

CHAPTER 2

THE THEORETICAL BACKGROUND

A firm is understood as an organisation which converts inputs, which it hires, into outputs, which it sells. The inputs, called factors of production, are mainly human resources (labour, entrepreneur) and capital resources (natural, man-made). The output of firms consists of goods and services they produce. It includes production in various sectors such as agriculture, industry, trade, transport, banking and communication.

Different firms have different objectives, like profit maximisation (short-run or long-run), sales maximisation subject to some level of pre-determined profit, size maximisation, long-run survival and some non-profit objectives. To a great extent, the objectives of a firm depend upon whether it is a business firm, private sector or public sector firm or a corporate firm.

The firms, while trying to achieve their objectives, often face certain constraints which may be internal or external viz., the resources constraints, legal constraints, environmental (economic, political and social) constraints and output quality constraints.

For a firm or an industry, the concept of economies of scale, derived from the theory of firm, could be used for determining the most appropriate production level (or range) for the entrepreneur, within the framework of the given objectives and

constraints. For providing a proper perspective to this understanding, we have discussed the production, cost and profit functions of a firm.

In the following sections, we discuss the theory of economies of scale, explained and dealt by production function, cost function and profit function, with a view to enquire how these economic theories and concepts can be used to answer the questions raised in Chapter I, regarding economies of scale.

2.1 The Production Function Approach to Economies of Scale

Production-function is a technical relationship between factor inputs and output of a firm. It gives the maximum possible output that can be produced from a given amount of various inputs, or alternatively, the minimum quantity of inputs necessary to produce a given level of output.

A production function, in its traditional sense, can be written as:

$$Q = f (L, M, K, T)$$

where

- Q = Output in physical units of good X
- L = Land units employed in the production of Q
- M = Manpower employed in the production of Q
- K = Capital units employed in the production of Q
- T = Technology employed in the production of Q
- f = Unspecified function
- $f_i > 0$
- f_i = Partial derivative of Q with respect to ith input

The production-function assumes that the output is an increasing function of all inputs. However, if an input is excessively applied in relation to other inputs, an increase in it, other inputs held constant, might lead to a decrease in output.

In the short-run, output may be increased by using more of variable factor(s) while other factor(s) is kept constant. As more and more quantities of the variable factors(s) is combined with the constant factor(s), the marginal product of the variable factor(s) declines due to operation of Law of Variable Proportions.

In other words, under this law, as more and more units of the variable input are employed in the production, with fixed inputs remaining unchanged, production first increases at an increasing rate (1st stage), and then at a diminishing rate (2nd stage) and eventually leading to a decline in total production (3rd stage). In practice, usually, the first stage is short, the second stage is long and third stage never occurs.

Alternatively, output may be increased by changing all factors in the same proportion and direction and the term, 'return to scale', thus, refers to the long-run production-function. If all inputs increase in a given proportion and output increases by more than that proportion, the law of increasing returns to scale is said to operate, implying, an increase in total factor productivity. If output increases by the same proportion as all inputs have increased, there are constant returns to scale and there is no change in total factor productivity. Finally, if output increases by a smaller proportion than a change in all inputs, there are decreasing returns to scale and a decrease in total factor productivity.

The all-input elasticity of output (e1.) concept related to the returns to scale is defined as follows:

$$e1. = \frac{\% \text{ change in output}}{\% \text{ change in all inputs}}$$

if

- e1. > 1, there are increasing returns to scale
- e1. = 1, there are constant returns to scale
- e1. < 1, there are decreasing returns to scale

The returns to scale concept has critical importance in the theory of production. If an industry or a firm is characterised by increasing returns to scale, there is further scope for expanding the size of the firm(s) and vice versa. Firms of all sizes would survive equally well in an industry characterised by constant returns to scale.

When the firm expands its size of production by employing more factors of production, there are internal and external economies/diseconomies of production which bring about increasing/decreasing returns to scale. The returns to scale include the technical economies alone whereas economies of scale are concerned with technical as well as monetary economies taking into consideration various types of efficiencies (inefficiencies) and hence, carries a wider concept.

The economies of scale can be examined through two approaches, the cost function and the profit function, both derived from a production function. In the next section, we discuss the theory of cost function and the theory of profit function and explain how these functions can be used to find out the economies/diseconomies of scale.

2.2 The Cost Function Approach to Economies of Scale

2.2.1 Introduction

In the classical theory of cost and production, the firm is assumed to face fixed technological possibilities and competitive input markets, and to choose an input bundle to minimise the cost of producing each possible output. For fixed input prices, this behaviour determines minimum cost as a function of output, yielding the standard cost curves of elementary textbooks. An immediate generalization is to allow input prices to vary and consider minimum cost as a function of both, input prices and output. With this minor modification, the cost function becomes a powerful analytical tool in the theory of production, particularly, in econometric applications.

The principal advantage of the cost function lies in its computationally simple relation to the cost minimising input demand functions: the partial derivatives of cost function with respect to input prices yield the corresponding input demand functions, and the sum of the values of the input demands equal cost. The useful analytical properties of the cost function are derived from fundamental duality between this function and the underlying production possibilities. The definition of the cost function, as the result of an optimization, yields strong mathematical properties, and establishes the cost function as a "sufficient statistics" for all the economically relevant characteristics of the underlying technology. (Fuss and Mcfadden, 1978)

The systematic analysis of the properties of price derivatives of the cost function originated in a paper of Hotelling (1932) on the mathematically equivalent problem of minimising consumer expenditure subject to a utility level constraint. The cost function and its properties were discussed in Samuelson (1947). Later, he developed the concept of factor-price frontiers, which is a level of a cost function.

The theory of establishing the dual relationship between the cost functions and production functions was introduced by Shephard (1953). Perhaps, as the theoretical results on cost functions were scattered and relatively inaccessible, their potential worth in econometric analysis was not recognised until Nerlove employed the Cobb-Douglas case in a study of returns to scale in electric utilities. Since the mid 1960's, a series of empirical studies, including paper Jorgenson and Lau (1974), have made systematic use of duality concepts.

2.2.2 Derivation of Cost Function

For a single product firm, which uses inputs whose quantity vector is $q = (q_1 \dots q_n)$, to produce a scalar output X , where each input quantity and the output quantity is positive, the technology can be represented by a production-function -

$$X = f(q_1, \dots, q_n)$$

or

$$X = f(q)$$

Which is interpreted as the maximum amount of output which can be produced by the firm using amounts q_1, \dots, q_n of the outputs. For a single product firm, this is the traditional way to represent production technology.

It has been shown by Shephard, McFadden and others that the technology of the firm can be represented equivalently by cost, revenue and profit functions under competitive conditions.

Accordingly, suppose that the firm faces an input price vector $p = (p_1, \dots, p_n)$ corresponding to its input quantity vector. Also, suppose that its objective is, for given input price and output quantity vectors, to minimise the cost $C = \sum p_i q_i$ of producing the given outputs by choosing optimum input quantities. Assuming that this objective is possible to achieve, we have cost as a function of the given input prices and output quantities :

$$C = C(X, p)$$

where

C = Cost
 X = Output
 p = Input price vector

The cost function has a number of highly interesting properties :

- (i) Domain : $C = f(X, p)$ is a positive real valued function defined for all positive prices and all positive producible outputs ; $(0, r) = 0$.
- (ii) Monotonicity : $C = f(X, p)$ is a non-decreasing function in output and tends to infinity as output tends to reach infinity. It is also non-decreasing in prices.
- (iii) Continuity : $C = f(X, p)$ is continuous from below in X and continuous in p .

- (iv) Concavity : $C = f(X, p)$ is a concave function in p .
 - (v) Homogeneity : $C = f(X, p)$ is linear homogeneous in prices.
 - (vi) Differentiability : $C = f(X, p)$ is twice differentiable in input prices. Under this property, the cost function possesses the important derivative property.
- a) $\partial C / \partial p_i = V_i$ [Shephard's Lemma]
 - b) $\partial^2 C / \partial p_i \partial p_j = \partial^2 C / \partial p_j \partial p_i$ [Symmetry]

Property (a) can be used to generate systems of factor demand functions. Property (b) is of use in reducing the number of parameters to be estimated, thus conserving degrees of freedom.

2.2.3 Comprehensive Cost Function

A comprehensive cost function can be written as follows :-

$$C = f(X, P_f, E_f, T)$$

$$f_1, f_2 > 0, f_3, f_4$$

where

- C = Total cost of production of commodity X
- X = Output of commodity X
- P_f = Vector of prices of all factor inputs used in the production of commodity X
- E_f = Vector of efficiency or productivity of all factor inputs used in the production of commodity X
- T = Existing technology for the production of commodity X
- f = Unspecified function
- f_i = partial derivatives of f with respect to the i th variable ($i = 1, \dots, 4$).

The total cost of production of a firm is the costs of inputs (i.e. of obtaining the use of the factors of production). These are expressed as money costs. Opportunity costs - what would have to be paid for a factor in its best alternative use (the foregone alternative) are not included, partially, because the firm does not pay these costs.

Output refers to the total output of the commodity produced per unit of time and refers to homogeneous products. Output can be measured either in physical terms (total quantity produced) or in monetary terms (money value of total output produced). The total cost varies directly with output.

When there is an increase in any one or more of factor prices, all other factors' prices, input requirements, technology, output etc. remaining constant, total production cost must increase. This is because more would have to be paid to those factor inputs whose prices have increased and there will be no simultaneous reduction in the costs from any other source. Thus, the cost of production varies directly with the prices of factors of production. But, if the price of one factor increases while that of the other decreases, and there is factor substitutability, its effects on total cost will be uncertain.

Production is synonymous to efficiency. Higher the productivity of an input factor, one needs lesser of that factor to produce a given output; while other factor inputs remain constant. Given the factor prices, technology and output level, an increase in factor productivity would decrease the total cost of production. But, if productivity of one input factor increases while that of other decreases, its effect on total cost will be uncertain, as in the case with the prices of factor inputs.

Technological change (improvement) leads to an increase in the efficiency or productivity of factors of production which in turn reduces the production cost.

2.2.4 Cost Function and Economies of Scale

The theory of economies of scale is the theory of the relationship between the scale of use of a properly chosen combination of all productive inputs services and the rate of output of the enterprise, Stigler (1958).

Accordingly, to quote Silberston (1972) "classical economies of scale relates to the effect on average cost of production of different growth rates of output per unit of time of a given commodity when all possible adaptations have been carried out to make production at each scale as efficient as possible."

Typically, the study of economies of scale is to find out the sensitivity of increments in output on costs. As production increases total cost increases too, but there is no definite relationship between cost and output. The direction in which costs will vary with output is shown by the shapes of average and total costs curves, relating costs to output. The reason for the variation is indicated by the theory of economies of scale.

There exist economies of scale if total cost increases less than proportionately with increase in output; diseconomies of scale exist if total cost increases more than proportionately with increase in output and neither exist, if the proportionate increase in cost and output is the same.

Economic theory argues that, in short-run, as production begins there are economies of scale upto a certain point, and thereafter diseconomies of scale set in. Underlying this is the economic

law of variable returns to factors of productions. As more and more units of a variable means of production are applied, the marginal production first increases and then, after a certain point, starts to decline. This is because, in the beginning, some fixed factors of production are underutilised and as the scale of operation increases, a point of their optimum is reached. Further expansion beyond this either results in inefficiency or requires a further addition of fixed factors of production.

The relationship between the production function and short-run cost function can be illustrated by defining the following symbols and by assuming that the production function contains only two inputs :-

V = number of units of variable factor
 p = price per unit of variable factor
 X = number of units of output
 MP = marginal productivity of the variable factor
 AP = average productivity of the variable factor
 MC = marginal cost
 AC = average variable cost per unit of output

then we have

$$AC = \frac{\text{Total Variable Cost}}{\text{Total Output}}$$

or

$$AC = PV/X = P[V/X] = P/AP$$

$$MC = \frac{d \text{ Total Variable Cost}}{d \text{ Total Output}} = \frac{d [PV]}{d [X]} = P/MP$$

So, if it is assumed that factor prices remain constant, the basic cost components are inversely related to the marginal and average products of the variable factor. If we assume the AP and MP to rise smoothly at first and then fall, the cost curves behave in the opposite fashion and we have the traditional U-shaped average and marginal cost curves. In this whole analytical framework it is assumed that there is only a single variable factor and the output is a single homogeneous product and the firm is operating in perfectly competitive product and factor markets.

The traditional theory of costs postulates that in the short-run average variable cost, average total cost and marginal cost are U shaped, reflecting the law of variable proportions.

In the long run, all the factors are assumed to become variable. The long-run average cost curve is derived from short-run cost curves. Each point on the long-run average cost curve (LAC) corresponds to a point on a short run cost curve, which is tangent to the LAC at that point. Each point of LAC curve shows the minimum cost for producing the corresponding level of output. In the traditional theory of the firm, the LAC curve is U-shaped and it is often called the "envelope" curve because it envelopes the short run cost curves. This U -shape of LAC reflects the laws of returns to scale in the firm's production and explained by the theory of economies of scale in a corresponding cost function. The economies of scale lead to a fall in average cost as output expands. The optimum size of the firm is reached when these economies of scale lead to a fall in average cost as output

expands. The optimum size of the firm is reached when these economies disappear and diseconomies of scale are about to set in. As the firm size becomes too large, the diseconomies of scale arrive and lead to a rise in average cost.

2.2.5 Methods of Measurement: Alternative Approaches

1. Accounting Approach

The accounting approach involves classification of expenses into fixed, variable and semi-variable costs. On the basis of inspection and experience, the fixed cost, the variable cost and the output range within which semi-variable cost is fixed are calculated. Then, for each output level, the total cost, average total cost, average variable cost and marginal costs are obtained using simple arithmetic.

There are three requisite conditions for the successful use of this method :

1. Experience with a wide range of fluctuations in output rate.
2. A detailed breakdown of accounts kept on the same basis over a period of years.
3. Relative constancy in wage rates, material prices, plant size, technology and so forth.

The Accounting approach does not provide any way out to correct data explicitly for changes in variables on conditions that affect cost behaviour.

2. Engineering Approach

Engineering method relies upon knowledge of physical relationships between factors of production supplemented by pooled judgements of practical operations. This method consists of systematic conjectures about what cost behaviour ought to be in the future on the basis of what is known about the rated capacity of equipment, modified by experience with manpower requirement and efficiency factors and past cost behaviour.

First, the physical units for an output level are determined, then they are multiplied by the respective current or expected factor prices and added together to yield cost estimates for that output level. This exercise is repeated for various quantities and thus the cost output relationship is obtained.

3. The Econometric Approach

Under the econometric approach, the historical data on cost and output are used to estimate the cost-output relationship. There are four popular hypotheses about the form of the cost-output relationship which can be deduced mathematically to find out the shape of the average and marginal cost curves, the economies of scale/diseconomies of scale, and also, the optimum size of a firm found in the literature [Gupta (1972)]. They are :-

1. Cubic Total Cost Function
2. Quadratic Total Cost Function
3. Linear Total Cost Function
4. Double-Log Total Cost Function

The Ordinary Least Square (OLS) method is the most popular technique to estimate the total cost function. A Cubic form gives U shaped AC and MC curve, a quadratic cost function gives a U shaped AC and monotonously rising MC curves, a linear form implies a constant MC and an L shaped AC curve. double-log functional form results in a constant, falling or rising AC and MC curve with a constant elasticity. All these results depend upon the 'a priori' signs of the coefficients.

All the four functional forms of total cost function should be tried for estimation and a particular form should then be chosen on the basis of both economic theory and statistical inference. Both, multi-variable time-series and cross-sectional data can be used for the estimation.

Once the appropriate total cost equation is estimated, it can be used to estimate the elasticity of total cost with respect to total output (economies/diseconomies of scale) and optimum size (output level where average cost is minimum) of a firm or an industry.

The elasticity of total cost with respect to output (e) is:-

$$e = \frac{\text{Proportionate change in Total Cost (TC)}}{\text{Proportionate change in Total Output (X)}}$$

or

$$e = \left\{ \partial TC / \partial X \right\} \times \left\{ X / TC \right\}$$

Here, the elasticity of total cost can be computed for each observation on Total Cost and corresponding observation on output.

if

$e > 1$; there are diseconomies of scale
 $e < 1$; there are economies of scale
 $e = 1$; neither economies nor diseconomies of scale

To find out the optimum size, average cost equation derived from total cost equation is partially derivated with respect to output and set equal to zero. The computed value of output (X) from the equation is the optimum size of a firm.

2.2.6 Basic Issues

1. The theory of economies of scale is concerned with internal economies or diseconomies of scale.
2. The theory of economies of scale deals with the economies at the firm level. It naturally includes plant economies.
3. The theory of economies of scale is concerned with the long-run. As Sandesara (1976) explains, "The theory of economies scale is concerned with changes in levels of production corresponding to the variation in the capacities. While the capacity to produce remains constant in the short period the capacity of the firm is variable in the long-run."
4. Scale or size in the theory of economies of scale relates to the total output per unit of time and to the homogeneous products.
5. Economies of scale examines the sensitivity of variation in output on costs where other factors influencing costs are held to be constant. These factors are : (1) Homogeneity of factors of production (2) Level of technology adopted by the firm and (3) Factor/output prices.
6. The theory of cost curves brings out the implications of various hypotheses about the production function on the assumption either that factor prices are constant, independent of the purchase of a firm or

that they are dependent on the firm's purchases. Actual cost observations frequently come from successive time periods during which factor prices may have changed significantly in response to influences other than the firm's purchases. The common methods of correcting these factor price changes are deflation of the actual cost figures by a factor price index or the recalculation of the cost figures by applying some related set of factor prices to the actual factor inputs of each period. Johnston (1960) observes that the removal of the assumption of constant factor prices would not materially alter the slope of cost curves. The only difference it would make is the rise in marginal, average variable and average total cost curves earlier and greater.

2.3 PROFIT FUNCTION APPROACH TO ECONOMIES OF SCALE

2.3.1 Introduction

The theory of profit function for competitive firms and the relationship of the profit function to the production function (duality) were developed by McFadden (1966) and Lau (1969) extended the theory to non-competitive situations. Lau and Yotopolous (1971) have used the profit function to test for differences in economic efficiency with an application to Indian agriculture. Lau (1972) proved that the elasticities of profit function with respect to fixed factors of production show the degree of returns to scale using Cobb-Douglas production function. Yotopolous and Lau (1973) attempted to estimate returns to scale in the Indian agriculture using a Cobb-Douglas profit function.

2.3.2. The Derivation of Profit Function

The production function for a firm with the neoclassical properties is -

$$V = f(X_1, \dots, X_m ; Z_1, \dots, Z_n) \quad \dots 1$$

where

V = Output
 X_i = Variable Inputs ($i = 1, \dots, m$)
 Z_j = Fixed Inputs ($j = 1, \dots, n$)

and

$$P^* = P \cdot f(X_1, \dots, X_m; Z_1, \dots, Z_n) \quad \dots\dots 2$$

where

P^* = Profit
 P = Unit price of output
 C = Unit price of i th variable input ($i=1, \dots, m$)

Profit (P^*) is defined as current revenue less current total variable costs. Fixed costs are ignored since they do not affect the optimal combination of variable inputs (Lau and Yotopolous; 1971).

If we assume that the firm maximises profits given the levels of its technical efficiency and fixed inputs. Then, the marginal productivity (MP) conditions for profit maximisation requires the partial derivations of equation (2) with respect to X_i equal to zero.

Accordingly

$$\partial P_i / \partial X_i = P \{ \partial f(X, Z) / \partial X_i \} - C_i = 0$$

By moving input price term to the right, we get -

$$P \{ \partial f(X, Z) / \partial X_i \} = C_i \quad \dots\dots 3$$

where

$$i = 1, \dots, m$$

The first order conditions for profit maximisation require each input to be utilised upto a point which at the value of its MP equals its price.

By dividing both the sides by P (the unit price of output) of equation (3), we get -

$$\partial f (X,Z) / \partial X_i = C_i^* \quad \text{.....4}$$

where

$C_i^* = C_i/P$ (normalised price of ith input deflated by output price).

From equation (4)-

$$\partial f (X,Z) / C_i^* = \partial X_i \quad \text{.....5}$$

Equation (5) may be solved for the optimal quantities of variable inputs, denoted X_i^* , as a function of the normalised prices of variable inputs and of the quantities of the fixed inputs.

hence

$$X_i^* = f (C^*, Z) \quad \text{.....6}$$

where

X_i^* = Optimal quantities of variable inputs
 C_i^* = Vector of normalised input prices
 Z = Quantities of fixed inputs

By substituting (6) into (2), we get the profit function -

$$p = P[F(X_1^*, \dots, X_m^*; Z_1, \dots, Z_n) - \sum_{i=1}^m C_i^* X_i^*]$$

$$p = G[P, C_1, \dots, C_m, Z_1, \dots, Z_n] \quad \text{.....7}$$

The profit function (7) gives the maximised value of the profit for each set of values of prices of output and variable inputs and of the quantities of fixed factors.

2.3.3 Assumptions and Characteristics

The derivation of profit function assumes that :-

1. Firms are profit maximising.
2. Firms are price takers in both, output and variable input markets.
3. Production function is concave in the variable inputs.
4. Quantities of fixed factors are given, implying profit maximisation in the short-run and the profit function, thus derived is called the "Restricted Profit Function."

The profit function has the following characteristics :

1. The profit function is non-negative, convex, increasing in output prices, decreasing in the input prices and increasing in the quantities of fixed factors. The profit function is homogeneous of degree one in input and output prices.
2. The duality, as McFadden (1966) has shown, explains a one-to-one correspondence between the set of concave profit function. Every concave production function has corresponding convex profit function and vice versa. Hence, without loss of generality, one can consider for profit maximising, price taking firms, only profit function in the analysis of these behaviours without an explicit specification of the corresponding production function. (Lau and Yotopolous, 1971, pp-97).
3. OLS estimation of a production function is known to yield inconsistent estimates of the parameters because of simultaneous problems. Hence, the profit function is clearly a superior estimating technique.
4. The value of certain estimated parameters of the profit function can be used to test the hypothesis concerning -
 - a. Economies of Scale
 - b. Differences in relative economic efficiency accross different types of banking organisation.
 - c. The impact of regularity environment of efficiency.
5. The theoretical formulation of the profit function assumes that firms are price takers in output and input markets. Lau (1969) has suggested that the profit

function can be used to test whether a firm is a price taker in a given market. A finding that output prices make no significant contribution to the empirical explanation of firm's profit would be consistent with the hypothesis that firms are not price takers in any of the market for their products or services. A firm may operate competitively for a subset of their products, in which case a subset of output prices would appear in the profit function.

2.3.4 Profit Function and Economies of Scale

Lau and Yotopolous (1972) proved that the elasticities of profit with respect to fixed factors of production show the degree of returns to scale. Yotopolus and Lau (1973) used this concept to test the hypothesis of constant returns to scale to factors of production in Indian agriculture using a Cobb-Douglas production function. The elasticities of profit function with respect to fixed factors of production show the degree of returns to scale.

We have the profit function -

$$p = G [P, C_1^*, \dots, C_m^*, Z_1, \dots, Z_n]$$

Considering a profit function which is Cobb-Douglas in input prices, output prices and the quantities of fixed factors of production :

$$\log p = a + \sum_{i=1}^m a_i \log P_i + \sum_{j=1}^n b_j \log Q_j + \sum_{k=1}^w c_k \log Z_k$$

where

P_i = Output prices ($i=1, \dots, m$)
 Q_j = Input prices ($j=1, \dots, n$)
 Z_k = Quantity of fixed factors ($k=1, \dots, w$)
 p = Profit

This follows from the fact [derived from Lau and Yotopolous (1972)] that necessary and sufficient condition for homogeneity of degree K of the underlying function is :-

$$\left[\frac{(K-1)}{K} \right] \sum_{j=1}^n E_{bj} + \frac{1}{K} \sum_{k=1}^w E_{ck} = 1$$

$$\text{or} \quad \sum_{k=1}^w E_{ck} = 1 - \left[\frac{(K-1)}{K} \right] \sum_{j=1}^n E_{bj}$$

$$\text{or} \quad \sum_{k=1}^w E_{ck} = \left[1 - \left\{ \frac{(K-1)}{K} \right\} \sum_{j=1}^n E_{bj} \right] K$$

$$\text{or} \quad \sum_{k=1}^w E_{ck} = K (K-1) \sum_{j=1}^n E_{bj}$$

By the monotonicity conditions on the profit function, we have :

$$\sum_{j=1}^n E_{bj} < 0$$

hence, if

- i) $K > 1$; there are increasing returns to scale
then; $ck > 1$
- ii) $K = 1$; there are constant returns to scale
then; $ck = 1$
- iii) $K < 1$; there are decreasing returns to scale
then; $ck < 1$

Here, ck is the elasticity of profit function with respect to the fixed factors of production.

2.3.5 The Functional Form

The full Cobb-Douglas specification of the profit function has major properties which makes it very convenient to estimate the economies of scale. Lau and Yotopolous, who proved a test of hypothesis of constant returns to scale used Cobb-Douglas profit function which allows to test for scale economies or diseconomies by simply adding the coefficients of all the fixed factors of production to see whether the sum is equal to, larger than or smaller than unity.

Yotopolous, Lau and Some1 (1970) tried other alternative functional forms also, but Cobb-Douglas function appeared to be more superior through tests.

A profit function which is Cobb-Douglas in input prices, output prices and the quantities of fixed factors of production can be written as :-

$$\log p = a + \sum_{i=1}^m a_i \log P_i + \sum_{j=1}^n b_j \log Q_j + \sum_{k=1}^w c_k \log Z_k$$

where

P_i = Output prices ($i=1, \dots, m$)
 Q_j = Input prices ($j=1, \dots, n$)
 Z_k = Quantity of fixed factors ($k=1, \dots, w$)
 p = Profit (defined as current revenue less current total variable costs)

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