CHAPTER III

Economic Design of Control Charts for Variables with Known and Unknown Sigma

3.1 In this chapter the economic design of control charts for variables is developed for a process subject to a single assignable cause. The cost model used is an adaptation of the cost model developed for np-control chart in chapter II.

Duncan (1956, 1971) and Knappenberger and Grandage (1969) developed the economic design of $\bar{x}$-control charts under the assumption that the process standard deviation σ is known. However, if σ is unknown, then the case is treated differently. In this situation one may use T^2-control chart to monitor the production process.

In Section 3.2 the economic design of $\bar{x}$-control charts is developed under the assumption that σ is known. In section 3.3 economic design of T^2-control chart is developed under the assumption that σ is unknown. Of course, the normality of the variable under study is assumed through out.

3.2 Economic Design of $\bar{x}$-Control Charts under σ Known

3.2.1 The Production Process and the Sampling Scheme

The production process starts in an in-control state in which the process mean is μ_o. A single assignable cause produces a shift of the process mean from μ_o to $\mu_o + \delta\sigma$. Thus there is only one out-of-control state in which the process mean is $\mu_o+\delta\sigma$.

The assignable cause is assumed to occur according to a Poisson process with an intensity of λ occurrences per operating hour. Hence the time until the process remains in the in-control state is an exponential random variable with mean $1/\lambda$ operating hours. Once the process is in the out-of-control state it stays there until the shift in the process is detected by the control chart. The process parameters μ_0, δ and σ are assumed to be known.

This process is monitored by an $\bar{x}$-control chart with central line μ_0 and the upper and lower control limits $\mu_0 \pm L\sigma/\sqrt{n}$. After every production of k units, n units are sampled and inspected. The sample mean $\bar{x}$ is calculated. If the value of $\bar{x}$ falls within the control limits, the process is declared to be in control and the production continues. If the value of $\bar{x}$ falls outside the control limits, the process is declared to be out of control. The production at this stage may or may not be stopped and a search for the assignable cause is undertaken.

The design variables n, k, L are to be determined such that the expected cost per unit of the product during the production cycle is minimized.

3.2.2 The Probability of Type I Error and the Power of $\bar{x}$-Control Chart

When the assignable cause occurs, the probability that it will be detected on any subsequent sample is

$$q_1 = P(\ \bar{x}<\mu_0 - L\sigma/\sqrt{n}\ |\ \mu=\mu_0+\delta\sigma\ ) + P(\ \bar{x}>\mu_0 + L\sigma/\sqrt{n}\ |\ \mu=\mu_0+\delta\sigma\ )$$

$$= \Phi\ (-L-\delta\sqrt{n}) + 1 - \Phi(L-\delta\sqrt{n}) \qquad ...(3.2.1)$$

where $\Phi(y)$ is the distribution function of the standard normal variate Y,

$$\text{i.e. } \Phi(y) = \int_{-\infty}^{y} (2\pi)^{-1/2} \exp(-z^2/2)\, dz.$$

The quantity q_1 is the power of the control chart.

Next, the probability of a false alarm is

$$q_0 = P(\bar{x} < \mu_0 - L\sigma/\sqrt{n} \mid \mu = \mu_0) + P(\bar{x} > \mu_0 + L\sigma/\sqrt{n} \mid \mu = \mu_0)$$

$$= \Phi(-L) + 1 - \Phi(L)$$

$$= 2[1-\Phi(L)] \qquad \ldots(3.2.2)$$

The quantity q_0 is the probability of type I error.

3.2.3 The Proportions of Nonconforming Units

A unit is considered to be nonconforming if its measurement falls outside the specification limits (u_1, u_2). Let $p_i(i=0,1)$ be the proportion of nonconforming units when the process is in state $\mu_i(i=0,1)$.

Then,

$$p_i = P(X < u_1 \mid \mu = \mu_i) + P(X > u_2 \mid \mu = \mu_i)$$

Thus,

$$p_0 = 1 - \Phi\left[\frac{u_2-\mu_0}{\sigma}\right] + \Phi\left[\frac{u_1-\mu_0}{\sigma}\right] \qquad \ldots(3.2.3a)$$

$$p_1 = 1 - \Phi\left[\frac{u_2-\mu_0}{\sigma} - \delta\right] + \Phi\left[\frac{u_1-\mu_0}{\sigma} - \delta\right] \qquad \ldots(3.2.3b)$$

It may be noted that both the proportions p_0 and p_1 are known constants since they are functions of known constants.

3.2.4 The Expected Cost Model.

We compute the expected cost per unit of controlling the process during the production cycle. The cost model used is an adaptation of the cost model for np-control chart developed in chapter II.

Recall the definitions of the following terms well explained there

(i) C_1, C_2, C_3 (iv) a_1, a_2

(ii) N, $N(O)$, B_o, θ (v) $a_{3,1}$, $a_{3,2}$

(iii) D, S, Δ (vi) $a_{4,1}$, $a_{4,2}$.

Then using the derivations of that section the expression for the expected total cost per unit (ECPU) of controlling the process is

$$\frac{(a_1+a_2n)[\theta/(1-\theta)+1/q_1]+a_{3,1}q_o\theta/(1-\theta)+a_{3,2}+a_{4,1}S+a_{4,2}(D-S)}{[\theta/(1-\theta)+1/q_1]k} \quad \text{...(3.2.4)}$$

For ready reference and continuity the expressions for D, S, Δ and θ are reproduced here.

$$D = [k\theta/(1-\theta)+\Delta k]p_o + [k/q_1-\Delta k]p_1 \quad \text{...(3.2.5)}$$

$$S = np_o\theta/(1-\theta) + np_1/q_1 \quad \text{...(3.2.6)}$$

$$\Delta = \frac{1-(1+\lambda k/R)\theta}{(1-\theta)\lambda k/R}$$

$$\theta = \exp(-\lambda k/R)$$

It should be noted that the expression (2.3.21), obtained after substitutions, for ECPU for np-control chart and the expression (3.2.4) given above look alike but are different in the sense that the expressions required for q_o and q_1 are different. In the evaluation of (3.2.4) one has to use (3.2.2) and (3.2.1) for q_o and q_1 respectively, whereas while evaluating (2.3.21) one has to use (2.3.10) and (2.3.4) for q_o and q_1 respectively. Minimization of the objective function ECPU with appropriate substitution of q_o and q_1 gives the optimal values of the design variables of $\bar{X}$-control chart or np-control chart as the case be. The reason for mentioning this point elaborately is explained in the next few lines.

The practitioners of the control charts quite often switch over from the control charts for the proportion of nonconforming units to the control charts for variables since the general feeling is that one requires a smaller number of units for inspection for the control chart for variables. However, one should not forget the point that the cost of inspection for variables is comparatively higher than the cost of merely classifying a unit as conforming-nonconforming unit. Hence it is thought to compare the performance of the two types of charts from the cost point of view. The similarity and the difference in the cost structure of these charts is useful to have a comparative study of these charts.

In the next section a numerical comparision is made between the performance of $\bar{X}$-control chart and np-control chart from the cost point of view.

3.2.5 Numerical Example

Let $\mu_o=0$, $\sigma=1$, $u_1=-2.58$, $u_2=2.58$, $\delta=1.3$.

Thereby $p_o=0.01(=1\%)$ and $p_1=0.10(=10\%)$.

Thus the problem of controlling the process average at zero (i.e. $\mu=\mu_o=0$) and detecting the shift of 1.3 on the process average (i.e. $\mu=\mu_1=\mu_o+\delta\sigma=1.3$) by $\bar{x}$-chart is comparable with controlling the process by np-chart for in-control state p_o at 1% and for out-of-control state p_1 at 10%. We consider the same set of cost coefficients and the set of systems parameters of the numerical example of np-control chat of Section 2.3.5 of Chapter II, except one change for the cost of sampling and inspection per unit. Four different values for a_2 are considered in addition to a_2=\$1. The maximum cost of a_2 considered for inspection by variables is three times the cost of inspection by attributes.

Thus taking

a_1 = \$ 10, a_2 = (\$1, \$1.5, \$2, \$2.5, \$3 one at a time),

$a_{3,1}$ = \$ 100, $a_{3,2}$ = \$ 100, $a_{4,1}$ = \$ 10, $a_{4,2}$ = \$ 15,

λ = 1, R = 1000.

the objective function ECPU given by (3.2.4) is minimized using the direct search technique explained in Section 2.3.4 of Chapter II. Since in this case only two design variables n and k are discrete the precaution for the proper step size and for the reduction factor is required only for two variables. The listing of the FORTRAN program developed for calculating the objective function ECPU using (3.2.2) and (3.2.1) for q_o and q_1 is given at the end of this chapter.

The optimal values of the design variables n, k, L along with the other findings are listed in the Table 3.1. The last row of the table gives the reproduction of the optimal values along with the other findings of the np-control chart from the numerical example of the Section 2.3.5 of Chapter II.

The following points are revealed from the Table 3.1.

(I) Comparision of raw(5) and row(6).

Though the cost of sampling and inspection per unit for $\bar{X}$-control chart is 3 times that for np-control chart, it is seen that $\bar{X}$-control chart leads to smaller ECPU as compared to that for np-control chart.

(II) Comparision of rows as listed (1) through (5).

As the cost of sampling and inspection per unit for $\bar{X}$-control chart increases, the optimal value of n decreases. However the total expected cost per unit (ECPU), as one expects, increases.

3.3 Economic Design under Unknown σ

We shall assume that the process standard deviation σ is unknown. Though unknown it is assumed to be attaining some constant value throughout the production cycle. All the other assumptions of the model and the system discussed in Section 3.2 are continued to be true throughout the present section.

3.3.1 T^2-Control Chart

It is proposed that the production process be monitored by T^2-control chart.

Table 3.1

Findings for $\bar{x}$-control chart

Row No.	a_2	Optimal Values n	k	L	ECPU	$\frac{E(C_1)}{Nk}$	$\frac{E(C_2)}{Nk}$	$\frac{E(C_3)}{Nk}$	N(0)	N	q_0	q_1	D	S
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	
1	1.0	11	252	2.30	0.4448	0.0833	0.0853	0.2761	4	5	0.0214	0.9778	23.70	1.51
2	1.5	9	254	2.20	0.4623	0.0925	0.0863	0.2835	4	5	0.0278	0.9554	24.42	1.25
3	2.0	9	254	2.20	0.4801	0.1102	0.0863	0.2835	4	5	0.0278	0.9554	24.42	1.25
4	2.5	7	257	2.05	0.4917	0.1070	0.0885	0.2962	4	5	0.0403	0.9176	25.71	1.00
5	3.0	7	257	2.05	0.5053	0.1206	0.0885	0.2962	4	5	0.0403	0.9176	25.71	1.00

Findings for np-control chart

6	1.0	37	350	2.00	0.5457	0.1343	0.0804	0.3310	3	4	0.0528	0.8964	33.00	5.00

Here the statistic T^2 based on n observations $x_i (i=1,2,...n)$ of the sample of size n is defined as

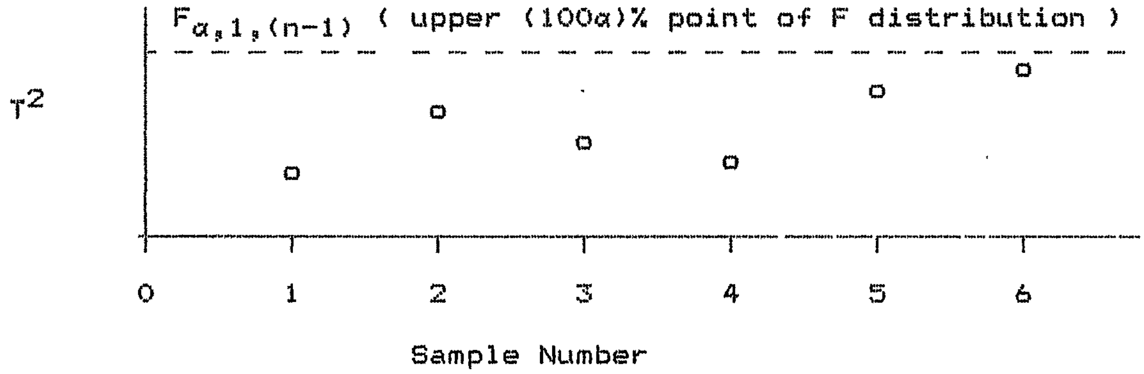
$$T^2 = n(\bar{x} - \mu_o)^2 / s_{n-1}^2 \qquad ...(3.3.1)$$

where $$s_{n-1}^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2/(n-1) \qquad ...(3.3.2)$$

and $$\bar{x} = \sum_{i=1}^{n} x_i/n \qquad ...(3.3.3)$$

It may be noted that T^2 has F distribution with 1 and (n-1) d.f.

A typical T^2-control chart is given below.



The sampling scheme and the control procedure are as follows. After the production of every k units, n units are sampled and examined. For each sample, sample mean $\bar{x}$, sample variance s_{n-1}^2 and T^2 are calculated. If $T^2 \leq F_{\alpha,1,n-1}$ then the process is declared to be in control and the production is continued. If $T^2 > F_{\alpha,1,n-1}$ the process is declared to be out of control and a search for the assignable cause is undertaken. Here $F_{\alpha,1,n-1}$ is the upper (100α)% point of F distribution such that

$$P(F > F_{\alpha,1,n-1}) = \alpha \qquad ...(3.3.4)$$

The design variables n, k, $F_{\alpha,1,n-1}$ are to be determined such that the expected total cost per unit of the product is minimum.

One may use T-control chart in place of T^2-control chart to monitor the production process. If T-control chart is used, one has to use t and noncentral t-distributions to calculate the probability of type I error (q_0) and the power of the test (q_1). (The expressions for q_0 and q_1 are derived in the next section). Whereas if T^2-control chart is used, one needs central and noncentral F-distributions for the calculation of q_0 and q_1. The subroutines developed for central and noncentral F-integrals can be used further for multivariate T^2-control chart also. Hence with a view to extending the present model for multivariate T^2-control chart, we prefer T^2-control chart rather than T-control chart in univariate case also.

3.3.2 The probability of Type - I Error and the Power of T^2-Control Chart

We recall that T^2-control chart is proposed to find whether the process is in the in-control state μ_0 or whether the process is in the out-of-control state $\mu_0+\delta\sigma$ due to assignable cause.

Hence when the assignable cause occurs, the probability that it will be detected by any subsequent sample is

$$q_1 = P[T^2 > F_{\alpha,1,n-1} \mid \mu_0+\delta\sigma] \qquad \ldots(3.3.5)$$

where T^2 has noncentral F distribution with 1 and (n-1) d.f. and the noncentrality parameter $n\delta^2$. The probability q_1 is known as the power of the T^2-control chart. It may be noted that q_1 is independent of σ.

The probability of a false alarm is

$$q_0 = P [T^2 > F_{\alpha,1,n-1} \mid \mu_0] \quad \ldots(3.3.6)$$

where T^2 has the F-distribution with 1 and (n-1) d.f.. The probability q_0 is known as the probability of type-I error.

3.3.3 The Proportions of Nonconforming Units

The definitions of p_0 and p_1 remain the same as given in Section 3.2.3 and are to be obtained by the expressions (3.2.3a) and (3.2.3b) given there. Immediately one can see that one has to face the problem of unknown σ in evaluating these expressions. This problem can be solved in the following way. One may calculate the sample variance on the basis of a preliminary sample of some suitable size taken when the process is in control. The square root of this sample variance will give a quick estimator of σ. Using this estimator in place of σ in the expressions (3.2.3a) and (3.2.3b) one gets the approximate values of p_0 and p_1.

However, if one wants to maintain some stipulated value of p_0 when the process is in the in-control state μ_0, then the value of p_1 can be obtained as follows. Making an appropriate break-up of p_0 and referring to standard normal tables one can obtain the values for $(u_1-\mu_0)/\sigma$ and $(u_2-\mu_0)/\sigma$ using (3.2.3a). Substituting these values in (3.2.3b) one can find p_1 for known δ. In many cases u_1 and u_2 are equally spaced from μ_0 on either side. In these situations $(u_1-\mu_0)/\sigma$ and $(u_2-\mu_0)/\sigma$ are numerically equal but opposite in signs so that $p_0 = 2\Phi[(u_1-\mu_0)/\sigma]$. For instance, if stipulated value of p_0 is 1 % then $(u_2-\mu_0)/\sigma = 2.58$ and

$(u_1-\mu_0)/\sigma$ = −2.58. It is easy to see that in the above method the knowledge of σ is not required.

3.3.4 The Expected Cost Model

Since we have assumed that all the assumptions of the model of the case of known σ prevail here also, the expression for the total expected cost per unit (ECPU) is the same as given by (3.2.4). While evaluating this expression, one has to use (3.3.6) and (3.3.5) for q_0 and q_1 respectively. Substitution for p_0 and p_1 is just discussed in Section 3.3.3.

3.3.5 Solution Method and Numerical Example

(A) Program

A computer program on FORTRAN is developed to calculate the expected total cost per unit of the product for the given values of n, k, $F_{\alpha,1,n-1}$. This program computes the probabilities q_0 and q_1 using central and noncentral F-distributions. A subroutine is developed for the calculation of central F-distribution using Trapezoid Rule. The noncentral F-integrals are calculated from central F-integrals using the following results given in a book by Abramowitz and Stegun (1972) pp. 946 - 947.

These results are as follows.

(1) Distribution Function of central F

$$P (F|v_1,v_2) = I_x \left(\frac{v_1}{2} , \frac{v_2}{2} \right) \qquad \ldots(3.3.7)$$

where $I_x\left(\frac{v_1}{2}, \frac{v_2}{2}\right)$ is incomplete beta integral with $x = \frac{v_1 F}{v_2 + v_1 F}$

(2) Distribution Function of noncentral F

$$P\left(F \mid v_1, v_2, \lambda\right) = \sum_{j=0}^{\infty} \frac{\exp(-\lambda/2)\ (\lambda/2)^j}{j!}\ I_x\left(\frac{v_1}{2} + j, \frac{v_2}{2}\right) \quad \ldots(3.3.8)$$

where $x = \frac{v_1 F}{v_2 + v_1 F}$ and λ is noncentrality parameter.

The proportions p_0 and p_1 are supplied externally.

This program is linked to Hooke-Jeeves search technique to find the optimal values of n, k, $F_{\alpha,1,n-1}$ which minimize ECPU. For the objective function understudy, two design variables n and k are discrete and the third design variable $F_{\alpha,1,n-1}$ is continuous. However, by giving the suitable initial values for (n, k, $F_{\alpha,1,n-1}$) and by choosing the proper step size and reduction factor, Hook-Jeeves' procedure works successfully and gives the optimal solution. The listing associated with all the programs is given at the end of this chapter.

(B) Numerical Example

Let a_1 = \$ 10.0, a_2 = \$ 1.0, $a_{3,1}$ = \$ 100, $a_{3,2}$ = \$ 100, $a_{4,1}$ = \$ 10, $a_{4,2}$ = \$ 15 .

Let $\lambda = 1$, $R = 1000$, $u_1 = -2.58$, $u_2 = 2.58$.

Let $\mu_0 = 0$, $\delta = 1.3$.

We take assessment of σ to be 1 to calculate p_0 and p_1.

For these values of the cost coefficients and systems parameters the search technique yielded the following optimal procedure.

$n = 11$, $k = 253$, $F = 5.3$ with minimum ECPU = \$ 0.4560 .

For these optimal design variables we give the values of some intermediate terms required in the calculation of ECPU.

$N(0) = 4$, $N = 5$, $q_0 = 0.0522$, $q_1 = 0.9653$

$D = 24.10$, $S = 1.52$

$$\frac{E(C_1)}{Nk} = \$\ 0.0830, \quad \frac{E(C_2)}{Nk} = \$\ 0.0934, \quad \frac{E(C_3)}{Nk} = \$\ 0.2796$$

Comparing the ECPU derived in this example with the corresponding example given by row (1) of Table 3.1 (when σ is known), one can see that the lack of knowledge of σ leads to cost penalty of (0.4560 - 0.4448 =) \$ 0.0112 per unit of the product.

```
C LISTING OF CHAPTER III
        SUBROUTINE XBC(RK,NSTAGE,SUM,A1,A2,A3,A3P,A4,A4P,ALEMDA,
     1 RATE,CPN,FNOT,FONE)
C    FILE NAME IS VCC
C   PROGRAM FOR ECPU OF XBAR-CHART
      DIMENSION RK(10)
      WRITE(*,2) A1,A2,A3,A3P,A4,A4P
    2 FORMAT(1X,'A1=',F10.4,'A2=',F10.4,'A3=',F10.4'A3P=',F10.4,
     1 'A4=',F10.4,'A4P=',F10.4)
      WRITE(*,4) ALEMDA,RATE,CPN
    4 FORMAT(1X, 'ALEMDA=',F10.4,'RATE=',F10.4,'CPN=',F10.4)
      N=RK(1)
      K=RK(2)
      CL=RK(3)
      WRITE(*,6) N,K,CL
    6 FORMAT(1X,'N=',I5,'K=',I5,'CL=',F10.4)
      WRITE(*,8) FNOT,FONE
    8 FORMAT(1X,'FNOT=',F10.6,'FONE=',F10.6)
      POWER=ALEMDA*K/RATE
      PPOWER=-POWER
      THEETA=EXP(PPOWER)
      WRITE(*,9) THEETA
    9 FORMAT(1X,'THEETA=',F10.6)
      X=CL
      CALL NDTR(X,P,D)
      QNOT=2*(1.0-P)
      WRITE(*,10) QNOT
   10 FORMAT(1X,'QNOT=',F10.6)
      AN=N
      X=CL-SQRT(AN)*CPN
      CALL NDTR(X,P,D)
      QONE=1-P
      WRITE(*,40) QONE
   40 FORMAT(1X,'QONE=',F10.6)
      TNOS=THEETA/(1-THEETA)+1/QONE
      NOS=TNOS+0.5
      WRITE(*,50)NOS
   50 FORMAT(1X,'NOS=',I5)
      EC1=(A1+A2*N)*NOS
      BNOT=QNOT*THEETA/(1-THEETA)
      EC2=A3*BNOT+A3P
      TAW=(1-(1+POWER)*THEETA)/(1-THEETA)
      WRITE(*,55) TAW
   55 FORMAT(1X,'TAW=',F10.4)
      H=K/RATE
      D=(RATE*FNOT/ALEMDA)+(H/QONE-TAW)*RATE*FONE
      S=THEETA*N*FNOT/(1-THEETA)+N*FONE/QONE
      WRITE(*,60) D,S
   60 FORMAT(1X,'D=',F12.4,'S=',F12.4)
      EC3=A4*S+A4P*(D-S)
      EC=EC1+EC2+EC3
      ECPU=EC/(NOS*K)
      WRITE(*,65)EC1,EC2,EC3,EC,ECPU
   65 FORMAT(1X,'EC1=',F12.4,'EC2=',F12.4,'EC3=',F12.4,'EC=',F12.4,
     1 'ECPU=',F12.4)
      SUM=ECPU
      RETURN
      END
```

```
C FILE NAME IS MAN3
C  CALCUATION OF NORMAL INTEGRAL
        SUBROUTINE NDTR(X,P,D)
        AX=ABS(X)
        T=1.0/(1.0+.2316419*AX)
        D=0.3989423*EXP(-X*X/2.0)
        P=1.0-D*T*((((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T
     1  +0.3193815)
        IF(X) 1,2,2
  1     P=1.0-P
  2     RETURN
        END


         SUBROUTINE XBC(RK,NSTAGE,SUM,A1,A2,A3,A3P,A4,A4P,ALEMDA,
     1  RATE,CPN,FNOT,FONE)
C    FILE NAME IS XBAR
C  COST MODEL FOR TSQR CHART FOR UNKNOWN VARIANCE
        DIMENSION P(100),Q(100),R(100),RK(10)
        WRITE(*,2) A1,A2,A3,A3P,A4,A4P
    2   FORMAT(1X,'A1=',F10.4,'A2=',F10.4,'A3=',F10.4'A3P=',F10.4,
     1  'A4=',F10.4,'A4P=',F10.4)
        WRITE(*,4) ALEMDA,RATE,CPN
    4   FORMAT(1X, 'ALEMDA=',F10.4,'RATE=',F10.4,'CPN=',F10.4)
        N=RK(1)
        K=RK(2)
        F=RK(3)
        WRITE(*,6) N,K,F
    6   FORMAT(1X,'N=',I5,'K=',I5,'F=',F10.4)
        WRITE(*,8) FNOT,FONE
    8   FORMAT(1X,'FNOT=',F10.6,'FONE=',F10.6)
        CPN1=N*CPN
        POWER=ALEMDA*K/RATE
        PPOWER=-POWER
        THEETA=EXP(PPOWER)
        WRITE(*,9) THEETA
    9   FORMAT(1X,'THEETA=',F10.6)
        Y1=0
        Y2=F/((N-1)+F)
        A=0.5
        B=(N-1)/2
        H=0.01
        CALL QR(Y1,Y2,A,B,H,BI)
        QNOT=1-BI
        WRITE(*,10) QNOT
  10    FORMAT(1X,'QNOT=',F10.6)
        DO 15 J=1,90
        Y1=0
        Y2=F/((N-1)+F)
        A=0.5+J
        B=(N-1)/2
        H=0.01
        CALL  QR(Y1,Y2,A,B,H,BI)
        P(J)=BI
        IF(P(J).LT.0.00001)GO TO 21
  15    CONTINUE
  21    ISTOP=J-1
```

```
      WRITE(*,32) (P(J),J=1,ISTOP)
      POW=CPN1/2.0
      PPOW=-POW
      R(1)=EXP(PPOW)*POW*P(1)
      IST=ISTOP-1
      DO 30 J=1,IST
 30   R(J+1)=POW*P(J+1)*R(J)/(P(J)*(J+1))
      WRITE(*,32) (R(J),J=1,ISTOP)
32    FORMAT(1X,7F10.6)
      RNOT=EXP(PPOW)*(1-QNOT)
      TEM=RNOT
      DO 35 J=1,ISTOP
      TEM=TEM+R(J)
 35   CONTINUE
      QONE=1-TEM
      WRITE(*,40) QONE
 40   FORMAT(1X,'QONE=',F10.6)
      TNOS=THEETA/(1-THEETA)+1/QONE
      NOS=TNOS+0.5
      WRITE(*,50)NOS
50    FORMAT(1X,'NOS=',I5)
      EC1=(A1+A2*N)*NOS
      BNOT=QNOT*THEETA/(1-THEETA)
      EC2=A3*BNOT+A3P
      TAW=(1-(1+POWER)*THEETA)/(1-THEETA)
      WRITE(*,55) TAW
55    FORMAT(1X,'TAW=',F10.4)
      H=K/RATE
      D=(RATE*FNOT/ALEMDA)+(H/QONE-TAW)*RATE*FONE
      S=THEETA*N*FNOT/(1-THEETA)+N*FONE/QONE
      WRITE(*,60) D,S
60    FORMAT(1X,'D=',F12.4,'S=',F12.4)
      EC3=A4*S+A4P*(D-S)
      EC=EC1+EC2+EC3
      ECPU=EC/(NOS*K)
      WRITE(*,65)EC1,EC2,EC3,EC,ECPU
65    FORMAT(1X,'EC1=',F12.4,'EC2=',F12.4,'EC3=',F12.4,'EC=',F12.4,
     1 'ECPU=',F12.4)
      SUM=ECPU
      RETURN
      END


      SUBROUTINE QR(Y1,Y2,A,B,H,BI)
C FILE NAME IS XBAR1
      Y1=Y1
      Y2=Y2
      A=A
      B=B
      H=H
      CALL BITA(Y1,Y2,A,B,H,PROB2)
      BIN=PROB2
      Y1=Y1
      Y2=1.0
      A=A
      B=B
      H=H
```

```
      CALL BITA(Y1,Y2,A,B,H,PROB2)
      BID=PROB2
      BI=BIN/BID
      WRITE(*,12) BI
 12   FORMAT(1X,'BI=',F10.6)
      RETURN
      END


      SUBROUTINE   BITA( Y1,Y2,A,B,H,PROB2)
C  COMPUTATION OF INCOMPLETE BETA INTEGRAL BY TRAPEZOID RULE
C  FILE NAME IS  TRALS
      NOY1=(Y2-Y1)/H
      NOY=NOY1+1
      IF ((NOY1*H) .EQ. (Y2-Y1)) GO TO 80
      GO TO 85
 80   WRITE(*,20)
 20   FORMAT(1X,'NOY1 AND NOY ARE REALLY INTEGER AND NOT BY COM
     1 TECH BOTH PROB AND PROB1 SAME')
      GO TO 90
 85   WRITE(*,22)
 22   FORMAT(1X,'NOY AND NOY1 ARE NOT REALLY INTEGER AND ARE MADE
     1 INTEGER , PROB AND PROB1 NOT EXPECTED SAME')
 90   WRITE(*,24) NOY
 24   FORMAT(1X,'NOY='I8)
      S1=0
      DO 100 M=1,NOY
      RM=M
      YSUB=Y1+(RM-1.0)*H
      IF(YSUB.EQ.0.)GO TO 200
      GO TO 210
 200  F=0.
      GO TO 212
 210  F=(YSUB**(A-1.0))*((1-YSUB)**(B-1.0))
 212  IF(M.EQ.1) GO TO 120
      IF((1 .LT. M) .AND. (M .LT. NOY)) GO TO 125
      IF (M .EQ. NOY) GO TO 130
 120  F=F/2.
      S1=S1+F
      GO TO 100
 125  S1=S1+F
      GO TO 100
 130  F=F/2.
      S1=S1+F
      FY2=(Y2**(A-1.0))*((1-Y2)**(B-1.0))
      T=(F+FY2)*((Y2-Y1)-(RM-1.)*H)
      S2=S1+T
 100  CONTINUE
      WRITE(*,26)FY2,T
  26  FORMAT(1X,'FY2=',F18.10,'T=',F18.10)
      PROB1=S1*H
      PROB2=PROB1+T
      WRITE(*,28) PROB1,PROB2
  28  FORMAT(1X,'PROB1=',F18.10,'PROB2=',F18.10)
      RETURN
      END
```