

Chapter I

Introduction

1.1 The Control Chart

1.1.1 Statistical Process Control (SPC)

If a product is to meet the consumer's satisfaction, it should be produced by a process which is stable. That is, the process must be capable of operating with a little variability around the nominal dimension of the product quality characteristic. All the individuals involved with the process including operators, engineers, quality assurance personnel and management must continuously seek to improve the process performance and reduce the variability of process parameters. Statistical process control (SPC) is a collection of tools useful for achieving this objective. The seven major tools of the statistical process control are as follows.

- (1) Histogram
- (2) Check Sheet
- (3) Pareto Chart
- (4) Cause and effect diagram
- (5) Defect concentration diagram
- (6) Scatter diagram
- (7) Control chart

Of these seven tools, the control chart is the simplest and the most widely used on-line statistical process control technique. It was developed in 1920s by Dr. W. A. Shewhart.

1.1.2 Shewhart's Theory of Variability

In any production process, how well designed or carefully maintained it is, a certain amount of natural variability always exists in the output of the process. This natural variability is generally small. It is the cumulative effect of many small, unavoidable causes known as chance causes. A process that is operating with only chance causes of variation is said to be in control.

The other kind of variability may occasionally be present in the output of the process. This variability usually arises from three sources; improperly adjusted machines, operator error, defective raw materials. We refer to these sources of variability as assignable causes. The variability due to assignable causes is generally large as compared to the variability due to chance causes. A process that is operating in the presence of assignable causes is said to be out of control.

The production process starts in the in-control state, producing acceptable product for a relatively long period of time. Occasionally the assignable causes will occur at random resulting in a shift to the out-of-control state. The objective of the statistical process control is to quickly detect the occurrence of assignable causes. The control chart is an on-line process control technique widely used for this purpose.

1.1.3 How the Control Chart Works ?

A typical control chart is shown in the Figure 1.1. It is a graphical display of the quality characteristic itself (or some function) that has been measured from the sample versus the sample number.

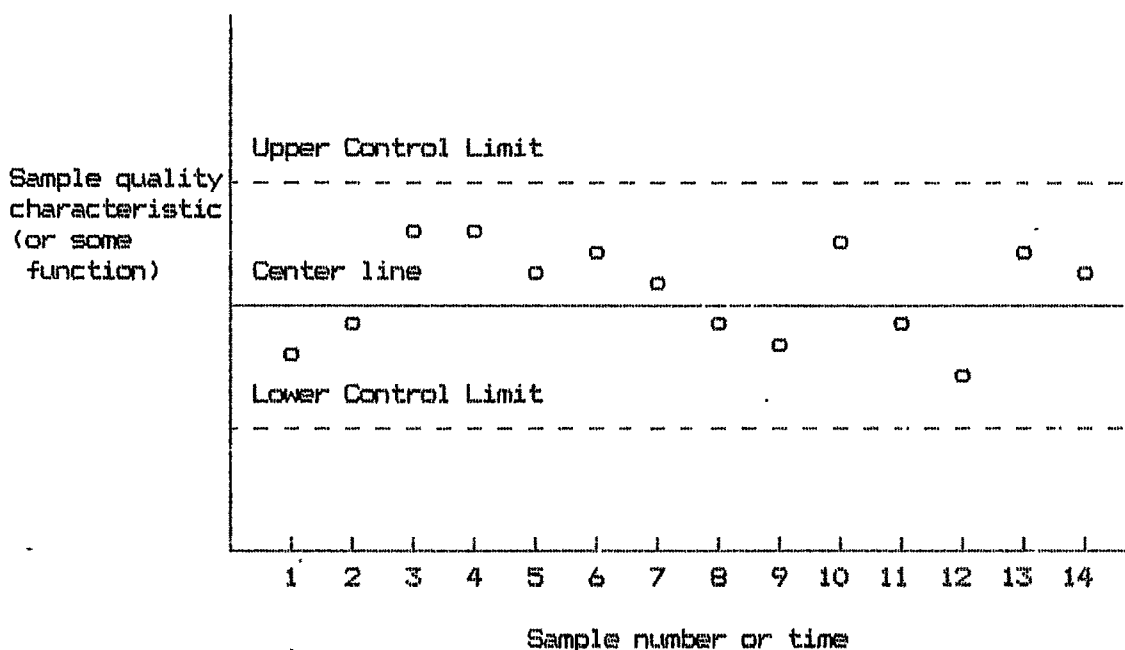


Figure 1.1

This chart contains a center line that represents average value of the quality characteristic corresponding to the in-control state. Two other horizontal lines are called upper control limit (UCL) and lower control limit (LCL). These control limits are chosen so that if the process is in control nearly all the sample points will fall between them. As long as points plot within the control limits the process is assumed to be in control. During this period no action is necessary since the

process is subject to unavoidable variation. A point that plots outside the control limits is interpreted as the evidence that the process is out of control and the investigative and corrective action is required to find and eliminate the assignable causes.

1.2 Economic Design of Control Chart

The use of the control charts requires that the engineer or the analyst selects the sample size, the interval between the samples and the control limits for the chart. Selection of these three parameters is called the design of the control chart. The traditional practice was to choose the parameters like the size of the sample and the interval between the samples from convenience point of view. Later on the control charts were designed with respect to statistical criteria. This involves selecting the sample size and the control limits such that the power of the chart to detect the shift in the process and the probability of type I error are equal to specified values. The frequency of sampling is rarely treated analytically.

The design of the control chart has the economic consequences. Three costs namely, (i) the cost of sampling and inspection, (ii) the cost associated with investigating and correcting the assignable causes, and (iii) the cost of producing nonconforming units are all affected by the control chart parameters. Therefore, it is logical to consider the design of a control chart from an economic viewpoint. In recent years

considerable research has been developed on the economic design of control charts.

1.3 The Formulation of the Cost Model

1.3.1 The Production Process

To formulate the cost model for the economic design of a control chart, it is necessary to make certain assumptions about the behavior of the production process. The process is assumed to be characterized by a single in-control state. If the process has a measurable quality characteristic then the in-control state would correspond to the mean of the quality characteristic when no assignable causes are present. When the quality characteristic is an attribute, the in-control state would be represented by a fraction nonconforming produced by the process when no assignable causes are present. The process may have s (≥ 1) out-of-control states. Quite often it is possible to assume that the assignable causes occur according to some probability law. The occurrence of these causes results into a shift in the process constants such as process average, process standard deviation, proportion of nonconforming units etc. being controlled. The nature in which the shift occurs is called the process failure mechanism. A few mechanisms are considered by the various authors and are described in Section 1.4.2.

Among these mechanisms the more popular is the Poisson Process. According to Poisson Process the length of the time the process remains in the in-control state, given that it starts in the in-control state, is an exponential random variable. This

assumption allows a considerable mathematical simplification and in some situations results in a Markov Chain model structure.

It is also assumed that the process is not self-correcting. Once a transition to the out-of-control state has occurred, external intervention is required to restore the process to the in-control state. In most of the models transitions between the different out-of-control states are allowed, provided the transitions are towards further quality deterioration.

1.3.2 The Cost Coefficients

Three categories of costs are customarily considered in the economic design of the control charts. These are the cost of sampling and inspection, the cost associated with the investigation and correction of the assignable causes, and the cost of producing nonconforming items.

The cost of sampling and inspection includes the expenses of the inspectors' and technicians' salaries, the cost of necessary test equipments, and in case of destructive testing the unit cost of the items sampled. Usually the cost of sampling and inspection is assumed to consist of both fixed and variable components a_1 and a_2 respectively. Hence the total cost of sampling and inspection is $(a_1 + a_2n)$ where n is the sample size.

The cost of investigating and possibly correcting the process following an out of control signal has been treated in several ways. Some authors have suggested that the cost of investigating the false alarms will differ from the cost of correcting the assignable causes. Consequently these two

situations must be represented in the cost model by different cost coefficients. Furthermore, the cost of repairing the process would depend on the type of the assignable cause present. Thus in the economic models having s out-of-control states, $(s+1)$ cost coefficients are necessary to model the search and repair procedures associated with the out of control signal. Some authors have argued that this level of modeling detail is unnecessary because in most of the cases small shifts are difficult to find but easy to correct whereas large shifts are easy to find but difficult to correct. Hence one can use a single cost coefficient, a_3 , to represent the average cost of investigating and possibly correcting the assignable causes.

The cost associated with producing the nonconforming items consists of the cost of scrap for internal failures, replacement or repair cost for units covered by warranties in case of external failures. With external failures, there may also be secondary effects. The consumer's dissatisfaction with the product may cause an alteration in the future purchases of the products manufactured by the company. Most authors represent these costs by a single average cost coefficient, a_4 , expressed on either per item or per unit time basis. However, it is more realistic to assign a higher cost to a nonconforming item that goes undetected to the customer than to the one which is detected during sampling.

1.3.3 The Cost Model

The cost models are generally formulated using a total cost

function which expresses the relationship between the design variables of the chart and the three types of costs discussed above. A production cycle begins with the process in the in-control state, and continues with the process monitoring via control chart, until the chart gives an out of control signal. Following an adjustment in which the process is returned to the in-control state, a new production cycle begins. Thus the production cycles may be thought as a series of independent cycles over time. Let $E(T)$ be the expected length of the production cycle, and let $E(C)$ be expected total cost incurred during the production cycle. Then the expected cost per unit time is

$$E(A) = \frac{E(C)}{E(T)} \quad \dots(1.3.1)$$

Optimization techniques are then applied to the expression (1.3.1) to determine the optimal values of the design variables of the chart which minimize $E(A)$. Some authors have elected to replace $E(T)$ in (1.3.1) by the expected number of units produced during the production cycle. The resulting expected cost is then expressed on a per item rather than per time unit base. In this case the optimization techniques are to be applied to

$$E(B) = \frac{E(C)}{E(U)} \quad \dots(1.3.2)$$

where $E(U)$ is the expected number of units produced during the production cycle.

In the expression (1.3.1) the variables C and T are dependent random variables and yet the expected value of their

ratio $E(\frac{C}{T})$ is represented by the ratio of their expectations

$\frac{E(C)}{E(T)}$. It is well known that the expected value of the ratio

is not equal to the ratio of expected values (even for independent random variables). However, the above formulation of the cost model represents a particular type of stochastic process called the renewal reward process. The stochastic process of this type has the property that their average time cost is given by the ratio of the expected cost per cycle to the expected length of the cycle as given by (1.3.1). This sort of justification is due to Ross(1970)

1.4 A Short History of the Work done on Economic Design of Control Charts.

1.4.1 Early Work and Semieconomic Designs

A fundamental paper in the area of cost modeling of the quality control systems was published by Girshick and Rubin (1952). They were the first researchers to propose the expected cost (or income) per unit time (1.3.1) criteria. Their results are of significant theoretical interest but had little practical application.

Economic design of Shewhart control charts was investigated by several early researchers. Most of their work can be classified as semieconomic design procedures, in that either the proposed cost model did not consider all the relevant costs

described earlier or no optimization techniques were applied to the cost function. Weiler (1952) obtained the optimum sample size for \bar{x} -control chart that minimizes the total amount of inspection required to detect the specified shift. This minimization does not take into consideration the various costs explained earlier. Cowden (1957) considered all the three types of costs but offered no optimization technique. Barish and Maurser (1963) employed Monte Carlo simulation to investigate the \bar{x} -control chart design. In addition to the expected cost per unit, they also evaluated the number of process adjustments required and the fraction nonconforming produced. But they did not employ any optimization method.

1.4.2 Economic Design of Usual Control Charts

(A) Process Failure Mechanism : Exponential

Duncan (1956) was the first to develop the fully economic model for a Shewhart type control chart. His paper was the stimulus for much of the subsequent research in this area. Duncan assumed that the production process starts in an in-control state μ_0 and that the single assignable cause of magnitude δ occurring at random results in a shift in the mean from μ_0 to either $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$. He further assumed that the time until the occurrence of the assignable cause is an exponential random variable with mean $1/\lambda$.

Later on two distinctly different models were developed for the process subject to multiple assignable causes : one by Duncan (1971) himself, and the other by Knappenberger and Grandage

(1969). There are advantages and the drawbacks in both these multiple assignable cause models. Naturally one should think of constructing a model which will eliminate the drawbacks and take care of the advantages of the existing models.

Chiu (1975, 1976) formulated the single and the multiple assignable cost models for np-control chart using Duncan's (1956,1971) models for \bar{x} -control charts. Montgomery, Heikes and Mance (1975) formulated the multiple assignable cause model for np-control chart using Knappenberger and Grandage's (1969) model for \bar{x} -control chart.

Gibra (1971) proposed a model for \bar{x} -chart similar to Duncan's (1956) model, where he introduced the concept of worst cycle quality level (WCQL). The WCQL is an upper limit on the expected number of nonconforming units produced during the out of control production.

Gibra (1967) developed the economic model for \bar{x} -control chart for a production process in which the mean exhibits a linear trend over time. This model is suitable for the process involving tool wear.

Gibra (1978 , 1981) developed the single and the multiple assignable cause models for np-control charts. The general modeling approach is similar to Duncan's (1956) model.

All the economic models mentioned above assume that the process failure mechanism (the time that the process remains in the in-control state) is an exponential random variable.

(B) Process Failure Mechanism : other than Exponential

We now list those economic models where the process failure mechanism is represented in a different way.

Baker (1971) developed two discrete time models. The first model uses the geometric distribution with range space $0,1,2,\dots$ to model the number of periods (the sampling intervals) the process remains in the in-control state. His second model allows the use of any discrete probability distribution to model the number of periods the process remains in the in-control state.

Banerjee and Rahim (1988) generalised Bakers (1971) model to treat the case of a Weibull process failure mechanism. They also incorporated the variable-sampling-interval strategy in the model. This model seems appropriate for many processes where failure mechanism is driven by fatigue consideration.

Montgomery and Heikes (1976) investigated the use of geometric, poisson and logarithmic series distributions to model the time duration in control. They have shown that the choice of the proper process failure mechanism is an important aspect of the economic design and the misspecification of this results in significant cost penalties.

(C) Three Interesting Models

(1) The Lorenzen and Vance Model

Lorenzen and Vances (1986) developed the economic model that uses a somewhat different modeling approach. This model is developed in terms of the in-control and the out-of-control average run lengths, rather than the α , β risks. This model has

the advantage that the control charts for variables as well as the control charts for attributes can be incorporated in the model simply by changing the probability distribution that generates the ARLs.

(2) The Tagaras and Lee Model

Tagaras and Lee (1988) proposed to use a control chart with different control limits for different assignable causes. The chart not only detects the shift in the process but also suggests which assignable cause has occurred. This is a new modeling approach.

In all the economic Models discussed in the part (A) as well as the part (B), the control chart only detects the shift in the process. Once the out of control signal is given by the chart, one has to search for the assignable cause in effect.

The search for the assignable cause in effect is very expensive and time consuming when there exist multiple assignable causes. Tagaras and Lee have shown that the control chart proposed by them always dominates the traditional single response control chart from cost point of view.

(3) The Williams, Looney and Peters Model

Williams, Looney and Peters (1985) introduced the use of curtailed sampling policy to develop the economic model. They have shown that curtailed sampling is no more expensive than the traditional complete sampling. Also one would certainly prefer curtailed sampling policy in case of destructive inspection.

1.4.3 Economic Design of other Control Charts

(A) Cumulative Sum Control Charts

The economic design of CUSUM charts was firstly investigated by Taylor (1968). Goel and Wu (1973) and Chiu (1974) developed the economic design of CUSUM Chart using the models similar to Duncan's (1956) model.

(B) Control Charts with Warning Limits

The economic design of control charts with warning limits have been studied by Tiago de Oliveira and Littaurr (1966), Gordon and Weindling (1975) and Chiu and Cheung (1977). Chiu and Cheung's (1977) model is similar in structure to Duncan's conventional model.

(C) Multivariate Control Chart

Montgomery and Klatt (1972a, 1972b) developed the economic design of Hotelling T^2 -control chart. The cost model used by them is the single assignable cause version of Knappenberger and Grandage's (1969) model.

(D) Joint \bar{x} and R Chart

Saniga (1978) presented a model for the joint economic design of \bar{x} and R control chart. His general modeling approach is similar to Knappenberger and Grandage's (1969) model.

1.5 The Criticism on the Economic Design of Control Charts

Woodall (1986, 1987) has criticized the economic design of control charts. He mentioned that in many economic designs the type I error of the control chart is considerably higher than it

would be in the statistical design. This will lead to more false alarms which is undesirable. Furthermore, if the type I error is high, then this would lead to excessive process adjustments which often increases the process variability. Woodall also notes that the economic models assign a cost to a defective item and this is counter to Deming's philosophy that these costs can not be measured and that consumer's satisfaction is necessary for staying in business.

These points are easily overcome. An economic design should always be checked for statistical properties such as type I and type II errors, the average run lengths and so on. If any of these properties are at undesirable level, this may indicate that inappropriate costs have been assigned, or that a constrained solution is necessary.

1.6 The Scheme of the Work presented in the Thesis and Achievements

In the Section 1.4 we have discussed the history of the work done on the economic design of control charts. While discussing the various aspects of the research work done on the economic design of control charts, we have also mentioned the situations where there is a scope of improvements and further research work.

The improvements in the economic models may be achieved in two dimensions.

(a) The economic model should be constructed in such a way that it does not involve unrealistic assumptions. In many situations, certain assumptions make the mathematical structure of the model

simple. But because of these unrealistic assumptions the application of the model becomes restricted.

(b) Another way of improving the economic model is to make it cheaper from cost point of view.

We have achieved the improvements in the economic models along both the dimensions.

We also study the following :-

- (i) The effects of increasing the power of the control chart on various expected costs,
- (ii) the development of the economic model for the multivariate control chart when several related variables are of interest,
- (iii) the construction of the economic model for the control chart with two sets of control limits for minor and major assignable causes,
- (iv) the graphic representation when curtailment in the inspection is proposed while maintaining the control charts.

We now give the scheme of the work presented in this thesis and the main results achieved chapterwise.

In Chapter II, the economic models for np-control charts are developed when the production process is affected by a single assignable cause and when it is affected by several assignable causes. The basis for the construction of these models is as follows.

We have mentioned in the Section 1.4.2, about two distinctly different modeling approaches to the multiple assignable cause

problem. One has been developed by Duncan (1971) and the other is developed by Knappenberger and Grandage (1969). Both these models differ considerably from each other. Both the models involve some unrealistic assumptions. At the same time there are some good points (advantages) involved in both models. We have constructed the model in such a way that our model does not involve any of the unrealistic assumptions involved in both the models. Also our model takes care of the advantages of both the models. Because of this type of improvements, the model constructed by us is likely to be more realistic and hence more applicable than the two models mentioned above.

The optimization technique used by us throughout our research work, is the direct search method given by Hooke and Jeeves (1961). We have to make some appropriate changes in Hooke-Jeeves method to make it suitable for optimizing the cost functions under our study. The changes and modifications made by us in the Hooke-Jeeves direct search method are also explained in this chapter.

In Chapter III, the economic design is developed for the control charts when the characteristic under study is measurable on continuous scale. This is done under two situations, (i) the process standard deviation σ known and (ii) the process standard deviation σ unknown. In the first situation \bar{X} -control chart is used to develop the expected cost model. In the second situation T^2 -control chart is used to develop the expected cost model. In both the situations the cost structure of the model is similar to the single assignable cause model developed by us in Chapter II.

We have observed that the absence of knowledge of σ leads to some cost penalty.

Furthermore, a comparison is made between the performance of \bar{x} -control chart and np-control chart, for the given values of cost coefficients and systems parameters. It is observed that \bar{x} -control chart leads to smaller expected cost as compared to np-control chart. While comparing the performance of these two charts, we have taken into consideration the fact that the cost of sampling and inspection is comparatively higher for measurable characteristic.

In Chapter IV, the economic design of np-control chart is developed using (i) fully-curtailed sampling policy and (ii) double sampling policy in Knappenberger and Brandages' (1969) model. Theoretical comparisons are made among the performance of the complete, the semi-curtailed and the fully-curtailed sampling policies. It is proved theoretically that the fully curtailed sampling is no more expensive than the semi-curtailed sampling, and the semi-curtailed sampling is no more expensive than the uncurtailed (complete) sampling.

Next, the numerical comparisons are made between the performance of double sampling and single sampling policies for the various cost coefficients and systems parameters. It is observed that the improvement in cost due to double sampling over complete single sampling is considerably large as compared to the improvement in cost due to both types of curtailed single sampling over complete single sampling. This ultimately leads to

a conclusion that the double sampling policy is better than the single sampling policy (complete as well as curtailed) from the cost point of view.

This type of cost improvement is also achieved for the economic model for np-control chart developed by us in Chapter II. It is given in a separate Section of this chapter, namely, chapter IV.

Woodall (1986) mentioned that the economic design balances the cost of poor quality against the cost of sampling and the cost of investigating and correcting the process. The statistical performance of economically designed control chart is ignored. In view of Woodall's comments it is decided to study the effect of using a more powerful decision rule on various expected costs occurring in Knappenberger and Grandage's (1969) model. This is studied in Chapter V. It is observed that the more powerful decision rule leads to higher expected total cost. This shows that if one wants to use a more powerful decision rule, he should be prepared to pay higher cost. Of course, the advantage in using a more powerful decision rule is that it detects the shift quickly, when the shift occurs.

Quality control problems in which several related variables are of interest are called multivariate quality control problems. This subject is particularly important today, as automatic inspection procedures make it relatively easy to measure many quality characteristics on each unit of the product manufactured. Taking into consideration the increasing applicability of multivariate quality control procedures, we develop the economic

design of Hotelling T^2 -control chart in Chapter VI. The cost model used is an adaptation of the single assignable cause model developed by us in the Chapter II.

One difficulty encountered with the use of T^2 -control chart is the practical interpretation of an out of control signal. Specifically, which subset of the p variables is responsible for the signal is an important question to be studied whenever someone controls the related variables simultaneously. The solution of this question needs further investigation. We have discussed a method for obtaining the solution in the present chapter, namely, Chapter VI.

In Chapter VII, we propose an np-control chart with two upper control limits. The lower control limit is assumed to be zero. The advantage of this chart is that the chart not only detects the shift in the process but also suggests whether a minor assignable cause has occurred or a major assignable cause has occurred. We construct the economic model for the proposed np-control chart. The modeling approach for np-control chart is similar to the one given by Tagaras and Lee (1988) for \bar{x} -control chart.

It is observed that the proposed control chart leads to an improvement in the cost as compared to the traditional single upper control limit chart. The further improvement in cost is achieved by using curtailed sampling policy in place of complete sampling policy in the cost model developed.

In the Chapter VIII we study several miscellaneous problems.

The first problem deals with the semieconomic design of np-control chart. We find the optimum sample size that minimizes the total amount of inspection to detect a specified shift. If the shift is from an in-control state p_0 to an out-of-control state p_1 ($p_1 > p_0$), it is observed that the optimum value of the sample size decreases as p_1 increases.

The second problem deals with some corrections in the paper by Knappenberger and Grandage (1969). In this paper the process failure mechanism is represented by an exponential distribution. This results in a Markov Chain cost model structure. Knappenberger and Grandage (1969) described the procedure for solving the equation $q'B = q'$, where B represents the transition probability matrix. In the development of their solution the matrix B^* is involved which is a function of matrix B. The expressions for B^* given by them are not correct. They are not in accordance with the procedure described by them. We have corrected the expressions for B^* . The numerical values of B^* are also calculated correctly.

The third problem deals with some modifications in the economic model for np-control charts developed by Montgomery, Heikes and Mance (1975). Montgomery et.al. (1975) have retained the cost structure of the model as it is, while developing the economic design for np-control charts using Knappenberger and Grandage's (1969) model for \bar{x} -control charts. As we have mentioned earlier, there is a scope of significant improvements in Knappenberger and Grandage's (1969) model. These improvements are already discussed and accordingly the model is constructed in

Chapter II. However if someone wants to use Knappenberger and Grandage's model as it is, we feel that the following modification is essential. We, therefore, modify Montgomery's (1975) model at least in one respect. We assign a higher cost to a nonconforming unit that goes undetected to the customer than to the one which is detected during sampling and inspection. This modification will make the model realistic at least in this respect and hence it will increase its applicability.

Lastly we give some graphical representation useful in curtailed sampling and inspection.

As we know, control chart is an on-line process control technique. The information given by the control chart as a time oriented sequence is very important for the analyst. While introducing curtailed sampling in the economic design of np-control charts, Williams, Looney and Peters (1985) ignored the graphical representation of the outcome of curtailed inspection. We feel that there is a need of graphical representation of this vital information while maintaining the control charts. We, therefore, suggest some graphical procedure for this purpose. This type of graphical procedure is introduced for the first time in the literature of control charts. This is done in the last Section of Chapter VIII. We also give the graphic device to make the execution of curtailed sampling easy. Both the points stated above regarding the graphical presentation are done for the curtailed sampling given by us for the np-control chart with two sets of control limits.

1.7 Some portion of the thesis is related to our papers either published or presented at the conferences whose details are as given below.

- [1] "A correction Note on Minimum Cost Quality Control Tests" (1988) (with D. K. Shah and A. G. Phatak), IIE Transactions, Vol.20, No. 2, 231-233. [Section 8.2].

- [2] "Use of Fully-Curtailed Sampling Plans in the Economic Design of np-control Charts" (1989) (with D. K. Shah and A. G. Phatak), presented at the First Asian Congress on Quality and Reliability at New Delhi. This paper is published in a book entitled Quality for Progress and Development, Wiley Eastern Limited New Delhi, 524-531. [Section 4.2].

- [3] "Use of Double Sampling Plans in the Economic Design of np-Control Charts" (1990) (with D. K. Shah and A. G. Phatak), presented at National Convention (1990) of National Institute for Quality and Reliability (NIQR) at Bangalore. This paper is published in a book entitled Technical Papers for NIQR National Convention 1990, 1.65-1.69. [Section 4.3].

- [4] "The Effects of Using a More Powerful Decision Rule on the Economic Design of \bar{x} -control Charts" (1990), (with A. G. Phatak), Sankhyā Vol. 52, Sr. B, Part 3, 361-367. [Chapter 5, excluding Section 5.5].

- [5] "Economic Design of np-Control Charts" (1990), Journal of the Indian Statistical Association, Vol. 28, 81-85. [Sections 2.2, 2.3].
- [6] "Some Modifications in the Expected Cost Model of np-Control Charts" (with A. B. Phatak), accepted for publication in Gujarat Statistical Review, [Section 8.3].