3.55)$$

These equations show the equivalence of the traditional concepts of active and reactive power and the new ones. Next, a three-phase balanced source supplying a non-linear load will be studied

3.5 Three Phase Sinusoidal Voltage Supplying a Non-Linear Load

Consider an electrical system with the voltage sources as given by Eq. 3.52 and with the following currents:

$$i_a = \sum_{n=1}^{\infty} \sqrt{2}\, I_n \sin(n\omega t - \emptyset_n) \quad (3.56)$$
$$i_b = \sum_{n=1}^{\infty} \sqrt{2}\, I_n \sin[\mathrm{n}(\omega\mathrm{t} + 120^{\circ}) - \emptyset_\mathrm{n}]$$
$$i_c = \sum_{n=1}^{\infty} \sqrt{2}\, I_n \sin\,[\mathrm{n}(\omega\mathrm{t} - 120^{\circ}) - \emptyset_\mathrm{n}]$$

Then,

$$i_\propto = \sum_{n=1}^{\infty} \frac{2}{\sqrt{3}} I_n sin(n\omega t - \emptyset_n)[1 - \cos\,(n120^{\circ})] \quad (3.57)$$
$$i_\beta = \sum_{n=1}^{\infty} 2\, I_n cos(n\omega t - \emptyset_n) \sin(n120^{\circ}),$$
$$i_0 = \frac{1}{\sqrt{3}}(i_a + i_b + i_c) = \sum_{n=1}^{\infty} \sqrt{6}\, I_{3n} \sin\,(3n\omega t - \emptyset_{3n})$$

It is interesting to note that the harmonics of order *"3n"* appear only in i_0.The power components$p$,q, p_0 and $p_{3\emptyset}$ are

$$p = v_\propto i_\propto + v_\beta i_\beta$$
$$= P_{\alpha p} + Q_{\beta p}$$
$$= 3VI_1 cos\emptyset_1 - 3VI_2 \cos(3\omega t - \emptyset_2) + 3VI_4 \cos(3\omega t - \emptyset_4) \quad (3.58)$$

$$q = v_\propto i_\beta - v_\beta i_\alpha$$
$$= 3VI_1 sin\emptyset_1 - 3VI_2 \sin(3\omega t - \emptyset_2) + 3VI_4 \sin(3\omega t - \emptyset_4) \quad (3.59)$$

$$p_0 = v_0 i_0 = 0\; and\; p_{3\emptyset} = p$$

Observing these equations, it is reasonable to write:

$$p = \bar{p} + \tilde{p}\; and\; q = \bar{q} + \tilde{q} \quad (3.60)$$

Where "-" indicates *"mean values"* and "~" indicates *"alternating components with mean-value equal to zero"*.

From Eq. 3.56 and Eq. 3.57 it is possible to conclude that:

$$\bar{p} = P_{3\emptyset}\; and\; \bar{q} = Q_{3\emptyset}$$
$$H = \sqrt{P^2} + Q^2 \quad (3.61)$$

Where P and Q are the root mean square values of $\tilde{p}$ and $\tilde{q}$ respectively.

The above equations give the relationship between the new and the conventional power theory. The average values of p that is $\bar{p}$ in this case, corresponds to the conventional average power. The real alternating power $\tilde{p}$ represents the energy per second that is being transported from source to load or vice versa at any time, due to current harmonic and its average value is zero. This pulsation of energy between the source and the load represents the energy being stored or released at the three-phase or two-phase ($\propto -\beta$) load or source. The average value of the imaginary power q that is $\tilde{q}$ corresponds to the conventional reactive power. The alternating part of q i.e $\tilde{q}$ is responsible from the harmonic (due to the current) reactive power in each phase, but vanishes instantaneously when added. The imaginary power $q = \bar{q} + \tilde{q}$ does not contribute for the instantaneous energy transport, although the reactive current exists in each phase and obviously occupies part of the conductor area.

From Eq. 3.59 it is possible to see that the conventional harmonic power H is composed by the alternating real and imaginary power.

3.6 Unbalanced Three-Phase 4-Wire System

To better understand how important the new concepts are an example considering an unbalanced three phase 4-wire voltage source supplying a linear load will be analyzed. For this analysis it will be assumed that the system is in steady state conditions and that $V_a, V_b, V_c,$ and $I_a, I_b, I_c,$ are respectively the voltage and current phasors for each phase. The symmetrical components of these currents are given by Eq. 3.62

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto & \propto^2 \\ 1 & \propto^2 & \propto \end{bmatrix}\begin{bmatrix} I_0 \\ I_+ \\ I_- \end{bmatrix} \qquad (3.62)$$

Where, *"0"*, *"+"* and *"-"*subscripts denotes zero, positive and negative sequences, respectively. The $\propto$ operator is equal to $e^{j2\pi/3}$. Similar relations hold for the load voltages. The inverse symmetrical components transformation of Eq. 3.62 is

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \propto & \propto^2 \\ 1 & \propto^2 & \propto \end{bmatrix}\begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix} \tag{3.63}$$

Now, it is possible to write the time functions corresponding to the phasors defined in Eq. 3.64. That is

$$\begin{aligned} v_a &= \sqrt{2}V_0 \sin(\omega t + \emptyset_0) + \sqrt{2}V_+ \sin(\omega t + \emptyset_{0+}) + \sqrt{2}V_- \sin(\omega t + \emptyset_-) \\ v_b &= \sqrt{2}V_0 \sin(\omega t + \emptyset_0) + \sqrt{2}V_+ \sin(\omega t - 120 + \emptyset_{0+}) + \sqrt{2}V_- \sin(\omega t + 120 + \emptyset_-) \\ v_c &= \sqrt{2}V_0 \sin(\omega t + \emptyset_0) + \sqrt{2}V_+ \sin(\omega t + 120 + \emptyset_{0+}) + \sqrt{2}V_- \sin(\omega t - 120 + \emptyset_-) \end{aligned} \tag{3.64}$$

The transformation of the instantaneous voltages in Eq. 3.64 in $\propto -\beta - 0$ axis as in (17) gives:

$$\begin{aligned} v_\propto &= \sqrt{3}V_+ \sin(\omega t + \emptyset_+) + \sqrt{3}V_- \sin(\omega t + \emptyset_+) \\ v_\beta &= \sqrt{3}V_+ \cos(\omega t + \emptyset_+) + \sqrt{3}V_- \cos(\omega t + \emptyset_+) \\ v_0 &= \sqrt{6}V_0 \sin(\omega t + \emptyset_0) \end{aligned} \tag{3.65}$$

Similarly, for the load currents $\dot{I}_a$, $\dot{I}_b$ and $\dot{I}_c$ it is possible to obtain their instantaneous $\propto -\beta - 0$ transformation, i.e

$$\begin{aligned} i_\propto &= \sqrt{3}I_+ \sin(\omega t + \delta_+) + \sqrt{3}I_- \sin(\omega t + \delta_-) \\ i_\beta &= \sqrt{3}I_+ \cos(\omega t + \delta_+) + \sqrt{3}I_- \cos(\omega t + \delta_-) \\ i_0 &= \sqrt{6}I_0 \sin(\omega t + \delta_0) \end{aligned} \tag{3.66}$$

In Eq. 3.65 and Eq. 3.66 it is possible to see that positive and negative sequences appear only in the $\propto$ and β axis. On the other hand zero sequence components appear only in the "*0*" axis. Using these voltages and currents it is possible to calculate the expressions for the real (p) imaginary (q) and zero sequence (p_0) instantaneous powers. These powers can be separated in their average values and alternating parts. That is

Instantaneous average real power:

$$\bar{p} = 3V_+I_+\cos(\emptyset_+ - \delta_+) + 3V_-I_-\cos(\emptyset_- - \delta_-) \quad (3.67)$$

Instantaneous average imaginary power:

$$\bar{q} = -3V_+I_+\sin(\emptyset_+ - \delta_+) + 3V_-I_-\sin(\emptyset_- - \delta_-) \quad (3.68)$$

Instantaneous alternating real power:

$$\tilde{p} = -3V_+I_+\cos(2\omega t + \emptyset_+ - \delta_-) + 3V_-I_+\cos(2\omega t + \emptyset_- - \delta_+) \quad (3.69)$$

Instantaneous alternating imaginary power:

$$\tilde{q} = -3V_+I_+\sin(2\omega t + \emptyset_+ - \delta_-) - 3V_-I_+\sin(2\omega t + \emptyset_- + \delta_+) \quad (3.70)$$

Instantaneous average zero-sequence power:

$$\bar{p}_0 = 3V_0I_0\cos(\emptyset_0 - \delta_0) \quad (3.71)$$

Instantaneous alternating zero-sequence power:

$$\tilde{p}_0 = -3V_0I_0\cos(2\omega t + \emptyset_0 - \delta_0) \quad (3.72)$$

From the above expressions, some important conclusions must be pointed out:

(1) The zero sequence power p_0 is exactly equal to the power in a single-phase circuit having an average part and an alternating component. Its characteristics is similar to the real power p that is the value of the zero sequence power at any time gives the amount of energy being transported per second from source to load and vice versa. The alternating component of p_0 has a relatively low frequency. In term of the new concepts there is no reactive power in p_0.

(2) Both positive and negative sequences produces p and q however their simultaneous presence produces alternating components p and q even though no current.

(3) Only positive and negative sequences can produce reactive power in terms of the new concepts that is the power that exists in each phase but their instantaneous sum is zero.

3.7 Principles of Four Quadrant Power Measurement

These meters used for measurement of power flows in E H V transmission system should be able to identify the direction of power flow also. Therefore, unlike in the case of meters used in the distribution sector, the measurement in this case is based only on the fundamental quantities. The input voltage and current signals are digitized and from these samples the fundamental component of voltage and current are evaluated. Using these components and the phase angle, the active and reactive power in 4 quadrants is computed. Integrating the powers, the power flow is monitored. To take care of the variable tariff, multiplication factors are changed depending upon the frequency and voltage. These multiplication factors change the actual energy measurement so that keeping the tariffs constant, the actual payment made will be higher or lower depending upon the system conditions.

Thus the active power drawn over and above the schedule when frequency is low is charged more and same is the case for reactive energy drawn under low voltage. The meter will have facility for storing and downloading the metered data. Meter also records voltage, current frequency and MVA demand.

3.8 Proposed Concept of Variable Power Tariff's

In an Inter-connected power system, the inter-state transmission links carry active and reactive power from generating stations (which include private and public power plants) to the state grids. The tariffs for these power exchanges should be frequency and voltage dependent unlike in the case of meters used in the distribution sector. These variable charges for active and reactive power are proposed with a view to make the inter-connected system function efficiently.

The main reason to go in for frequency dependent tariffs is for the improvement of performance of integrated power system. The tariff for active power is frequency dependent and that for reactive power is voltage dependent. The idea is to force the state grids to become self-sufficient as for active and reactive power generation is concerned. The state grid receives some specified share of active power from the Central and private generating stations. The power transfer from these stations to the state grid takes place through E H V lines of the central and state transmission system. In an inter-connected system if the available generation is less than the prevailing load, the frequency falls below the normal. In such cases, the state generation has to pick up to bring back the frequency. If there is no extra generation available then the state begins to draw more than its share of power from, the central and private generating stations thus depriving other states from their share of power. Therefore, the state has to pay extra for drawing more than its share of power. To ensure that the state plan their own generation growth properly the extra power transfer over the normal share from the central and private power plants is priced at a much higher level than the usual charges.

If the state draws more than its share of power from the outside generating stations when the grid frequency is above normal, then the extra power drawn should be charged at a much lower rate than normal tariff. This is because the state is helping these generating stations to operate at a high plant load factor (PLF).

Similarly the reactive power transfer on the E H V lines takes place due to difference in bus voltage levels. To maintain proper voltage in the grid, the state should provide enough reactive power generation. If the available reactive power generation in the state grid is not enough, the voltage level falls at high loads. The grid draws more reactive power from the central and private generators over the inter-connecting EHV lines. This results in increased losses and lower stability limits. Therefore, the reactive power drawn when the voltage is low should be charged at a higher rate. The reactive power flow under normal voltage level is not to be charged. Similarly if the state draws reactive power (lag) from the central system when the voltage level is high, the state would get subsidy for improving the stability of the inter-connected power system by ensuring lagging power factor at the generator terminals of central and private stations.

These higher payments will force the state sector to plan their active and reactive power sources properly.

3.9 Mathematical Analysis

The reason for selecting instantaneous Power theory [30] is that the sign of the active and reactive power can be computed easily.

In the case of four quadrant power measurement active and reactive power is computed on-line and from the information of the sign of active and reactive power the director of power vector is computed i.e in which quadrant. If the vector for active power is in quadrant 1^{st} and 4^{th} then it indicates that the active power is exported i.e. from generator to load and if the vector for active power is in quadrant 2^{nd} and 3^{rd} then it indicates that the active power is imported i.e. from load to generator. If the vector for reactive power is in quadrant 1^{st} and 2^{nd} then it indicates that the reactive power is exported i.e. from generator to load. If the vector for reactive power is in 3^{rd} and 4^{th} then the reactive power is imported i.e. from load to generator.

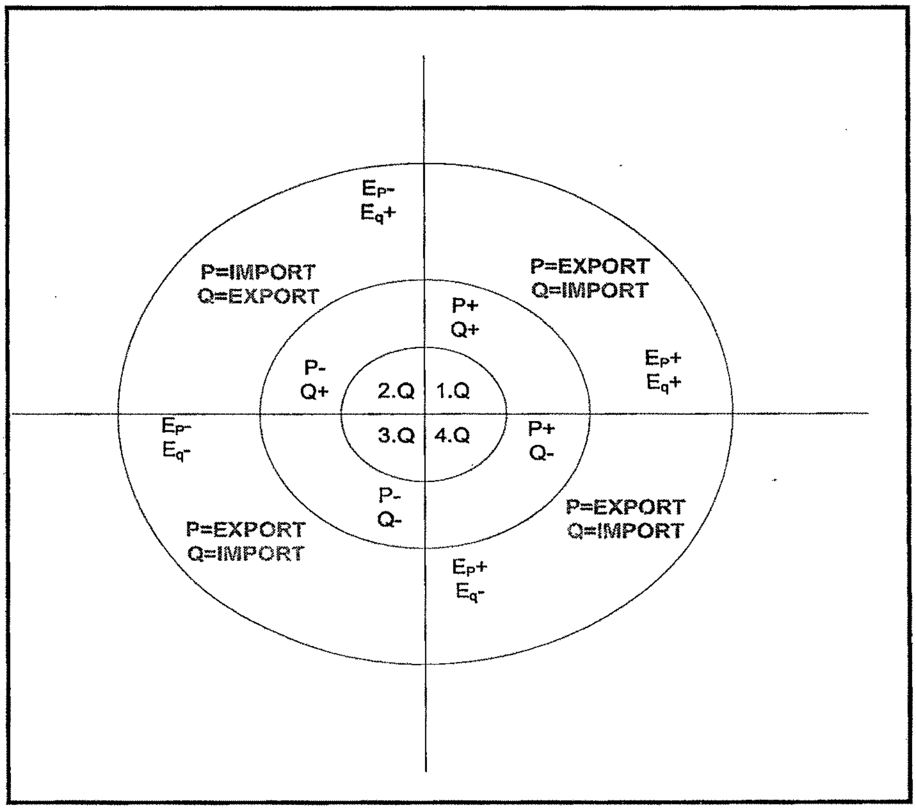


Fig. 3.9-1 Four quadrant plane

Using the generalized active reactive power theory [1, 2] active and reactive power vector are computed and then the flow of power (import/export) is calculated In the case of 4-quadrant power measurement the frequency and system voltage is also monitored If the system frequency is lower than the normal frequency and the active power is imported then the tariff rate should be increased. Similarly if the system voltage is reduced and reactive power is imported then the tariff rate should be increased. The use of 4-quadrant power measurement and flexible tariff structure would increase the stability of the power system in an interconnected power system. It also enables the state grid to plan their power consumption and plan load shedding accordingly

3.10 Simulation

The proposed method for four quadrant power measurement is tested under sinusoidal and non-sinusoidal condition. The method is first mathematically tested using Microsoft Excel and then simulated in MATLAB™ software and later deployed on a hardware platform using TMS320F2806 processor.

3.10.1 Simulation on Excel

In order to test the suitability of method for four quadrant power measurement all the equations are simulated on Excel. The equations on excel are simulated by providing various input parameters such as frequency, voltage, current and power factor. Active, reactive and apparent power values are obtained by using the Eq 3.48 and Eq 3.49. Since in case of interconnected power system, any system connected for measurement of power must be suitable under sinusoidal, non sinusoidal and harmonic conditions. These studies are carried on Excel and various case studies and results are tabulated below.

- CASE-1

The method is first tested under sinusoidal conditions at various power factors. In this case, calculations are done mathematically irrespective of load in order to test feasibility of the method for four quadrant power measurement. As shown in the Table 3.10.1-1 the result obtained at various voltages, current and power factor.

	Condition: Sinuosidial Conditions										
Sr.No	Voltage (Volts)			Current(Amps)			Angle	P.F	Power (W,W,VA)		
	Vr	Vy	Vy	Ir	Iy	Ib	Deg	COS(Φ)	Active	Reactive	Apparent
1	240	240	240	10	10	10	0	1.00	2084.92163	0	2084.922
2	240	240	240	10	10	10	30	0.87	1805.5951	1042	2084.922
3	240	240	240	10	10	10	60	0.50	1042.46082	1805.595	2084.922
4	240	240	240	10	10	10	90	0.00	0	2084.922	2084.922
5	240	240	240	10	10	10	120	-0.50	-1042.46082	1805.595	2084.922
6	240	240	240	5	10	10	150	-0.87	-1805.5951	1042.461	2084.922
7	240	240	240	10	10	10	180	-1.00	-2084.92163	0	2084.922
8	240	240	240	10	10	10	210	-0.87	-1805.5951	-1042.46	2084.922
9	240	240	240	10	10	10	270	0.00	0	-2084.92	2084.922
10	240	240	240	10	10	10	310	0.64	1340.16179	-1597.14	2084.922

Table 3.10.1-1 Results under Sinusoidal conditions

As shown in Table 3.10.1-1 three phase voltage and currents are given as input parameter and the angle represents angle between voltage and current. The power value represents values calculated using instantaneous alpha-beta theory at different power factor. It can be seen that depending on power factor there is bidirectional flow of power for both active and reactive power. The positive sign indicates that power is imported from source to load and negative sign indicates that power is drawn by the source. It can be seen from results that under sinusoidal conditions this method can be used for bidirectional power measurement and is very useful to implement in case of an interconnected power system to compute total import and export of power.

❖ CASE-2

The feasibility of the system under various unbalanced conditions is studied.

Sr.No	Condition: Sinuosidial Unbalanced Conditions										
	Voltage (Volts)			Current(Amps)			P.F	COS(Φ)	Power (W,W,VA)		
	Vr	Vy	Vy	Ir	Iy	Ib	Deg		Active	Reactive	Apparent
1	240	100	240	10	2	10	0	1.00	173.74347	0	173.7435
2	240	100	240	10	2	10	30	0.87	150.466258	56.87173	173.7435
3	240	100	240	10	2	10	60	0.50	86.8717348	150.4663	173.7435
4	240	100	240	10	2	10	90	0.00	0	173.7435	173.7435
5	100	240	150	2	10	5	120	-0.50	-1042.46082	1805.595	2084.922
6	100	240	150	2	10	5	150	-0.87	-1805.5951	1042.461	2084.922
7	100	240	150	2	10	5	180	-1.00	-2084.92163	0	2084.922
8	100	240	150	2	10	5	210	-0.87	-1805.5951	-1042.46	2084.922
9	100	240	150	2	10	5	270	0.00	0	-2084.92	2084.922
10	100	240	150	2	10	5	310	0.64	1340.16179	-1597.14	2084.922

Table 3.10.1-2 Results under Sinusoidal unbalance conditions

The results of simulation under unbalanced conditions are given in Table 3.10.1-2. The unbalance is created both in voltage and current magnitudes at different phase angels. The results of various power values as obtained by instantaneous alpha-beta theory and are as shown in Table 3.10.1-2. In this case the bidirectional flow of power can be seen and hence proved its suitability in case of unbalanced condition for interconnected power system.

❖ CASE-3

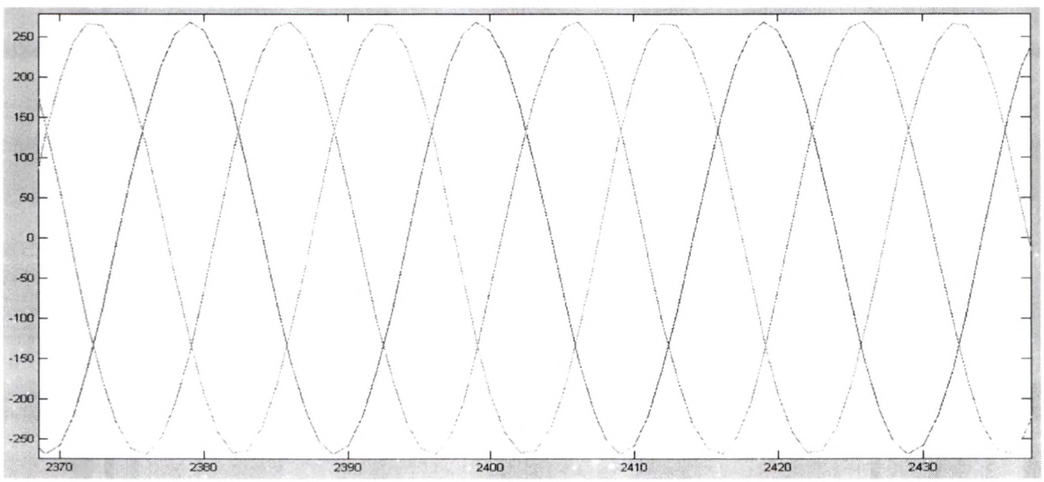
The system is simulated under various harmonic conditions to study its suitability. In this case various order of harmonics such as third, fifth and seventh are added to voltage and current signal and then power components are calculated. Also the magnitude of harmonic is varied from 5% to 10% of the fundamental magnitude and then results are verified. Along with harmonic, power factor is varied and results are as mention in Table 3.10.1-3.

Condition: Harmonics												
	Voltage	Vr, Vy,Vb	240 V							POWER		
	Current	Ir, Iy,Ib	10A									
Sr.No	Harmonic		Occurrence							Active	Reactive	Apparent
1	Order	Mag (%)	Vr	Vy	Vb	Ir	Iy	Ib	COS(Φ)	W	Va	VA
2	3	5							1.00	2084.92163	0	2084.922
1	3	5							1.00	2084.92163	0	2084.922
2	3	5							1.00	2084.92163	0	2084.922
3	3 & 5	5							1.00	2084.92163	0	2084.922
4	3	10							1.00	2084.92163	0	2084.922
5	3	10							0.50	1042.46082	1805.595	2084.922
6	3	10							0.87	1805.5951	1042.461	2084.922
7	3	10							-0.50	-1042.46082	1805.595	2084.922
8	3	10							-0.87	-1805.5951	1042.461	2084.922
9	3 & 5 & 7	5							-0.50	-1042.46082	1805.595	2084.922
10	3 & 5 & 7	5							0.5	1042.46082	1805.595	2084.922

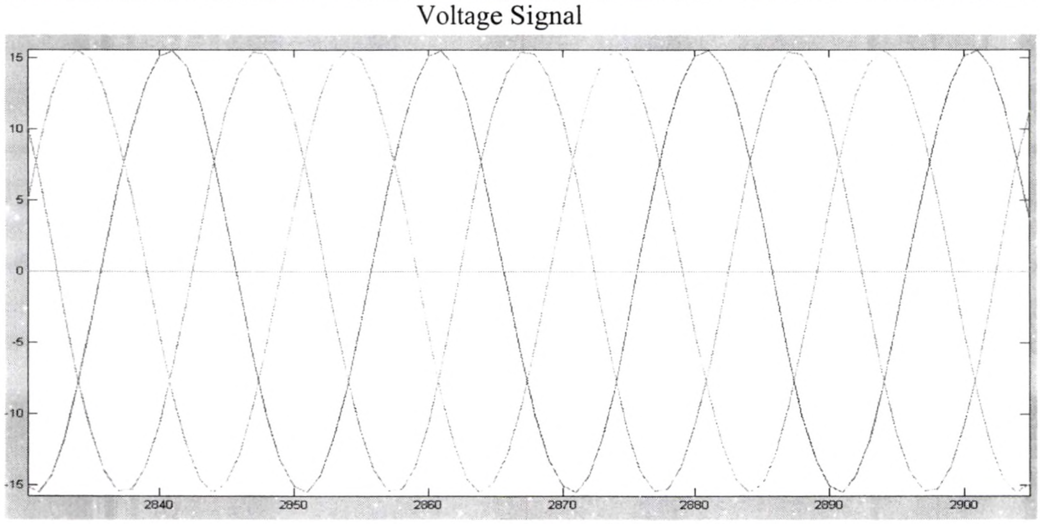
Table 3.10.1-3 Results under harmonic conditions

In this case magnitudes of Voltage and Current mentioned are the peak magnitude and representing the harmonic order to be added in respective voltage. Example 3 represents third order harmonic i.e 150Hz if the fundamental freq is 50 Hz. The harmonic signal is added in phase with fundamental signal. The occurrence (i.e green shaded) represents that particular order of harmonic is added to the respective voltage, colored represent that the harmonic is added in the respective voltage or current.

3.11.2. Simulation On Matlab^{TM}



Voltage Signal



Current Signal

Fig. 3.10.1-1 Three phase voltage and current waveforms on simulink

The system is simulated using simulation software MATLAB^{TM} for difference cases. In case of MATLAB, system is simulated using *m-file editor,* Eq 3.58 and Eq 3.59 are modeled in *m-file editor.* The system is simulated for three phase voltage and current signals. From these components active power and reactive power are computed. As shown in

Fig. 3-16 simulated *m-file* with GUI capabilities are added so that user can directly add values and can simulate the various conditions as were done on excel. Fig. 3.10.1-1 shows input signal given to the system and Fig. 3.10.1-2 shows various plots obtained for four quadrant power measurements.

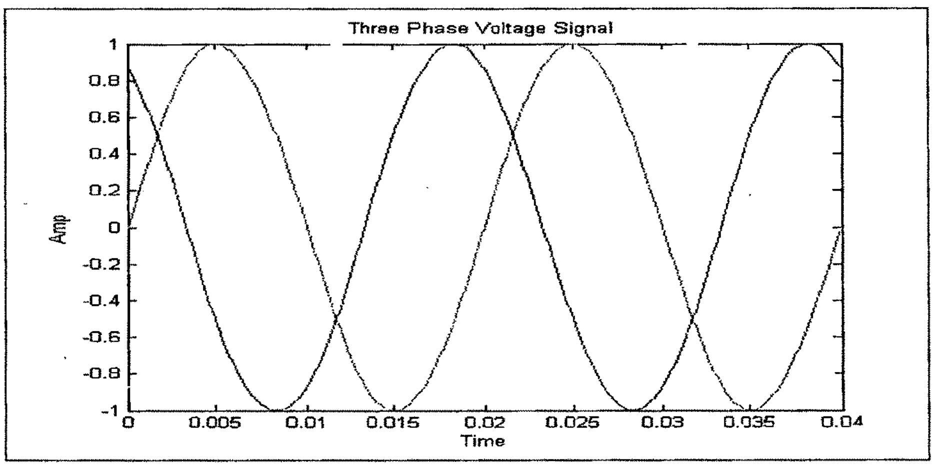


Fig. 3.10.1-2 Three phase voltage signals

The alpha and beta components are computed and phase angle is varied for different cases.

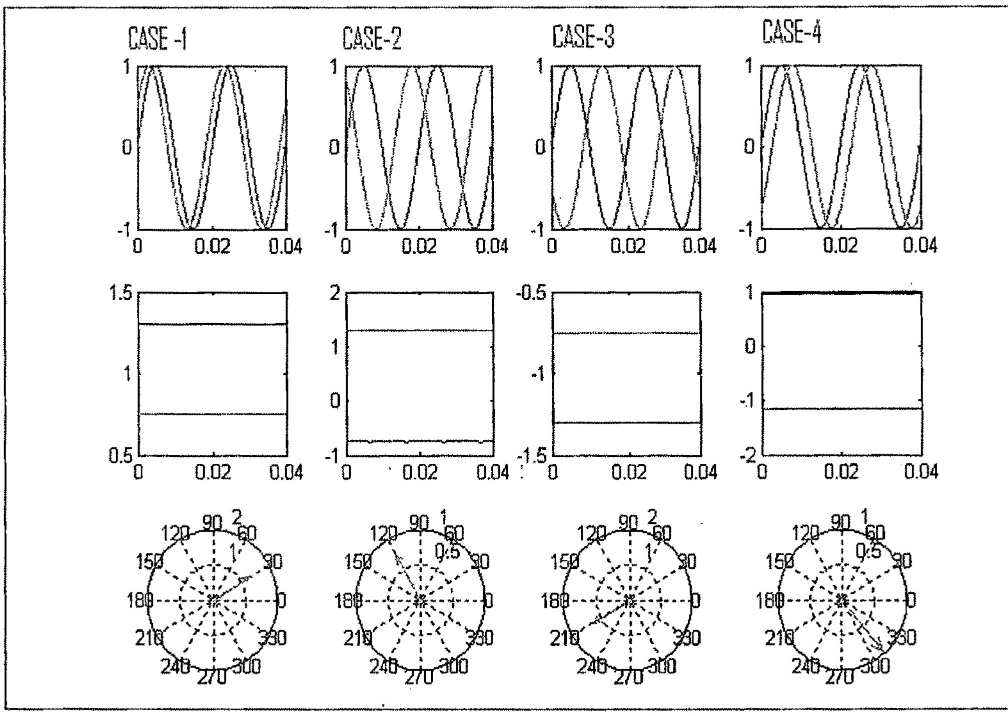


Fig. 3.10.1-3 Simulation Results for Four quadrant power

The system is simulated for four different cases for calculating active and reactive power. In all four cases voltage and current magnitude are unity. The phase angle is varied in each case and then active and reactive power is calculated and then quadrant is plotted so that the import and export of power can be evaluated. The results obtained under various conditions are as shown in Table 3.10.1-4.

Case 1	Angle	30
	Active power	1.3002
	Reactive power	0.7507
	Quadrant	First
Case 2	Angle	120
	Active power	-0.7507
	Reactive power	1.3002
	Quadrant	Second
Case 3	Angle	210
	Active power	-1.3002
	Reactive power	-0.7507
	Quadrant	Third
Case 4	Angle	310
	Active power	0.9650
	Reactive power	-1.1501
	Quadrant	Fourth

Table -3.10.1-4 Simulation Results of four quadrant power

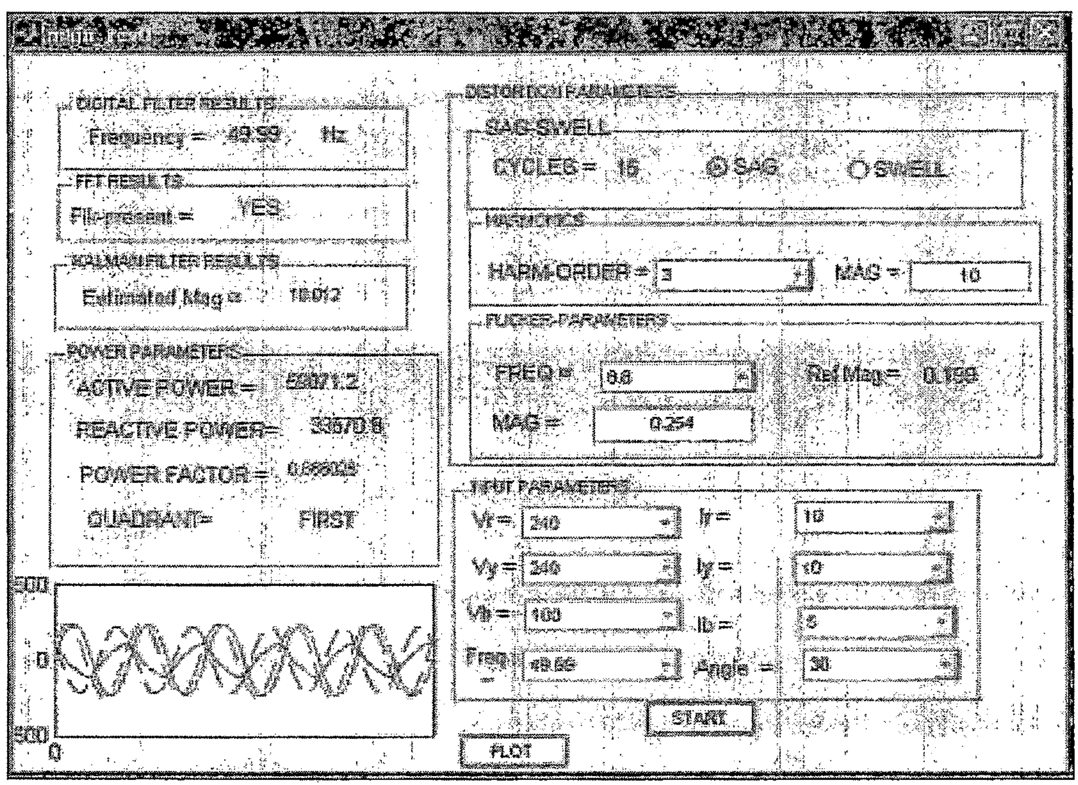


Fig. 3.10.1-4 Screen Shot of power measurement window

❖ SIMULINK

The four quadrant power measurement is also modeled in the SIMULINK software of MATLAB™. In this case the model is developed as shown in Fig. 3.10.1-5. The mode is tested for various loads such as resistive, inductive and under non-linear condition.

❖ CASE-1-Resistive Load

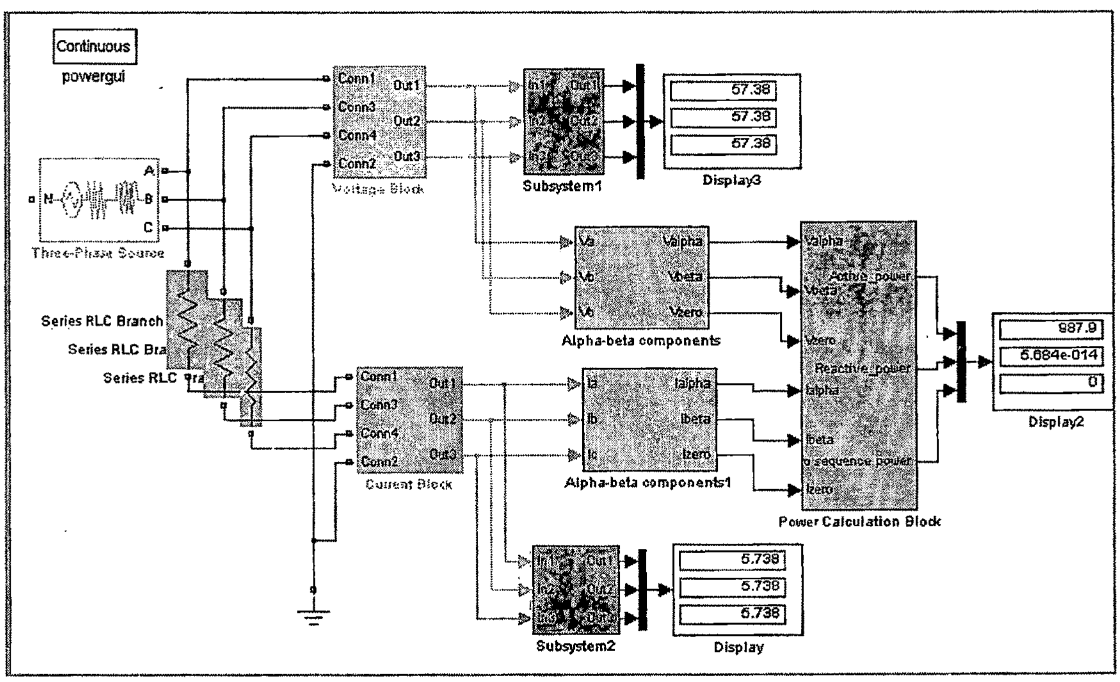


Fig. 3.10.1-5 Screen Shot for power measurement at resistive load

As shown in Fig.3.10.1-5 the model is developed on a SIMULINK platform. A three phase voltage source is used and then voltage and current measurement block are modeled. The three phase to two phase conversion is done by equation and is inserted in sub-block alpha-beta components. The output of voltage, current and power output are connected to a digital display. The detailed description of the sub-block is given in Fig. 3.10.1-6, Fig. 3.10.1-7. The system is tested for various voltages and results are formulated in Table 3.10.2-5.

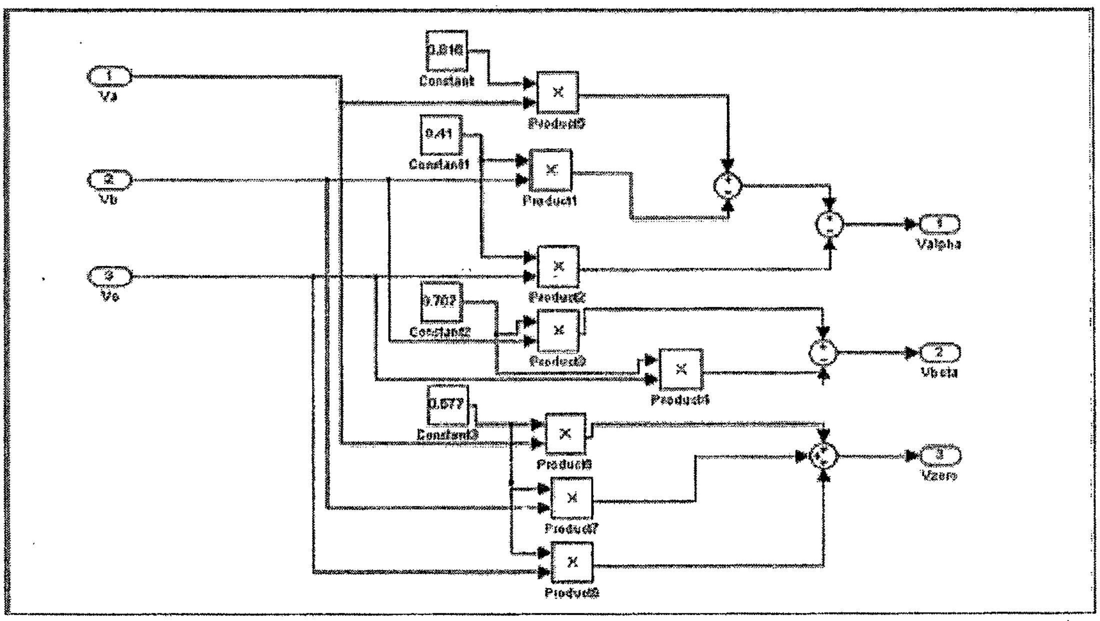


Fig. 3.10.1-6 Three Phase to Two-phase conversation

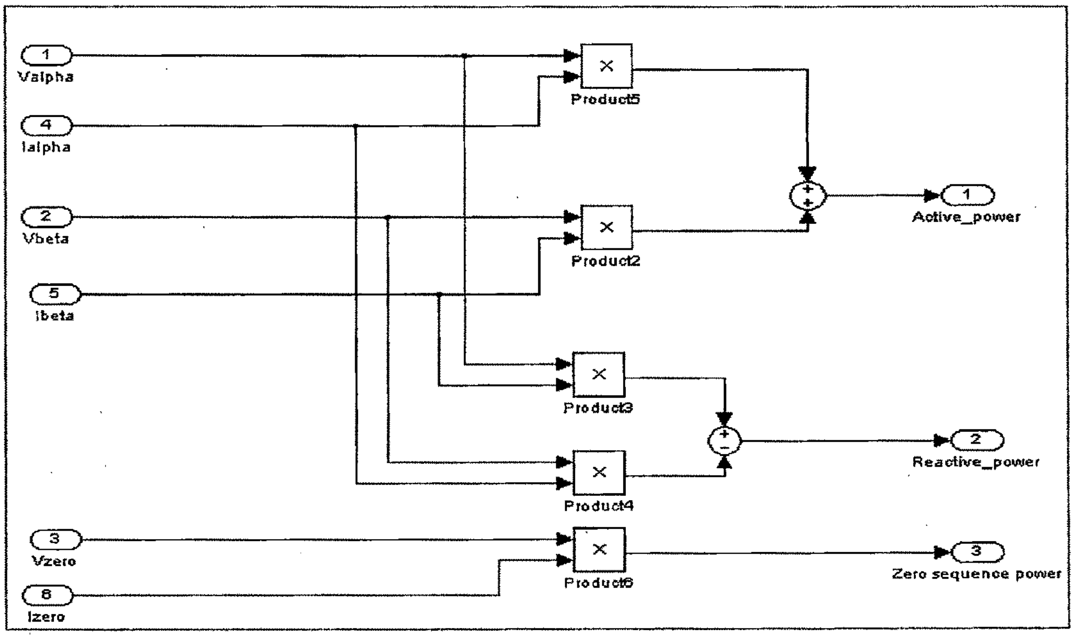


Fig. 3.10.1-7 Power computation Block

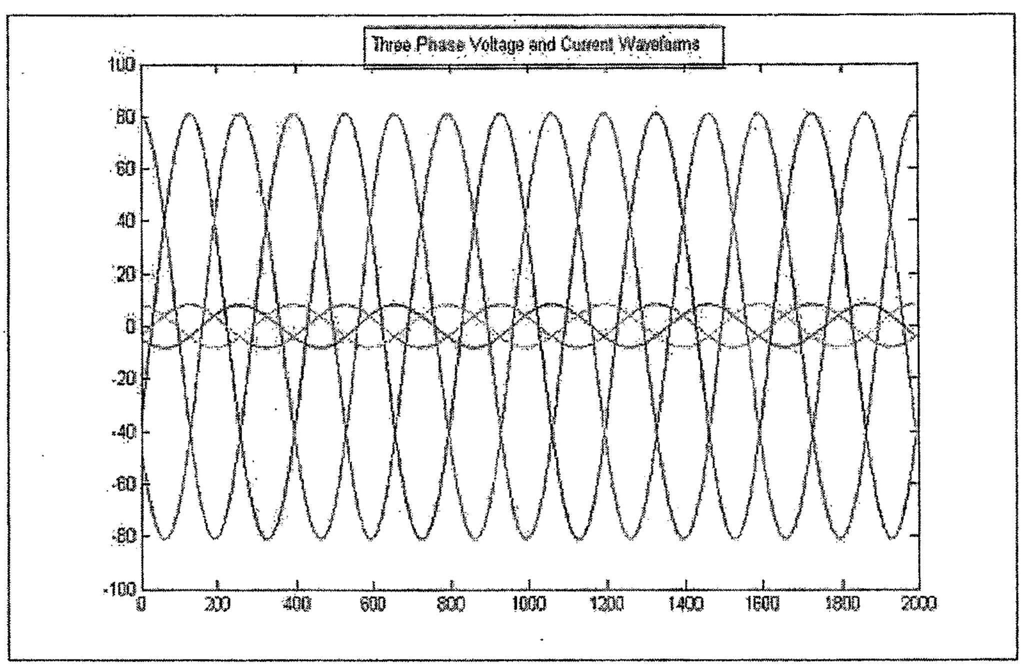


Fig. 3.10.1-8 Three phase Voltage and Current Waveforms at resistive load

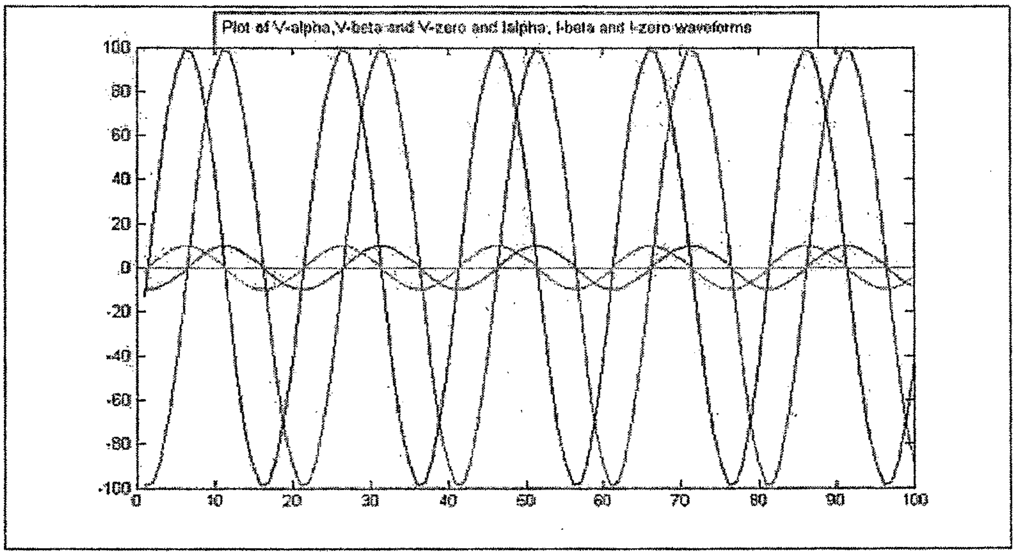


Fig. 3.10.1-9 Voltage and Current Alpha and Beta components

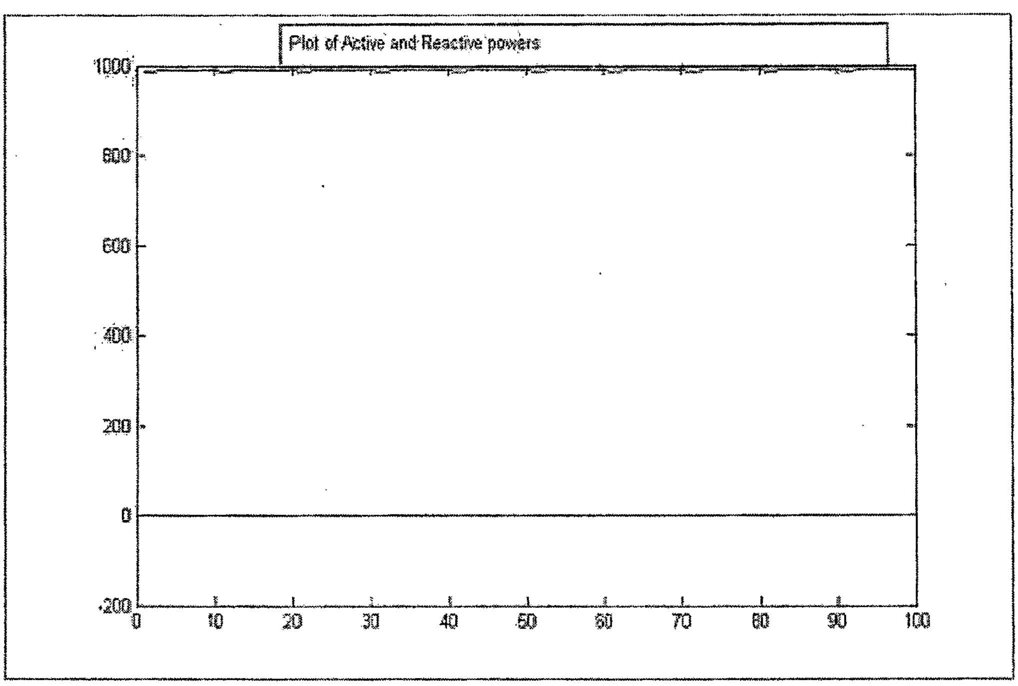


Fig. 3.10.1-10 Active and Reactive power components

Condition: Resistive load R= 10Ohm								
	Voltage			Current			Power	
	Vr	Vy	Vb	Ir	Iy	Ib	P	Q
Sr.No	Volts			Amps			W	Var
1	23.9	23.9	23.91	2.39	2.39	2.39	171.5	0
2	47.8	47.8	47.82	4.78	4.78	4.78	686	0
3	57.4	57.4	57.38	5.74	5.74	5.738	987.9	0
4	198	198	198.4	19.8	19.8	19.84	11820	0
5	239	239	239.1	23.9	23.9	23.91	17150	0

Table -3.10.1-5 Results of power measurement at resistive load

Table 3.10.1-5 shows the result of the proposed theory under resistive load conditions. Fig. 3.10.1-8 shows the V_α ,V_β and V_0 and I_α ,I_β and I_0 waveforms respectively. Also the active and reactive power plot is also shown in Fig. 3.10.1-9.

- CASE 2- INDUCTIVE LOAD

The system is simulated for the inductive load and the various power parameters are computed and evaluated.

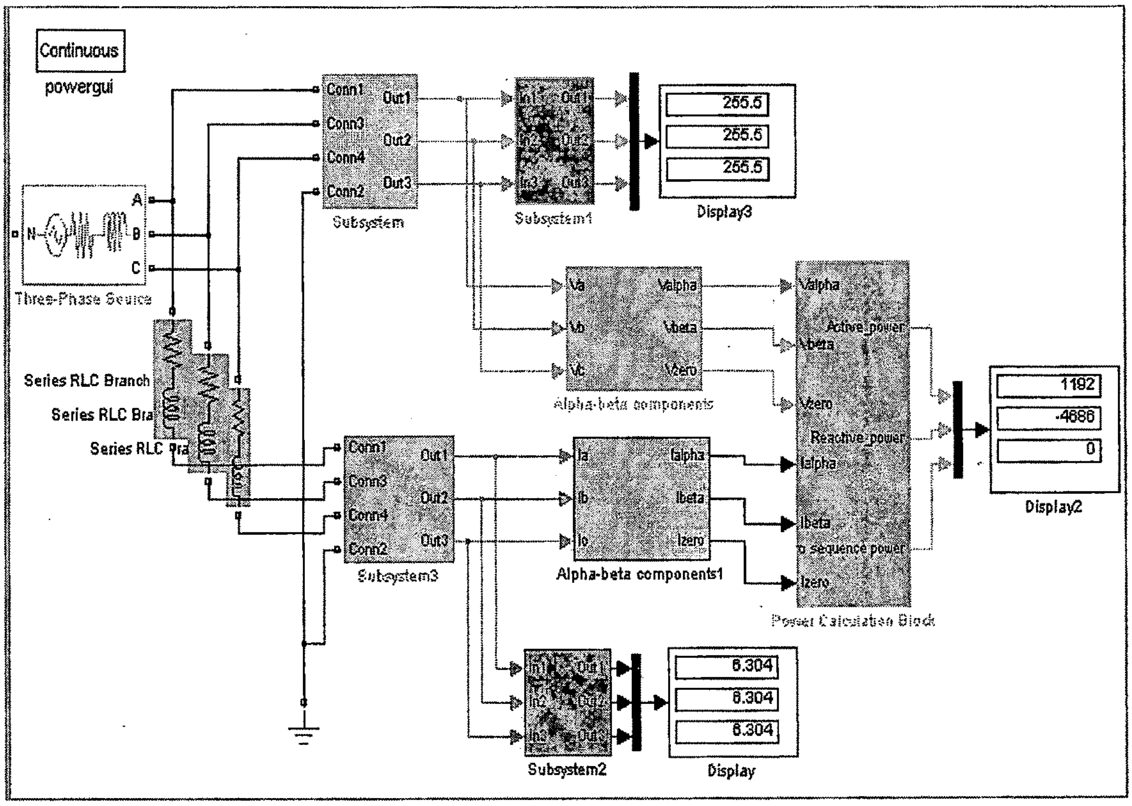


Fig. 3.10.1-11 Screen Shot for power measurement at inductive load

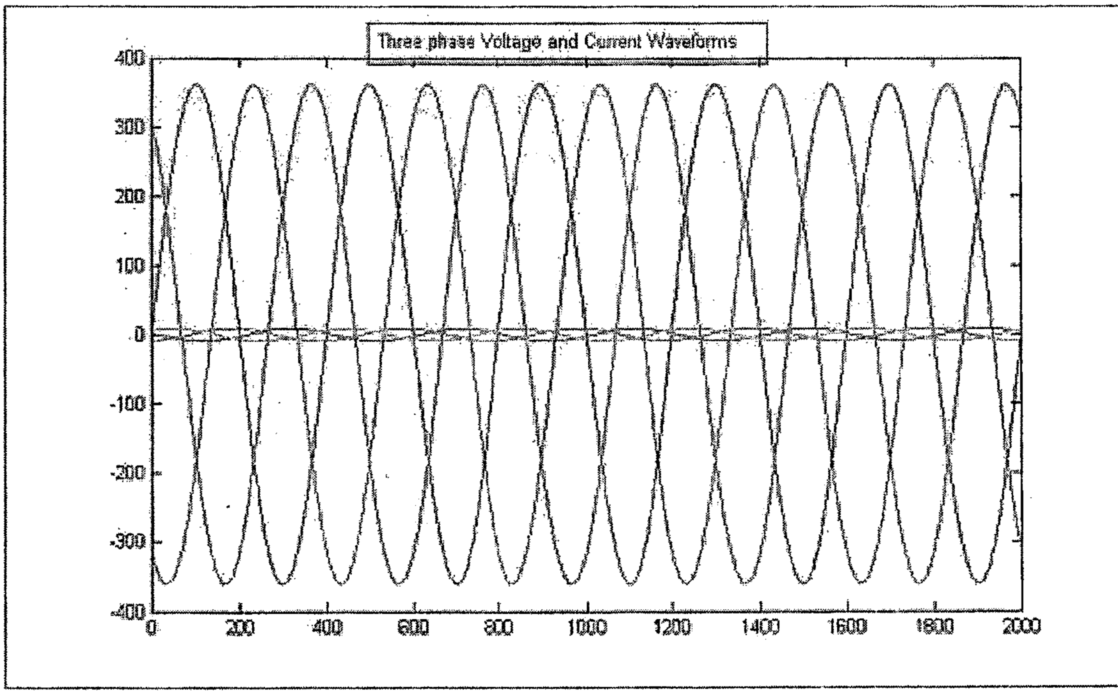


Fig. 3.10.1-12 Three phase Voltage and Current Waveforms at inductive load

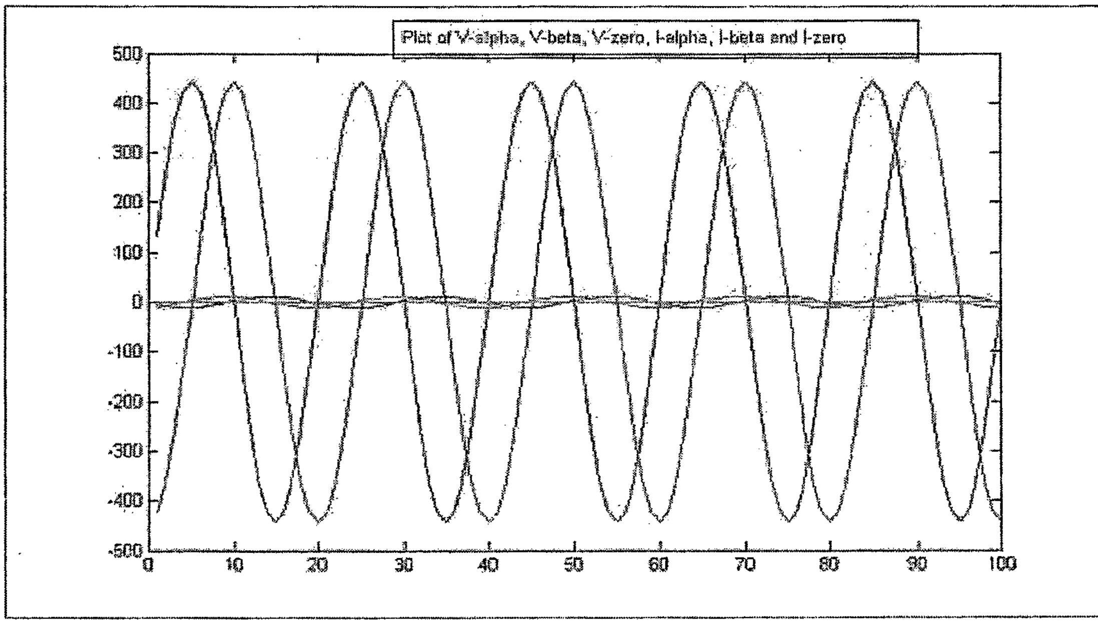


Fig. 3.10.1-13 Voltage and Current Alpha and Beta components

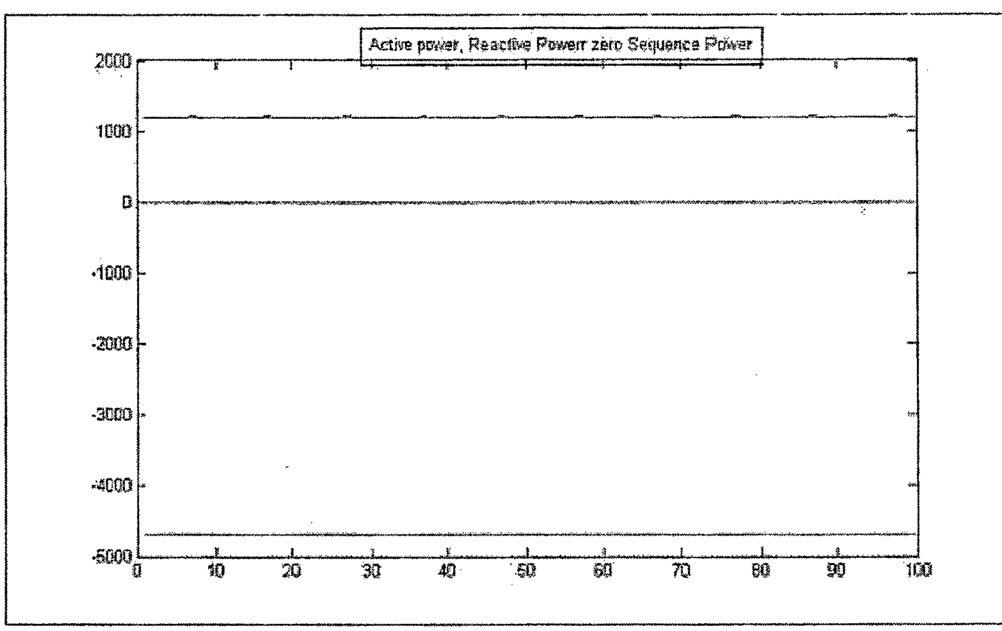


Fig. 3.10.1-14 Active and Reactive power components

Condition: Inductive load R= 10 Ohm and L=0.125H								
	Voltage			Current			Power	
	Vr	Vy	Vb	Ir	Iy	Ib	P	Q
Sr.No	Volts			Amps			W	Var
1	25.6	25.6	25.55	0.63	0.63	0.63	11.92	-46.86
2	51.1	51.1	51.09	1.26	1.26	1.261	47.68	-187.4
3	76.6	76.6	76.64	1.89	1.89	1.891	107.3	-421.7
4	212	212	212	5.32	5.32	5.32	821.1	-3228
5	256	256	255.5	6.3	6.3	6.304	1192	-4686

Table 3.10.1-6 Results of power measurement at inductive load

Table 3.10.2-6 shows the result of proposed theory under inductive load conditions. Fig. 3.10.1-13 shows the V_α, V_β and V_0 and I_α ,I_β and I_0 waveforms respectively. Also the active and reactive power plot is also shown in Fig. 3.10.1-14.

- CASE 2- CAPACITIVE LOAD

The system is also simulated for leading power factor i.e for capacitive load and power components are evaluated.

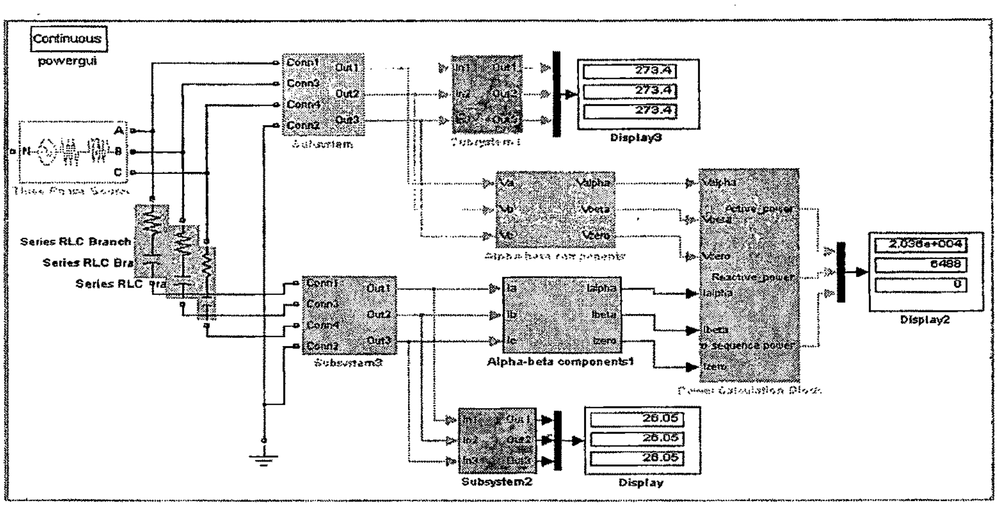


Fig. 3.10.1-15 Screen Shot for power measurement at capacitive load

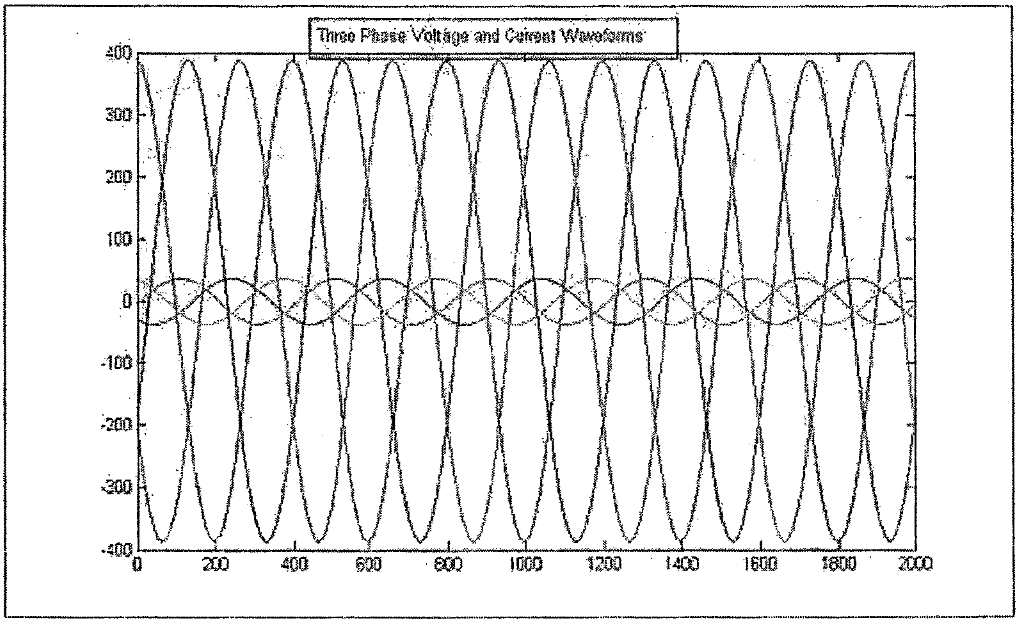


Fig. 3.10.1-16 Three phase Voltage and Current Waveforms at capacitive load

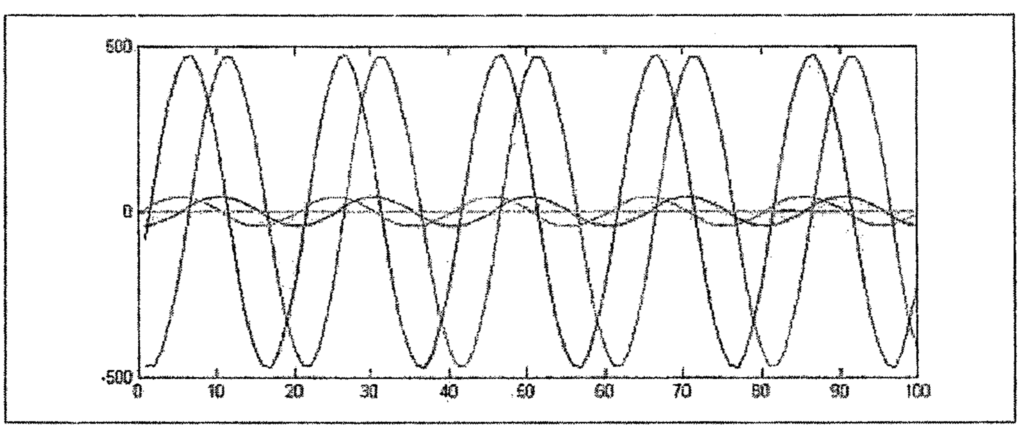


Fig. 3.10.1-17 Voltage and Current Alpha and Beta components

Condition: Inductive load R= 10 Ohm and C=0.001F								
	Voltage			Current			Power	
	Vr	Vy	Vb	Ir	Iy	Ib	P	Q
Sr.No	Volts			Amps			W	Var
1	27.3	27.3	27.3	2.61	2.61	2.605	203.6	64.88
2	54.7	54.7	54.69	5.21	5.21	5.211	814.5	259.5
3	82	82	82.03	7.82	7.82	7.816	1833	583.9
4	227	227	226.9	21.6	21.6	21.63	14030	4470
5	273	273	273.4	26.1	26.1	26.05	20360	6488

Table-3.10.1-7 Results of power measurement at capacitive load

Table 3.10.2-7 shows the result of proposed theory under capacitive load conditions. Fig. 3.10.1-17 shows the V_α ,V_β and V_0 and I_α ,I_β and I_0 waveforms respectively.

- CASE 3-NON-LINEAR LOAD

The system is also simulated under non-linear load.

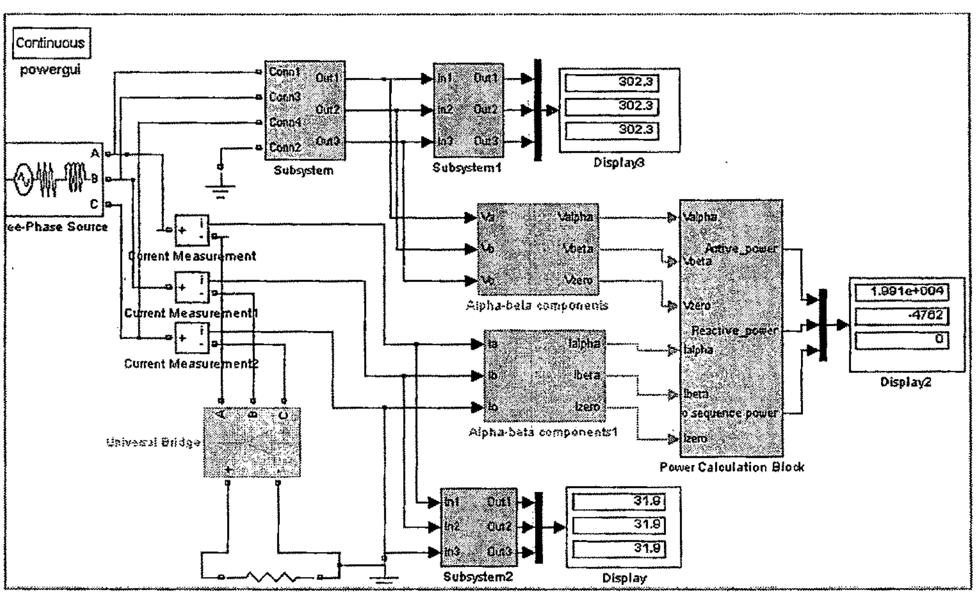


Fig. 3.10.1-18 Screen Shot for power measurement at non-linear load

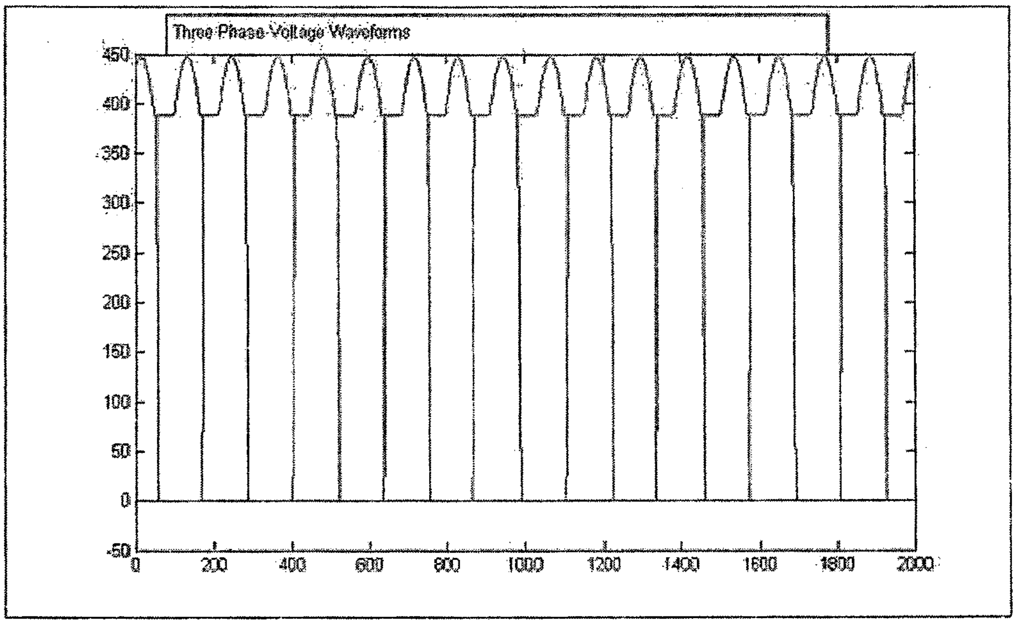


Fig. 3.10.1-19 Three phase Voltage Waveforms at non-linear load

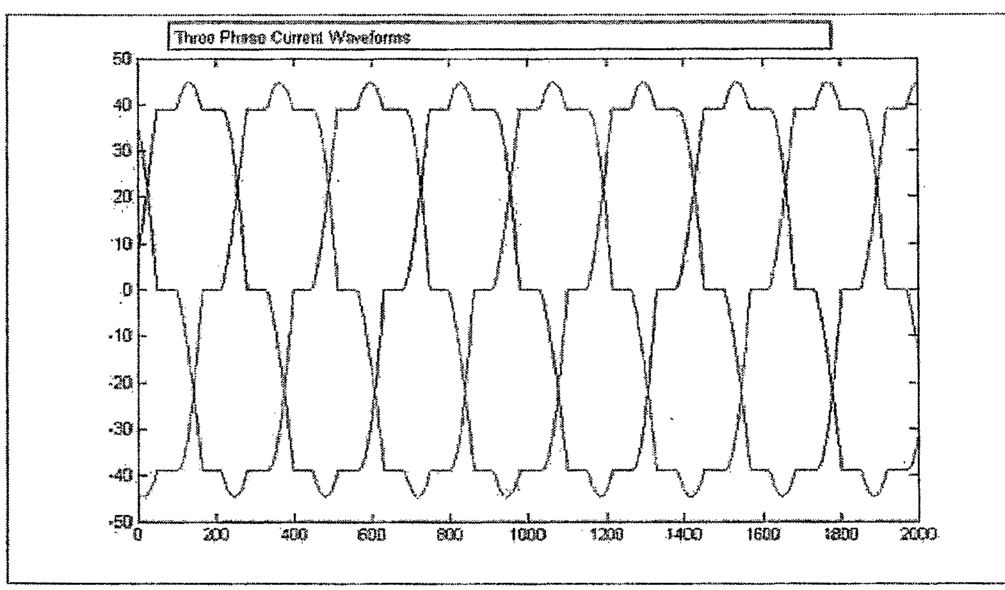


Fig. 3.10.1-20 Three phase current waveforms at non-linear load

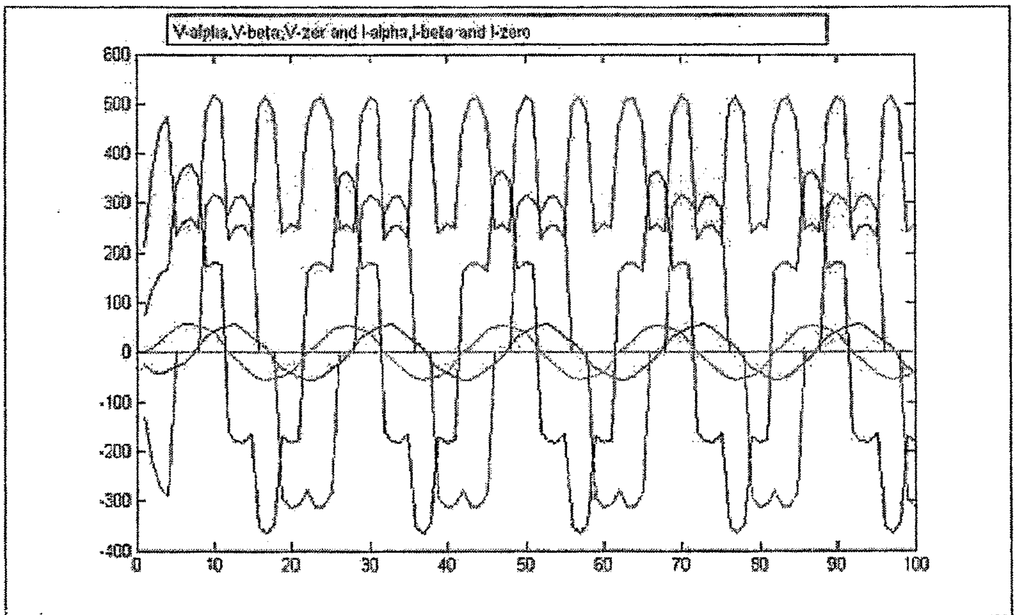


Fig. 3.10.1-21 Voltage and Current Alpha and Beta components

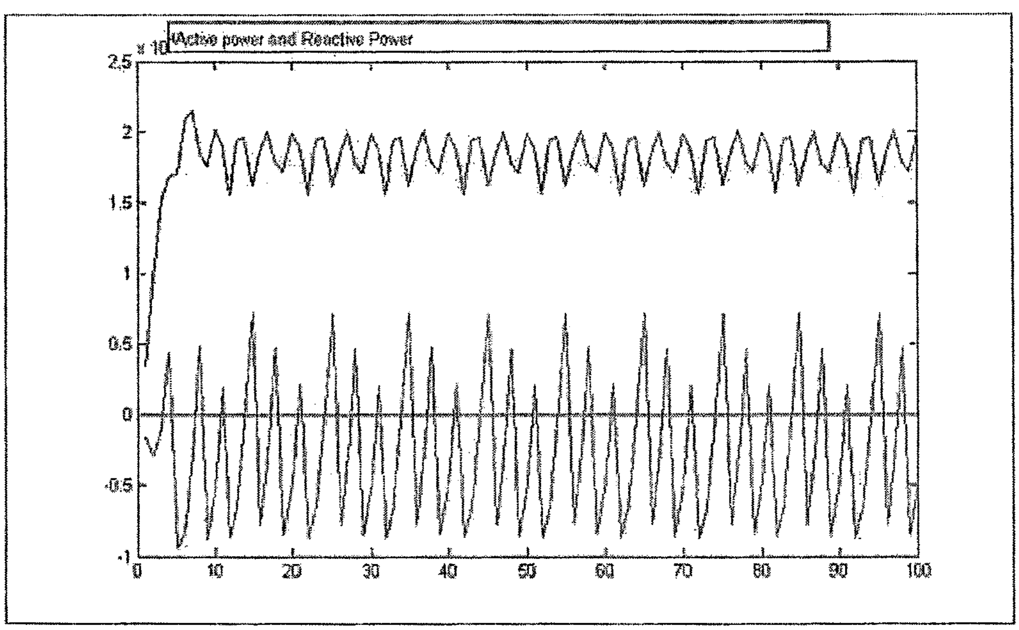


Fig. 3.10.1-22 Active and reactive power components

Condition: 3-phase Diode bridge R=10 Ohm								
	Voltage			Current			Power	
	Vr	Vy	Vb	Ir	Iy	Ib	P	Q
Sr.No	Volts			Amps			W	Var
1	30.2	30.2	30.23	3.19	3.19	3.19	199.1	-47.62
2	60.5	60.5	60.46	6.38	6.38	6.379	796.6	-190.5
3	90.7	90.7	90.69	9.57	9.57	9.569	1792	-428.6
4	251	251	250.9	26.5	26.5	26.47	13720	-3281
5	3023	3023	3023	31.9	31.9	31.9	19910	-4762

Table 3.10.1-8 Results of power measurement at non-linear load

Table 3.10.2-8 shows the result of proposed theory under non-linear load conditions. Fig. 3.10.2-19 and Fig. 3.10.2-20 shows the three phase voltage and current waveforms. Fig. 3.10.2-21 shows the V_α ,V_β and V_0 and I_α ,I_β and I_0 waveforms respectively. Also the active and reactive power plot is also shown in Fig. 3.10.2-22.

From the above simulation it can be concluded that instantaneous active reactive power theory [29] can be applied for four quadrant power measurement. The active and reactive power can be positive or negative in an interconnected power system. Such system can be used to determine the total active power import/export and total reactive power import/export. Such system will help the utility to determine total flow of active/reactive power in system so that each grid network can plan their consumption properly.

3.11 Block Diagram for Four Quadrant Power Measurement

As shown in Fig. 3.11-1 the complete block diagram of system. The algorithm for frequency measurement is deployed on TMS320F2806 platform. The Three phase voltage and current signals are scaled down to the voltage level of the DSP by using a three potential Transformers (240/1V) and three current transformers (5/1 Amp).

The Digital Signal Processor (DSP) used to deploy the algorithm is TMS320F2806. DSP is 16-bit fixed point DSP with internal 12-bit Analog to Digital Converter. The ADC in DSP is unipolar ADC. The bipolar signal from the P.T and C.T are converted to unipolar by using a level shifter circuit.

The output of P.T is connected to a Level Shifter circuit. The unipolar signals (Vr, Vy, Vb) are then given to DSP board. The DSP samples all six channels at a sampling frequency of 1000Hz. The Three phase voltage is then converted into two phase signals using instantaneous alpha-beta theory. Form alpha and beta components active and reactive power is computed. This system is implemented in a multiprocessor based system when one processor continuously measure the frequency cycle by cycle and send's the same data to the other processor using the SPI interface.

The DSP calculates active and reactive power continuously and send's the data on SPI interface, details of which are covered in chapter 6.

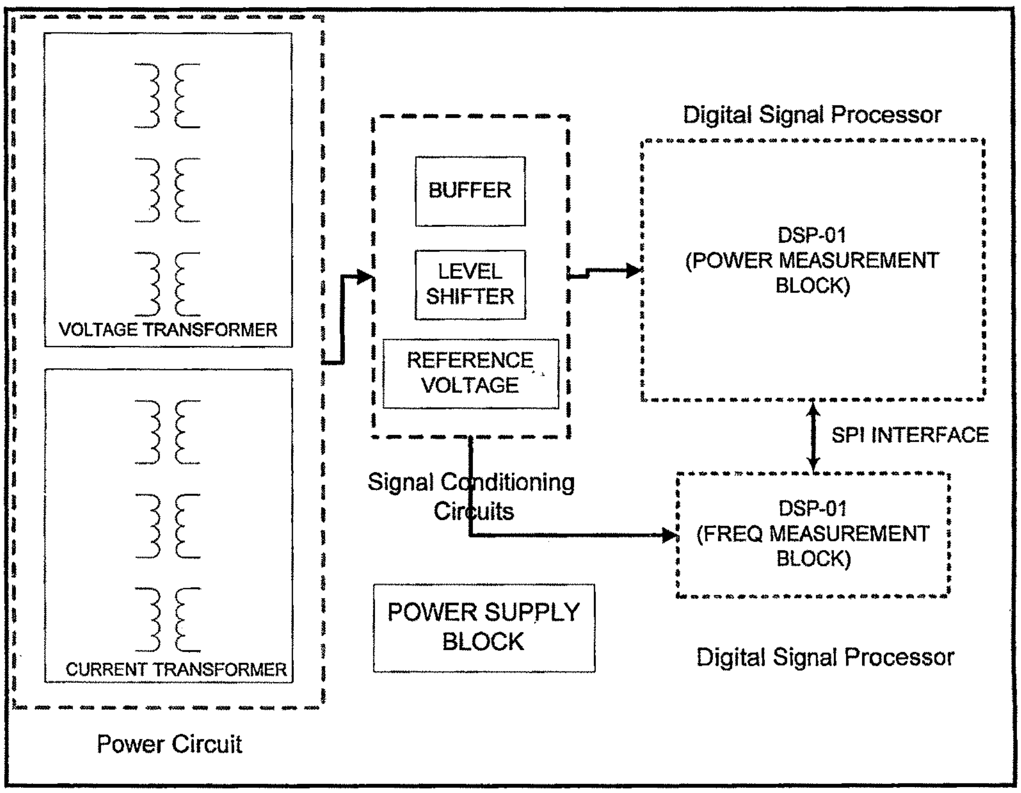


Fig. 3.11-1 Block Diagram for four quadrant power measurement

3.12 Hardware and Software

The Hardware consist of the 3 Potential Transformer, 3 Current Transformer, analog card and a DSP card. The details of analog and DSP card design can be found in the Chapter 6. The DSP card for four quadrant power measurement is connected to other cards via SPI interface. The details of SPI and its waveforms can be found in chapter 6. The DSP card calculates the active and reactive power and then computes direction of power flow. All these data is sent to master card via SPI interface. The master card is interfaced to PC via RS232 interface. In house Visual Basic software is developed for real time monitoring of various power quality parameters. A screen shot for power displayed on computer screen is shown in Fig. 3.12-1.

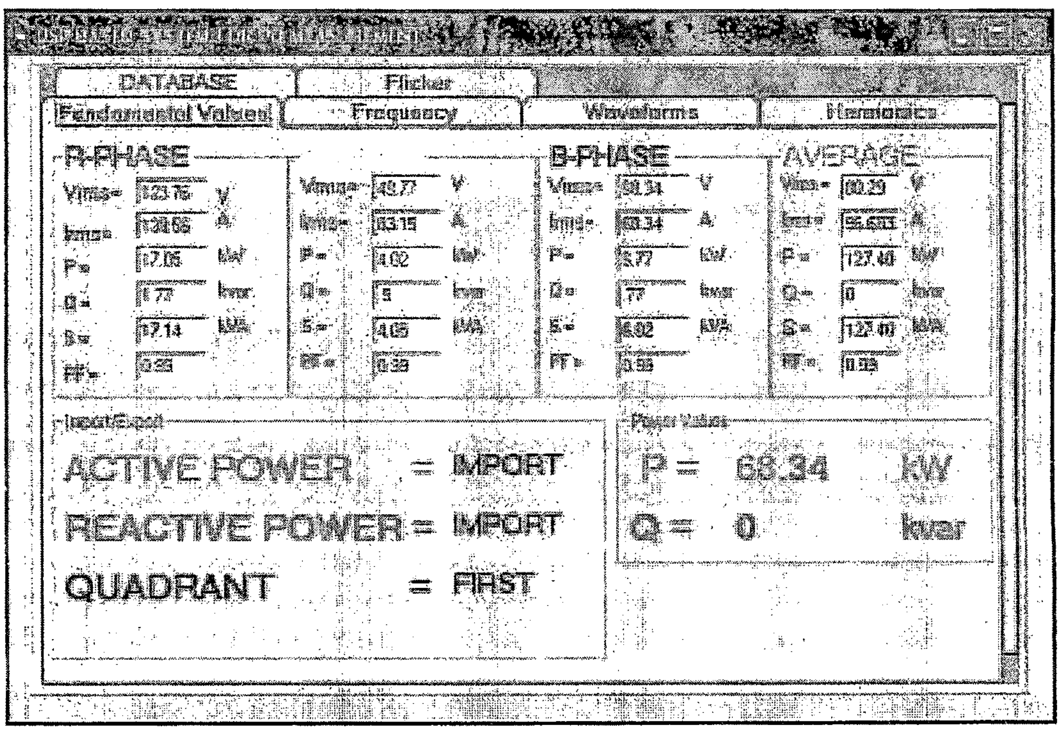


Fig. 3.12-1 VB Screen Shot for Power Measurement

3.13 Flowchart

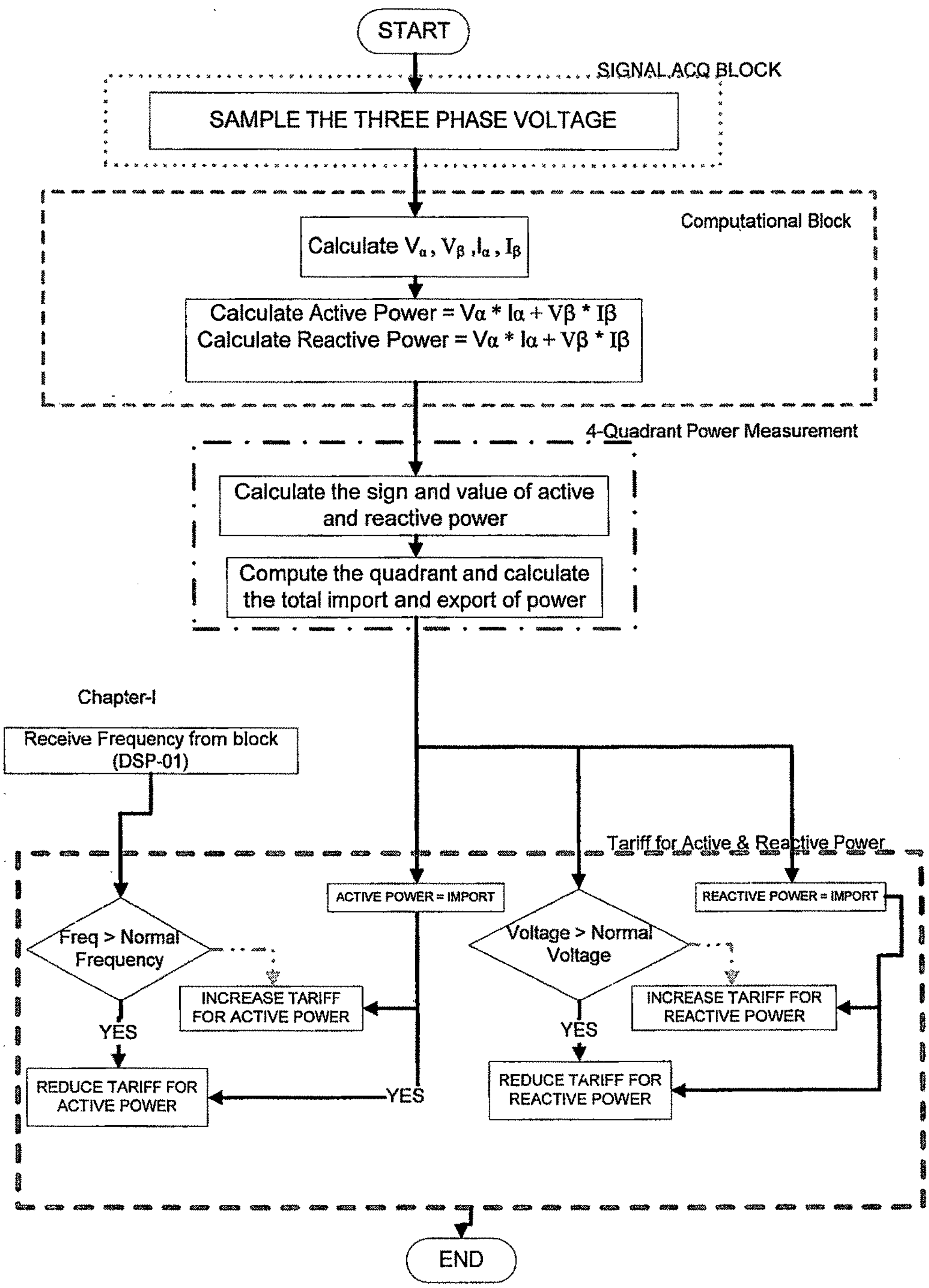


Fig. 3.13-1 Flow-Chart for four-quadrant power measurement

3.14 Results

In order to validate various computations, system is tested on actual hardware. The system is tested with available 3-phase Variac source (240Vac, 15Amp). The developed system is connected to PC via RS232 connector for viewing and analysis of data. The hardware developed for four quadrant power measurement is tested at resistive lamp load, 3-phase AC induction motor and under harmonic conditions.

Case 1-Resistive Load (200W lamp load)

The hardware model developed for the project is tested for resistive load. Three lamps of 200 W each are connected to each phase as shown in Fig. 3.14-1. The results obtained at various voltages are given in Table 3.14-1.

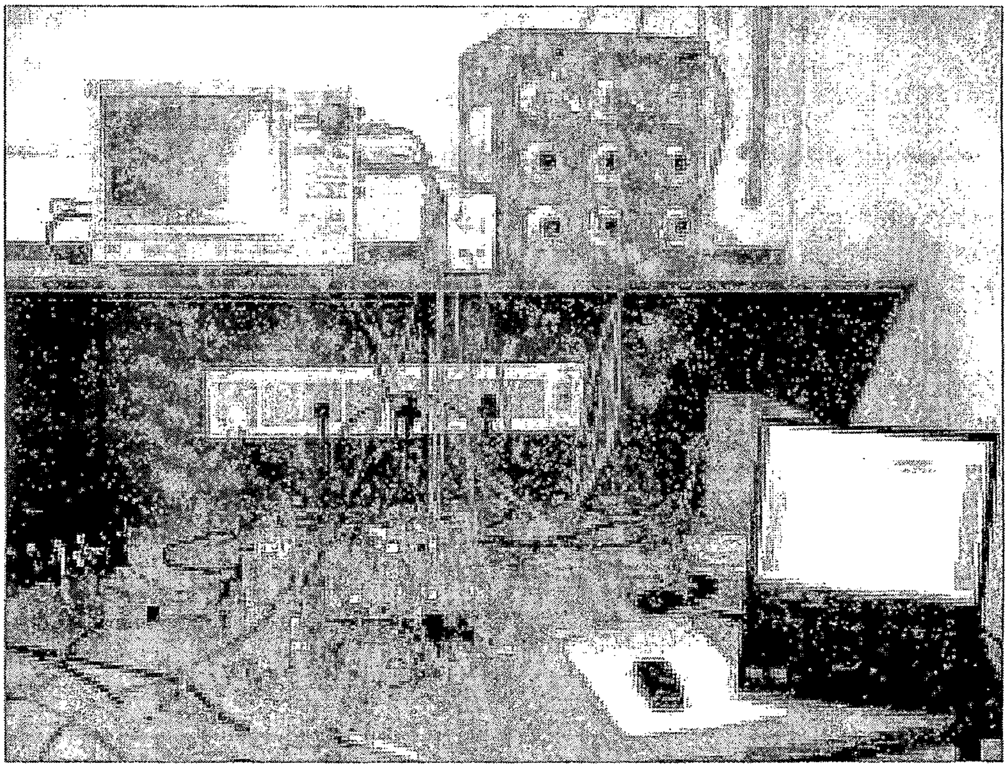


Fig. 3.14-1 Screen shot of the DSP hardware interfaced to PC

Parameter	R-phase	Y-Phase	B-Phase
U	160	167	165
I	0.604	0.642	0.656
P	96.5	107	108
S	96.5	107	108
Q	-0.375	-0.42	-0.467
PF	1	1	1

Table 3.14-1 Actual Results of Power Measurement at resistive load

Case 2-Inductive load

The hardware model developed for the project is also tested for inductive load. The details of motor connected as load is given in Table 3.14-2 and as shown in Fig. 3.14-2. The results obtained at various voltages are given in Table 3.14-3

Sr.No	Parameter	Details
1	Type	3-phase quirrel cage Induction Motor
2	Voltage	415V, 14.8 Amp
3	Power Rating	10 Hp, 50Hz, 7.5kW,1440 rpm
4	Connection Type	Delta

Table 3.14-2 Details of the Induction Motor

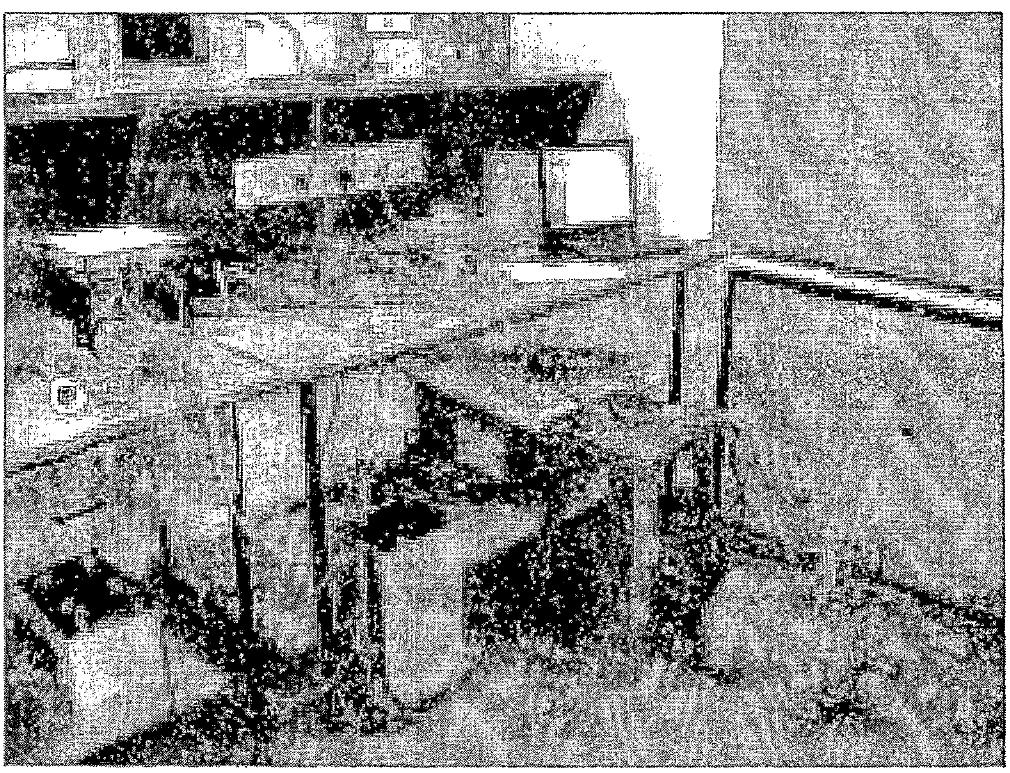


Fig. 3.14-2 Experimental photo of power measurement using inductive load

Parameter	R-phase	Y-Phase	B-Phase
U	23.074	23.9628	23.2656
I	1.22	1.29	1.23
P	22.99	24.59	21.98
S	28.19	30.99	28.78
Q	16.32	18.86	18.58
PF	0.815	0.793	0.763

Table 3.14-3 Actual results of power measurement at inductive load

Case 2-Harmonic Conditions

The hardware model developed for project is also tested under harmonic conditions. Source used is a 3-phase Variac, since it was not possible to generate harmonics. A single phase shift rectifier was connected in serial with the lamp load and firing angle was varied in order to generate harmonics in voltage and current. The single phase shift rectifier is as shown in Fig. 3.14-3. Fig. 3.14-4 shows a phase shift rectifier connected to system with a lamp load. The results obtained at various voltages are given in Table 3.14-4

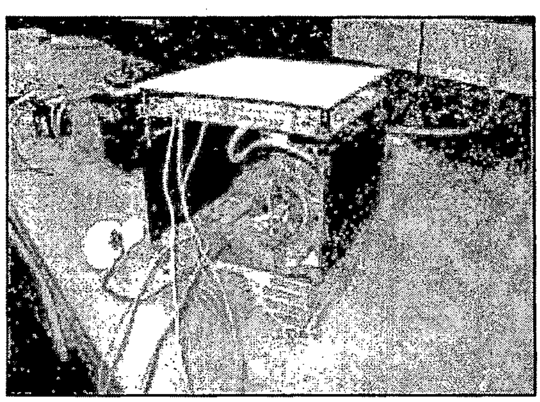


Fig. 3.14-3 Phase shift rectifier

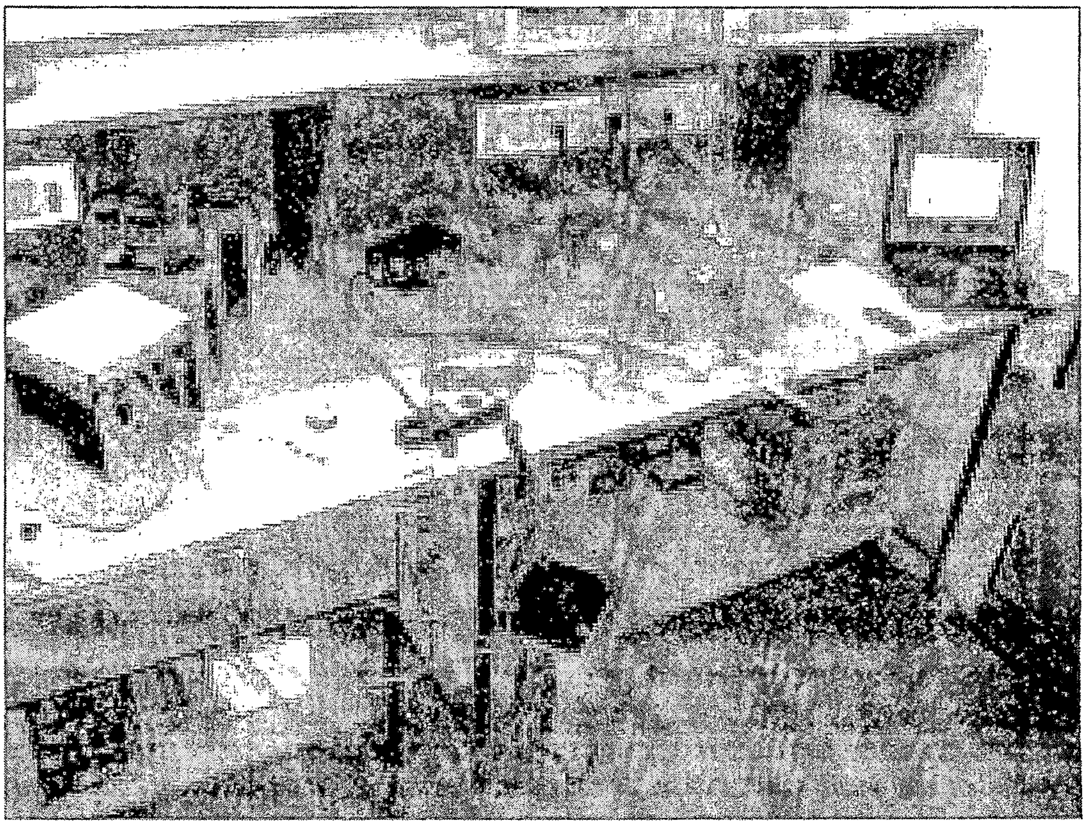


Fig. 3.14-4 Experimental photo of power measurement using PSR

Parameter	R-phase	Y-Phase	B-Phase
U	43.67	167.98	165.32
I	0.306	0.64	0.65
P	13.38	108.09	108.55
S	13.38	108.09	108.55
Q	-0.053	0.446	0.459
PF	0.99	0.99	0.99

Table 3.14-4 Actual results of power measurement in harmonics

3.15 Conclusions:

In this chapter, algorithm for power measurement in case of interconnected power system has been derived using instantaneous active reactive power theory and a new tariff based on four quadrant power measurement is proposed for interconnected power system. The performance of the proposed algorithm under harmonics is discussed in detailed. conditions such as harmonics, sub-harmonics, noise and multiple zero harmonic conditions. Also a method to compute frequency sample by sample has been derived so as to compute frequency faster and accurately as compared to conventional technique. The Mathematical model has been developed for single phase and three phase condition. Error obtained under sinusoidal and non-sinusoidal has been discussed under various conditions. The algorithm is mathematically validated and then it is simulated under various balance and unbalance conditions. The proposed system is also tested under various load conditions such as resistive, inductive and capacitive load in order to prove its validity for interconnected power system. In order to prove the validity of the proposed system for actual implementation, the system is tested on a prototype hardware using three potential and current transformers and its performance is validated under various load conditions. Based on the computation and findings of the proposed algorithm, following conclusions have been drawn:

- A comparison of the various proposed method is done which are used for power measurement. The chapter describes the advantage and disadvantage of the various methods and reasons for selecting instantaneous active reactive power theory for 4-quadrant power measurement.

- The frequency is measured using the concept given in Chapter 1 and the value of frequency is used in the proposed system for implementing variable tariff system.

- The chapter overcomes the traditional method of tariff based on active power consumption and penalty on reactive power support. The chapter proposed a new concept of tariff implementation, which would help the state utility to plan for the total import and export of the power.

- The use of 4-quadrant power measurement and flexible tariff structure would increase the stability of the power system in an interconnected power system. It also enables the state grid to plan their power consumption and plan load shedding accordingly

- The proposed tariff system can be implemented in interconnected tariff system for various grids. The proposed uses frequency, active and reactive power and based on the values and sign of these quantities takes the decision.

- The study has revealed that –quadrant power measurement is effective for interconnected power systems and the experimental and simulation results justify the same. The system can be implemented on today digital hardware.