Chapter 4

On Line Tracking of Harmonics in Power System

4.0 Introduction

In the view of past scenario of power system applications, due to increase in applications of nonlinear loads, power systems are often subject to harmonic injections [40-42]. The presence of harmonics in power systems could cause serious problems such as voltage distortion, increased losses and heating, and wrong operation of protective equipment [43]. Hence, electric utility's are more concerned about power system harmonics and voltage distortion in recent years. Usually, nonlinear loads or harmonic sources occur possibly everywhere in power systems and operated at a continuous variable power. The locations and magnitudes of harmonic source injection depend on placements of nonlinear load devices in the systems and their ratings. Considering these reasons, it is beneficial to estimate time-varying magnitudes of harmonic injection, to eliminate them and provide high-quality and reliable electricity

However, due to economical considerations, number of harmonic analyzers in sub-station is limited and most of the times harmonic measurement is done once a year. The limitation in the number of harmonic meters makes the harmonic state estimation an underdetermined problem . The quality of estimations is a function of the number and locations of the harmonic measurements. So, for given few harmonic analyzers, it is very difficult to track harmonics present in sub-station. Furthermore, in power distribution system, three phase nonlinear loads may be unbalanced to some extent and they may even exist in just one phase or two phases of the system. As a result, the unbalance of harmonic sources further complicates the harmonic sources tracking problems.

Many algorithms are available to evaluate harmonics of the system where Fast Fourier Transforms WT) used by Cooley and Tukey[41] which is widely used. Other algorithms include, recursive DFT, spectral observer, Hartley transform [44] for selecting the range of harmonics This thesis presents Kalman filtering [42] based technique for optimal harmonic measurement locations and dynamic estimation of unbalanced three-phase harmonic injections in power systems. The method is dynamic, has the capability of identifying, analyzing and tracking each harmonic injection and does not need redundant harmonic measurements.

4.1 Applications of Harmonic Measurement

Estimation of harmonic components in a power system is a standard approach for the assessment of quality of delivered power. There is a rapid increase in harmonic currents and voltages in the present AC systems due to huge introduction of solid state power switching devices. Transformer saturation in a power network produces a huge amount of harmonic currents. Consequently, to provide quality of the delivered power, it is imperative to know the harmonic parameters such as magnitude and phase. This is essential for designing filters for eliminating and reducing the effects of harmonics in power system

In fact, it is considered that harmonic distortion is the type of disturbance which it is most necessary to control. This means imposing limitations on the emission levels of equipment, and filtering the inevitable harmonic components present. During the past few years, several international organizations have made a considerable effort to elaborate norms and recommendations on the measurement and limitation of harmonics in power systems [45]. The distributed nature of harmonics-generating loads and their randomness means that it is essential to have a supervision system spread over the whole network which is capable of undertaking a global assessment for existing harmonic contamination

Most electronically switched industrial loads found in mining, refining and melting processes, paper mills, etc. are dynamic in nature. In normal operation, repeated stop / start and braking acceleration cycles tend to generate significant speed variations resulting in time-varying current amplitudes having substantial amount of non stationary harmonics. An increasing number of high voltage transmission systems have static VAR compensators placed at strategic locations which can inject time varying harmonics to the systems. The advent of FACTS devices suggests their use in future power transmission and distribution systems. This gives rise to the possibility of generation of non stationary harmonic voltages and currents in the power system.

The series compensation can also produce low frequency oscillations that interact with SVCs to produce amplitude-modulated harmonics. Several disturbances further complicate this phenomenon by modulating the fundamental frequency, which ultimately yields harmonics with changing amplitudes and frequency. Thus accurate measurement of harmonic levels is essential for designing harmonic filters, monitoring the stress to which the power system devices are subjected due to harmonics and specifying digital filtering techniques for phasor measurements and for relaying.

4.2 Various Harmonic Measurement Methods

In an ideal power system, voltage and current waveforms are pure sinusoidal. However, in practice under various circumstances, voltage and current waveform distortions occur. These waveform distortions are further discussed in terms of harmonics, being integer multiples of the fundamental power frequency. The measurement of these harmonics is important to derive power quality. However, despite that a harmonic is a steady-state phenomenon, in practice the measurements have to be performed in dynamic conditions. In such cases, the classical frequency domain-based methods (e.g. FFT) may fail and have to be replaced by alternatives running in the time domain (Kalman estimators) or time frequency domain (wavelet filters).

4.2.1 Frequency domain Method

Harmonic analysis algorithms in frequency domain are based either on the discrete Fourier transform (DFT) or on the Fast Fourier transform (FFT) in order to obtain the spectra of the voltage and the current signals using the discrete time samples. The DFT and FFT transform is a useful analytical tool that has been applied to power system for phasor measurement and harmonic analysis [41].The DFT is computed using Eq. 4.1

$$H\left(\frac{k}{NT}\right) = \sum_{n=0}^{N-1} h(nT)e^{\left(\frac{-j2\pi kn}{N}\right)} \qquad (4.1)$$

Where,

$H(f)$ is a function of frequency
$h(t)$ is a function of time
T is the time interval between the samples.
N is the number of samples in the window.
n = 01,2,..............N-1.

There are basic assumptions embodied in implementing the DFT

(i) The sampling frequency is equal to the number of samples multiplied by the fundamental frequency assumed by the algorithm
(ii) The sampling frequency is greater than twice the highest frequency in the signal to be analyzed
(iii) The sampling frequency in the signal is an integer multiple of the fundamental frequency. The results of the DFT are accurate when these assumptions are true.

However misapplication of the FFT algorithm can have destructive side-effects that would lead to incorrect results. The major destructive side-effects which have to be taken care in the case of

FFT are leakage effect, picket fence effect and aliasing error. Aliasing error can be alleviated by increasing the sampling frequency (f_s) and properly designed anti-aliasing filters which completely cut-off frequencies above ($f_s/2$). The FFT process is discrete, it evaluates the frequency content of a time signal in terms of discrete points in the frequency domain. The picket-fence effect offers if the analyzed waveform includes a frequency which is not an integer multiple of fundamental frequency. For example in a 400 line FFT analyzer on a 20 kHz span there will be magnitude of the signal at every 50Hz, at 50,100,150.... Suppose the signal had a one frequency component of say 125Hz than this will be absent in the spectrum since it falls between 2 frequencies. This is Picket-fencing effect. The term leakage refers to the apparent spreading of energy from one frequency into adjacent ones. It arises due to discontinuities at the edge of the window that leads to false results. These discontinuities modulate the original signal. Modulation causes sidebands, appearing at ($f \pm f_s$) .Some of the energy signal goes in side lobes. This effect is termed as leakage effect. Hence selection of appropriate window is required when DFT technique is applied to time varying signal

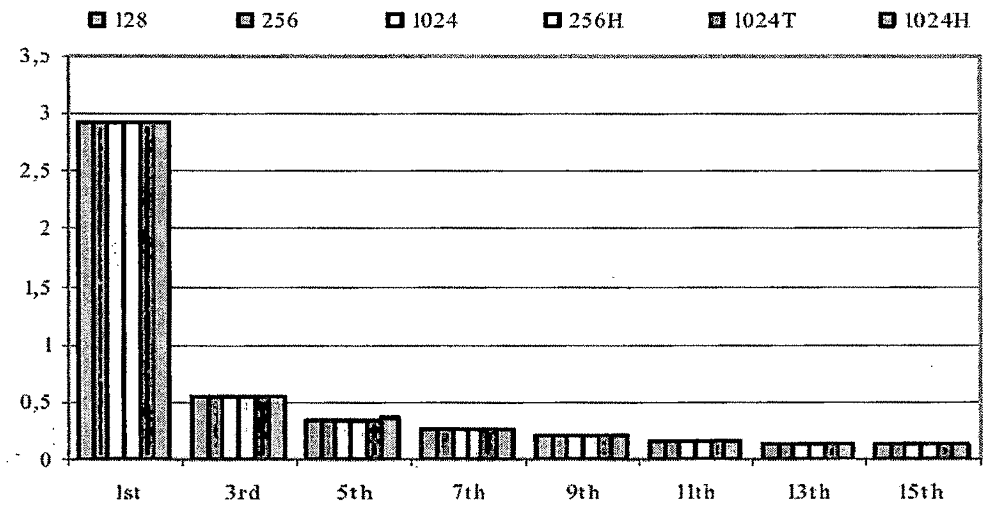


Fig. 4.2.1-1 Comparison plot for different techniques

Sr.No	FFT Method	Irms Magnitude (A)	THD (%) Computed
1	128-point FFT	3.03	25.97
2	256-point FFT	3.034	25.85
3	1024 point FFT	3.034	26.03

Table 4.2.1-1 Result for different FFT window

4.2.2 Wavelet Based Harmonic Estimation

For the method based on the (real) wavelet transform of the analytic representation of the signals, the following main items need to be considered the type of the (real) wavelet transform should be ortho-normal.

- The wavelet (filter) order, generally related to the frequency separation characteristic of the selected wavelet: good frequency separation reduces the amount of leakage energy to the adjacent frequency bands;
- The number of levels, related to the input frame size, e.g., if the number of input samples is N=2D then a maximum of D levels can

be performed; the wavelet levels are from 0 to D-1 and the scaling level is 0*.

All features above have an impact on the performance and the accuracy of harmonic measurements and consequently, need to be chosen careful.

4.3 Kalman Filter

The Kalman filter [46] is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of squared error. The filter is powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown.

Let x be a scalar quantity that is constant in time. If n measurements of x are available which are corrupted by noise drawn from uncorrelated zero mean Gaussian distributions of equal variance, the best estimate of x is the mean of all the measurements. Denoting the i^{th} measurement by y_i, and the best estimate after n measurements by x_n, we thus have x_n given by Eq. 4.2

$$x_n = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{4.2}$$

If a further measurement, denoted by y_{n+1} becomes available the new best estimate obviously follows as Eq. 4.3

$$x_{n+1} = \frac{1}{n+1}\sum_{i=1}^{n+1} y_{i+1} \tag{4.3}$$

One can express this new estimate in terms of the previous one, to obtain the following update equation as given by Eq. 4.4

$$x_{n+1} = \left[\frac{n}{n+1}\right] x_n + \frac{y_{n+1}}{n+1} \tag{4.4}$$

This equation can be rearranged to yield as shown in Eq. 4.5

$$x_{n+1} = x_n + \frac{1}{n+1}[y_{n+1} - x_n] \tag{4.5}$$

We may interpret this equation as follows. If we have an estimate x_n of a quantity x, and a new measurement of x becomes available, we may use it to update the estimate x_n to an improved estimate, x_{n+1} by adding to x_n the difference between the new measurement and the previous estimate scaled by a weighting factor. This idea of updating an estimate in the light of a new measurement to form an improved estimate is central concept of kalman filtering.

- ## The Time Varying Case:

Consider a scalar quantity x, which is observed at discrete points in time. Rather than being time independent we now assume that the change in x from the time of nth to (n+1)th measurement can be modeled by an Eq. 4.6 of the form.

$$x_{n+1} = \Phi_n x_n + w_n \tag{4.6}$$

Where w_n is drawn from a zero mean distribution. Then w_n effectively allows for the uncertainty in our model of the time variation of x as expressed by Φ_n, and is known as process noise.

If we have an estimate of x_n denoted by $\hat{x}_n$ (+) the best estimate of x_{n+1}, which we will denote by $\hat{x}_{n+1}(-)$ is obviously given by Eq. 4.7

$$\hat{x}_{n+1}(-) = \Phi_n \hat{x}_n(+) \tag{4.7}$$

If $\hat{x}_n(+)$ is characterized by a variance $P_n(+) = E\{(\hat{x}_n(+) - x_n)^2\}$, where E {........} denotes the expectation operator, the variance of $\hat{x}_{n+1}(-)$ will have a contribution of $\Phi_n^2 P_n(+)$ from the original uncertainty in $\hat{x}_n(+)$. It will have a further contribution of $Q_n = E[w_n^2]$ from the unknown w_n. Denoting the variance of $\hat{x}_{n+1}(-)$ by $P_{n+1}(-)$, we thus have Eq. 4.8 given as

$$P_{n+1}(-) = \Phi_n^2 P_n(+) + Q_n \tag{4.8}$$

Suppose a measurement is made of the scalar y_{n+1} which is linearly related to x_{n+1} and given by Eq. 4.9

$$y_{n+1} = H_{n+1} x_{n+1} \tag{4.9}$$

Let the actual measurement be be given by Eq. 4.10

$$\hat{y}_{n+1} = H_{n+1} x_{n+1} + v_{n+1} \tag{4.10}$$

Where v_{n+1} is drawn from a zero mean distribution and represent measurement noise. If we denote the variance of this measurement by Eq. 4.11

$$R_{n+1} = E\{V_{n+1}^2\} \tag{4.11}$$

We may, as in the time invariant case, seek an update estimate of $\hat{x}_{n+1}(-)$ which we denote $\hat{x}_{n+1}(+)$ of the form given by Eq. 4.12

$$\hat{x}_{n+1}(+) = \hat{x}_{n+1}(-) + k_{n+1}[\hat{y}_{n+1} - H_{n+1}\hat{x}_{n+1}(-)] \tag{4.12}$$

We seek the Kalman Gain K_{n+1} which minimizes the variance of $\hat{x}_{n+1}(+)$ which we will denote by $P_{n+1}(+)$, and hence gives rise to an updated estimate with the minimum possible uncertainty.

If we use the notation Δ to represent a change in a variable, we have Eq. 4.13

$$\Delta\hat{x}_{n+1}(+) = (1 - k_{n+1}H_{n+1})\Delta\hat{x}_{n+1}(-) + k_{n+1}\Delta\hat{y}_{n+1} \tag{4.13}$$

Hence

$$P_{n+1}(+) = (1 - k_{n+1}H_{n+1})^2 P_{n+1}(-) + k_{n+1}^2 R_{n+1} \tag{4.14}$$

By differentiating the Eq. 4.14 with respect to K_{n+1} and setting the result equal to zero, we may solve for the value k_{n+1} that minimizes $P_{n+1}(+)$ we get Eq. 4.15

$$k_{n+1} = \frac{P_{n+1}(-)H_{n+1}}{H_{n+1}^2 P_{n+1}(-) + R_{n+1}} \tag{4.15}$$

It then follows that, with this value of k_{n+1},

$$P_{n+1}(+) = (1 - k_{n+1}H_{n+1})P_{n+1}(-) \tag{4.16}$$

The Kalman filter estimates a process by using a form of feedback control: filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for Kalman filter fall into two groups: time update equations and measurement update equations as shown in Fig 4.3-1. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain apriori estimates for the next time step. The measurement update equations are responsible for feedback—i.e. for incorporating a new.

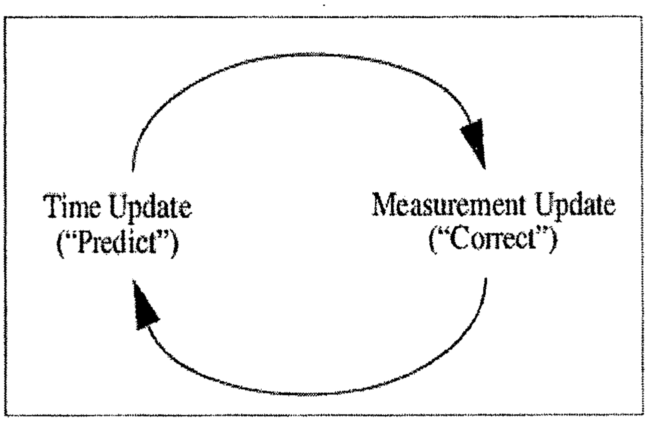


Fig. 4.3-1 Discrete Kalman filter cycle

The time update equations, can also be thought of as predictor equations, while the measurement update equations can be thought of as corrector equations. Indeed the final estimation algorithm resembles that of a predictor-corrector algorithm for solving numerical problems as shown below in Fig 4.3-2.

The specific equations for the time and measurement updates are presented below in Eq. 4.17 and Eq. 4.18.

$$\hat{x}_k = A\hat{x}_{k-1} + Bu_k \quad (4.17)$$

$$P_k^- = AP_{k-1}A^T + Q \quad (4.18)$$

The Discrete kalman filter measurement update equations are given by Eq. 4.19

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \quad (4.19)$$

$$\hat{x}_k = x_k^- + K_k (z_k - H x_k^-)$$

$$P_k = (1 - K_k H) P_k^-$$

The first task during the measurement update is to compute the Kalman gain, K_k. The next step is to actually measure the process to obtain z_k, and then to generate an state estimate by in corporating the measurement as in Eq. 4.19. The final step is to obtain an error covariance estimate.

After each time and measurement update pair, the process is repeated with the previous estimates used to project or predict the new estimates. This recursive nature is one of the very appealing features of the Kalman filter-it makes practical implementations much more feasible. Fig 4.3-2 shows below a complete picture of the operation of the filter combining the high level diagram with the equations.

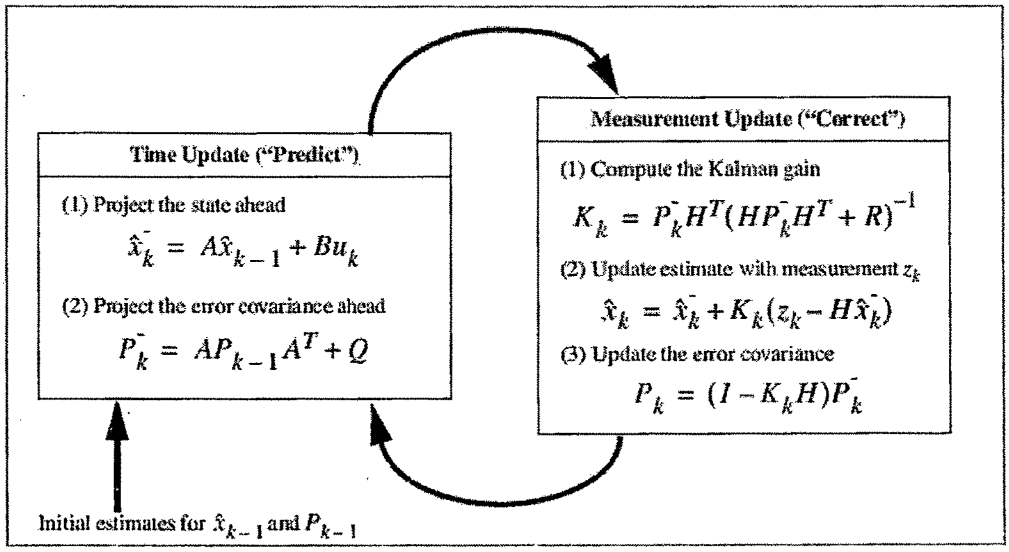


Fig. 4.3-2 High Level diagram of the Kalman filter

4.4 Harmonic Estimation using Kalman Filter

Kalman filtering technique is used in this thesis to calculate the amplitude and phase angle of power system harmonics up-to 25th order. The real time tracking of the frequency component of line voltage and current harmonics is performed by means of the application of Kalman filter to the samples of voltage and current signals. A mathematical model of the signals in state variable form is implemented including all the possible spectral components which may be associated with signal to be analyzed.

Kalman filtering provides means for estimating the parameters of time-varying signals. In case of voltage signals a natural choice of the model is that consists of the fundamental frequency component and a certain number of harmonics N

Consider a signal with a frequency ω and a magnitude of $A\ (t)$, where $A\ (t)$, represent a combination of a constant value plus a time-variant component. Considering a reference rotating at ω, the noise free signal may be expressed by Eq. 4.20.

$$s(t) = A(t)\cos(\omega t + \theta) = A(t)cos\theta cos\omega t - A(t)sin\theta sin\omega t \qquad (4.20)$$

Let x_1 be $A(t)\cos(\theta)$ and x_2 be $A(t)\sin(\theta)$; therefore, each x_1 and $x_2$includes two componens. One component is constant but unknown. The other component may be time-varying. The variables x_1 and x_2 represent the in-phase and quadrature-phase components and referred to as state variables. This leads to the following state equations given by Eq. 4.21.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_k \qquad (4.21)$$

Where w_1 and w_2 allow the state variables for random walk (time variation). The measurement equation would include the signal and noise and it can be represented as Eq. 4.22.

$$z_k = [\cos(\omega t_k) \quad -sin\ (\omega t_k)] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v_k \qquad (4.22)$$

Where v_k represent the high frequency noise.

State Variable Representation of a Signal that Includes n Harmonics

A noise-free current or voltage signal $s(t)$ that includes n harmonic may be represented by Eq. 4.23.

$$s(t) = \sum_{i=1}^{n} A_i\,(t)\cos\,(i\omega t + \theta_i) \qquad (4.23)$$

Where

$A_i(t)$ is the amplitude of the phasor quantity representing the i^{th} harmonic at time t,

θ_i is the phase angle of the ith harmonic releative to a reference rotating at $i\omega$,

n is the harmonic order.

As indicated in the previous subsection, each frequency components requires two state variables. Thus, the total number of state variables is 2n. These state variable are defined as Eq. 4.24

$$\begin{aligned} &x_1(t) = A_1(t)cos\theta_1 \;\; , x_2(t) = A_1(t)sin\theta_1 \qquad (4.24)\\ &x_3(t) = A_2(t)cos\theta_2 \;\; , x_4(t) = A_2(t)sin\theta_2\\ &\cdots\cdots\cdots \qquad \cdots\cdots\cdots\\ &\cdots\cdots\cdots \qquad \cdots\cdots\cdots\\ &x_{2n-1}(t) = A_n(t)cos\theta_n \;\; , x_{2n-1}(t) = A_n(t)sin\theta_n \end{aligned}$$

These state variables represent the in-phase and quadrature phase components of the harmonic with respect to a rotating reference. Thus, the state variable equations may be expressed as Eq. 4.25:

$$\begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & \vdots & 1 & 0 \\ 0 & 0 & \vdots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix} + \begin{bmatrix} \propto_1 \\ \propto_2 \\ \cdots \\ \propto_{2n-1} \\ \propto_{2n} \end{bmatrix} w_k \qquad (4.25)$$

The measurement equation can be then expressed as Eq. 4.26

$$z_k = H_k x_k + v_k = \begin{bmatrix} \cos(\omega k\Delta t) \\ -\sin(\omega k\Delta t) \\ \cdots \\ \cos(n\omega k\Delta t) \\ -\sin(n\omega k\Delta t) \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{bmatrix} + v_k \qquad (4.26)$$

It should be indicated here that H_k in this case is a time varying vector. A constant H_k vector can be obtained if a stationay reference is used in the state variable representation.

Selection of Kalman Filter parameters

1. Initial process vector ($\hat{x}_0$)

As the Kalman filter model started with no past measurement, the initial process vecot was selected to be zero. Thus, the first half cycle (8 milliseconds) is considered to be the initialization period.

2. Initial covariance matrix (P_0^-)

The initial covariance matrix was selected to be a diagonal matrix with the diagonal values equal to 10 pu^2.

3. Noise variance (R)

The noise variance was selected to be constant at a value of 0.05 pu^2. This was based on the background noise variance at field measurement.

4. State variable covariance matrix (Q).

The matrix Q was selected to be 0.05 pu^2.

The Kalman filtering technique is used here for the estimation of the fundamental and harmonic components of the three-phase system.

The design of a Kalman filter required a state-space model of the signal to be estimated in the form

$$x_{k+1} = \emptyset x_k + w_k \qquad (4.27)$$
$$z_k = Hx_k + v_k$$

Where,

x_k is the $r \times 1$ process state vector at time t_k

$\emptyset$ is the $r \times r$ state transition matrix.

w_k is a $r \times 1$ noise vector-assumed to be white sequence with known covariance matrix Q

z_k is a $m \times 1$ vector measurement at t_k

H is the $m \times r$ matrix giving the noiseless connection between the measurement and the state vector.

v_k is a $m \times 1$ vector measurement error-assumed to be white noise sequence with known covariance matrix R and uncorrelated with w_k sequence.

To start the Kalman filter recursive estimation, an initial process vector $(\hat{x}_0)$ and the associated initial covariance matrix (P_0^-) are needed. The initial covariance matrix describes in a statistical sense, the range of variations of the state vector x from the initial process vector $\hat{x}_0$. In general, the error covariance matrix (P_0^-) associated with an apriori estimate $\hat{x}_k^-$ is defined by Eq. 4.28:

$$P_k^- = E[e_k^- e_k^{-T}] = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \qquad (4.28)$$

Having an *apriori* estimate, $\hat{x}_k^-$, and the associated error covariance matrix, P_k^-, we now wish to optimally approve the estimate using the measurement z_k. This is achieved by a linear blending of the noisy measurement and the *prior* estimate according to the Eq. 4.29

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H_k\hat{x}_k^-) \tag{4.29}$$

Where

$\hat{x}_k$ is the updated estimate,

K_k is the blending factor.

The idea is to find the particular blending factor that yields an optimal updated estimate. This is achieved by forming first the expression for the error covariance matrix associated with the updated estimate as Eq. 4.30:

$$P_k = E[e_k\, e_k^T] = E\left[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\right] \tag{4.30}$$

Now, we wish to find the particular K_k that minimizes the diagonal elements of the matrixP_k, because thse elements represent the estimation error variances of the state vector components. This particular blending factor is called the Kalman gain and is given as Eq. 4.31:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{4.31}$$

The covariance matrix associated with the optimal estimate may now be computed as shown in Eq. 4.32

$$P_k = (1 - K_k H_k)P_k^- \tag{4.32}$$

Now there is a means of assimilating the measurement at t_k ,by the use of optimal K_k, $\hat{x}_k^-$ and P_k^-. At the next step, we need $\hat{x}_{k+1}^-$ and P_{k+1}^-

to make an optimal use of z_{k+1}. First, the update estimate $\hat{x}_k$ is projected ahead via state transition matrix ($\emptyset_k$) to obtain the apriori estimate $\hat{x}_{k+1}^{-}$. As given in Eq. 4.33

$$\hat{x}_{k+1}^{-} = \emptyset_k \hat{x}_k \tag{4.33}$$

The error covariance matrix associated with, $\hat{x}_{k+1}^{-}$ is then obtained by forming the expression for the apriori error given in Eq. 4.34

$$e_{k+1}^{-} = x_{k+1} - \hat{x}_{k+1}^{-} = \emptyset_k e_k + w_k \tag{4.34}$$

Thus,

$$P_{k+1}^{-} = E[e_{k+1}^{-} e_{k+1}^{-T}] = \emptyset_k P_k \emptyset_k^T + Q_k \tag{4.35}$$

It should be noted that the Kalman gain, in usual linear recursive Kalman filter, is independent of measurements. Thus, only equation B-6 and B-10 needs to be computed on-line. The Kalman gain vector, which is the key parameters, can be computed off-line.

Kalman Model Development

For the purpose of model development it is assumed that the voltage and current signals are band-limited and strictly periodic. The discrete-time state-space representation of a periodic signal having harmonic components up to nth order with samples z_k at time c can be given by Eq. 4.36

$$x_{k+1} = \emptyset x_k \tag{4.36}$$
$$z_k = H x_k$$

Where x_k is a (2n+) state vectorgiven by Eq. 4.37

$$\emptyset = \begin{bmatrix} f(1\varphi) & 0 & 0 \\ 0 & f(n\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.37)$$

Where $\emptyset = \omega T$, ω is the fundamental supply frequency in randians per sec and T is the sampling interval in sec.

$$\emptyset(i\varphi) = \begin{bmatrix} \cos(i\varphi) & -\sin(i\varphi) \\ \sin(i\varphi) & \cos(i\varphi) \end{bmatrix} \; \mathrm{I} = 1,2,\ldots\ldots n \qquad (4.38)$$

And

$H = [1 \quad 0 \cdots 0 \quad 1]$

The harmonic components h_i(rms) are given by Eq. 4.39

$$h_i^2 = \left(x_k^2(2i-1) + x_k^2(2i)\right)/2 \;, \mathrm{I} = 1,\ldots\ldots n \qquad (4.39)$$

$h_0 = x_k(2n+1)$

Where i-th element of x_k is represented by $x_k(i)$

The design of a Kalman filter involves computation of the model parameters $\emptyset, H, Q, R, P_0^-$ and the resulting Kalman gain K_k. The matrices $\emptyset$ *and* H are obtained by assuming $\hat{x}_0^-$. In order to find the matrices Q, R and P_0^-, it is required to have voltage and current signals under various harmonic conditions. The initial estimation error covariance matrix P_0^- is assumed to be diagonal with non zero elements equal to the squares of the standard deviations (variances) of state variables. The measurement noise variance R is assumed to be constant and it obtained as the sum of the variances of all the high frequency components.

An twenty-six (r = 26, m=1) Kalman filter is designed to estimate the dc, fundamental and up to twenty-fifth harmonic components of current signals. The real-time current and voltage signals are sampled and processed through this twenty six state Kalman to estimate the magnitude of various frequency components such as fundamental, second, third and so on.

Various input parameters for Kalman filter estimation are as follows:
Sampling frequency = 100kHz
Initial state x_0 = [0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0] ,
Measurement noise variance (Sigma n) = 0.01
Process noise variance (Sigma v) = 0.0000005
Error Covariance

$$M=\begin{bmatrix} 0.48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.48 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.48 \end{bmatrix} \quad (4.40)$$

Decaying DC parameter = 1000

$$P=\begin{bmatrix} \cos(h1) & -\sin(h1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sin(h1) & \cos(h1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos(h3) & -\sin(h3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(h3) & \cos(h3) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(h23) & -\sin(h23) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin(h23) & \cos(h23) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(h25) & -\sin(h25) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sin(h25) & \cos(h25) \end{bmatrix} \quad (4.41)$$

Where,

```
h1 = 2*pi*f*T;
h3 = 3*2*pi*f*T;
h25 = 25*2*pi*f*T;
```

f is the fundamental frequency

T = 1/f_s is the sampling time

```
B = [sigman^2]
```

$$U=\begin{bmatrix} sigma^2 & 0 & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 & 0 & 0 \\ 0 & sigma^2 & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 & 0 & 0 \\ .. & .. & ..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&..&.. & .. & .. \\ 0 & 0 & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 & sigma^2 & 0 \\ 0 & 0 & 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 & 0 & sigma^2 \end{bmatrix} \quad (4.42)$$

The input signal values are sampled and then given to the Kalman filter, Since it is required to calculate amplitude and phase angle of dc, fundamental and upto 25th order of harmonic. Each model of Kalman filter is tuned for a particular parameter. For example one model is for fundamental amplitude estimation, other model for 2nd harmonic and so on.

Using these values the Kalman gain and the error covariance matrix values are computed and the magnitudes of fundamental and harmonic components were estimated.

4.5 Simulation

The Kalman filter is modeled in MATLAB software for estimation of harmonics up-to 25th order. Fig 4.5-1 shows the functional diagram of the system.

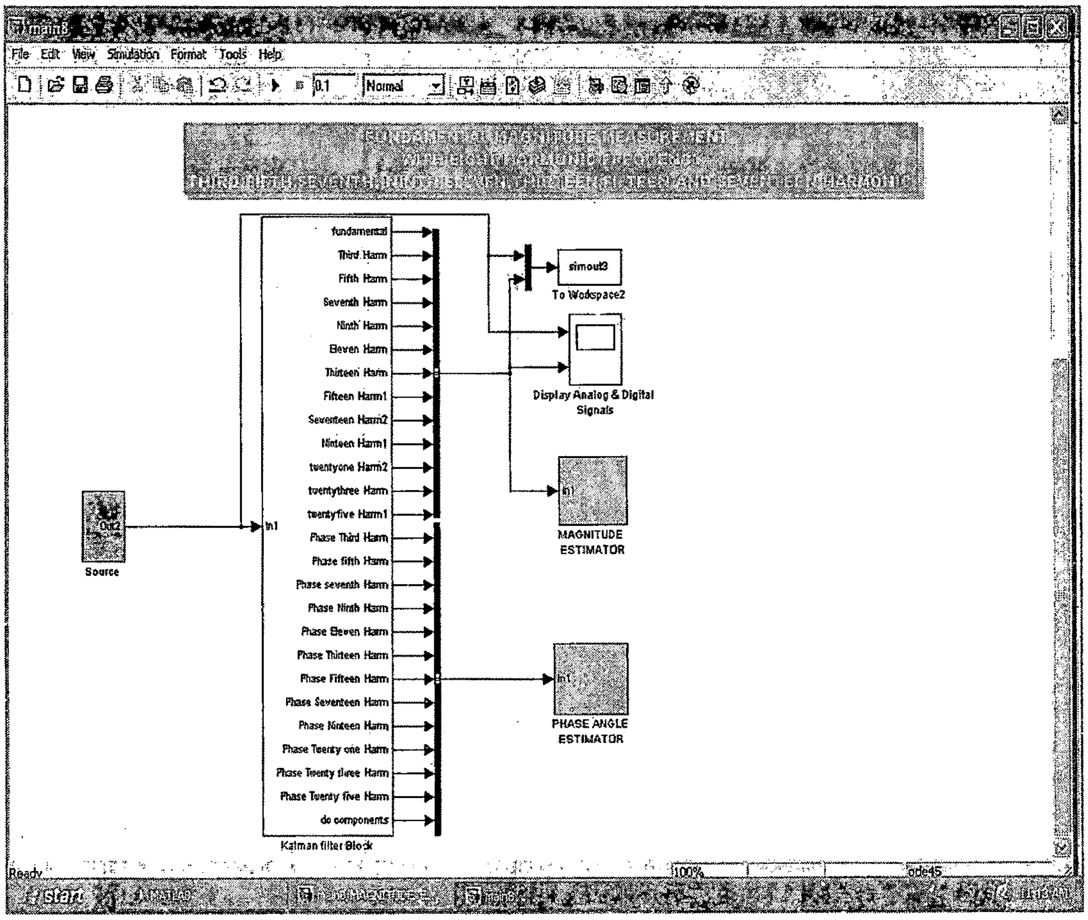


Fig. 4.5-1 Screen shots for harmonic measurement using Matlab

The total system is divided into three minor sub-blocks. The first subsystem consist of all the input elements to be modeled into system, the second block consist of Kalman filter block and the third block is display block which displays harmonic magnitudes of the various harmonics.

Fig 4.5-2 shows the detailed of block-1. As shown in Fig 4.5-1 the various order of harmonics are feed into system using switch and magnitude, frequency were set accordingly, and can be varied during run time.

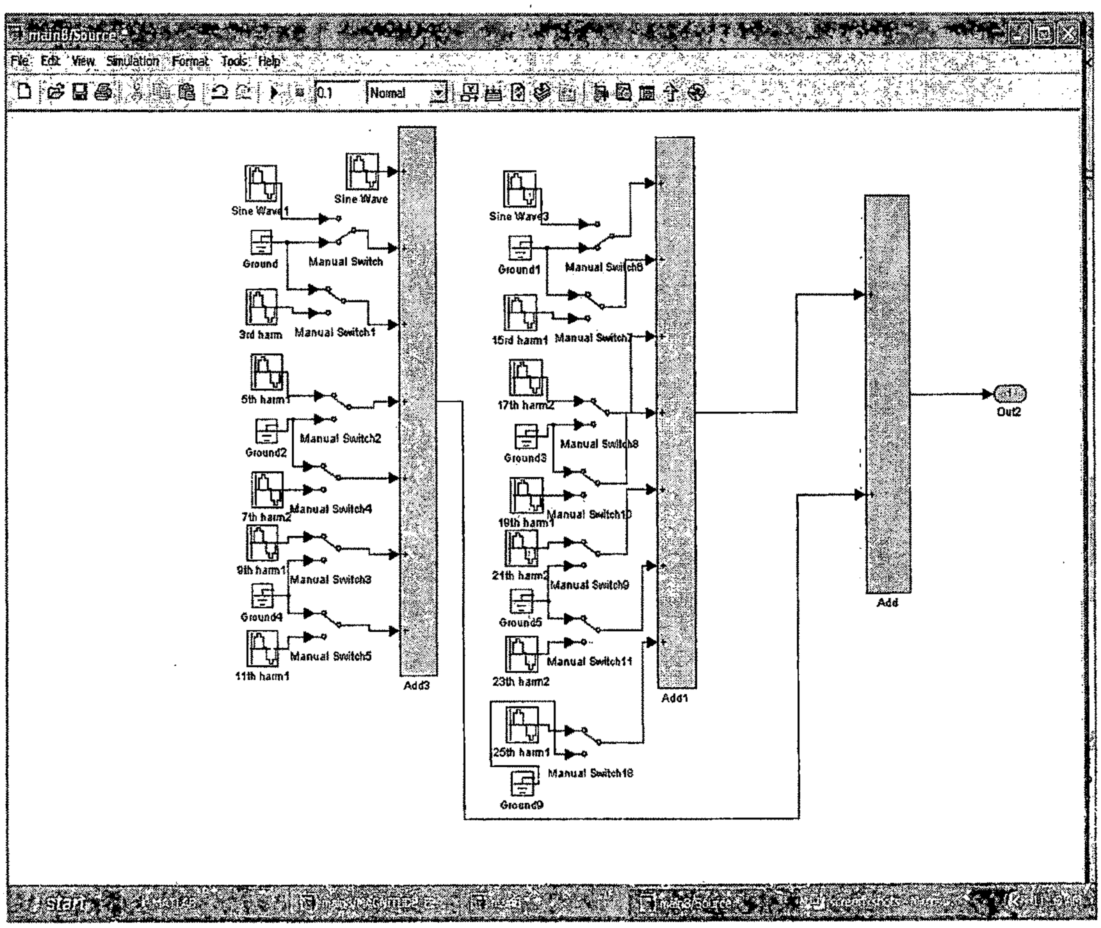


Fig. 4.5-2 Screen Shot of the input block

The system is modeled for the odd harmonics, since they are more frequently present in the system. The magnitude of the harmonics is increased from 0.1 to 50% of fundamental value, the angle with which the harmonic component is added to fundamental is not considered in this thesis.

The Kalman filter is added in block 2, Fig 4.5-3 shows the detailed version of block 2. As shown in Fig., program for Kalman filter is in *m-file* which is called from block using the parameter module as shown in Fig 4.5-4 and Fig 4.5-5.

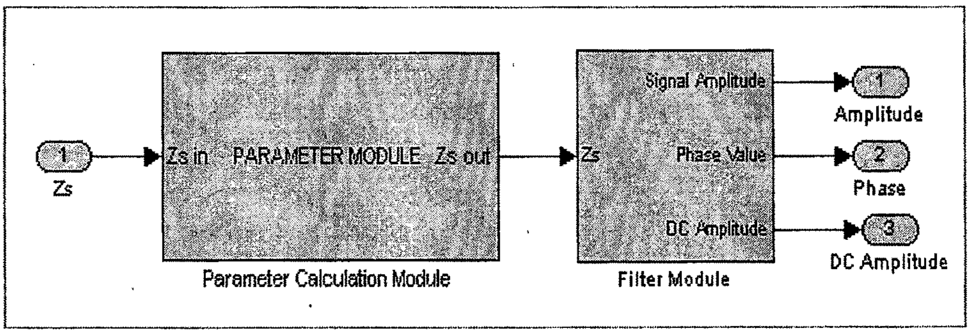


Fig. 4.5-3 Screen Shot of the Kalman block

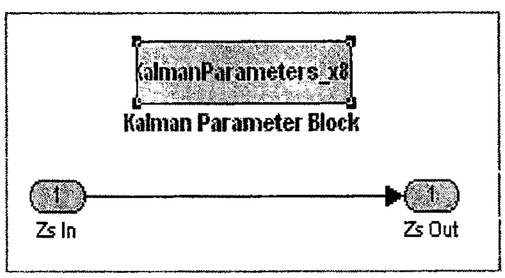


Fig. 4.5-4 Screen Shot of the m-file linker block

As shown in the Fig 4.5-5 the model extracts magnitude and phase components from the block.

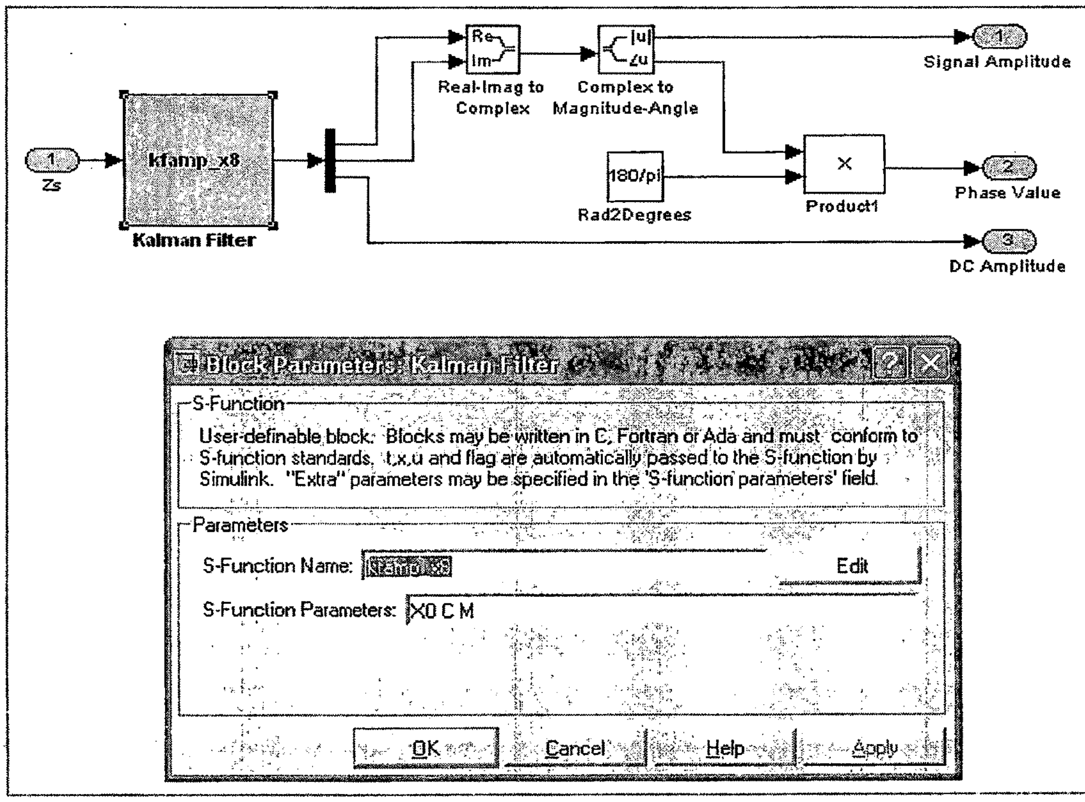


Fig. 4.5-5 Screen Shot of the output block

Fig 4.5-6 shows tracking of fundamental component and Kalman filter output. It can be seen from Fig 4.5-6, that magnitude of fundamental computed by Kalman filter and actual change in fundamental value. The time needed for Kalman filter to track the change in fundamental is less than 0.01 sec.

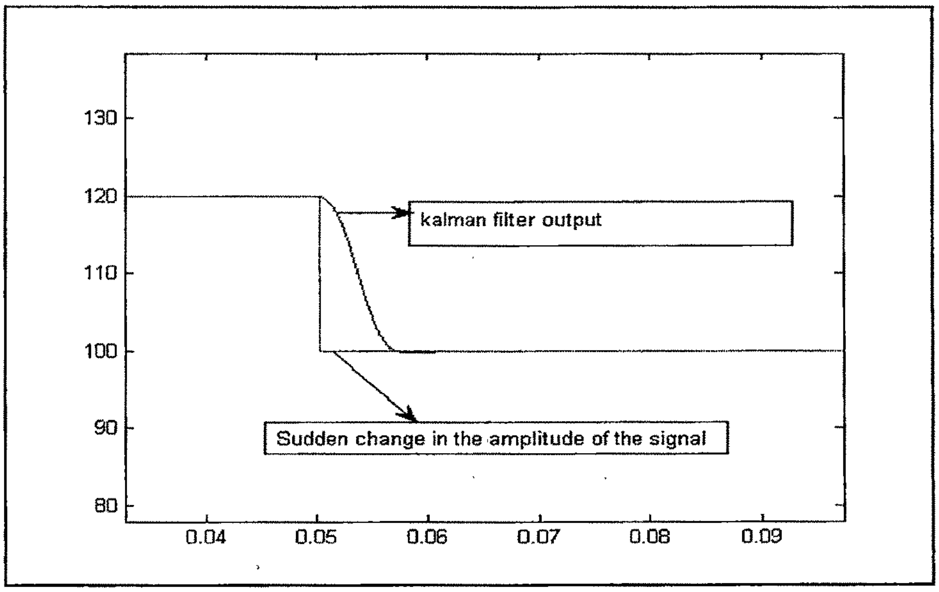


Fig. 4.5-6 Simulation results of the Kalman filter

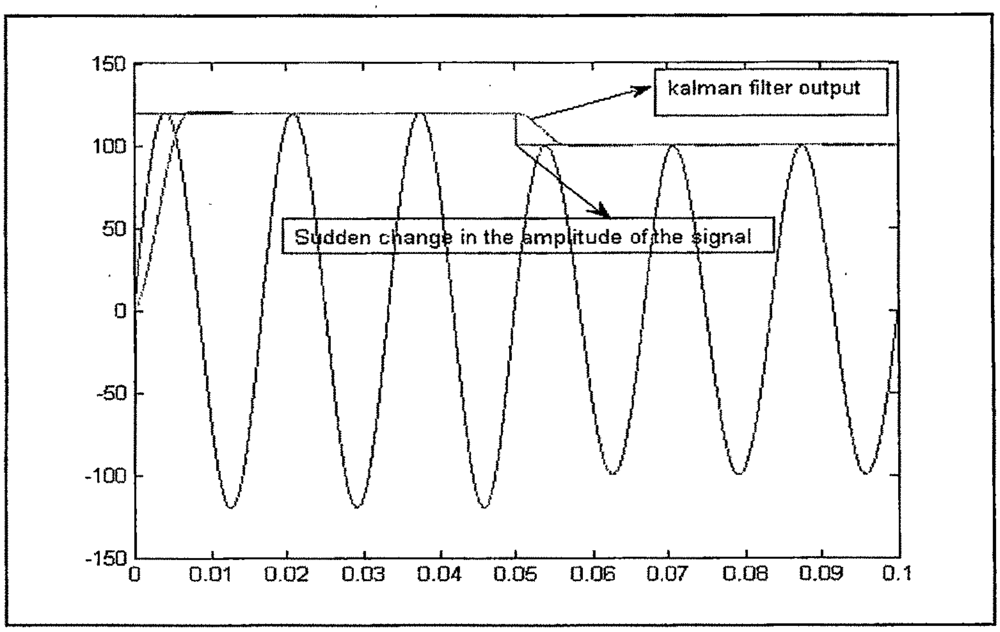


Fig. 4.5-7 Kalman filter estimated and actual signal

As described before the system is modeled for estimation of various harmonics. Fig 4.5-7 shows the estimated magnitude of third harmonic and fundamental component. It can be seen from the Fig. that after initial estimation, output of Kalman filter tracks magnitude continuously sample by sample.

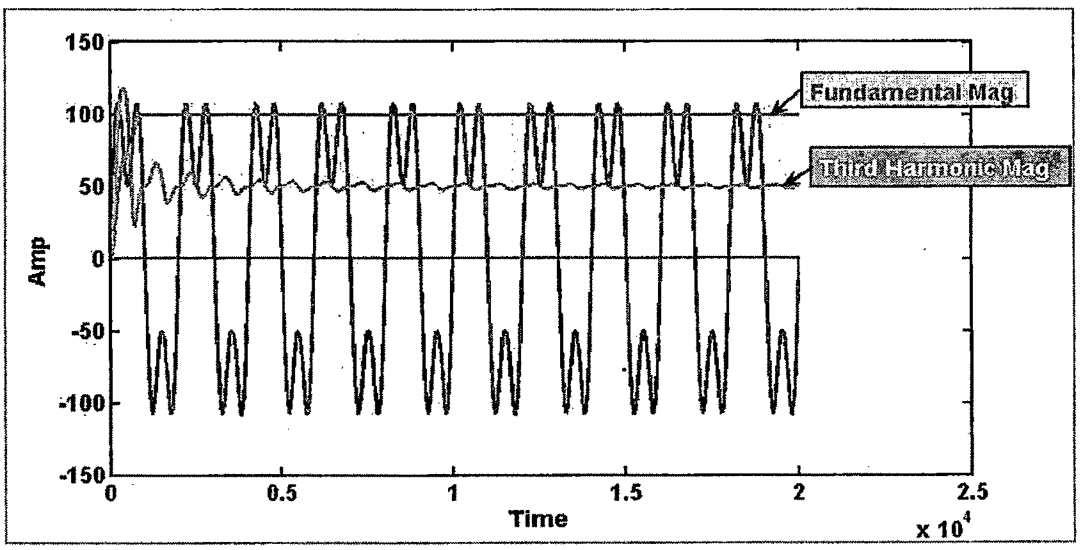


Fig. 4.5-8 Estimated Magnitude of Fundamental and Third Harmonic

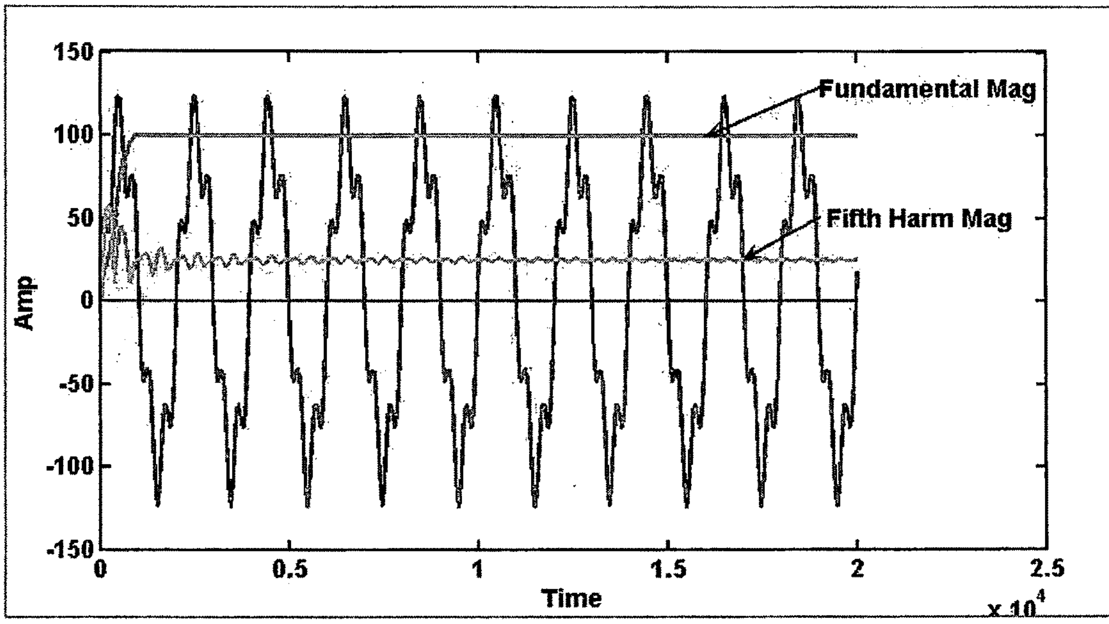


Fig. 4.5-9 Estimated Magnitude of Fundamental and Fifth Harmonic

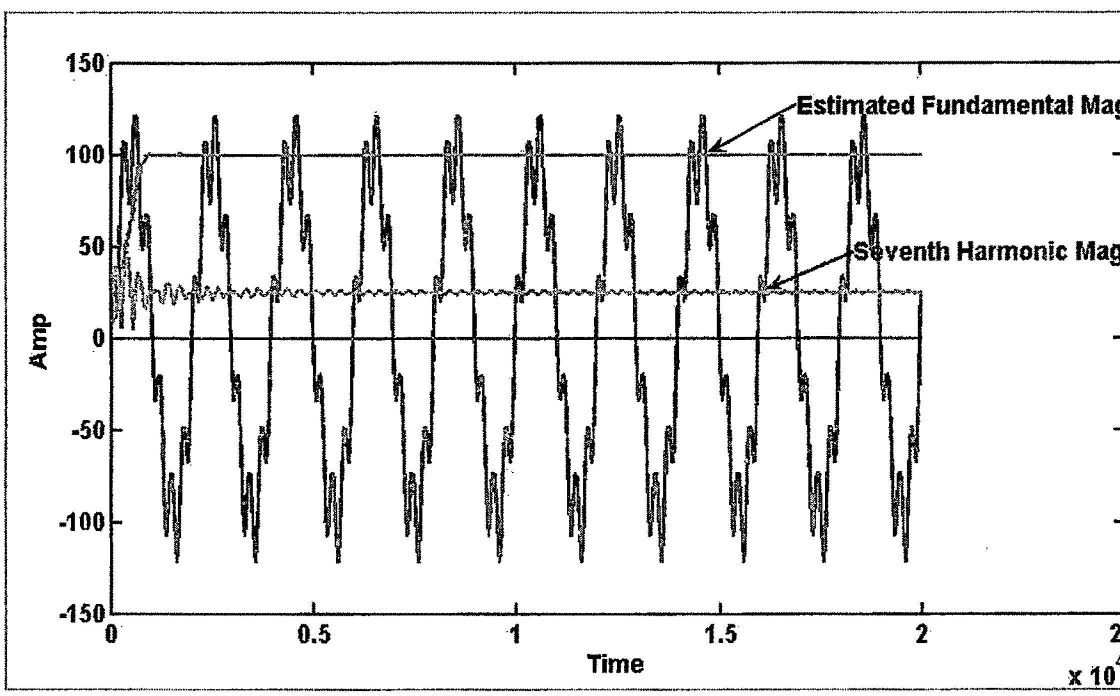


Fig. 4.5-10 Estimated Magnitude of Fundamental and Seventh Harmonic

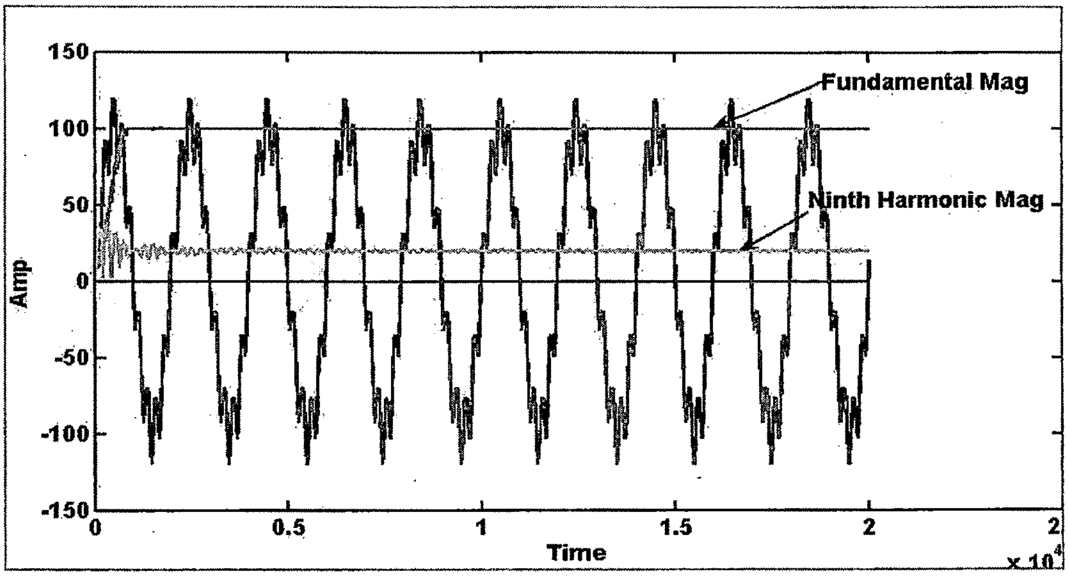


Fig. 4.5-11 Estimated Magnitude of Fundamental and Ninth Harmonic

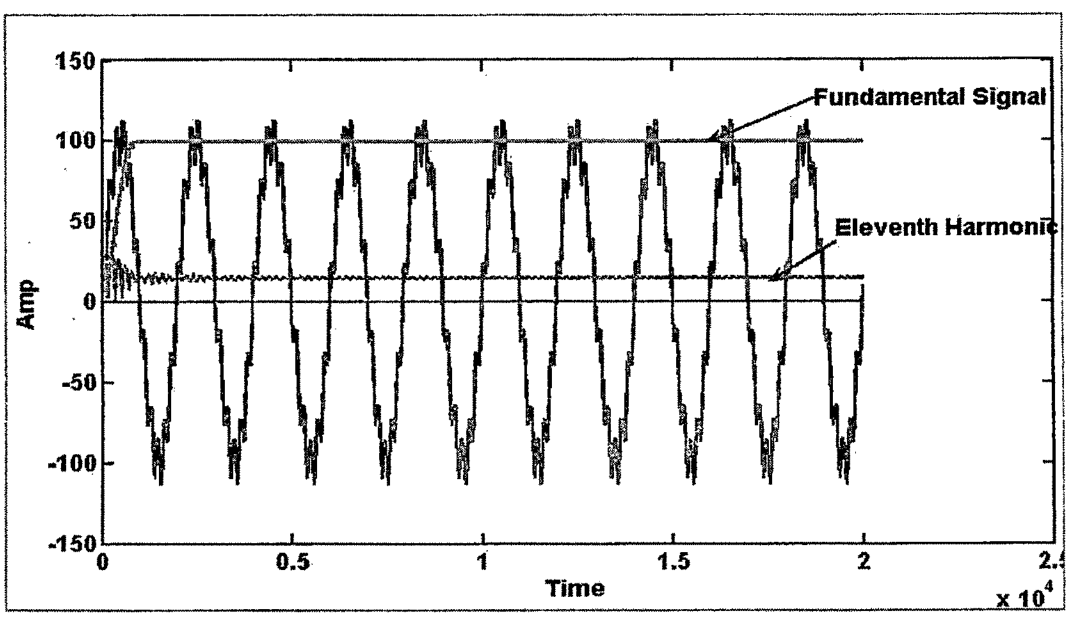


Fig. 4.5-12 Estimated Magnitude of Fundamental and Eleventh Harmonic

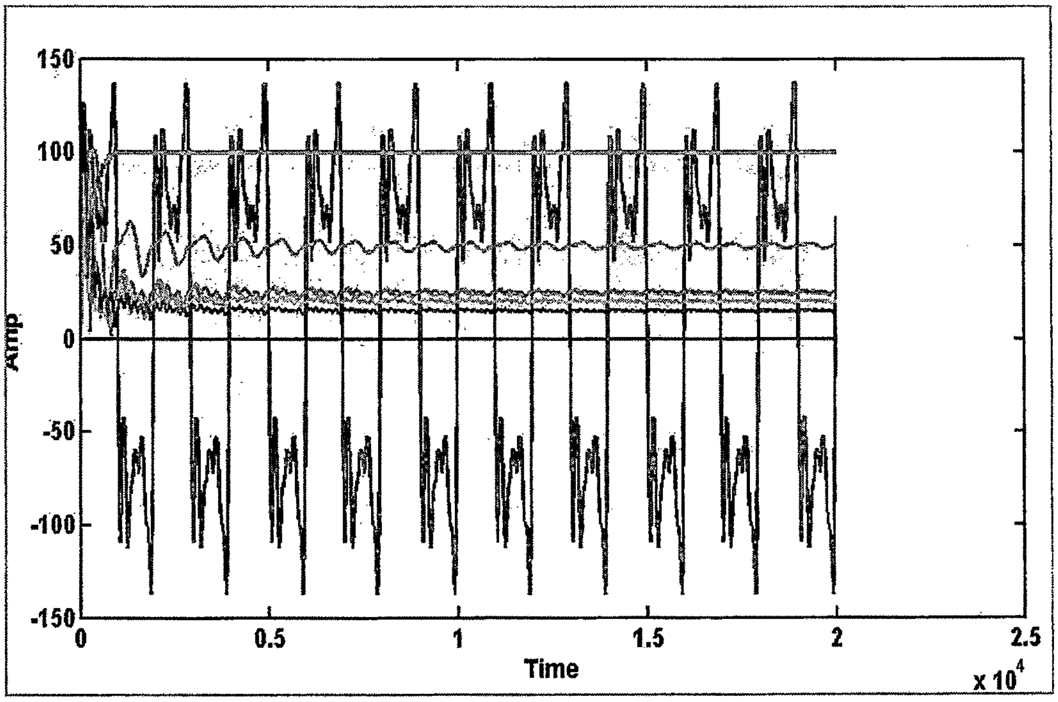


Fig. 4.5-13 Estimated Magnitude of Fundamental and various Harmonic

On –line estimation of various harmonics is done by kalman filter which enables to determine the magnitude of harmonic components at any instant of time. The system is also tested with harmonics up-to 25th order. The computation burden for such a large model has been increased tremendously. The signal is shown in Fig 4.5-13

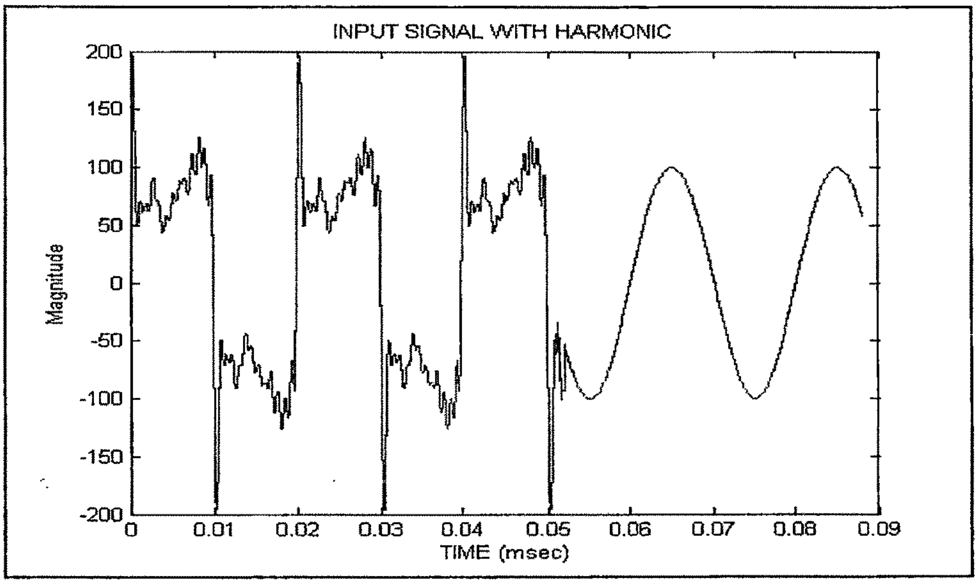


Fig. 4.5-14 Input signal with harmonic upto 25th order

The system was tested by injecting harmonic up-to 25 orders and then suddenly harmonic components were removed. The kalman filter was able to track magnitudes of harmonics. The harmonic magnitudes computed by Model are as shown in Table 4.5-1.

Harmonic Order	Actual Magnitude	Estimated Magnitude	% Error
1	100	99.99	0.01
3	50	49.74	0.52
5	25	19.76	1.16
7	25	19.43	0.32
9	20	19.97	0.005
11	15	14.53	1.2
13	15	14.74	1.2
15	15	14.68	0.6
17	15	14.94	0.004
19	10	10.01	0.001
21	10	10.01	0.001
23	10	10.01	0.001
25	10	10.01	0.001

Table 4.5-1 Simulation results

4.6 Hardware and Software

The proposed hardware system for harmonic tracking using kalman filter will be as shown in the Fig 4.6-1. The system voltage and current signal are converted into voltage range suitable for measurement using potential transformer and current transformer. The output of P.T and C.T are converted into unipolar using a level shifter circuit. Three voltages and current channels are then given to Channel A and Channel B of the internal ADC of DSP. The Kalman filter is implemented into DSP. The hardware designed is a multiprocessor based system in which one processor continuously measure frequency and send's the data to the SPI bus. The SPI bus is interconnected to all five DSP boards. The frequency data is used by Kalman filter to determine the various magnitude and phase of various orders of harmonics. The details of the hardware can be found in chapter 6.

The Software program is written in C language to model Kalman filter. As shown in fig the multiprocessor system developed along with the VB based software interface for display of various parameters on the system.

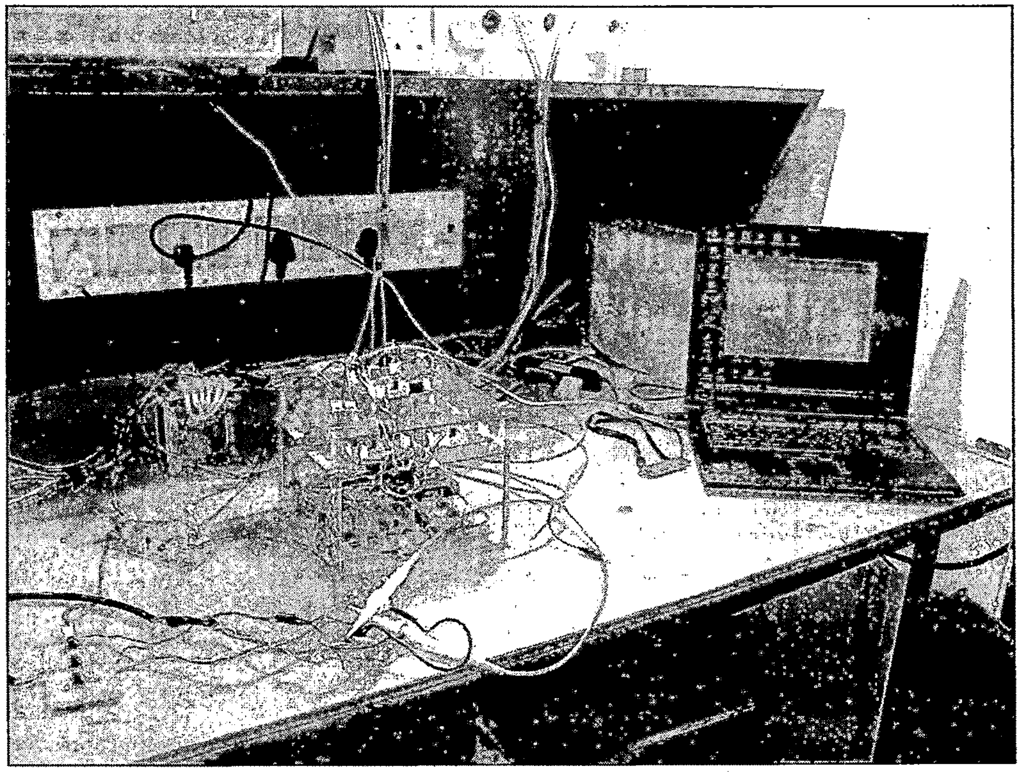


Fig. 4.6-1 Experimental Set-up for harmonic measurement

4.7 Flowchart

The flow chart for high level software for on-line tracking of harmonics using Kalman filter will be as shown in Fig.4.7-1. The detailed description of the software can be found in chapter 6.

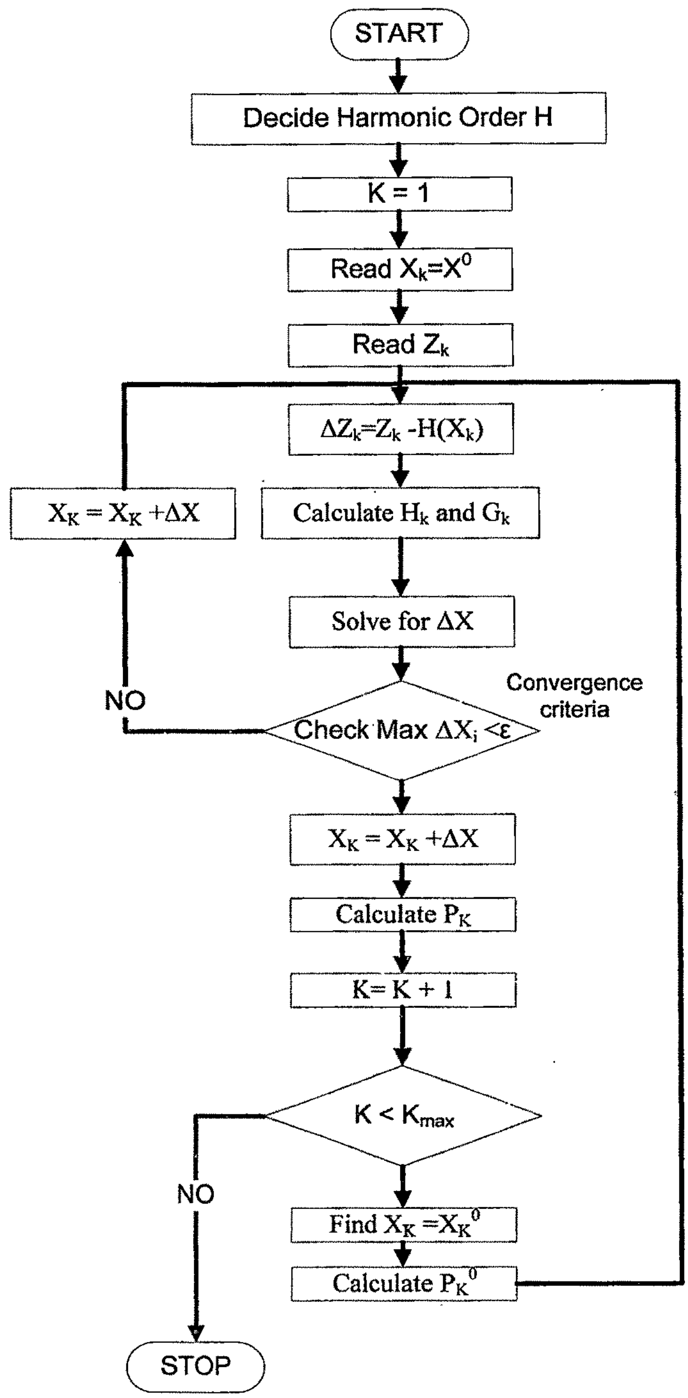


Fig. 4.7-1 Flowchart for harmonic estimation using Kalman filter

4.8 Results

VB based software has been developed in-house for displaying the various parameters computed by multiprocessor system. As shown in Fig 4.8-1 the waveforms of the three phase voltages and current signals and Fig shows the estimated magnitude of harmonics up to 25th harmonics.The harmonic plot as computed by the system is shown in Fig 4.8-2.

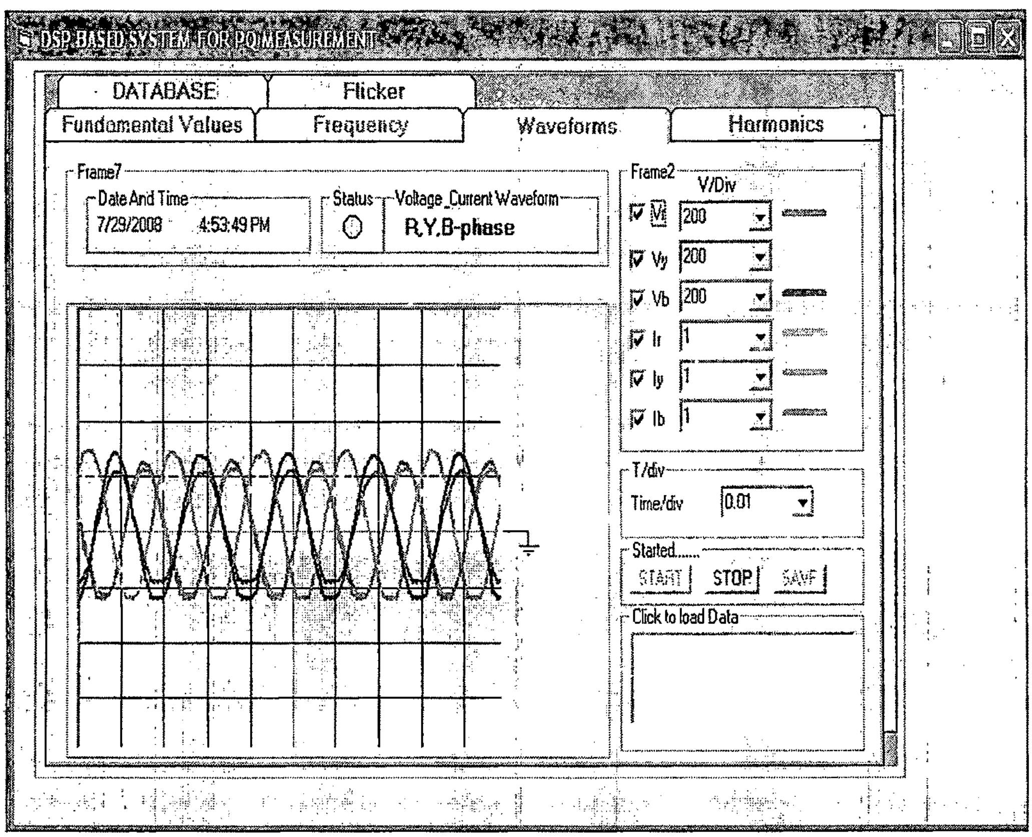
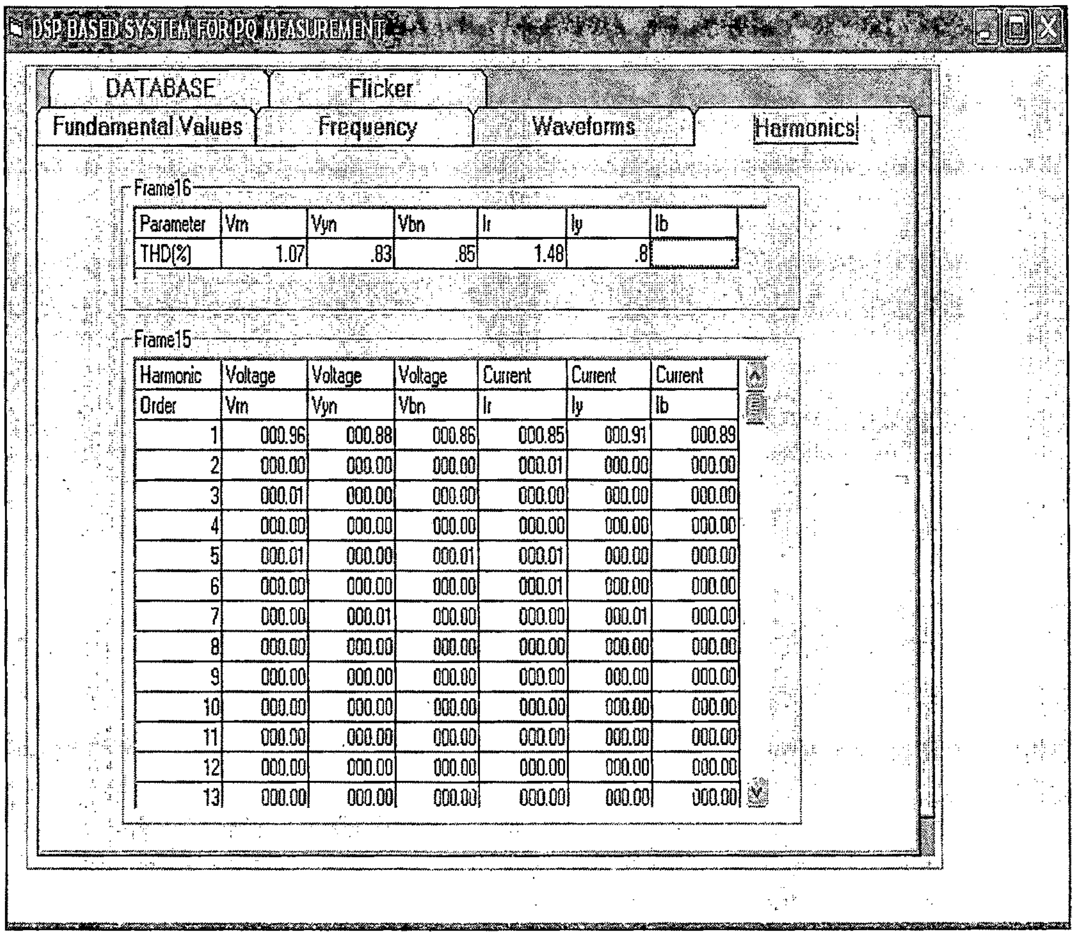


Fig. 4.8-1 VB Screen shot for harmonic measurement



Parameter	Vrn	Vyn	Vbn	Ir	Iy	Ib
THD(%)	1.07	.83	.85	1.48	.8	

Harmonic Order	Voltage Vrn	Voltage Vyn	Voltage Vbn	Current Ir	Current Iy	Current Ib
1	000.96	000.88	000.86	000.85	000.91	000.89
2	000.00	000.00	000.00	000.01	000.00	000.00
3	000.01	000.00	000.00	000.00	000.00	000.00
4	000.00	000.00	000.00	000.00	000.00	000.00
5	000.01	000.00	000.01	000.01	000.00	000.00
6	000.00	000.00	000.00	000.01	000.00	000.00
7	000.00	000.01	000.00	000.00	000.01	000.00
8	000.00	000.00	000.00	000.00	000.00	000.00
9	000.00	000.00	000.00	000.00	000.00	000.00
10	000.00	000.00	000.00	000.00	000.00	000.00
11	000.00	000.00	000.00	000.00	000.00	000.00
12	000.00	000.00	000.00	000.00	000.00	000.00
13	000.00	000.00	000.00	000.00	000.00	000.00

Fig. 4.8-2 Results displayed on VB Software

4.9 Conclusions:

The chapter presents one more digital technique for real time estimation of harmonics. The chapter covers the traditional technique used for measurement of harmonics and reveals the problems in these technique. The chapter describes the use of adaptive filter for harmonic estimation in power systems. The use of adaptive filter has an added advantage for dynamic parameter such as harmonics. Since harmonics are dependent on load and in an industrial/ interconnected system load varies frequently and Kalman filter describes is able to track the change of harmonics level. Based on the computation and findings of the proposed algorithm, following conclusions have been drawn:

- A comparison of the various methods is done which are used for harmonic estimation. The chapter describes the advantage and disadvantage of the various methods and proposes an adaptive and its implementation in power system.
- The chapter models the Kalman filter for real time estimation of harmonics using a modeling matrix. The mathematical modeling and the time taken for estimation of harmonics is compared with Kalman filter. The chapter also covers the simulation results obtained under various conditions.
- The chapter also implements the Kalman filter on a digital platform and proves its validity.