*Appendix A*

# DIFFERENTIATED INSTRUCTIONAL DESIGNS

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# DIFFERENTIATED INSTRUCTIONAL DESIGNS

Differentiated instruction means creating multiple paths so that students of different abilities or learning needs get equally appropriate ways to experience, conceptualize, apply and reflect in the learning process. It allows students to take greater responsibility and ownership for their own learning. The researcher followed a constructivist approach as the differentiated instruction for high ability students, integrated technology approach as the differentiated instruction for average ability students and scaffolding approach as the differentiated instruction for low ability students. According to different ability level classrooms, the principles of classroom instructions were also different. It is detailed in each lesson plan for the different ability groups.

## Topic: PERIMETER OF A CIRCLE

A polygon is solely made up of straight lines and as such it is easy to find its perimeter. Since any polygon can be split in to triangles, it is also not difficult to compute the areas of polygons. But what about a circle? We can place a string around a circle and try to find its perimeter. But this is not very convenient; and the result may not be accurate. In many practical contexts, we do have to find out the perimeter or area of circles. Ever since geometrical figures were studied, there were efforts to determine these using mathematical techniques. Ancient Egyptians and Babylonians devised mathematical methods to determine such measures approximately. They were refined and made accurate by the Greeks.

# DIFFERENTIATED INSTRUCTION FOR HIGH ABILITY STUDENTS

## LESSON PLAN – PERIMETER OF A CIRCLE

### CONCEPTS

- Perimeter of a circle
- Π, the constant

### PRINCIPLES OF CLASSROOM INSTRUCTION

- High ability students are expected to go above and beyond the objectives for the lesson.
- High ability students are gifted, excel in specific areas and/or have a strong knowledge base.
- High ability students require a high pace learning environment and it should be challenged.
- High ability students learn with minimal instruction.
- High ability students conduct independent study.
- High ability students learn and grasp basic concepts quickly.
- High ability students will create a class presentation with models, graphs and discussion points.

### OBJECTIVES

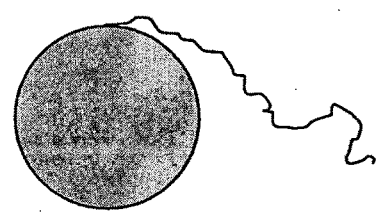
- To understand the perimeter of a circle is proportional to the diameter.
- To understand the perimeter of a circle divided by the diameter is an irrational number denoted by π and it is equal to 3.14.
- To understand the perimeter of a circle of radius r is $2\pi r$.

**Step 1** (Introduction to the Topic)

High ability students have a strong base in basic concepts. So they already know about the perimeter of a square, rectangle etc. and they can easily define perimeter of a circle. Teachers' role is to elicit these facts by asking proper heuristic questions.

**Step 2** (Development of the topic)

This is the first stage to nurture the abilities of the high ability students. They are able to construct knowledge themselves. Teacher should provide an appropriate situation for learning by giving very less guided activity. Here teacher gives the situation as to find the relationship between perimeter and diameter. For this teacher provide different circular discs to different group, tied with a string in one end as shown in the figure and an activity sheet with necessary instruction. When we are giving activity to high ability students, make sure that they will get an opportunity to do some thing by themselves.



**Activity Sheet**

| Sl. No. | Perimeter | Diameter | Perimeter/Diameter |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Since the high ability students can read the pattern and to identify the relationship between the variable, they will very soon

come out with the result that the perimeter divided by diameter of any circle is a constant. With the help of teacher students identifies that perimeter divided by diameter of any circle is a constant called π.

*The number obtained by dividing the perimeter by the diameter is the same for all circles. It is called the number π and is approximately equal to 3.14*

From the above relation, we want to find a new relation. As said earlier, since high ability students are good in identifying the patterns and relationship, teacher pause the question to them as from the above relation can you find the formula for finding the perimeter of a circle. Since they are pace of learning is very high, they easily come out with the formula for finding the perimeter of circle as follows:

Perimeter / Diameter = π

i.e. Perimeter = Diameter × π

Perimeter = 2 × radius × π

Perimeter, C = 2πr (Perimeter of a circle is also called as circumference of a circle)

*Perimeter of a circle of radius r is 2πr*

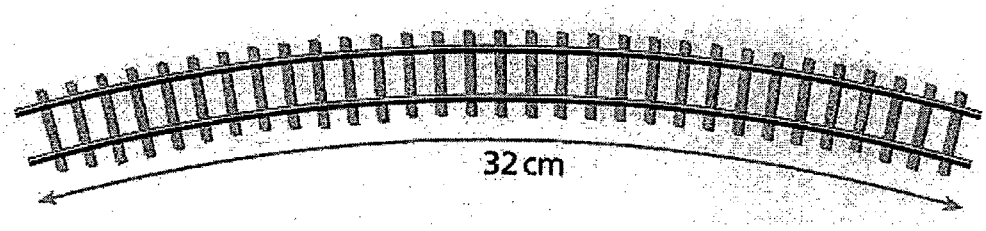
**Step 3** (Application)

This is the second stage to nurture and explore the abilities of the high ability students. They should provide with different levels of objective based questions in the increasing order. Otherwise they will get frustrated. The questions should be challenging them.

Question: What is the approximate circumference of a circle of radius 12 cm?

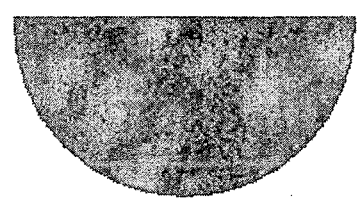
Question: A piece of wire 6 meters long is bent into the shape of a circle. What is the radius of the circle?

Question: 12 pieces of model railway track like this make a circle.




Calculate the radius of the circle.

Question: This shape is a semicircle. Its perimeter is 12.3 cm.
What is its diameter?
Explain how got your answer.



**Assignment**

Since high ability students have strong base of old concepts and have a strong memory, teacher can interrelated the new concepts with old concepts and ask question of that kind.

Question: The wheels of a bicycle have diameter 40 cm, including the tyres.

- What is their circumference?
- How far forward, in meters, does a wheel go in 100 turns?
- What is the bicycle's speed in meters per second if the wheels are turning at 100 turns per minute?

Question: Prepare a presentation on the constant π.

# DIFFERENTIATED INSTRUCTION FOR AVERAGE ABILITY STUDENTS

(Soft copy is attached with the thesis and hard copy is given at the end of the lesson plan)

## LESSON PLAN – PERIMETER OF A CIRCLE

### CONCEPTS

- Perimeter of a circle
- Π, the constant

### PRINCIPLES OF CLASSROOM INSTRUCTION

- Average ability students are expected to master all of the objectives.
- Average ability students are expected to perform at grade level.
- Average ability students can learn from modeling.
- Average ability students can do independent work.
- Average ability students need review and practice.
- Average ability students are expected to complete a three-page, individual final project.

### OBJECTIVES

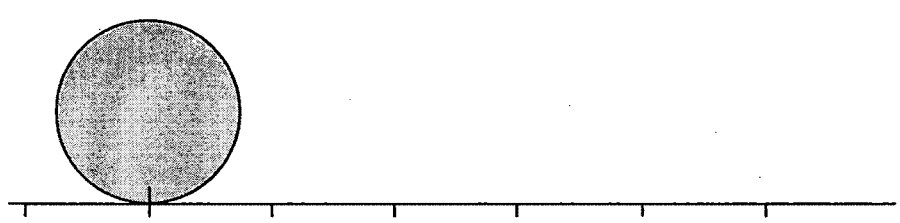
- To understand the perimeter of a circle is proportional to the diameter.
- To understand the perimeter of a circle divided by the diameter is an irrational number denoted by π and it is equal to 3.14.
- To understand the perimeter of a circle of radius r is $2\pi r$.

**Step 1** (Introduction to the Topic)

Average ability students have a mediocre base in basic concepts. So they know about the very simple concepts like perimeter of a square, rectangle etc. Since average ability students are learn from modeling, with help of slide show teachers can easily make them to understand the concept perimeter of a circle.

**Step 2** (Development of the topic)

The average ability students are lack in attention. So the slide show and the lecture will help them to stick on attention. With the help of slide show, teacher explained the following activity in detail. Teacher asks them to roll the model of circle on a scale to measure its perimeter and to record the data in the following activity sheet given below. Since this particular activity demands less precision, they are able to achieve this task.



**Activity Sheet**

| Sl. No. | Perimeter | Diameter | Perimeter/Diameter |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Since the average ability students are not good in read the pattern and to identify the relationship between the variables,

teacher himself/herself collect the data from students and shows through the slide show. Through dialogue teacher conclude that the perimeter divided by diameter of any circle is a constant. With the help of teacher students identifies that perimeter divided by diameter of any circle is a constant called π.

*The number obtained by dividing the perimeter by the diameter is the same for all circles. It is called the number π and is approximately equal to 3.14*

From the above relation, with the help of slide show and dialogue teacher makes them to understand that the formula for finding the perimeter of circle as follows:

Perimeter / Diameter = π

i.e. Perimeter = Diameter × π

Perimeter = 2 × radius × π

Perimeter, C = 2πr (Perimeter of a circle is also called as circumference of a circle)

Since low ability students retain less information and for shorter periods the slide show help them to overcome this deficiency to an extent.

*Perimeter of a circle of radius r is 2πr*

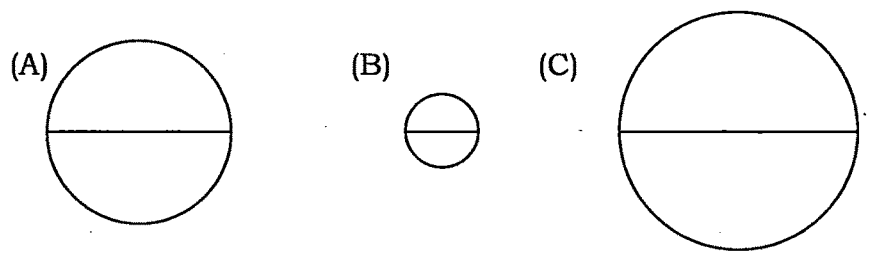
**Step 3** (Application)

Average ability students are expected to perform at grade level. So they should provide with different levels of objective based questions in the increasing order up to the grade level. Remember that, teacher should provide opportunities for those who are going for higher order questions

Question: What is the approximate circumference of a circle of radius 5 cm?

Question: What is the approximate circumference of a circle of radius 12 cm?

Question: For each of the circle measure the diameter and find the circumference.




**Step 3** (Assignment)

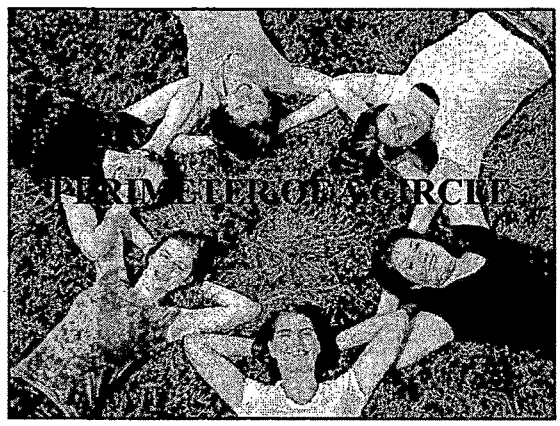
Average ability students need review and practice; teacher should encourage them to do more problems of same kind.

Question: Calculate the circumference of the following circles having diameter is,

| | | | |
|---|---|---|---|
| (a) 10cm | (2a) 20cm | (3a) 30cm | (4a) 40cm |
| (b) 12cm | (2b) 18cm | (3b) 28cm | (4b) 36cm |
| (c) 5.2m | (2c) 8.4m | (3c) 6.6m | (4c) 7.6m |

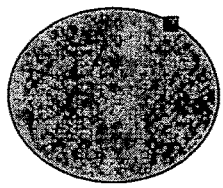
Question: A piece of wire 6 meters long is bent into the shape of a circle. What is the radius of the circle?

Question: Write a page about the constant π




## PERIMETER OF A CIRCLE

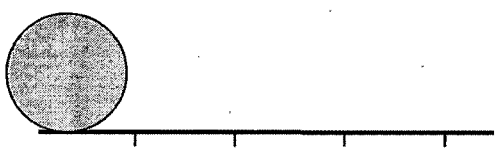
- The distance travelled in starting from a certain point on the circle, making one complete circuit around it and coming back to the starting point



## RELATIONSHIP BETWEEN PERIMETER AND DIAMETER OF A CIRCLE

- Measure the perimeter and diameter of the given circles. When you divide perimeter of a circle by its diameter, did you notice any relation

?



## ACTIVITY SHEET NO.1

| No. | Perimeter | Diameter | Perimeter / Diameter |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

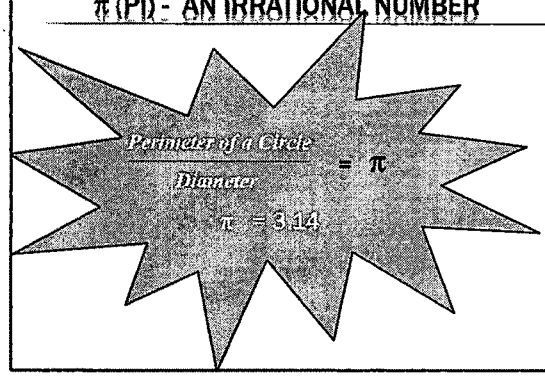
## INFERENCE !!!

*The number obtained by dividing the perimeter by diameter is the same for all circles*

## π (PI) - AN IRRATIONAL NUMBER

- The number obtained by dividing the perimeter by diameter of any circle is called pi and its value approximately is equal to 3.14159. For our practical purposes, it suffices to take its value as 3.14

## π (PI) - AN IRRATIONAL NUMBER




## FORMULA FOR PERIMETER OF A CIRCLE

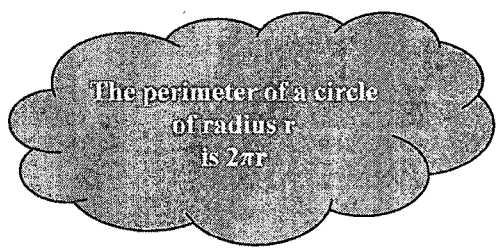
- If the perimeter of a circle (also Known as circumference)of diameter d is C, then

$C / d = \pi$

Therefore $c = \pi d$

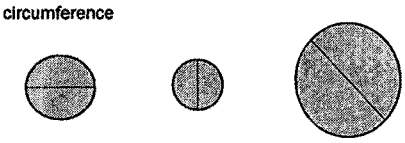
ie. $C = 2\pi r$

## FORMULA FOR PERIMETER OF A CIRCLE




## PROBLEM

- Question: What is the approximate circumference of a circle of radius 5 cm?
- Question: What is the approximate circumference of a circle of radius 12 cm?
- Question: For each of the circle measure the diameter and find the circumference



## ASSIGNMENT

- Question: Calculate the circumference of the following circles having diameter is,

| | | | |
|---|---|---|---|
| (a) 10cm | (2a) 20cm | (3a) 30cm | (4a) 40cm |
| (b) 12cm | (2b) 18cm | (3b) 28cm | (4b) 36cm |
| (c) 5.2m | (2c) 8.4m | (3c) 6.6m | (4c) 7.6m |

- Question: A piece of wire 6 meters long is bent into the shape of a circle. What is the radius of the circle?
- Question: Write a page about the constant π

# DIFFERENTIATED INSTRUCTION FOR LOW ABILITY STUDENTS

## LESSON PLAN – PERIMETER OF A CIRCLE

### CONCEPTS

- Perimeter of a circle
- Π, the constant

### PRINCIPLES OF CLASSROOM INSTRUCTION

- Low ability students require guided instruction and low pace learning environment.
- Low ability students' lack conceptual understanding of mathematics and therefore classroom instruction should begin with simple relevant concepts and link them with the new learning.
- Low ability students have a capacity to succeed in rote learning.
- Low ability students show interest in learning where relationships are clearly demonstrated.
- Low ability students need individualized instruction.
- Low ability students needs review, practice and re-teaching.
- Low ability students provide a group project for evaluation.

### OBJECTIVES

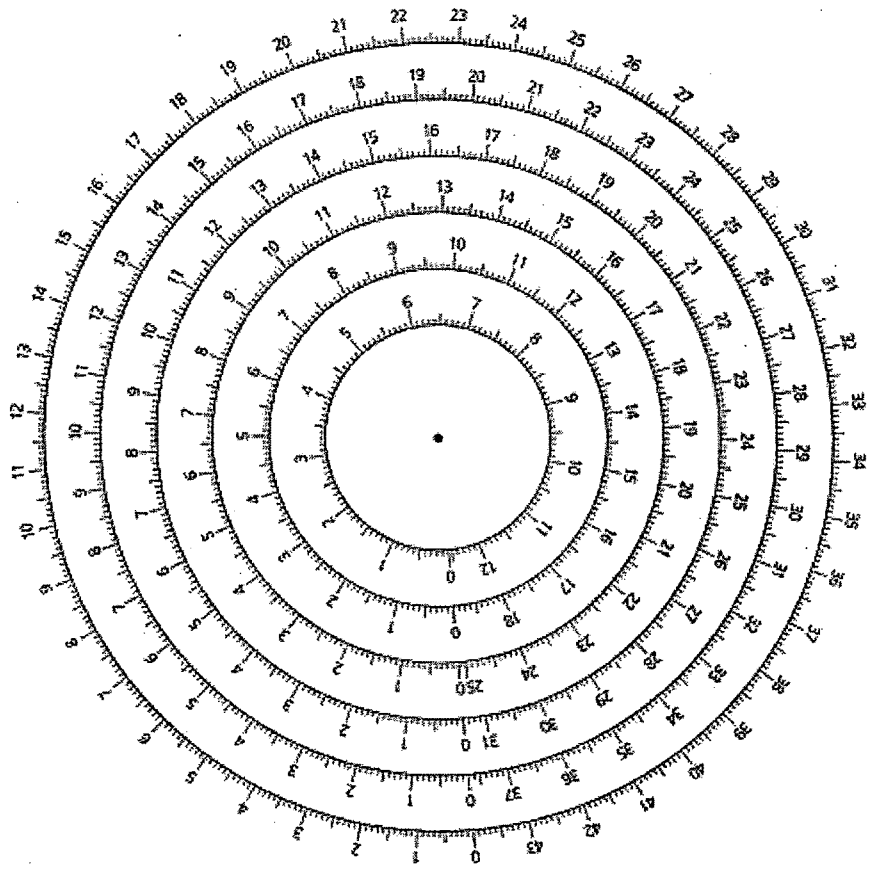
- To understand the perimeter of a circle is proportional to the diameter.
- To understand the perimeter of a circle divided by the diameter is an irrational number denoted by π and it is equal to 3.14.
- To understand the perimeter of a circle of radius r is 2πr.

**Step 1** (Introduction to the Topic)

One of the pertinent characteristics of low ability students is poor memory. So their pre-requisite may not be up to the mark. Teacher makes sure that they understand the concepts like, perimeter of a rectangle, perimeter of a square, perimeter of a circle by asking repeated questions to them. If they need more help teacher should draw these things on the black board and makes them to understand the perimeter of a circle. Teacher should create an environment in the class in such a way that the emotions of the students are protected and interest be nurtured in mathematics.

**Step 2** (Development of the Topic)

Teacher shows the model of concentric circles as shown in the figure below. Teacher makes them to understand that the outer circle is scaled. Teacher gives each model of concentric circles to different groups. Teacher asks them to measure the perimeter by counting the units and diameter of each six circles and direct them to record in the given activity sheet. In this activity the things are required to do are in concrete nature and very less precise in nature. Low ability students are able to do these kinds of small activities. Therefore, a feeling of success in them can be developed and this ultimately increases self esteem. This will increase their self esteem and attitude towards mathematics. Also small activities make them free from boring lecture.



**Activity Sheet**

| Sl. No. | Perimeter | Diameter | Perimeter/Diameter |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

Next task is to observe the pattern in the activity sheet and to identify the relationship between them. Since low ability students are lack in identifying the patterns and relationship, teacher should

play a role for this task. For this teacher should summarize the values in the black board and through dialogue makes them to understand that perimeter of circle divided by its diameter is always a constant and that constant we denoted as π (pie) and the value is approximately equal to 3.14. Since low ability students have a capacity to succeed in rote learning, teacher should repeat this fact at least 10 times by asking the same question to different students.

*The number obtained by dividing the perimeter by the diameter is the same for all circles. It is called the number π and is approximately equal to 3.14*

From the above relation, we want to find a new relation. As said earlier, since low ability students are lack in identifying the patterns and relationship, teacher should illustrate it in the black board and through dialogue makes them to understand that formula for finding the perimeter of circle as follows:

- Perimeter / Diameter = π

i.e. Perimeter = Diameter × π

Perimeter = 2 × radius × π

Perimeter, C = 2πr (Perimeter of a circle is also called as circumference of a circle)

As said earlier, since low ability students have a capacity to succeed in rote learning, teacher should repeat this fact at least 10 times by asking the same question to different students.

*Perimeter of a circle of radius r is 2πr*

**Step 3** (Application of the Topic)

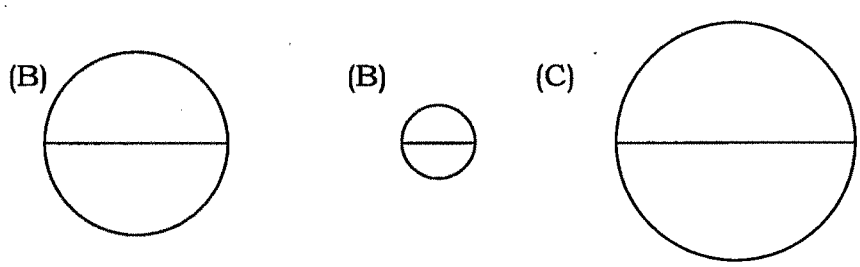
Question: What is the approximate circumference of a circle of radius 5 cm?

Since low ability students have limited cognitive capacity they are unable to do this task. So teacher should demonstrate the task in the black board by writing each step by step in detail.

Question: What is the approximate circumference of a circle of radius 12 cm?

Second question teacher should attempt with the help of low ability students. This will help them gain some confidence to do these kinds of problems.

Teacher should give a problem as group activity as follows:
Question: For each of the circle measure the diameter and find the circumference.




This kind of group activity helps low ability students to understand the concept to an extent. Now they are able to do sole problems individually. We can give more questions as assignments. Note that same type of questions should repeat to four or five times. This will encourage rote learning for them. Also the expected level of attainment is lower application level, because it is the attainable level for them. So don't go beyond that level. When we are giving only attainable task low ability students change from failure oriented to win oriented.

**Step 4** (assignment)

Question: Calculate the circumference of the following circles having diameter is,

| | | | |
|---|---|---|---|
| (d) 10cm | (2a) 20cm | (3a) 30cm | (4a) 40cm |
| (e) 12cm | (2b) 18cm | (3b) 28cm | (4b) 36cm |
| (f) 5.2m | (2c) 8.4m | (3c) 6.6m | (4c) 7.6m |

If they won in these kinds of problems then teacher can discuss little higher level problems in the very next day.