APPENDIX

Some of the integrals used frequently throughout the thesis are given below. They are evaluated using standard techniques (Grandshteyn and Ryzhik 1965).

(A)
$$I_1(q^2,\lambda_1^2,\lambda_2^2) = \int \frac{d\underline{P}}{(P^2+\lambda_1^2)\ (|\underline{q}-\underline{p}|^2+\lambda_2^2)}$$
$$= \frac{\pi}{E} \log \left[\frac{(\lambda_2^2+q^2)\ (\lambda_2^2+q^2+E) - \lambda_1^2\ (\lambda_2^2-q^2)}{\lambda_1^2\ (E+\lambda_2^2-\lambda_1^2-q^2)} \right]$$

where $E^2 = \lambda_1^4 - 2\lambda_1^2\ (\lambda_2^2 - q^2) + (\lambda_2^2+q^2)^2$

The special case of $I_1\ (q^2,\ \lambda_1^2,\ \lambda_2^2)$ with $\lambda_1^2 = \beta^2 + \lambda^2$ and $\lambda_2^2 = \beta^2$ has been denoted as $I_1\ (\beta^2,\ \lambda^2)$ (Yates 1979).

(B)
$$I_2\ (q^2,\ \beta^2,\ \lambda^2) = \mathcal{P} \int d\underline{P} \int_{-\infty}^{\infty} \frac{dP_z}{(P_z-\beta)\ (|q-\underline{P}|^2+P_z^2)(P^2+P_z^2+\lambda^2)}$$
$$= \frac{-\pi^3}{[(q^2+\lambda^2)^2+4q^2\beta^2]^{1/2}} \left[1-\mathrm{Sgn}(\lambda^2-q^2)\left\{\frac{1}{2} - \frac{\sin^{-1} A_1}{\pi}\right\}\right]$$

where $A_1 = 1 - \dfrac{2\beta^2(\lambda^2-q^2)^2}{(\beta^2+q^2)^2(\lambda^2+\beta^2)}$

and $\mathrm{Sgn}\ X = \dfrac{X}{|X|}$

(C) $$I_3\ (\beta,\lambda) = \mathcal{P} \int d\underline{P} \int_{-\infty}^{\infty} \frac{dP_z}{(P_z-\beta)\ (P^2+P_z^2+\lambda^2)}$$

$$= -2\pi^2\left[\frac{\pi}{2} - \tan^{-1}\frac{\lambda}{\beta}\right]$$

(D) $$I_{21}\ (z) = \int \frac{d\underline{P}}{P^2/\underline{z} - \underline{P}/^2}$$

$$= \lim_{k \to 0} \frac{2\pi}{z^2} \ln \frac{z^2}{k^2}$$

(E) $$I_{22}\ (z) = \int \frac{d\underline{P}}{(1+P^2)/\underline{z}-\underline{P}/^2}$$

$$= \lim_{k \to 0} \frac{2\pi}{1+z^2} \ln \frac{1+z^2}{k}$$

(F) $$I_{st}\ (q^2, \lambda_i^2, \lambda_j^2) = \mathcal{P}\int dP \int_{-\infty}^{\infty} \frac{dP_z}{(P_z-\beta)\ (/q-P/^2 + P_z^2 + \lambda_j^2)\ (P^2+P_z^2+\lambda_i^2)}$$

$$= -\pi^2\left[\mathrm{Sgn}\ (y+q^2) \left\{ \frac{1}{2S_1} - \frac{\mathrm{Sin}^{-1}A_1}{\pi S_1} \right\} - \mathrm{Sgn}\ (y-q^2) \left\{ \frac{1}{2S_2} - \frac{\mathrm{Sin}^{-1}A_2}{\pi S_2} \right\}\right][$$

where $y = \lambda_i^2 - \lambda_j^2$

$$S_1^2 = (y+q^2)^2 + 4q^2(\beta^2 + \lambda_i^2)$$

$$S_2^2 = (y-q^2)^2 + 4q^2(\beta^2 + \lambda_j^2)$$

$$A_1 = 1 - \frac{2\beta^2 \ (y+q^2)^2}{[(y+q^2)^2 + 4q^2\lambda_i^{\ 2}] \ (\beta^2+\lambda_i^2)}$$

$$A_2 = 1 - \frac{2\beta^2 (y-q^2)^2}{[(y-q^2)^2 + 4q^2\lambda_j^{\ 2}] \ (\beta^2+\lambda_j^{\ 2})}$$

(G) $$I_4 \ (\beta^2, \lambda_i^{\ 2}, \lambda_j^{\ 2}) = \int \frac{d\underline{P}}{(P^2+\beta^2+\lambda_i^{\ 2}) \ (/q-P/^2+\beta^2+\lambda_j^{\ 2})}$$

is obtained by putting $\lambda_1^2 = \beta^2 + \lambda_i^{\ 2}$

and $\lambda_2^2 = \beta^2 + \lambda_j^{\ 2}$ in $I_1(q^2, \lambda_1^{\ 2}, \lambda_2^{\ 2})$ given above(A).

(H) $I_5 \ (\beta^2, K_{1k}^{\ \ 2}, \ K_{2k}^{\ \ 2}) =$

$$\mathcal{P} \int d\underline{P} \int_{-\infty}^{\infty} \frac{dP_z}{(P_z-\beta) \ (P^2+P_z^{\ 2}+K_{1k}^{\ \ 2}) \ (/q-p/^2+P_z^2 + K_{2k}^{\ \ 2})}$$

which is the same as $I_{St} \ (q^2, \lambda_i^{\ 2}, \lambda_j^{\ 2})$ defined above (F) with $\lambda_i = K_{1k}$ and $\lambda_j = K_{2k}$.

(I) Evaluation of U_{pol} for ESEH process:

To evaluate the U_{pol} for ESEH, the polarised orbital method of Temkin and Lamkin (1961) is followed.

Accordingly,

$$(\nabla_1^{\ 2} + \frac{2}{r_1} - \frac{1}{4}) \ U_{pol} = \frac{2r_1 \ \ Cos\theta}{r_2^{\ 2}} \ U_1 \qquad \text{(a)}$$

Let $$U_{pol} = \frac{1}{4\sqrt{2\pi}} \ \frac{Cos \ \theta \ R^1(r_1)}{r_2^2 \ \ r_1}$$

Substituting $$U_1 = \frac{1}{4\sqrt{2\pi}} \ (2-r_1) \ \ e^{-r_1/2} ,$$

(a) becomes

$$\frac{d^2 R^1}{dr_1^{\ 2}} - \left(\frac{2}{r_1^2} - \frac{2}{r_1} + \frac{1}{4}\right) R^1 = 2r_1^2 \ (2-r_1) \ e^{-r_1/2}$$

Let $R^1(r_1) = e^{-r_1/2} f(r_1)$

Then $\dfrac{d^2f}{dr_1^{\ 2}} - \dfrac{df}{dr_1} - \left(\dfrac{2}{r_1^{\ 2}} - \dfrac{2}{r_1}\right) f = 4r_1^{\ 2} - 2r_1^{\ 3}$ (b)

Let $f = \sum_{n=1}^{\infty} a_n r_1^{\ n}$

Substitution in equation (b) gives

$$f = \frac{2}{5} r_1^{\ 4} - \frac{1}{15} r_1^{\ 5}$$

$$\therefore R^1(r_1) = \frac{1}{5} \left(2r_1^{\ 4} - \frac{r_1^{\ 5}}{3}\right) e^{-r_1/2}$$

and $U_{pol} = \dfrac{1}{4\sqrt{2\pi}} \dfrac{\cos\theta}{r_2^{\ 2}} \dfrac{1}{5} \left(2r_1^{\ 4} - \dfrac{r_1^{\ 5}}{3}\right) \dfrac{e^{-r_1/2}}{r_1} \epsilon(r_1, r_2)$