Chapter VIII

The Miscellaneous Problems

8.1 The Semi-Economic Design of np-Control Chart

8.1.1 Early Work on Semi-Economic Design

The economic design of conventional Shewhart type control charts was investigated by several early researchers. Most of their work can be classified as semi-economic design procedures in that either the proposed model did not consider all relevant costs or no optimization techniques were applied.

Weiler (1952) obtained for an $\bar{x}$-chart the optimum sample size that would minimize the total amount of inspection required to detect a specified shift. Weiler (1952) did not formally consider costs; the implication is that minimizing total inspection will minimize total cost. Another type of semi-economic design was studied by Cowden (1957) who considered all three major catagories of costs but offered no optimization technique. Cowden's (1957) work is also on $\bar{x}$-chart.

In this section we obtain, for np-control chart, the optimal sample size that minimizes the total amount of inspection required to detect a specified shift in the process.

8.1.2 The Most Economical Sample Size for np-Control Chart

The production process starts in an in-control state in which it produces a known acceptable proportion, p_o, of nonconforming units. As time passes the process may deteriorate and start producing a nonacceptable proportion, p_1, of noncconforming units. The out-of-control state of the process is represented by p_1.

This production process is monitored by an np-chart with the upper control limit $np_o+3\sqrt{np_o(1-p_o)}$. The lower control limit is assumed to be zero. The inspection procedure is as follows. At fixed intervals of time the samples of size n are taken. Let d be the nonconforming units found in the sample. The various values of d are entered in the chart in cronological order, and as soon as one such value falls outside the control limits, the process is declared to be out of control. The production is stopped to allow investigation.

We want to find the optimal value of n which would minimize the average total inspection required to detect the shift.

8.1.3 The Average Amount of Inspection

When the shift occurs, the probability that it will be detected on any subsequent sample is

$$q_1 = \sum_{d=m}^{n} \binom{n}{d} p_1^d (1-p_1)^{n-d} \qquad \ldots(8.1.1)$$

where $m = np_o + 3\sqrt{np_o(1-p_o)}$.

We know that the number of samples required to detect the shift given that the shift has occured is a geometric random

variable. Hence the expected number of samples required to detect the shift is $1/q_1$. This follows that on an average n/q_1 units have to be examined to detect the shift. Therefore the average total inspection required to detect the shift is

$$A(n) = n/q_1 \qquad \ldots(8.1.2)$$

8.1.4 To Minimize the Average Total Insepction

We find the value of n which would minimize

$$A(n) = \frac{n}{\sum_{d=m}^{n} \binom{n}{d} p_1^d (1-p_1)^{n-d}} \qquad \ldots(8.1.3)$$

where $$m = np_o + 3\sqrt{np_o(1-p_o)} \qquad \ldots(8.1.4)$$

The difference between the two successive terms A(n) and A(n+1) can be written as

$$A(n+1)-A(n) = \frac{n+1}{\sum_{d=m'}^{n+1} \binom{n+1}{d} p_1^d (1-p_1)^{n+1-d}} - \frac{n}{\sum_{d=m}^{n} \binom{n}{d} p_1^d (1-p_1)^{n-d}} \qquad \ldots(8.1.5)$$

where $$m' = (n+1)p_o + 3\sqrt{(n+1)p_o(1-p_o)} \qquad \ldots(8.1.6)$$

Since p_o represents the proportion of nonconforming units produced when the process is in the in-control state, the value of p_o is likely to be very small. Assuming that p_o is very small we have $m' \simeq m$. We therefore have,

$$A(n+1)-A(n) \propto \frac{n+1}{\sum_{d=m}^{n+1} \binom{n+1}{d} p_1^d (1-p_1)^{n+1-d}} - \frac{n}{\sum_{d=m}^{n} \binom{n}{d} p_1^d (1-p_1)^{n-d}} . \quad \ldots(8.1.7)$$

Let $B(m, n, p) = \sum_{x=m}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

and $b(x, n, p) = \binom{n}{x} p^x (1-p)^{n-x}$.

Using the relation

$$B(m, n+1, p_1) = p_1 B(m-1, n, p_1) + (1-p_1) B(m, n, p_1) \quad \ldots(8.1.8)$$

in the expression (8.1.7) we derive the following result:

$$\begin{aligned} A(n+1) - A(n) &> 0 \quad \text{if} \quad n < B(m, n, p_1)/[p_1 b(m-1, n, p_1)] \\ &= 0 \quad \text{if} \quad n = B(m, n, p_1)/[p_1 b(m-1, n, p_1)] \\ &< 0 \quad \text{if} \quad n > B(m, n, p_1)/[p_1 b(m-1, n, p_1)] \end{aligned} \quad \ldots(8.1.9)$$

From (8.1.9) it follows that the minimum value of A(n) is attained when

$$n = B(m, n, p_1)/[p_1 b(m-1, n, p_1)] \quad \ldots(8.1.10)$$

The above result helps in finding the optimal value of n by successive approximation method.

8.1.5 Poisson Approximation

Assuming that the value of n is large and the value of p_1 is relatively small we may write

$$q_1 = \sum_{x=m}^{n} \exp(-\lambda)\lambda^x/x! \quad \ldots(8.1.11)$$

where $\lambda = np_1$, $m = np_o + 3\sqrt{np_o}$

Under Poisson approximation the expression for A(n) is

$$A(n) = \frac{n}{\sum_{x=m}^{n} \exp(-\lambda)\lambda^x/x!} \qquad \ldots(8.1.12)$$

Differentiating A(n) with respect to n and equating to zero we have

$$n = F(m, \lambda) \,/\, [p_1 f(m-1, \lambda)] \qquad \ldots(8.1.13)$$

where $$F(m, \lambda) = \sum_{x=m}^{\infty} \exp(-\lambda)\lambda^x/x! \qquad \ldots(8.1.14)$$

and $$f(m-1,\lambda) = \exp(-\lambda)\lambda^{m-1}/(m-1)! \qquad \ldots(8.1.15)$$

The expressions for optimal n obtained by using the binomial law and the Poisson law as given by (8.1.10) and (8.1.13) are almost similar in the nature. The numerator on the right side in both the expressions is 1 minus the distribution function and the denominator is the product of p_1 and the individual probability mass function.

Numerical Example

A computer program on Fortran is developed for the computation of the R.H.S of the equation (8.1.10) for n = 1,2,3,... . This program uses a subroutine for the computation of cummulative and individual binomial probabilities. The values p_0 and p_1 are assumed to be known. We choose that value of n as optimal for which equation (8.1.10) is satisfied. While evaluating $B(m,n,p_1)$involved in (8.1.10) we have taken the value of m to be the integer part of $np_0+3\sqrt{np_0(1-p_0)}$. The listing of the program is given at the end of this chapter. The following

Table 8.1 gives the optimal values of n in column (3) for various values of p_o and p_1. The values of q_1 and the avarage amount of inspection derived at the optimal stage are given as columns (4) and (5).

Table 8.1

p_o	p_1	Optimal Sample Size(n)	q_1	A(n)
(1)	(2)	(3)	(4)	(5)
0.01	0.06	30	0.8437	35.56
0.01	0.08	22	0.8403	26.18
0.01	0.10	18	0.8499	21.18
0.03	0.15	32	0.8782	36.44
0.03	0.17	10	0.8448	11.84
0.05	0.20	51	0.9077	56.19

We illustrate the use of the above Table 8.1 as given below.

Suppose, using the np-control chart with the usual control limits, it is decided to control the proportion of nonconforming units at a level p_o = 0.01(1%) and to detect the shift in the proportion of nonconforming units at a level p_1 = 0.1(10%). Then referring to the table, one can see that one has to take a sample of 18 units. Thereby the probability of detecting the above stated shift is 0.8499 and the average amount of inspection is 21.18.

8.2 Some Corrections in Knappenberger and Grandage's (1969) Model for $\bar{x}$-Chart

In this section some corrections are made in Knappenberger and Grandage's (1969) model. We correct some expressions and numerical results which are not consistent with the procedure desiribed by the authors(1969).

Knappenberger and Grandage's (1969) model is well explained in the Section 5.2 of chapter V and hence it is not described again. The total expected cost per unit of the product under this model is

$$E(C) = (a_1+a_2n)/k + a_3\underline{\alpha}'\underline{q}/k + a_4\underline{r}'\underline{f} \qquad \ldots(8.2.1)$$

The cost coefficients a_i(i = 1,2,3,4) and the probability vectors $\underline{q}$, $\underline{\alpha}$, $\underline{r}$, $\underline{f}$ are as explained in the section 5.2of chapter V. The cost coefficients a_i (i = 1,2,3,4) are assumed to be known and are independent of the design variables (n, k, L). The probability vector $\underline{f}$ is also independent of the design variables. The probability vectors $\underline{q}$, $\underline{\alpha}$, $\underline{r}$ are functionally related to the design variables (n, k, L). We have observed some fallacies in the evaluation of vector $\underline{\alpha}$ which are corrected in this section.

The evaluation of the vector $\underline{\alpha}$ depends upon the transition probability matrix B. The element b_{ij} (i,j = 0,1, ..., s) represents the probability of the process shifting from state μ_i to the state μ_j during the production of k units between two successive samples. The expressions for b_{ij} are given by (4.2.11), (4.2.12), (4.2.13) and (4.2.14). The expressions b_{ij} (i,j = 0,1, ..., s) depend upon the transition probability matrix (p_{ij}) and the probability vector $\underline{q}$. The expressions p_{ij} (i,j =

0,1, ..., s) are given by (4.2.9) and (4.2.10). The probability vector $\underline{q}$ is given by (5.3.1).

Once B matrix is determined, the vector $\underline{\alpha}$ is obtained by solving the equation

$$\underline{\alpha}'B = \underline{\alpha}' \qquad \ldots(8.2.2)$$

This procedure for obtaining $\underline{\alpha}$ is described by the authors and is correct.

From the equation (8.2.2) we write

$$\underline{\alpha}'(B - I) = \underline{0}' \qquad \ldots(8.2.3)$$

where I is (s+1)x(s+1) identity matrix and $\underline{0}$ is a vector of (s+1) zeros.

The equation (8.2.3) along with the condition $\sum_{i=0}^{s} \alpha_i = 1$ can be rewritten as

$$\underline{\alpha}'(B - I \mid \underline{1}) = (\underline{0}, 1)' \qquad \ldots(8.2.4)$$

where $(B - I \mid \underline{1})$ on the left side of (8.2.4) is (B - I) matrix augmented by column vector $\underline{1}$ of (s+1) ones and 1 appearing on the right side is just scaler 1.

It may be noted that the sum of any s of the first (s+1) restrictions given by (8.2.4) implies the remaining restriction of the first (s+1) restrictions. Hence to solve for $\underline{\alpha}$ any one of the first (s+1) restrictions given in (8.2.4) can be omitted. This can be done by omitting any one of the columns of matrix (B - I) on the left side and the corresponding zero on the right side of (8.2.4). Arbitrarily omitting the first column labelled "0" on the left side and the corresponding zero on the right side of (8.2.4) as suggested by the authors we get the following

equation for determining the value of the vector $\underline{\alpha}$

$$\underline{\alpha}' B^* = (\underline{0}\ ,\ 1)' \qquad \ldots(8.2.5)$$

where $\underline{0}$ is now a vector of s zeros.

The elements of the matrix B^* are derived using the process described above. They are related to elements of B and I. The exact expressions for b^*_{ij} $(i = 0, 1, \ldots, s;\ j = 0, 1, \ldots, s)$ are

$$\begin{aligned} b^*_{i,j} &= b_{i,j+1} && \text{for all } i \text{ and for all } j,\ j \neq i-1, s \\ b^*_{i,i-1} &= b_{ii}{}^{-1} && \text{for } i \neq 0 \\ b^*_{is} &= 1 && \text{for all } i \end{aligned} \qquad \ldots(8.2.6)$$

The expressions for b^*_{ij} given by the authors (1969) are wrong.

Lastly having obtained B^*, one should obtain $(B^*)^{-1}$. The last row of $(B^*)^{-1}$ gives $\underline{\alpha}'$.

For the sample problem with n = 2, k = 40, L = 3, λ = 1, R = 1000 and π = 0.376 of Knappenberger and Grandage, we have

p_{oo} = 0.96079	q_o = 0.00270
p_{o1} = 0.00889	q_1 = 0.05639
p_{o2} = 0.01340	q_2 = 0.43189
p_{o3} = 0.01076	q_3 = 0.89300
p_{o4} = 0.00486	q_4 = 0.99606
p_{o5} = 0.00117	q_5 = 0.99998
p_{o6} = 0.00013	q_6 = 1.00000

The B^* matrix is found using the corrected formulae (8.2.6) and is as given below :

$$B^{*} = \begin{bmatrix} .00889 & .01340 & .01076 & .00486 & .00117 & .00112 & 1 \\ -.78547 & .32317 & .25964 & .11734 & .02828 & .00284 & 1 \\ .00384 & -.67123 & .16061 & .07258 & .01749 & .00176 & 1 \\ .00794 & .01196 & -.90018 & .01762 & .00425 & .00043 & 1 \\ .00886 & .01335 & .01072 & -.99134 & .00129 & .00013 & 1 \\ .00889 & .01340 & .01076 & .00486 & -.99880 & .00012 & 1 \\ .00889 & .01340 & .01076 & .00486 & .00117 & -.99988 & 1 \end{bmatrix}$$

As stated, one can verify that the matrix B^{*} given above and given by the authors (Knapenberge and Grandage) are totally different. Not only that B^{*} given by them is not in accordance with the formulas stated by them.

We have calculated inverse of B^{*} given above and the last row of $(B^{*})^{-1}$ is

$\underline{a}' = (.93547, .01101, .02452, .01885, .00803, .00193, .00019)$

This gives the solution for $\underline{a}$.

8.3 Some Modifications in the Montgomery's (1975) Expected Cost Model for np-Control Chart.

8.3.1 Montgomery, Heikes and Mance (1975) developed the economic design of np-control chart using Knappenberger and Grandage's (1969) cost model for $\bar{x}$-chart. One of the drawbacks of Knapenberger and Grandage's (1969) model is that the cost of producing a nonconforming unit remains the same whether the unit is detected during sampling or it goes undetected to the customer. This is not realistic. Surely an undetected nonconforming unit which goes to the customer is more costly than the one which is detected during sampling. In this section we modify Montgomery's (1975) expected cost model for np-control chart in such a way that it assigns a higher cost to a

nonconforming unit that goes undetected to the customer than to the one which is detected during sampling.

8.3.2 The Modified Expected Cost Model.

Montgomery's (1975) expected cost model is well explained in the Section 4.2.3 of chapter IV. The total expected cost per unit, E(C), derived under this model is as follows.

$$E(C) = E(C_1) + E(C_2) + E(C_3) \qquad \ldots(8.3.1)$$

where

$E(C_1)$ = the expected cost per unit of sampling and inspection

$$= (a_1 + a_2 n)/k, \qquad \ldots(8.3.2)$$

$E(C_2)$ = the expected cost per unit of finding the assignable causes and repairing the process

$$= \frac{a_3 \sum_{i=0}^{m} \alpha_i q_i}{k}, \qquad \ldots(8.3.3)$$

$E(C_3)$ = the expected cost per unit of producing nonconforming units

$$= a_4 \sum_{i=0}^{m} r_i p_i \qquad \ldots(8.3.4)$$

Hence the expression for E(C) under Montgomery's model is

$$E(C) = \frac{a_1 + a_2 n}{k} + \frac{a_3 \sum_{i=0}^{m} \alpha_i q_i}{k} + a_4 \sum_{i=0}^{s} r_i p_i \qquad \ldots(8.3.5)$$

For the modified expected cost model developed by us the expressions for $E(C_1)$ and $E(C_2)$ remain the same as those given by Montgomery et. al. (1975) which are given by the expressions

(8.3.2) and (8.3.3) respectively. The expression $E(C_3)$ is developed in such a way that a higher cost is assigned to a nonconforming unit which goes undetected to the customer than to the one which is detected during sampling. The modified expression for $E(C_3)$ is developed in the next Section 8.3.3.

8.3.3 Computation of $E(C_3)$

The expected cost of producing nonconforming units between two successive samples is computed under two possibilities.

(1) The process is in state p_i at the time of taking a sample and remains in the same state at the time of taking the next sample.

(2) The process is in state p_i at the time of taking a sample and shifts to another state p_j ($p_j > p_i$) before the next sample is taken.

It is assumed that at most one shift can occur between two successive samples.

Under the situation (1) the process remains in the same state p_i during the production of k units between two successive samples. Hence the expected number of nonconforming units produced is kp_i. Among these k units only the first n units are sampled and remaining (k-n) go undetected to the customer. Hence the expected cost of producing nonconforming units is

$$a_{4,1}np_i + a_{4,2}(k-n)p_i \qquad \ldots(8.3.6)$$

where

$a_{4,1}$ = the cost of a nonconforming unit that is detected during sampling and inspection,

$a_{4,2}$ = the cost of a nonconforming unit that goes undetected to the customer.

clearly $a_{4,1} < a_{4,2}$.

Multiplying the expression (8.3.6) by the required probabilities and adding for i = 0,1, ..., s, the expected cost of producing nonconforming units under the situation (i) is given by

$$E(C_{3.1}) = \sum_{i=0}^{s} \alpha_i p_{ii} [\, a_{4,1} n p_i + a_{4,2}(k-n)p_i] \qquad ...(8.3.7)$$

where α_i (i = 0,1, ..., s) is the steady state probability that the process is in state p_i (i = 0,1, ...,s) at the time of taking a sample, and p_{ii} is the probability that the process remains in the same state p_i until the next sample is taken. The expression for α_i (i = 0,1,...,s) and the transition probability matrix (p_{ij}) are given by (4.2.15) and (4.2.10) respectively.

Under the situation (2) the process shifts from state p_i to state p_j ($p_j > p_i$) during the production of k units between two successive samples. Let Δ be the average fraction of time the process remains in the state p_i before shifting to p_j, given that the shift occurs between two successive samples. The expression for Δ is given by (4.2.17) and hence not reproduced here. The total time spent between two successive samples is k/R hours. Hence the fraction of time the process remains in the state p_i is $\Delta k/R$ hours and in the state p_j is $(1-\Delta)k/R$ hours. Hence the number of units produced in the state p_i is Δk and the number units produced in the state p_j is $(1-\Delta)k$. Therefore the expected number of nonconforming units produced is $\Delta k p_i + (1-\Delta)k p_j$. Among the Δk unit produced in the state p_i, only first n units are

sampled and inspected. Hence the expected number of nonconforming units detected during sampling is np_i. The remaining $(\Delta k-n)p_i+(1-\Delta)kp_j$ nonconforming units are undetected due to nonsampling. Hence the expected cost of producing nonconforming units is

$$a_{4,1}np_i + a_{4,2}\,[(\Delta k-n)p_i + (1-\Delta)kp_j] \qquad \ldots(8.3.8)$$

Multiplying (8.3.8) by the required probability and adding for $i = 0,1, \ldots, s-1$, $j = i+1, i+2, \ldots, s$, the expected cost of producing nonconforming units under the situation (2) is

$$E(C_{3.2}) = \sum_{i=0}^{s-1} \sum_{j=i+1}^{s} \alpha_i p_{ij}\,[a_{4,1}np_i + a_{4,2}\,\{(\Delta k-n)p_i + (1-\Delta)kp_j\}] \qquad \ldots(8.3.9)$$

where p_{ij} is the probability that the process shifts from the state p_i to the state p_j between two successive samples. The expression for p_{ij} is given by (4.2.10).

Hence the expected cost per unit $E(C_3)$, of producing nonconforming units is given by

$$E(C_3) = E(C_{3.1}) + E(C_{3.2}) \qquad \ldots(8.3.10)$$

When $a_{4,1} = a_{4,2}$ it can be verified that the expression for $E(C_3)$ given by (8.3.10) reduces to the expression for $E(C_3)$ given by (8.3.4). This means that when $a_{4,1} = a_{4,2}$, $E(C_3)$ under the present modified model is the same as $E(C_3)$ under the Montgomery's original model.

8.3.4 Numerical Example

The Hooke-Jeeves search technique is used to find the optimal values of n, m, k which minimize the expected cost per

unit of the product.

We consider an example presented in Montgomery et. al. (1975).

Let a_1 = \$ 5.0, a_2 = \$ 0.5, a_3 = \$ 100, a_4 = \$ 10, λ = 1, R = 1000, π = 0.597, p = (.01, .02, .04, .08, .16, .32, .64).

For this combination of cost coefficients and systems parameters, the search technique yielded the following optimal control procedure. n=4, m=1, k=52 with minimum E(C) = \$ 0.6313. This optimal procedure and the minimum E(C) are derived for Montgomery's original cost model.

Under the modified cost model it is assumed that $a_{4,1} < a_{4,2}$.

Let a_1 = \$ 5.0, a_2 = \$ 0.5, a_3 = \$ 100, $a_{4,1}$ = \$ 10.

We take three possible values of $a_{4,2}$.

(1) $a_{4,2}$ = \$ 20 (2) $a_{4,2}$ = \$ 15 (3) $a_{4,2}$ = \$ 10

Let λ = 1, R = 1000, π = 0.597.

Let p = (.01, .02, .04, .08, .16, .32, .64).

Under the possibility (1) the optimal control procedure yielded is n=12, m=2, k=36 with minimum E(C) = \$ 0.8580.

Under the possibility (2) the optimal procedure yielded is n=11, m=2, k=41 with minimum E(C) = \$ 0.7567.

Under the possibility (3) the optimal procedure yielded is n=4, m=1, k=52 with minimum E(C) = \$ 0.6313.

The results of possibility (3) show that, when $a_{4,1} = a_{4,2}$, the optimal control procedure and the minimum E(C) derived under the modified model are same as those derived under the original Montgomery's (1975) model. This is already established theoretically in the last para of the Section 8.3.3.

8.4 Graphical Presentation used in Curtailed Sampling and Inspection

8.4.1 Diagnostic charts

A control chart is the graphic display of the quality characteristic measured from the sample versus the sample number or time. The sample points are plotted as time oriented sequence on the chart. The pattern of sample points plotted on the chart contains information of diagnostic value to an experienced operator or analyst. Also control charts provide information about the values of the important process parameters and their stability.

Williams, Looney and Peters (1985) introduced the use of curtailed sampling to develop the economic model for np-control chart, where sampling is stopped as soon as m nonconforming units are observed or n units are examined. There is no mention about the graphical display of the information collected during the sampling and inspection by Williams et.al. (1985), while introducing curtailed sampling policy. In view of the above defficiency in graphical presentation while using curtailed sampling, we make an effort to exhibit the outcome of the inspection from time to time. One of the vital information in the use of curtailed sampling is the number of units inspected to make a decision. Hence we propose to plot this information as a time oriented sequence on the chart. It is hoped that the pattern of the sample points plotted on the proposed charts will give some sort of information of a diagnostic value for the analyst.

The proposed charts are not the usual control charts since they do not contain the control limits as such. Hence the proposed charts will be named as "diagnostic charts".

The construction of the proposed charts is explained in detail for the curtailed sampling used by us while developing the economic model for np-control chart with two upper control limits in the Section 7.7 of Chapter VII.

Recall, for continuity, the sampling procedure and the decisions associated with the procedure.

After the production of every k units, inspect the units one by one till one of the following occurs.

(1) m_2 nonconforming units are observed.

(2) $n-m_1+1$ conforming units are observed.

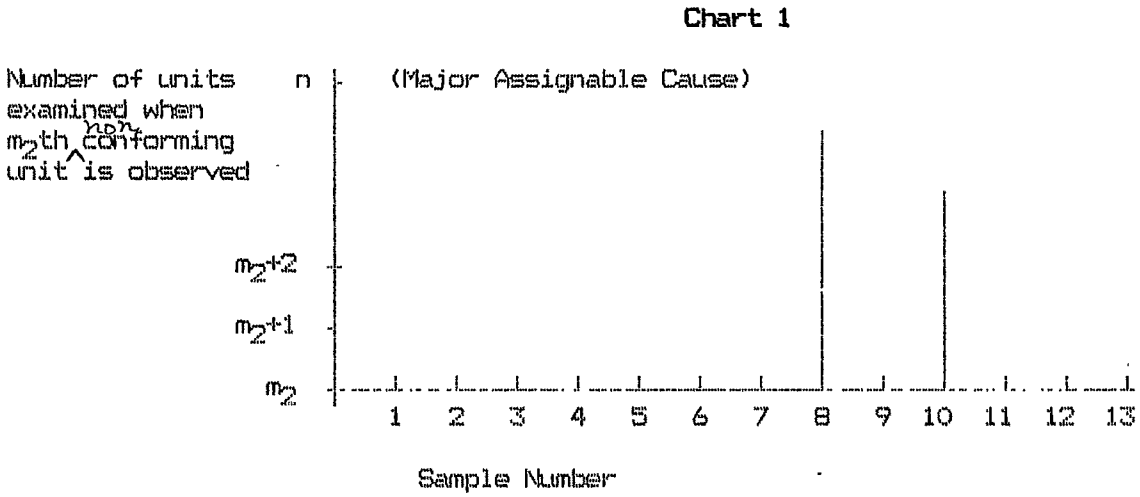
(3) n units are inspected.

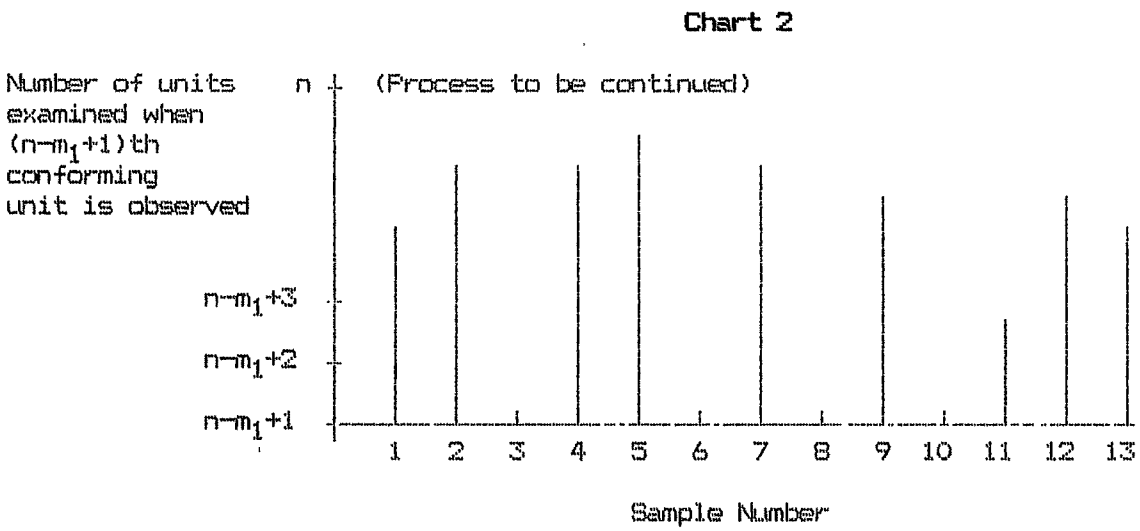
If (1) happens, the process is declared to be out of control and a major action is taken. If (2) happens, the process is declared to be in control and no action is taken. If (3) happens, the process is declared to be out of control and a minor action is taken to repair the process.

We now explain the proposed diagnostic chart for the above curtailed sampling procedure using the Figure 8.1. The important feature is that the proposed diagnostic chart consists of three charts such that the chart j represents the above mentioned possibility (j) (j = 1,2,3), and these charts are plotted one below the other. The sample points are plotted as a time oriented sequence on the chart such that at each point of time, the sample point will fall either on chart 1 or chart 2 or on

chart 3. Chart 1 represents the number of units inspected when the m_2th nonconforming unit is observed. Chart 2 represents the number of units inspected when $(n-m_1+1)$th conforming unit is observed. Chart 3 represents the number of nonconforming units X where X lies in (m_1, m_2-1). When the analyst finds that a sample point is plotted on chart 1, he concludes that the major assignable cause has occurred and hence he suggests a major action to repair the process. When a sample point is plotted on chart 2, it indicates that the process is operating in control and no action is required. When a sample point is plotted on chart 3, it indicate that a minor assignable cause has occurred and a minor action is suggested to repair the process.

The plotting on these charts for a long period of time will help the analyst to know more about the state of the production process. For instance, a large number of plots on chart 3 in the past indicates that most of the times the minor action was required to restore the process.





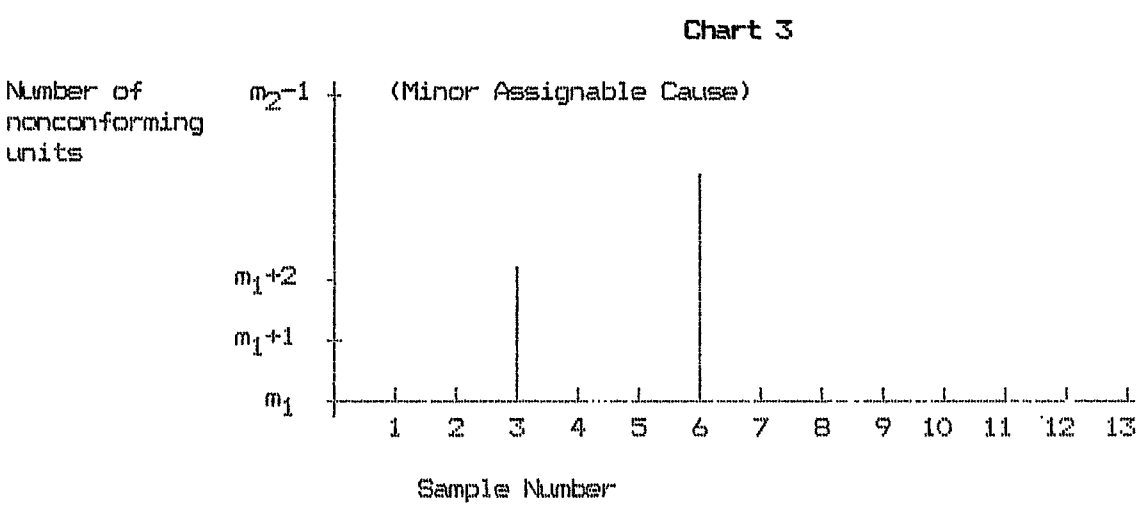


Figure 8.1 191

8.4.2 Graphic Devise for Execution

Another problem arising with the use of curtailed inspection is that quite often the inspector experiences difficulty in its execution. Hence there is a need for some special device which can reduce the complexity and makes the execution simple. One such device is to use a grid (graph) in two dimensional space whose ordinates denote nonconforming units observed and abscissa denote the number of units inspected. A point with coordinates (Y, X) represents the occurrence of X nonconforming units in the inspection of Y units.

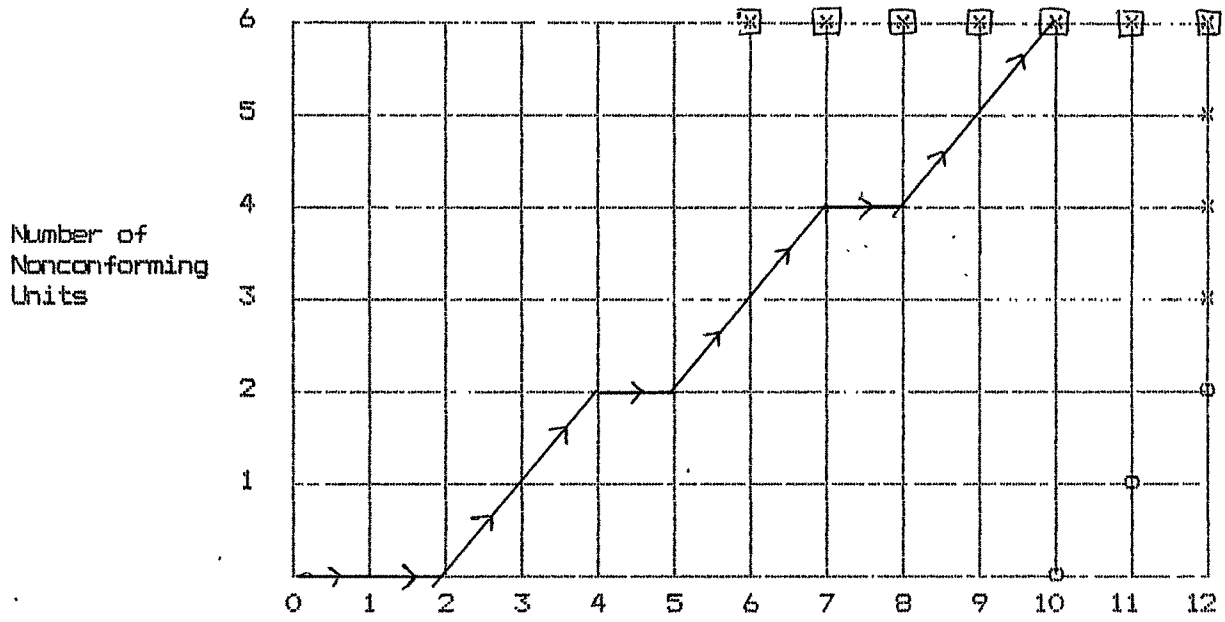
We give the graphical procedure for the execution of the curtailed sampling policy used by us to develop the economic model of the np-control chart with two upper control limits. Firstly, we plot the boundary points of the region in accordance with the statement of the sampling policy given in the Section 8.4.1. These boundary points are classified into three mutically exclusive classes. The boundary points belonging to a class corresponding to possibility (1) are denoted by $\boxed{X}$. The boundary points belonging to a class corresponding to the possibility (2) are denoted o. The boundary points belonging a class corresponding to possibility (3) are denoted by x. As the units are inspected one by one, starting from the origin we plot a unit horizontally to the right when a conforming unit is observed, and we plot a unit diagonally up when a nonconforming unit is observed. This process of inspection of units and of plotting the points on the graph continues till the path joining the points

plotted reaches one of the boundary points. When the path reaches the boundary point [X] , the major action is suggested to restore the process. When the path reaches the boundary point o, no action is taken. When the path reaches the boundary point X, minor action is taken to restore the process.

We illustrate the above procedure for the execution of curtailed inspection when $n = 12$, $m_1 = 3$, $m_2 = 6$. The boundary points are shown in the Figure 8.2. Consider the following sequence of inspection.

g g d d g d d g d d

where g represents a conforming unit and d represents a nonconforming unit. The path of this sequence is given in the Figure 8.2. The path reaches the boundary point [X] whose coordinates are (10,6). Hence we stop the inspection at 10th unit with a decision to take a major action to repair the process.



Curtailed Sampling Policy with

$n = 12,\ m_1 = 3,\ m_2 = 6$

Figure 8.2

```
C LISTING OF CHAPTER VIII
C  PROGRAM ON MINIMIZATION OF ATI
C  FILE NAME IS ESTN1
       READ(*,1) PNOT,PONE
  1    FORMAT(1X,2F10.4)
       WRITE(*,1) PNOT,PONE
       DO 10 I=4,100
       N=I
       WRITE(*,2) N
  2    FORMAT(1X,'N=',I3)
       IUCL=N*PNOT+3.0*SQRT(N*PNOT*(1.0-PNOT))
       M1=IUCL+1
       WRITE(*,3) M1
  3    FORMAT(1X,'M1=',I4)
       NN=N
       MM=M1
       NT=NN-MM+1
       PROB=PONE
       CALL BIN(PROB, MM, NT, PR1,PA1, PI1)
       WRITE(*,5) PR1,PI1
   5   FORMAT(1X,'PR1=',F10.6,'PI1=',F10.6)
       RATIO=PR1/(PONE*PI1)
       WRITE(*,7) RATIO
   7   FORMAT(1X,'RATIO=',F12.6)
       D=N-RATIO
       WRITE(*,9) D
   9   FORMAT(1X,'D=',F10.6)
       IF(ABS(D).LT.0.5) GO TO 12
  10   CONTINUE
  12   NSTOP=I
       STOP
       END
```

```
C   FILE NAME IS MDF
C   PROGRAM FOR E(C) OF MONTGOMERYS MODEL WITH MODIFIED E(C3)
       SUBROUTINE OBJ7 (AKE,NSTAGE,SUMN,A1,A2,A3,A4,ALEMDA,RATE,
     1 PIE,NSTAT,PIN)
       DIMENSION PIN(10),PZ(10),P(10,10),QR(10),ZP(10),BZ(10),ZB(10)
     1 ,B(10,10),BST(10,10),CZ(10),ZC(10),C(10,10),D(10,10),
     1 DST(10,10),ALPHA(10),GAMMA(10),A(10,10),BSTZ(10),T(10,10),ZBST
     1 (10),S(10,10),U(10,10),V(10,10),BB(10,10),AKE(5),T7(10)
       WRITE(*,5)
 5      FORMAT(4X,'COST COEFFICIENTS')
       WRITE(*,1)A1,A2,A3,A4,A5
 1     FORMAT(1X,4F10.4)
       WRITE(*,3)ALEMBDA,RATE,PIE,NSTAT
 3     FORMAT(1X,3F12.4,I3)
       WRITE(*,6)(PIN(I),I=1,NSTAT)
 6      FORMAT(1X,7F8.4)
       SNOT = AKE(1)
       SRNOT = AKE(2)
       REJNOT = AKE(3)
       WRITE(*,2)SNOT,SRNOT,REJNOT
 2      FORMAT(1X,'SAMPLESIZE =',F10.2,'INT SAM RANGE =',F10.2,'REJ
     1 NUM=',F10.2)
```

```
      POWER = ALEMDA*SRNOT/RATE
      PPOWER =-POWER
      PZZ=EXP(PPOWER)
      WRITE(*,7) PZZ
7     FORMAT(1X,F10.4)
      NSTATE = NSTAT-1
      DENO=1.-(1.-PIE)**NSTATE
      MSNOT = SNOT
      DO 10 J=1,NSTATE
      M1 = J+1
      M2 = NSTATE-J
      CALL BIN(PIE,M1,M2,CPR,CPL,PI)
      WRITE(*,8) CPR,CPL,PI,J
8     FORMAT(1X,'CPR=',F10.6,'CPL=',F10.6,'PI=',F10.6,'J=',I2)
10    PZ(J) = PI*(1.-PZZ)/DENO
      DO 600 I = 1,NSTATE
600    ZP(I)=0.
      DO 20 I=1,NSTATE
      DO 20 J=1,NSTATE
      IF(I-J)30,31,32
 30   P(I,J) = PZ(J)/(1.-PZZ)
      GO TO 20
 31   SPZ=0.
      DO 40 KK=1,I
 40   SPZ = SPZ+PZ(KK)
      P(I,J) = SPZ/(1.-PZZ)
      GO TO 20
 32   P(I,J)=0.
 20  CONTINUE
      T(1,1)=PZZ
      DO 12 I =2,NSTAT
      K=I-1
 12   T(1,I) = PZ(K)
      DO 13 J=2,NSTAT
      K=J-1
 13   T(J,1)=ZP(K)
      DO 14 I=2,NSTAT
      K=I-1
      DO 14 J=2,NSTAT
      K1 = J-1
 14   T(I,J) = P(K,K1)
      WRITE(*,15)
 15   FORMAT(1X,'TRANSITION MATRIX')
      WRITE(*,11)((T(I,J),J = 1,NSTAT),I=1,NSTAT)
 11   FORMAT(1X,7F10.6)
      S1 = SNOT
      S2 = SRNOT
      S3 = REJNOT
      CALL PROBR(S1,S2,S3,PIN,NSTAT,QR)
      WRITE(*,301)(QR(I),I = 1,NSTAT)
301   FORMAT(1X,7F10.6)
      DO 60 I=2,NSTAT
      DO 60 J=1,NSTAT
      IF(I-J)61,62,63
 61     S(I,J)=QR(I)*T(1,J)+(1-QR(I))*T(I,J)
      GO TO 60
 62   S(I,J)=QR(I)*T(1,I)+(1-QR(I))*T(I,I)
```

```
      GO TO 60
 63   S(I,J)=QR(I)*T(1,J)
 60   CONTINUE
      DO 326 J=1,NSTAT
326   S(1,J)=T(1,J)
      WRITE(*,302)
302   FORMAT(1X,'MATRIX S(I,J)')
      WRITE(*,303) ((S(I,J),J=1,NSTAT),I=1,NSTAT)
303      FORMAT(1X,7F10.6)
      DO 330 I=1,NSTAT
      DO 330 J=1,NSTAT
      IF(I-J)331,332,331
331      U(I,J)=S(I,J)
      GO TO 330
332      U(I,J)=S(I,J)-1
330      CONTINUE
      WRITE(*,311)
311      FORMAT(1X,'MATRIX U(I,J)')
      WRITE(*,312)((U(I,J),J=1,NSTAT),I=1,NSTAT)
312    FORMAT(1X,7F10.6)
      DO 321 I=1,NSTAT
      DO 321 J=1,NSTATE
321     V(I,J)=U(I,J+1)
      DO 322 I=1,NSTAT
322     V(I,7)=1
      WRITE(*,323)
323     FORMAT(1X,'MATRIX V(I,J)')
      WRITE(*,324) ((V(I,J),J=1,NSTAT),I=1,NSTAT)
324   FORMAT(1X,7F10.6)
      DO 325 I=1,NSTAT
      DO 325 J=1,NSTAT
325      A(I,J)=V(I,J)
      N=NSTAT
      CALL INVRS(A,BB,N)
      WRITE(*,97)
97    FORMAT(1X,'INVERSE MATRIX')
      WRITE(*,98) ((BB(I,J),J=1,NSTAT),I=1,NSTAT)
98    FORMAT(1X,7F10.6)
C   BB(I,J) IS INVERSE OF A(I,J)
      DO 81 J=1,NSTAT
81    ALPHA(J)=BB(NSTAT,J)
      WRITE(*,150)
150     FORMAT(1X,'VECTOR ALPHA')
      WRITE(*,82) (ALPHA(J),J=1,NSTAT)
82    FORMAT(1X,7F10.6)
C COMPUTATION OF GAMMA
      ADALTA=(1.-(1.+POWER)*PZZ)/(POWER*(1.-PZZ))
      WRITE(*,160)
160       FORMAT(1X,'ADALTA')
      WRITE(*,82) ADALTA
      GAMMAZ=ALPHA(1)*PZZ+ALPHA(1)*ADALTA*(1-PZZ)
      WRITE(*,170)
170      FORMAT(1X,'GAMMAZ')
      WRITE(*,82) GAMMAZ
      DO 90 I=2,NSTAT
      I3=I-2
      TERM3=0
```

```
      TERM4=0
      I1=I-1
      I2=I+1
      IF(I1-1) 101,102,101
101      DO 100 J=1,I3
      K=J+1
100    TERM3=TERM3+ALPHA(K)*P(J,I1)
      IF(I1-6) 102,104,102
102    DO 110 K=I,NSTATE
110    TERM4=TERM4+P(I1,K)
104    GAMMA(I1)=ALPHA(I)*P(I1,I1)+(1.-ADALTA)*ALPHA(1)*PZ(I1)+
     1 (1.-ADALTA)*TERM3+ALPHA(I)*TERM4*ADALTA
      WRITE(*,82)GAMMA(I1)
90     CONTINUE
      EC1=(A1+A2*SNOT)/SRNOT
      TERM5=0
      DO 120 I=1,NSTAT
120    TERM5=TERM5+QR(I)*ALPHA(I)
      EC2=A3*TERM5/SRNOT
      TERM6=0
      DO 151 I=1,NSTAT
151    TERM6=TERM6+ALPHA(I)*T(I,I)*(A4*SNOT*PIN(I)+A5*(SRNOT-SNOT)*
     1 PIN(I))
      DO 155  I=1,NSTAT
      T7(I)=0
      I2=I+1
      DO  161 J=I2,NSTAT
161    T7(I)=T7(I)+ALPHA(I)*T(I,J)*(A4*SNOT*PIN(I)+A5*(ADALTA*SRNOT
     1 -SNOT)*PIN(I)+A5*(1-ADALTA)*SRNOT*PIN(J))
155    CONTINUE
      TERM8=0
      DO 165 I=1,NSTAT
165    TERM8=TERM8+T7(I)
      EC3=(TERM6+TERM8)/SRNOT
      TC=EC1+EC2+EC3
      SUMN=TC
      WRITE(*,140)TC,EC1,EC2,EC3
140    FORMAT(1X,'TOTAL COST=',E18.8,'EC1=',E18.8,'EC2=',E18.8,
     1 'EC3=',E18.8)
      RETURN
      END
```