APPENDIX

The $\int d\underline{p}$, $\int dp_z$ integrals occurred in the second Born approximation of the present calculations in chapter 3 are evaluated using standard integral techniques [Gradshiteyn and Ryhik, 1965]. The closed forms of those typical integrals are as follows:

A 1 $\quad I_1(\beta_i^2, \lambda)$ **integral :**

$$I_1(\beta_i^2, \lambda_i^2) = \int \frac{d\underline{p}}{(|\underline{q}-\underline{p}|^2 + \beta_i^2)(p^2 + \beta_i^2 + \lambda_i^2)}$$

$$= \int_0^{\infty}\int_0^{2\pi} \frac{p\,dp\,d\varphi}{(|\underline{q}-\underline{p}|^2 + \beta_i^2)(p^2 + \beta_i^2 + \lambda_i^2)}$$

$$= \frac{\pi}{\xi}\ln\left[\frac{(q^2+\beta_i^2)(\xi + q^2 + \lambda_i^2) + 2\beta_i^2(q^2 - \lambda_i^2)}{(\beta_i^2 + \lambda_i^2)(\xi - q^2 - \lambda_i^2)}\right],$$

where $\xi^2 = (\lambda_i^2 + q^2)^2 + 4q^2\beta_i^2$.

$$\therefore I_1(\beta_i^2, 0) = \int \frac{d\underline{p}}{(|\underline{q}-\underline{p}|^2 + \beta_i^2)(p^2 + \beta_i^2)} = I_1(\beta_i^2, \lambda_i^2)\Big|_{\lambda_i = 0}$$

A 2 $\quad I_4(\beta_i^2, \lambda_i^2, \lambda_j^2)$ **integral :**

$$I_4(\beta_i^2, \lambda_i^2, \lambda_j^2) = \int \frac{d\underline{p}}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + \lambda_i^2)(p^2 + \beta_i^2 + \lambda_j^2)}$$

$$= \int_0^{\infty}\int_0^{2\pi} \frac{p\,dp\,d\varphi}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + \lambda_i^2)(p^2 + \beta_i^2 + \lambda_j^2)}$$

$$= \frac{\pi}{E}\ln\left[\frac{(q^2+v^2)(q^2+v^2+E) - u^2(v^2-q^2)}{u^2(E^2+v^2-u^2-q^2)}\right],$$

where $E^2 = u^4 + (q^2+v^2)^2 - 2u^2(v^2-q^2)$,

where $v^2 = \beta_i^2 + \lambda_i^2$ and $u^2 = \beta_i^2 + \lambda_j^2$.

<u>A 3</u> ***$I_2(\beta_l, \lambda^2)$* integral :**

$$I_2(\beta_l,\lambda_l^2) = \mathrm{P}\int d\underline{p} \int_{-\infty}^{\infty} \frac{dp_z}{(p_z-\beta_l)(|\underline{q}-\underline{p}|^2+p_z^2)(p^2+p_z^2+\lambda_l^2)}$$

$$= -\frac{\pi^3}{\xi}\left\{1-\mathrm{sgn}(\lambda_l^2-q^2)\left[\frac{1}{2}-\frac{1}{\pi}Sin^{-1}\left(1-\frac{2\beta_l^2(\lambda_l^2-q^2)^2}{(\lambda_l^2-q^2)(\beta_l^2+\lambda_l^2)}\right)\right]\right\}$$

where $\xi^2=(\lambda_l^2+q^2)^2+4q^2\beta_l^2$. and $\mathrm{sgn}(\lambda^2-q^2)=+1\,;\lambda^2>q^2$

$=-1\,;\lambda^2<q^2$.

$$\therefore I_2(\beta_l^2,0) = \mathrm{P}\int d\underline{p} \int_{-\infty}^{\infty} \frac{dp_z}{(p_z-\beta_l)(|\underline{q}-\underline{p}|^2+p_z^2)(p^2+p_z^2)} = I_2(\beta_l^2,\lambda_l^2)\Big|_{\lambda_l=0}$$

<u>A 4</u> ***$I_5(\beta_l^2, \lambda_l^2, \lambda_j^2)$* integral :**

$$I_5(\beta_l^2,\lambda_l^2,\lambda_j^2) = \mathrm{P}\int d\underline{p} \int_{-\infty}^{\infty} \frac{dp_z}{(p_z-\beta_l)(|\underline{q}-\underline{p}|^2+\beta_l^2+\lambda_l^2)(p^2+\beta_l^2+\lambda_j^2)}$$

$$= \pi^2\left[\mathrm{sgn}(Y+q^2)\left\{\frac{1}{2E_1}-\frac{Sin^{-1}A_1}{\pi E_1}\right\}-\mathrm{sgn}(Y-q^2)\left\{\frac{1}{2E_2}-\frac{Sin^{-1}A_2}{\pi E_2}\right\}\right];$$

where $E_1^2=(Y+q^2)^2+4q^2(\beta_l^2+\lambda_j^2)$; $Y=\lambda_j^2-\lambda_l^2$

$$E_2^2=(Y-q^2)^2+4q^2(\beta_l^2+\lambda_l^2);$$

$$A_1=1-\frac{2\beta_l^2(Y+q^2)^2}{[(Y+q^2)^2+4q^2\lambda_j^2](\beta_l^2+\lambda_j^2)}$$

$$A_2=1-\frac{2\beta_l^2(Y-q^2)^2}{[(Y-q^2)^2+4q^2\lambda_l^2](\beta_l^2+\lambda_l^2)}$$

$\mathrm{sgn}(Y+q^2)=+1\,;Y+q^2>0$ $\quad$ $\mathrm{sgn}(Y-q^2)=+1\,;Y>q^2$

$=-1;\,Y+q^2<0$ $\quad$ $=-1;\,Y<q^2$

<u>A 5</u> ***$I_3(\beta_l, \lambda)$* integral :**

$$I_3(\beta_l,\lambda_l^2) = \mathrm{P}\int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z}{(p_z-\beta_l)(p^2+p_z^2+\lambda_l^2)}$$

$$= -\pi^3\left[1-\frac{2}{\pi}\tan^{-1}\frac{\lambda}{\beta_l}\right].$$

$$\therefore I_3(\beta_l,0) = \mathrm{P}\int d\underline{p} \int_{-\infty}^{+\infty} \frac{dp_z}{(p_z-\beta_l)(p^2+p_z^2)} = I_3(\beta_l,\lambda_l^2)\Big|_{\lambda_l=0}.$$