CHAPTER - IV

SCATTERING OF ELECTRONS BY HELIUM ATOMS

4.1 Introduction :

Inspired by the success of the hydrogen related problems, which are presented in the previous chapter, now we turn our attention to the scattering of electron by helium (He) ($z = 2$) atom, where there is a very large amount of data (Bromberg , 1969 ; Crooks and Rudd, 1971 ; Oda *et al*, 1972 ; Bromberg, 1974 ; Jansen *et al*, 1974 ; Sethuraman *et al*, 1974 ; McConkey and Preston, 1975 ; Jansen *et al*, 1976 ; Byron and Joachain, 1977) available for the comparison of present theory . From the theoretical point of view the situation is nearly identical to that of atomic hydrogen, with the only important difference being that for helium we must rely on approximate wave functions in the various models described in chapter II. If one uses a Hartree - Fock functions to describe the ground and excited states of helium, then all the quantities evaluated for atomic hydrogen (Chapter - III) can be handled in the same manner as for helium. In the present helium related problems we consider the following $\bar{e}$ - He collision processes.

i) Elastic scattering of electrons by the ground state of helium atom using the Hartree - Fock wave functions and with the two types of closure approximations in the second Born approximation (equations 2.55, 2.56) (1s - 1s).

ii) Elastic scattering of electrons by helium atom in the ground state using Hylleraas wave function.

iii) Inelastic scattering of electrons by helium atom using Hartree - Fock wave functions (1s - 2s).

In these $\bar{e}$ - He interaction processes, we calculate DCS (equation 3.2),TCS (equation 3.3) at incident energies 100 to 700 eV for the elastic process and at 200 and 400 eV for inelastic process. In the derivation of scattering amplitudes (equations 2.12, 2.57, 2.36, 2.45) for these problems, first we consider a typical exponential term from the product of the wave functions. For this term we construct the scattering amplitudes, from these closed forms, we extend these results to the remaining exponential terms of the product of the wave functions. Since the helium atom is a two electron system, the interaction (equation 2.24) between the incident electron and the target is complicated than the hydrogen (equation 3.7) atom. The evaluation of the volume integrals are difficult due to the coupling of the radial coordinates r_1 and r_2 of the target electrons. These difficulties are removed in the present study.

4.2.1 Elastic scattering of electrons by the ground (1s) state of helium atom (ESGHe) :

(Rao and Desai, 1981)

$$\bar{e} + He\ (1s) \longrightarrow \bar{e} + He\ (1s) \qquad (4.1)$$

Similar to hydrogen atom (Sec. 3.2.1) here also the final Hartree - Fock state function of the target helium atom is same as the initial ground state function. The well known Hartree - Fock wave function for the ground state of helium atom can be written as

$$\psi_{1s}(r_1, r_2) = \phi_o(r_1)\ \phi_o(r_2)$$

$$= \frac{(P + Q)}{\sqrt{4\ \pi}}\ \frac{(R + S)}{\sqrt{4\ \pi}} \qquad (4.2)$$

where $P = A \exp(-y' r_1)$, $Q = B \exp(-y'' r_1)$,

and $R = A \exp(-y' r_2)$, $S = B \exp(-y'' r_2)$.

The normalization constants, and exponential parameters of these defined quantaties for (equation 4.2) can be given as

$A = 2.60505$, $B = 2.08144$, $y' = 1.41$ and $y'' = 2.61$.

Now the product of the initial and final states is

$$\psi_i (r_1 , r_2) \ \psi_f^* (r_1 , r_2) = \frac{1}{16 \pi^2} [(P + Q)^2 (R + S)^2]$$

$$= \frac{1}{16 \pi^2} [(P^2 R^2 + Q^2 S^2 + 4 P Q R S) + (P^2 S^2 + Q^2 R^2) + 2 (P Q S^2 + Q^2 R S) + 2 (P R^2 Q + P^2 R S)]$$

$$= \frac{1}{16 \pi^2} [(P^2 R^2 + Q^2 S^2 + 4 P Q R S) + (2 P^2 S^2) + 2 (2 P Q S^2) + 2 (2 P R^2 Q)] \qquad (4.3)$$

All the terms in this expression can be written in the derivative form like equation (3.6) . Now consider a typical term of equation (4.3).

$$2 P R^2 Q = 2 A^3 B \ \exp (- (y' + y'') r_1) \ \exp (- 2 y'' r_2)$$

$$= k \exp (- y (I) r_1) \ \exp (y (J) r_2)$$

$$= k \ D^n (y_1) \frac{\exp (-y_1 r_1)}{r_1} D^m (y_2) \frac{\exp (- y_2 r_2)}{r_2} \qquad \ldots\ldots(4.4)$$

where $m = n = 1$, $k = 2 A^3 B$, $y_1 = y' + y''$, $y_2 = 2 y''$, like this all the terms in (equation 4.3) can be obtained. Throughout this chapter , we will consider type of equation like (4.4) for the evaluation of scattering amplitudes (equations 2.12, 2.57 , 2.36). Finally we extend these results to equation (4.3).

The interaction between the incident electron and target helium atom can be written as

$$V_d = - \frac{2}{r_o} + \frac{1}{|\underline{r}_o - \underline{r}_1|} + \frac{1}{|\underline{r}_o - \underline{r}_2|} \qquad (4.5)$$

where $\underline{r}_o$, $\underline{r}_1$ and $\underline{r}_2$ are the position vectors of the incident electron and target electrons with respect to the target nuclei. Substituting the equations (4.3) and (4.5) in the scattering amplitudes (eqations 2.12, 2.57, 2.36) we will obtain the DCS (equation 3.2) O (k_i^{-2}) . The evaluation of these amplitudes is as follows .

$$f^{(1)}_{i \to f} = - \frac{1}{2\pi} \int d\,\underline{r}_o \ \exp(i\,\underline{q}.\underline{r}_o)\ V_{fi}(r_o) \qquad (4.6)$$

Where

$$V_{fi}(r_o) = \frac{1}{16\pi^2} \int\int d\,\underline{r}_1\ d\,\underline{r}_2\ \left[- \frac{2}{r_o} + \frac{1}{|\underline{r}_o - \underline{r}_1|} + \frac{1}{|\underline{r}_o - \underline{r}_2|} \right] ((P^2 R^2 + Q^2 S^2 + 4 P Q R S) + (2 P^2 S^2) + 2 (2 P Q S^2)$$

$$+ 2 (2 P R^2 Q)) \qquad (4.7)$$

$$= \frac{1}{16 \pi^2} \int \int d \underline{r}_1 \; d \underline{r}_2 \; \Psi_i (r_1 , r_2) V_d \; \Psi_f^* (r_1 , r_2)$$

where

$$P^2 R^2 = A^4 \exp (- y y' (r_1 + r_2))$$

$$Q^2 S^2 = B^4 \exp (- y y''(r_1 + r_2))$$

$$P^2 S^2 = A^2 B^2 \exp (- y (y' r_1 + y'' r_2))$$

$$4 P Q R S = 4A^2 B^2 \exp (- y y''' (r_1 + r_2))$$

$$2 P Q S^2 = 2B^3 A \exp (- y (y'' r_2 + y''' r_1))$$

$$2 P R^2 Q = 2A^3 B \exp (- y (y' r_2 + y''' r_1))$$

here $y = 2$, and $y''' = y' + y''/2$. For the evaluation of $d \underline{r}_o$, $d \underline{r}_1$ and $d \underline{r}_2$ integrals in equations (4.6) and (4.7) consider a typical exponential term of equation (4.7).

$$P^2 S^2 = A^2 B^2 \exp (- y (y' r_1 + y'' r_2))$$

$$= V \exp (- y (M r_1 + N r_2)) \qquad (4.8)$$

Substituting this term for (------) terms in equation

(4.7), we will obtain the closed form of equation (4.6) for equation (4.8).

$$f_i^t = \frac{-V}{2\pi\, 16\, \pi^2} \int d\underline{r}_0 \exp(i\, \underline{q}\cdot\underline{r}_0) \int\int d\underline{r}_1\, d\underline{r}_2$$

$$\left[\frac{-2}{r_0} + \frac{1}{|\underline{r}_0 - \underline{r}_1|} + \frac{1}{|\underline{r}_0 - \underline{r}_2|} \right]$$

$$\exp(-y(M\, \underline{r}_1 + N\, r_2))$$

$$= \frac{+V}{2\pi\, y^4\, M^3\, N^3} \int d\underline{r}_0 \exp(i\, \underline{q}\cdot\underline{r}_0) \left[\left(M + \frac{1}{r_0}\right) \right.$$

$$\left. \exp(-y\, M\, r_0) + \left(N + \frac{1}{r_0}\right) \exp(-N\, r_0\, y) \right]$$

$$= \frac{2V}{y^4\, M^3\, N^3} \left[\frac{(8M^2 + q^2)}{(4M^2 + q^2)^2} + \frac{(8N^2 + q^2)}{(4N^2 + q^2)^2} \right] \quad .....(4.9)$$

if $N = M$

$$= \frac{4V}{y^4\, M^3\, M^3} \left[\frac{(8M^2 + q^2)}{(4M^2 + q^2)^2} \right] \Bigg|_{y=2} \qquad (4.10)$$

Using equations (4.9) and (4.10) we can get the closed form of equation (4.6) . The reduced form of this Born amplitude can be obtained as

$$f_{i-f}^{(1)} = \left[\frac{A^4}{y^2\, y'^6} + \frac{A^2\, B^2}{y^2\, y'^3\, y''^3} + \frac{A^3\, B}{y\, y'^3\, y'''^3} \right] \left[\frac{(\,8\, y'^2 + q^2\,)}{(\,4\, y'^2 + q^2\,)^2} \right] +$$

$$\left[\frac{A^2\, B^2}{y^2\, y'^3\, y''^3} + \frac{B^4}{y^2\, y''^6} + \frac{B^3\, A}{y\, y'''^3\, y''^3} \right] \left[\frac{(\,8\, y''^2 + q^2\,)}{(\,4\, y''^2 + q^2\,)^2} \right] +$$

$$\left[\frac{B^3\, A}{y\, y'''^3\, y''^3} + \frac{A^2\, B^2}{y'''^6} + \frac{A^3\, B}{y\, y'^3\, y'''^3} \right] \left[\frac{(\,8\, y'''^3 + q^2\,)}{(\,4\, y'''^2 + q^2\,)^2} \right]$$

$$= \sum_{k=1}^{3} C_k \left[\frac{(\,8\, y_k^2 + q^2\,)}{(\,4\, y_k^2 + q^2\,)} \right] \qquad (4.11)$$

where C_k's and y_k's are constants given as

$C_1 = 2.420884$; $C_2 = 0.2336732$; $C_3 = 1.33543$

$y_1 = 1.41$; $y_2 = 2.61$; $y_3 = 2.01$

Equation (4.11) is the first Born approximation for the ESGHe process.

Now the imaginary part of the second Born amplitude equation (2.60) can be written as

$$\mathrm{Im}\ f^{(2)}_{HEA} = \frac{4\ \pi^3}{k_i} \int d\underline{p}\ U^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{p} + \beta_i \hat{y}) \qquad \ldots(4.12)$$

where

$$U^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{p} + \beta_i \hat{y}) = \langle \psi^*_f (r_1 , r_2) | \bar{V} (\underline{q} - \underline{p} - \beta_i \hat{y} ; r_1 , r_2) \bar{V} (\underline{p} - \beta_i \hat{y} ; r_1 , r_2) | \psi_i (r_1 , r_2) \rangle$$

$\bar{V}$ (------)'s can be substituted from equation (2.53) in the above expression

$$= \frac{1}{4\ \pi^4 (| \underline{q} - \underline{p} |^2 + \beta_i^2) (p^2 + \beta_i^2)} \int\int d\underline{r}_1\ d\underline{r}_2\ \psi^*_f (r_1 , r_2) \psi_i (r_1 , r_2)$$

$$[\exp (i (\underline{q} - \underline{p}) . \underline{b}_1 - i \beta_i Z_1) + \exp (i (\underline{q} - \underline{p}) . \underline{b}_2 - i \beta_i Z_2) - 2]$$

$$[\exp (i \underline{p} . \underline{b}_1 + i \beta_i Z_1) + \exp (i \underline{p} . \underline{b}_2 + i \beta_i Z_2) - 2]$$

$$= \frac{1}{4\pi^4 (| \underline{g} - \underline{p} |^2 + \beta_i^2)(p^2 + \beta_i^2)} \int\int d\,\underline{r}_1 \; d\,\underline{r}_2$$

$$\psi_f^* (r_1 , r_2) \; \psi_i (r_1 , r_2)$$

$$[\exp (i \underline{g} \cdot \underline{b}_1) - 2 \exp (i (\underline{g} - \underline{p}) \cdot \underline{b}_1 - i \beta_i Z_1)$$

$$- 2 \exp (i \underline{p} \cdot \underline{b}_1 + i \beta_i Z_1) + \exp (i \underline{g} \cdot \underline{b}_2) -$$

$$2 \exp (i (\underline{g} - \underline{p}) \cdot \underline{b}_2 - i \beta_i Z_2) - 2 \exp$$

$$(i \underline{p} \cdot \underline{b}_2 + i \beta_i Z_2) + \exp (i (\underline{g} - \underline{p}) \cdot \underline{b}_1 +$$

$$i \beta_i Z_2 + i \underline{p} \cdot \underline{b}_2 - i \beta_i Z_1) + 4 + \exp (i$$

$$(\underline{g} - \underline{p}) \cdot \underline{b}_2 + i \beta_i Z_1 + i \underline{p} \cdot \underline{b}_1 - i \beta_i Z_2)]$$

...(4.13)

Substituting a typical exponential term (equation 4.4)for the product of the wave functions (equation 4.3), we can evaluate the $d\,\underline{r}_1$ and $d\,\underline{r}_2$ integrals in equation(4.13).

$$U^t_{fi} (\text{------}, \text{------}) = \frac{k \; D (y_1) \; D (y_2)}{4\pi^4 (| \underline{g} - \underline{p} |^2 + \beta_i^2)(p^2 + \beta_i^2)}$$

$$\int\int \frac{\exp (- y_1 r_1)}{r_1} \frac{\exp (- y_2 r_2)}{r_2}$$

$$[\text{- - - - - - - - - - - - -}] \; d\,\underline{r}_1 \; d\,\underline{r}_2$$

The integral procedure for $d\,\underline{r}_1$ and $d\,\underline{r}_2$ is similar to that of ESGH process (given under equation 3.10). The closed form of the above integrals can be obtained as

$$= \frac{k\,(4\pi)^2}{4\pi^4\,(|\underline{q}-\underline{p}|^2+\beta_i^2)\,(p^2+\beta_i^2)}\Big[\,D(y_1)\frac{1}{y_1^2}\,D(y_2)\left\{\frac{1}{q^2+y_2^2}+\frac{1}{y_2^2}-\frac{D(y_2)}{(|\underline{q}-\underline{p}|^2+\beta_i^2+y_2^2)}-\frac{D(y_2)}{(p^2+\beta_i^2+y_2^2)}\right\}+D(y_2)\frac{1}{y_2^2}\,D(y_1)$$

$$\left\{\frac{1}{q^2+y_1^2}+\frac{1}{y_1^2}-\frac{D(y_1)}{(|\underline{q}-\underline{p}|^2+\beta_i^2+y_1^2)}-\frac{D(y_1)}{(p^2+\beta_i^2+y_1^2)}\right\}+\left\{\frac{D(y_1)\;D(y_2)}{y_1^2\;\;y_2^2}-\right.$$

$$\frac{D(y_1)\;D(y_2)}{(|\underline{q}-\underline{p}|^2+\beta_i^2+y_1^2)\,y_2^2}-\frac{D(y_1)\;D(y_2)}{y_1^2\,(p^2+\beta_i^2+y_2^2)}$$

$$\left.+\frac{D(y_1)\;D(y_2)}{(|\underline{q}-\underline{p}|^2+\beta_i^2+y_1^2)\,(p^2+\beta_i^2+y_2^2)}\right\};$$

$$+ \left\{ \frac{D(y_2)}{y_2^2}\frac{D(y_1)}{y_1^2} - \frac{D(y_2)\, D(y_1)}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + y_1^2)} - \right.$$

$$\left. \frac{D(y_2)\, D(y_1)}{y_2^2\,(p^2 + \beta_i^2 + y_1^2)} + \frac{D(y_2)\, D(y_1)}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + y_2^2)(p^2+\beta_i^2+y_1^2)} \right\}]$$

.....(4.14)

Substitution of equation (4.14) in equation (4.12), we will obtain this closed form for a typical exponential term equation (4.4) of the product of the wave functions equation (4.3).

$$\text{Im } f^{t}_{HEA} = \frac{(4\pi)^2 k}{\pi k_i} \int \frac{d\,\underline{p}}{(p^2+\beta_i^2)(|\underline{q}-\underline{p}|^2+\beta_i^2)}$$

$$\left[D(y_1) \frac{1}{y_1^2} D(y_2) \left\{ \frac{q^2 + 2y_2^2}{q^2 + y_2^2} - \right. \right.$$

$$\left. \frac{D(y_2)}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + y_2^2)} - \frac{D(y_2)}{(p^2+\beta_i^2+y_2^2)} \right\}$$

$$+ D(y_2) \frac{1}{y_2^2} D(y_1) \left\{ \frac{q^2 + 2y_1^2}{q^2 + y_1^2} - \right.$$

$$\left. \frac{D(y_1)}{(|\underline{q}-\underline{p}|^2 + \beta_i^2 + y_1^2)} - \frac{D(y_1)}{(p^2+\beta_i^2+y_1^2)} \right\}$$

$$+ \left\{ D(y_1) \left(\frac{1}{y_1^2} - \frac{1}{(| \underline{g} - \underline{p} |^2 + \beta_i^2 + y_1^2)} \right) D(y_2) \left(\frac{1}{y_2^2} - \frac{1}{p^2 + \beta_i^2 + y_2^2} \right) \right\} + \left\{ D(y_2) \left(\frac{1}{y_2^2} - \frac{1}{(| \underline{g} - \underline{p} |^2 + \beta_i^2 + y_2^2)} \right) D(y_1) \left(\frac{1}{y_1^2} - \frac{1}{p^2 + \beta_i^2 + y_1^2} \right) \right\}] \quad (4.15)$$

Using the results of ESGH (equation 3.11) and ESEH (equation 3.24) processes this can be obtained as

$$= \frac{(4\pi)^2 k}{\pi k_i} \left[- \frac{2}{y_1^3} D(y_2) \frac{H(y_2)}{y_2^2} - \frac{2}{y_2^3} D(y_1) \frac{H(y_1)}{y_1^2} + 2 D(y_1) D(y_2) \frac{I_4'(y_1^2, y_2^2)}{y_1^2 y_2^2} \right] \quad (4.16)$$

if $y_1 = y_2$

$$= \frac{(4\pi)^2 k}{\pi k_i} \left[- \frac{4}{y_2^3} D(y_2) \frac{H(y_2)}{y_2^2} \right.$$

$$\left. + 2 D(y_2) D(y_2) \frac{I_4'(y_2^2, y_2^2)}{y_2^2 \; y_2^2} \right] \qquad (4.17)$$

where the defined quantaties $H(y_1)$ and $I_4'(y_1^2, y_2^2)$ are given as

$$H(y_1) = 2 I_1 (\beta_i^2, y_1^2) - \frac{q^2}{(q^2 + y_1^2)} I_1' (\beta_i^2, 0)$$

and
$$I_4'(y_1^2, y_2^2) = \int \frac{d\underline{p}}{(|\underline{q} - \underline{p}|^2 + \beta_i^2 + y_1^2)(p^2 + \beta_i^2 + y_2^2)}$$

These are given in ESGH (equation 3.11) and ESEH (equation 3.24) processes and the closed form of these will be derived in appendix using the results (equations 4.13 to 4.17) of the typical exponential term (equation 4.4) we can obtain the closed form of (equation 4.12) through the equations (4.3 and (4.13) . The final form of the imaginary contribution in this ESGHe process can be obtained as

$$\mathrm{Im}\, f^{(2)}_{HEA} = \frac{1}{\pi k_i} \left[- \left(\frac{4 A^4}{y_1^3} + \frac{4 A^2 B^2}{y_2^3} + \frac{8 A^3 B}{y_3^3} \right) \right.$$

$$D(y_1)\frac{H(y_1)}{y_1^2} - \left(\frac{4B^4}{y_2^3} + \frac{4A^2B^2}{y_1^3} + \frac{8B^3A}{y_3^3}\right)$$

$$D(y_2)\frac{H(y_2)}{y_2^2} - \left(\frac{16A^2B^2}{y_3^2} + \frac{8A^3B}{y_1^3} + \frac{8AB^3}{y_2^3}\right)$$

$$D(y_3)\frac{H(y_3)}{y_3^2} + 2A^4 D(y_1)\ D(y_1)\frac{1}{y_1^2\ y_1^2}$$

$$I'_4(y_1^2, y_1^2) + 2B^4 D(y_2)\ D(y_2)\ \frac{1}{y_2^2\ y_2^2}$$

$$I'_4(y_2^2, y_2^2) + 8A^2B^2 D(y_3)\ D(y_3)\frac{1}{y_3^2\ y_3^2}$$

$$I'_4(y_3^2, y_3^2) + 4A^2B^2 D(y_1)\ D(Y_2)\frac{1}{y_1^2\ y_2^2}$$

$$I'_4(y_1^2, y_2^2) + 8B^3A\ D(y_2)\ D(y_3)\frac{1}{y_2^2\ y_3^2}$$

$$I'_4(y_2^2, y_3^2) + 8A^3B\ D(y_3)\ D(y_1)\frac{1}{y_1^2\ y_3^2}$$

$$I'_4(y_1^2, y_3^2)\ ]$$

$$= \frac{1}{\pi k_i} \sum_{\substack{k=1,2,3\\ J=2,3,1}} \left[- A_k D(y_k)\frac{H(y_k)}{y_k^2} + B_{kk} D(y_k)\right.$$

$$D(y_k)\ \frac{I_4'(y_k^2, y_k^2)}{y_k^2\ y_k^2} + B_{kJ}\ D(y_k)\ D(y_J)$$

$$\frac{I_4'(y_k^2, y_J^2)}{y_k^2\ y_J^2}\ ] \qquad (4.18)$$

A_k' s , B_{kk}'s and y_k' s , B_{kJ}' s are constants given as

y_1 = 2.82, A_1 = 13.573, B_{11} = 92.1074 , B_{12} = 117.604

y_2 = 5.22 , A_2 = 8.665 , B_{22} = 37.5392 , B_{23} = 187.931

y_3 = 4.02, A_3 = 21.689, B_{33} = 235.208, B_{31} = 294.376

Now the real part of order k_i^{-1} of the second Born amplitude (equation 2.58) can be written as

$$\text{Rel}\ f^{(2)}_{HEA} = \frac{-4\pi^2}{k_i}\ \mathcal{P} \int d\underline{p} \int_{-\infty}^{\infty} \frac{d\, p_Z}{(p_Z - \beta_i)}$$

$$U^{(2)}_{fi}\ (\underline{q} - \underline{p} - p_Z\,\hat{y}\ ;\ \underline{p} + p_Z\,\hat{y}) \qquad (4.19)$$

The basic difference between this real part and imaginary part equation (4.12) is only the principal value integral $d\,p_Z$, the evaluation of $d\,\underline{r}_1$ and $d\,\underline{r}_2$ integrals are same as imaginary. Replacing β_i in equation (4.15) by p_Z and following the results of ESGH (equation 3.13) process, we will obtain the closed form of equation

(4.19) for a typical exponential term (equation 4.4) of the wave function product equation (4.3).

$$\mathrm{Rel}\ f^{t}_{HEA} = \frac{(4\pi)^2 k}{\pi^2 k_i} \Big[\frac{2}{y_1^3} D(y_2) \frac{H'(y_2)}{y_2^2} + \frac{2}{y_2^3} D(y_1) \frac{H'(y_1)}{y_1^3} - 2 D(y_1) D(y_2) \frac{I_4(y_1^2, y_2^2)}{y_1^2\ y_2^2} \Big] \qquad (4.20)$$

If $y_1 = y_2$ a similar type of expression can be obtained as equation (4.17) for equation (4.20). Where the $H'(y_1)$ and $I_4(y_1^2, y_2^2)$ are given as

$$H'(y_1) = 2 I_2(\beta_i^2, y_1^2) - \frac{q^2}{(q^2 + y_1^2)} I_2'(\beta_i^2, 0)$$

and
$$I_4(y_1^2, y_2^2) = \mathcal{P} \int_{-\infty}^{\infty} \frac{d p_Z}{(p_Z - \beta_i)} \int \frac{d \underline{p}}{(|\underline{q} - \underline{p}|^2 + p_Z^2 + y_1^2)(p^2 + p_Z^2 + y_2^2)}$$

$I_2(\beta_i^2, y_1^2)$ and $I_2'(\beta_i^2, 0)$ are given in ESGH

(equation 3.13) process. And $I_4 (y_1^2, y_2^2)$ is similar to $I_2 (\beta_i^2, y_1^2)$, the closed form of this is given in appendix. Now using the results of equation (4.20), and substituting the equation (4.3) in equation (4.19), we will obtain the real part contribution in this ESGHe process as

$$\mathrm{Re1}\ f^{(2)}_{HEA} = \frac{1}{\pi^2 \cdot k_i} \sum_{\substack{k=1,2,3 \\ J=2,3,1}} \Big[A_k\, D (y_k)\, \frac{H' (y_k)}{y_k^2} - B_{kk}\, D (y_k)\, D (y_k)\, \frac{I_4(y_k^2, y_k^2)}{y_k^2\, y_k^2} - B_{kJ}\, D(y_k)\, D (y_J)\, \frac{I_4 (y_k^2 , y_J^2)}{y_k^2\, y_J^2} \Big] \quad .....(4.21)$$

All the constants in this amplitude are some as given under equation (4.18).

The real part of order k_i^{-2} of the second Born approximation equation (2.59) can be written as

$$\mathrm{Re2}\ f^{(2)}_{HEA} = -\frac{2\pi^2}{k_i^2}\, D'\, \mathcal{P} \int d\underline{p} \int_{-\infty}^{\infty} \frac{d p_Z\, (p^2 + p_Z^2)}{(p_Z - \beta_i)}\, U^{(2)}_{fi} (\underline{q} - \underline{p} - p_Z \hat{y} ; \underline{p} + p_Z \hat{y}) \qquad (4.22)$$

By comparing the ESGHe amplitudes (equations 4.18, 4.21) with the ESGH amplitudes (equations 3.11 , 3.13) we can directly obtain the closed form of equation (4.22) for a typical exponential term (equation 4.4) of the wave functions product equation (4.3).

$$\mathrm{Re}\,2\, f^{t}_{HEA} = \frac{(4\pi)^2 k}{2\pi^2 k_i^2} D' \Big[\frac{2}{y_1^3} D(y_2) H''(y_2) + \frac{2}{y_2^3} D(y_1) H''(y_1) - D(y_1) D(y_2) \frac{I_5(y_1^2, y_2^2)}{y_1^2 \; y_2^2} \Big] \qquad (4.23)$$

where $H''(y_2)$ and $I_5(y_1^2, y_2^2)$ are given as

$$H''(y_1) = \frac{I'_3(\beta_i, 0)}{(q^2 + y_1^2)} + \frac{I_3(\beta_i, y_1^2)}{y_1^2} - I_2(\beta_i^2, y_1^2)$$

and

$$I_5(y_1^2, y_2^2) = \mathcal{P} \int_{-\infty}^{\infty} \frac{d p_Z}{(p_Z - \beta_i)} \int \frac{d \underline{p}\,(p^2 + p_Z^2)}{(|\underline{q} - \underline{p}|^2 + p_Z^2 + y_1^2)(p^2 + p_Z^2 + y_2^2)}$$

$$+ \mathcal{P} \int_{-\infty}^{\infty} \frac{d\,p_Z}{(p_Z - \beta_i)} \int \frac{d\,\underline{p}\ (p^2 + p_Z^2)}{(|\underline{q} - \underline{p}|^2 + p_Z^2 + y_2^2)(p^2 + p_Z^2 + y_1^2)}$$

$$= I_3(\beta_i^2, y_1^2) - I_4(y_1^2, y_2^2)(y_1^2 + y_2^2)$$

$$+ I_3(\beta_i^2, y_2^2)$$

The above three terms are obtained by adding and deducting y_1^2 and y_2^2 in the preceding two terms of $I_5(y_1^2, y_2^2)$, and making use of the previous results . Using equation (4.23), the real part (4.22) can be obtained through the equations (4.3, 4.12).

$$\mathrm{Re}\,2\, f^{(2)}_{HEA} = \frac{1}{2\pi^2 k_i^2} \sum_{\substack{k=1,2,3 \\ J=2,3,1}} D' \Big[A_k D(y_k) H''(y_k) - \frac{B_{kk}}{2} D(y_k) D(y_k) \frac{I_5(y_k^2, y_k^2)}{y_k^2\, y_k^2} - \frac{B_{kJ}}{2} D(y_k) D(y_J) \frac{I_5(y_k^2, y_J^2)}{y_k^2\, y_J^2} \Big] \quad \ldots.(4.24)$$

After obtaining these scattering amplitudes (equations 4.11, 4.18, 4.21 and 4.24) for Hartree Fock wave functions, we can extend these results to the Hylleraas wave function. The product of the ground and final Hylleraas wave functions for helium atom is given as

$$\Psi_i (r_1 , r_2) \Psi_f^* (r_1 , r_2) =$$

$$\left[\frac{y^3}{\pi} \exp (- y (r_1 + r_2)) \right]^2$$

$$= \frac{y^6}{\pi^2} \exp (- y' (r_1 + r_2)) \quad (4.25)$$

where $y = 1.69$, $y' = 2 y = 3.38$. Substituting $M = N = y$ and $V = 16 y^6$ in equation (4.10) we will obtain the first Born amplitude for Hylleraas wave function.

$$f^{(1)}_{i \to f} = 4 \left[\frac{(8 y^2 + q^2)}{(4 y^2 + q^2)^2} \right] \quad (4.26)$$

Similarly the imaginary and real parts can be obtained by substituting $k = y^6 / \pi^2$ and $y_1 = y_2 = y'$ in equations (4.17, and 4.20 , 4.23). The corresponding scattering amplitudes for Hylleraas wave function are

$$\text{Im } f^{(2)}_{HEA} = \frac{1}{\pi k_i} [- A1 \, D (y') \frac{H (y')}{y'^2} + A2$$

$$D(y')\,D(y')\,\frac{I_4'(y'^2,\,y'^2)}{y'^2\;y'^2}\Big] \qquad (4.27)$$

$$Re1\ f^{(2)}_{HEA} = \frac{1}{\pi^2 k_i}\Big[\ A1\ D(y')\,\frac{H'(y')}{y'^2} - A2$$

$$D(y')\,D(y')\,\frac{I_4(y'^2,\,y'^2)}{y'^2\;y'^2}\Big]$$

.... (4.28)

$$Re2\ f^{(2)}_{HEA} = \frac{D'}{2\pi^2 k_i}\Big[\ A1\ D(y')\,H''(y') - \frac{A2}{2}$$

$$D(y')\,D(y')\,\frac{I_5(y'^2,\,y'^2)}{y'^2\;y'^2}\Big]$$

....(4.29)

where A1 = 38.615 ; A2 = 745.539 ; y' = 3.38 . All these derived amplitudes equations (4.11, 4.18, 4.21, 4.24, and 4.26 , 4.27, 4.28, 4.29) correspond to the Hartree - Fock and Hylleraas wave functions in this ESGHe process.

The TCS (equation 3.3) can be obtained for these wave functions by substituting equations (4.18, 4.27) in equation (3.3) . For the Hartree - Fock wave function the TCS can be derived through the equations (4.17, 4.18)

$$\sigma^{He}_{tot} = \frac{4\pi}{k_i} \, \mathrm{Im} \, f^{(2)}_{HEA} \, (q = 0)$$

$$= \frac{4\pi}{k_i^2} \sum_{\substack{k=1,2,3 \\ J=2,3,1}} \Big[- 2A_k \, D(y_k) \frac{1}{y_k^4} \log\frac{y_k^2 + \beta_i^2}{\beta_i^2} + B_{kk} \, D(y_k) \, D(y_k) \frac{1}{y_k^2 \, y_k^2 \, (\beta_i^2 + y_k^2)} + B_{kJ} \, D(y_k) \, D(y_J) \frac{1}{y_J^2 \, y_k^2 (y_k^2 - y_J^2)} \log\Big(\frac{y_k^2 + \beta_i^2}{y_J^2 + \beta_i^2} \Big) \Big] \qquad (4.30)$$

Using this expression TCS are calculated at incident energies 100 to 1000 eV (Rao and Desai, 1983c)and are tabulated in Table (4.2) . Similar type of TCS expression can be obtained for Hylleraas wave function through the equations (4.17, 4.27). These TCS results are also given in Table (4.2).

The first order exchange (equation 2.36) scattering amplitude can be obtained by using the Ochkur approximation, through the equations (3.17, 4.2 , 4.3).

$$g_{och} = - \frac{2}{k_i^2} \int \int d\,\underline{r}_1 \; d\,\underline{r}_2 \; \exp (i \, \underline{q} \cdot \underline{r}_1)$$

$$| \psi_i (r_1 , r_2) \psi_f (r_1 , r_2) |$$

$$= - \frac{2}{4 \pi k_i^2} \int d\,\underline{r}_1 \exp (i \, \underline{q} . \underline{r}_1) \; p^2 + Q^2 + 2 PQ)$$

$$= - \frac{8}{k_i} \sum_{k=1}^{3} A (k) / (q^2 + y (k)^2)^2 \qquad (4.31)$$

where $A (1) = A^2 y'$, $A (2) = B^2 y''$ $A (3) = AB (y' + y'')$ and $y (1) = 2.82$, $y (2) = 5.22$, $y (3) = 4.02$.

The third GES term can be obtained by using the equations (2.43) , (4.3), and (4.5). This term was evaluated by Singh and Tripathi , (1980) for elastic and inelastic scattering of electrons by helium atoms. The simplified form of this third GES term was given as

$$f^{(3)}_{GES} = \frac{2}{3 \pi^2 k_i^2} [X_{31} + 3 X_{32}] \qquad (4.32)$$

where X_{31} and X_{32} are defined quantities (Singh and Tripathi , (1980).

So far we obtained the second Born scattering amplitudes (equations 4.18, 4.21, 4.24) originated from equations (2.55 to 2.60), where the distortion term (n = 0) was included in the infinity summation over the intermediate atomic states, for the simplification of the calculations. In the ESGHe process we would like to exclude this distortion term (n = 0) in the second Born approximation (equation 2.47).

$$f^{(2)}_{i\to f} = -\frac{1}{\pi} \sum_{n}^{1\,\prime} \int d\,\underline{r}_o \exp(i\,\underline{q}\cdot\underline{r}_o)\, V_{on}(\underline{r}_o) \int d\,\underline{r}_o' \exp(i\,\underline{k}_i\cdot\underline{r}_o')\, V_{no}(\underline{r}_o - \underline{r}_o')\, G_n(\underline{r}_o') \quad \ldots(4.33)$$

where the prime on summation denotes that the distortion term (n = 0) is excluded. Now following the procedure for the evaluation of the $d\,\underline{r}_o$, $d\,\underline{r}_o'$ (Sec. 2.3.5), we will obtain the corresponding

$$U'^{(2)}_{fi}\,(\text{-----},\ \text{-----})$$

in the real, and imaginary parts of the above second Born approximation (equation 4.33). In the case of ESGHe process this function can be obtained as

$$U'^{(2)}_{fi}\,(\underline{q} - \underline{p} - \beta_i\,\hat{y}\ ;\ \underline{p} + \beta_i\,\hat{y},\ r_1, r_2)$$

$$= \langle\, U_f\,(r_1, r_2)\,|\,\bar{V}\,(\underline{q} - \underline{p} - \beta_i\hat{y}\ ;\ r_1\, r_2)$$

$$\bar{V} (\underline{p} + \beta_i y , r_1 r_2) \mid U_i (r_1 r_2) > -$$

$$- < \psi_f (\underline{r}_1, \underline{r}_2) \mid \bar{V} (\underline{q} - \underline{p} + \beta_i \hat{y} , \underline{r}_1) \mid$$

$$< \psi_i (\underline{r}_1, \underline{r}_2) > < \psi_f (\underline{r}_1, \underline{r}_2) \mid \bar{V}$$

$$(\underline{p} + \beta_i \hat{y}, \underline{r}_2) \mid \psi_f (\underline{r}_1, \underline{r}_2) > \qquad (4.34)$$

Now the imaginary (equation 4.12) and real parts (equations 4.19, 4.22) can be derived for equation (4.33). Using equation (4.34) in the ESGHe process. Making use of symmetry and normalization of the wave functions (equations 4.2, 4.3), the imaginary (equation 2.60) and real parts (equations 2.58, 2.59) of the second Born term can be simplified . The closed form of the imaginary part in the ESGHe process can be obtained as

$$\text{Im } f^{(2)}_{HEA} = \frac{4 \pi^3}{k_i} \int d \underline{p} \; U'^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} ; \; p + \beta_i \hat{y} , r_1, r_2) \qquad (4.35)$$

$$= \frac{1}{\pi k_i} \int \frac{d \underline{p}}{(| \underline{q} - \underline{p} |^2 + \beta_i^2) (p^2 + \beta_i^2)}$$

$$\int d \underline{r}_1 \int d \underline{r}_2 \; [\; \exp (i \underline{q} \cdot \underline{b}_1) - \exp (i$$

$$(\underline{p} \cdot \underline{b}_1 + \beta_i Z_1) \ \psi_i \ \psi_f^* \int d\, \underline{r}_1 \int d\, \underline{r}_2 \ \exp (i$$

$$(\underline{q} - \underline{p}). \underline{b}_2 + i \beta_i Z_2)] \ \psi_i \ \psi_f^* \qquad (4.36)$$

Substituting the product of initial and final states of Hartree- Fock wave functions (equations 4.3, 4.4) and following the procedure for evaluation of $d\,\underline{r}_1$, $d\,\underline{r}_2$ and $d\,\underline{p}$, we will obtain the above equation (4.36) as

$$\mathrm{Im}\ f^{(2)}_{HEA} = - \frac{1}{\pi k_i} \sum_{i=1}^{3} D (y_i) \frac{A_i I_1' (\beta_i^2 , 0)}{(q^2 + y_i^2)} -$$

$$\frac{1}{\pi k_i} \sum_{\substack{i=1,3 \\ J=1,3}} [A_{iJ} D (y_i) D (y_J)$$

$$\frac{1}{y_i^2 \ y_J^2} \left\{ I_1' (\beta_i^2 , 0) - I_1 (\beta_i^2, y_i^2) \right.$$

$$\left. - I_1 (\beta_i^2 , y_J^2) + I_4' (y_i^2 , y_J^2) \right\}]$$

.....(4.37)

Similarly real parts of the second Born can be derived through the equations (4.3, 4.4, 4.36) . The closed form of these can be obtained as

$$\mathrm{Rel}\ f^{(2)}_{HEA} = \frac{1}{\pi^2 k_i} \sum_{i=1}^{3} D (y_i) \frac{A_i I_2' (\beta_i^2 , 0)}{(q^2 + y_i^2)} -$$

$$\frac{1}{\pi^2 k_i} \sum_{\substack{i=1,3\\J=1,3}} \Big[A_{iJ} \, D(y_i) \, D(y_J) \frac{1}{y_i^2 \; y_J^2}$$

$$\Big\{ I_2'(\beta_i^2, 0) - I_2(\beta_i^2, y_i^2) - I_2(\beta_i^2, y_J^2)$$

$$+ I_4(y_i^2, y_J^2) \Big\} \Big] \qquad (4.38)$$

and

$$\mathrm{Re2}\; f^{(2)}_{HEA} = \frac{1}{\pi^2 k_i^2} \sum_{i=1}^{3} A_i \, D' \, D(y_i) \frac{I_3'(\beta_i^2, 0)}{(q^2 + y_i^2)}$$

$$- \frac{1}{2\pi^2 k_i^2} \sum_{\substack{i=1,3\\J=1,3}} A_{iJ} \, D' \, D(y_i) \, D(y_J)$$

$$\frac{1}{y_J^2} \Big[I_2(\beta_i^2, y_i^2) - I_4(y_J^2, y_i^2) \Big]$$

$$\ldots\ldots(4.39)$$

All the constants A_i, y_i and A_{iJ}, y_J can be obtained from equation (4.2).

A_1	=	6.7863,	A_2	=	4.3324,	A_3	=	10.845
y_1	=	2.82,	y_2	=	5.22,	y_3	=	4.02
A_{11}	=	46.0537,	A_{22}	=	18.7696	A_{33}	=	117.60341
A_{12}	=	29.4009,	A_{21}	=	29.4009,	A_{31}	=	73.5939
A_{13}	=	73.5939,	A_{23}	=	46.9827,	A_{32}	=	46.9827 .

4.2.2 Comparison of present ESGHe results with the recent theoretical and experimental data :

Using the scattering amplitudes equations (4.11, 4.18, 4.21, 4.24, 4.31, 4.32) we have calculated the DCS (equation 3.2) at incident energies E = 200 and 400 eV and the TCS (equation 4.30) are calculated in the incident energy range E = 100 to 1000 eV. We have used an average excitation energy DE = 1.20 a.u (Byron and Joachain, 1977). These calculated results for ESGHe process are presented in the Tables (4.1, 4.2) as well as in the form of graphs shown in figures (4.1, 4.2) along with the recent theoretical and experimental data. Our DCS results are found to be in good agreement with the compared data in the angular range $\theta \leq 50^{\circ}$, TCS results are also found to be in very good agreement with the compared theoretical and experimental data. The details of our comparisons with the present DCS and TCS are as follows .

Fig. (4.1) shows two sets of DCS results at incident energies E = 200 and 400 eV. In set A (E = 200 eV) present DCS (solid curve a) are compared with recent theoretical results, + — EBS results of Byron and Joachain (1973a,b), and experimental results, ● — Register et al (1980), O — Crooks and Rudd (1971), at incident energy

E = 200 eV. Set B (E = 400 eV) shows present DCS at E = 400 eV along with the GES results (dashed curve) of Singh and Tripathi (1980). The rest of the representations in this set B are same as in set A. It can be noted from these comparisons that the present DCS are in better agreement with the experimental data than the compared EBS and GES results in the angular range $\theta \leq 50^{o}$.

Fig. (4.2) shows the comparison of present imaginary part ((solid curve) equation 4.18) with the imaginary part of GES (dashed curve) of Singh and Tripathi (1980). This comparison was a checking of our present calculations, and also shows that the effect of β_i in imaginary part (equation 4.18) was negligible, which was the basic difference between the corresponding terms in GES and HEA approximations (Secs. 2.3.4 , 2.3.5).

Our present Hartree - Fock scattering amplitudes (equations 4.11, 4.18, 4.21, 4.24, 4.31, 4.32) are listed in Table (4.1) at incident energies E = 200 and 400 eV in the angular range $\theta \leq 130^{o}$. Similar to ESGH (Sec.3.2.1, Table 3.1) process here also fluctuations are observed in the real part of the second Born term (equation 4.24). The DCS are listed along with the GES results of Singh and Tripathi (1980). Very slight variation was observed between the DCS's calculated by using Hartree - Fock (equations 4.2, 4.3) and Hylleraas (equation 4.25) wave functions (Rao and Desai, 1981). Table (4.2) shows the present TCS (equation 4.30) at incident energies E = 100 to 1000 eV (Rao and Desai 1983c) along with the compared theoretical (Byron and Joachain , 1975, 1977) and experimental (de Heer , 1975 ; Dalba et al , 1979 ; Blaauw, 1980) data. Our TCS results are found in good agreement with the measured values of Blaauw (1980) and nearer to the results of Byron and Joachain (1975).

It can be noted from the Figs. (4.1, 4.2) and Tables (4.1, 4.2), that the present ESGHe results are better than GES and nearer to EBS and experimental data.

As in the case of ISH process (Sec. 3.4.1, Tables 3.5, 3.6) here also we have observed the sensitivity of second Born amplitudes (equations 2.58, 2.59, 2.60) and DCS with respect to the choice of the excitation energy DE . The ESGHe scattering amplitudes corresponding to the equations (4.37, 4.38, 4.39) are used at incident energies E = 100 to 800 eV with DE = 1.20 (Byron and Joachain, 1977) and $DE' = 1.10$ (Byron and Joachain, 1977) respectively for those observations. And the corresponding DCS (neglecting, exchange and third GES) are calculated at incident energies E = 100 to 800 eV. These DCS results are listed in the Table (4.4) and the scattering amplitudes (equations 4.37, 4.38, 4.39) at E = 200 eV are given in Table (4.3). The DCS at E = 200 eV along the recent theoretical and experimental data was shown in Fig. (4.3). The details of these Tables and Fig. are as follows.

Fig. (4.3) shows, the present DCS (solid curve and dashed curve) at incident energy E = 200 eV obtained by using the scattering amplitudes (equations 4.11, 4.37, 4.38, 4.39) with DE = 1.20 and DE' = 1.10 respectively, and the recent theoretical (+ — EBS (Byron and Joachain 1973b) and experimental (● —— Register et al, 1980 ; ▲ — Bromberg , 1974) results. These present DCS are found to be in good agreement with the compared data.

Table (4.3) shows the second Born, real parts (equations 4.38, 4.39) and imaginary part (equation 4.37) due to excitation energy DE = 1.20 and DE' = 1.10 respectively at incident energy E = 200 eV. The DCS with these type of scattering amplitudes are listed in Table (4.4) in the energy range E = 100 to 800 eV . In the Table (4.4), we listed two DCS (upper and lower) results at each scattering angle θ and incident energy E, corresponding to the excitation energy DE = 1.20 and 1.10 respectively. It can be observed from this table that the variation in the DCS was more in the small angle region than at large angles (fixed E), and this variation was negligible at higher incident energies (fixed Θ). It can also be noted from Table (4.3) that this choice of excitation energy was more effective in the real parts (equations 4.38, 4.39) than in the imaginary part (equation 4.37).

A careful comparison of Tables (4.4 and (3.6) shows that the excitation energies plays an important role in the inelastic process (ISH Sec. 3.4.1) than in the elastic process (present ESGHe Sec. 4.2.1). And comparison of Tables (4.1) and (4.3) for ESGHe process shows the effect of distortion term (n = 0) in summing over the atomic states on the scattering amplitudes equations (4.18, 4.21, 4.24) and equations (4.37, 4.38, 4.39) respectively.

4.3.1 Inelastic scattering of electrons by helium atom (ISHe) :

$$\bar{e} + \text{He}(1s) \longrightarrow \bar{e} + \text{He}^{*}(2s) \qquad (4.40)$$

Various transitions in the helium atom are of great interest, not only because there is in this case a rather large body of precise experimental data which can be compared with theoretical predictions, but also because much more detailed information has been obtained so far from the experiments performed in helium than from those carried out in atomic hydrogen.

Now we consider inelastic scattering (equation 1.2) of electrons by helium atom. Like in the case of ISH (Sec. 3.4.1) here also final state of the helium atom is different than the initial state. The scattering amplitudes of ESGHe process (equations 4.11, 4.18, 4.21, 4.24, 4.32) can be extended to this ISHe process. In this inelastic process the momentum transfer to the target is

$$q = (k_i^2 + k_f^2 - 2 k_i k_f \cos \theta)^{1/2} \qquad (4.41)$$

Using the energy conservation the final momenta k_f of the scattered electron can be derived as,

$$k_f = (k_i^2 - 3)^{1/2} \qquad (4.42)$$

The initial ground state and final excited state of Hartree - Fock wave functions for the helium atom can be given as

$$\psi_i (\underline{r}_1 , \underline{r}_2) = \phi_{1s} (\underline{r}_1) \phi_{1s} (\underline{r}_2)$$

$$= \frac{1}{4 \pi} (P + Q) (R + S) \qquad (4.43)$$

P, Q, R and S are defined in ESGHe process equation (4.2).

$$\psi_f(\underline{r}_1, \underline{r}_2) = \frac{1}{(2(1+d))^{1/2}} [\phi_1(\underline{r}_1)\phi_2(\underline{r}_2) + \phi_1(\underline{r}_2)\phi_2(\underline{r}_1) \quad (4.44)$$

where

$$d = 0.00534$$

$$\phi_1(\underline{r}) = \frac{M}{(4\pi)^{1/2}} \exp(-2r)$$

$$\phi_2(\underline{r}) = \frac{N}{(4\pi)^{1/2}} [\exp(-x_1 r) - y \exp(-x_2 r) r]$$

Now the product of the initial and final wave functions can be written as

$$\psi_i(\underline{r}_1, \underline{r}_2)\psi^*_f(\underline{r}_1, \underline{r}_2) = \frac{MN}{(4\pi)^2 d'} [(PR + PS + QR + QS)((T_1 V_2 + T_2 V_1) - y(T_1 W_2 + T_2 W_1))]$$

$$= \frac{MN}{(4\pi)^2 d'} [PRT_1 V_2 + PST_1 V_2 + QRT_1 V_2 + QST_1 V_2 + PRT_2 V_1 + PST_2 V_1 + QRT_2 V_1 + QST_2 V_1] -$$

$$\frac{MN\ y}{(4\pi)^2\ d'} [\ PRT_1\ W_2 + PST_1\ W_2 + QRT_1\ W_2 + QST_1\ W_2 +$$

$$PRT_2\ W_1 + PST_2\ W_1 + QRT_2\ W_1 + QST_2\ W_1\ ]$$

....(4.45)

where $T_1 = \exp(-2\ r_1)$, $T_2 = \exp(-2\ r_2)$

$V_1 = \exp(-x_1\ r_1)$, $V_2 = \exp(-x_1\ r_2)$

$W_1 = r_1 \exp(-x_2\ r_1)$, $W_2 = r_2 \exp(-x_2\ r_2)$

and $M = 5.656854$, $N = 0.61280$, $x_1 = 0.865$, $x_2 = 0.522$

$y = 0.432784$, $d' = (2(1+d))^{1/2} = 1.41799$

By symmetry of the terms in equation(4.45), this can be reduced as

$$= \frac{2\ M\ N}{(4\pi)^2\ d'} [\ (PRT_1\ V_2 + PST_1\ V_2 + QRT_1\ V_2 + QST_1\ V_2) -$$

$$y\ (PRT_1\ W_2 + PST_1\ W_2 + QRT_1\ W_2 + QST_1\ W_2)\ ]$$

....(4.46)

Similar to ESGHe process (sec. 4.2.1) here also we will consider a typical terms of equation (4.46) for the evaluation of all the scattering contributions to the DCS (equation 3.2). And then we extend these results to equation (4.46). Now a typical term of the above equation can be given as

$$PRT_1 V_2 - y (PRT_1 W_2) = k_1 \exp (- y_1 r_1) \exp (- y_2 r_2)$$

$$- y k_1 \exp (- y_1 r_1) r_2 \exp (- y_2' r_2)$$

$$= k_1 \left[D (y_1) \frac{\exp (- y_1 r_1)}{r_1} D (y_2) \frac{\exp (- y_2 r_2)}{r_2} + y D (y_1) \frac{\exp (- y_1 r_1)}{r_1} D^2 (y_2') \frac{\exp (- y_2' r_2)}{r_2} \right] \quad .....(4.47)$$

where $k_1 = A^2$, $y_1 = y' + 2$; $y_2 = y' + x_1$, $y_2' = y' + x_2$. In this way all the terms in equation (4.46) can be written as equation (4.47). It can be noted from equation (4.47) the first term of this is similar to (equation 4.4) and the second term is similar to first with only one additional derivative . Except for the constants all the ESGHe process amplitudes (equations 4.11, 4.18, 4.21, 4.24) can be used to obtain the ISHe scattering amplitudes respectively.

The first Born approximation in this ISHe process can be obtained easily through the equations (4.6, 4.9 and 4.45).

$$f^{(1)}_{i \to f} = - \frac{1}{2\pi} \int d\,\underline{r}_0 \; \exp(\, i\,\underline{q} \cdot \underline{r}_0\,) \int\int d\,\underline{r}_1$$

$$d\,\underline{r}_2 \left[\frac{-2}{r_0} + \frac{1}{|\,\underline{r}_0 - \underline{r}_1\,|} + \frac{1}{|\,\underline{r}_0 - \underline{r}_2\,|} \right]$$

$$\psi_i(\,\underline{r}_1\,,\,\underline{r}_2\,)\,\psi^*_f(\,\underline{r}_1\,,\,\underline{r}_2\,) \qquad (4.48)$$

$$= - \frac{MN}{(4\pi)^2\, 2\pi d'} \int d\,\underline{r}_0 \; \exp(\, i\,\underline{q} \cdot \underline{r}_0\,) \int\int$$

$$d\,\underline{r}_1 \; d\,\underline{r}_2 \left[- \frac{2}{r_0} + \frac{1}{|\,\underline{r}_0 - \underline{r}_1\,|} + \frac{1}{|\,\underline{r}_0 - \underline{r}_2\,|} \right]$$

$$\Big\{ (\, PRT_1 V_2 + PST_1 V_2 + QRT_1 V_2 + QST_1 V_2 +$$

$$PRT_2 V_1 + PST_2 V_1 + QRT_2 V_1 + QST_2 V_1\,) - y$$

$$(\, PRT_1 W_2 + PST_1 W_2 + QRT_1 W_2 + QST_1 W_2 +$$

$$PRT_2 W_1 + PST_2 W_1 + QRT_2 W_1 + QST_2 W_1\,) \Big\}$$

$$= \frac{-MN}{(4\pi)^2\, 2\pi d'} \sum_{\substack{I=1 \\ J=1}}^{8} \int d\,\underline{r}_0 \; \exp(\, i\,\underline{q} \cdot \underline{r}_0\,)$$

$$\int\int d\,\underline{r}_1 \; d\,\underline{r}_2 \left[- \frac{2}{r_0} + \frac{1}{|\,\underline{r}_0 - \underline{r}_1\,|} + \frac{1}{|\,\underline{r}_0 - \underline{r}_2\,|} \right]$$

$$\left\{ A(I) \exp(-(y_1(I) + y_2(I))) + B(J) \right.$$

$$\left. D(y_1'(J)) \exp(-(y_1'(J) + y_2'(J))) \right\}$$

In this expression all the constants $A(I)$'s and $B(J)$'s and exponentials values $y_1(I)$, $y_2(I)$, $y_1'(J)$, $y_2'(J)$ can be obtained from equations (4.43 , 4.45) The $d\underline{r}_o$, $d\underline{r}_1$ and $d\underline{r}_2$ integrals are evaluated for a typical term (equation 4.9) in the ESGHe process, these results can be used to get the closed form of equation (4.48).

$$f^{(1)}_{i \to f} = F_1 + F_2 \qquad (4.49)$$

where

$$F_1 = \frac{2MN}{d'} \sum_{I=1}^{8} \frac{A(I)}{(y_1(I)\, y_2(3))^3} \left[\frac{(8 y_1(I)^2 + q^2)}{(4 y_1(I)^2 + q^2)^2} + \frac{(8 y_2(I)^2 + q^2)}{(4 y_2(I)^2 + q^2)^2} \right]$$

$$F_2 = \frac{2MN}{d'} \sum_{J=1}^{8} \frac{B(J)\; D(y_1'(J))}{y_2'(J)^3\; y_1'(J)^3} \left[\frac{(8 y_1'(J)^2 + q^2)}{(4 y_1'(J)^2 + q^2)^2} + \frac{(8 y_2'(J)^2 + q^2)}{(4 y_2'(J)^2 + q^2)^2} \right]$$

The imaginary part in the ISHe process can be obtained as

$$\text{Im } F^{(2)}_{HEA} = \frac{4\pi^3}{k_i} \int d\underline{p} \; U^{(2)}_{fi} (\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{p} + \beta_i \hat{y}) \qquad (4.50)$$

$$= \frac{4\pi^3}{k_i} \int d\underline{p} \; \langle \psi^*_f (r_1 , r_2) | \bar{V} (\underline{q} - \underline{p} - \beta_i y ; \underline{r}_1, \underline{r}_2) \; \bar{V} (\underline{p} + \beta_i \hat{y} ; \underline{r}_1, \underline{r}_2) | \psi_i (\underline{r}_1 , \underline{r}_2) \rangle$$

$$= \frac{4\pi^3}{k_i} \int d\underline{p} \int d\underline{r}_1 \int d\underline{r}_2 \; \bar{V} (\underline{q} - \underline{p} - \beta_i \hat{y} ; \underline{r}_1 , \underline{r}_2) \; \bar{V} (\underline{p} + \beta_i \hat{y} ; \underline{r}_1 , \underline{r}_2) \; \psi_i (\underline{r}_1 , \underline{r}_2) \; \psi^*_f (\underline{r}_1 , \underline{r}_2) \qquad (4.51)$$

Substituting equation (4.47) instead of equation (4.46) in the above expression , and following the procedure (equations 4.13 to 4. 17) for the evaluation of $d\underline{p}$, $d\underline{r}_1$, $d\underline{r}_2$ we will obtain the closed form of equation (4.50) for a typical, term (equation 4.47) of equation (4.46).

$$\operatorname{Im} F^{t}_{HEA} = \frac{(4\pi)^2 k_1}{\pi k_i}\left[\left(\frac{D(y_1)}{y_1^2} D(y_2) \frac{H(y_2)}{y_2^2} \right.\right.$$

$$+ \frac{D(y_2)}{y_2^2} D(y_1) \frac{H(y_1)}{y_1^2} + 2 D(y_1)$$

$$\left. D(y_2) \frac{I_4'(y_1^2, y_2^2)}{y_1^2 \; y_2^2} \right) + y \left(\frac{D(y_1)}{y_1^2} \right.$$

$$D^2(y_2') \frac{H(y_2')}{y_2'^2} + \frac{D^2(y_2')}{y_2'^2} D(y_1)$$

$$\frac{H(y_1)}{y_1^2} + 2 D(y_1) \; D^2(y_2')$$

$$\left.\left. \frac{I_4'(y_1^2, y_2'^2)}{y_1^2 \; y_2'^2} \right) \right]$$

$$= \frac{(4\pi)^2 k_1}{\pi k_i}\left[\left(-\frac{2}{y_1^3} D(y_2) \frac{H(y_2)}{y_2^2} \right.\right.$$

$$- \frac{2}{y_2^3} D(y_1) \frac{H(y_1)}{y_1^3} + 2 D(y_1) D(y_2)$$

$$\left. \frac{I_4'(y_1^2, y_2^2)}{y_1^2 \; y_2^2} \right) + y \left(-\frac{2}{y_1^3} D^2(y_2') \right)$$

$$\frac{H(y_2')}{y_2'^{2}} + \frac{6}{y_2'^{4}} \; D(y_1) \; \frac{H(y_1)}{y_1^{2}} + 2\, D(y_1)$$

$$D^2(y_2') \; \frac{I_4'(y_1^2, y_2'^2)}{y_1^2 \; y_2'^2} \Big\} \Big] \qquad (4.52)$$

where $H(y_k)$'s and $I_4(y_k^2, y_J^2)$'s are defined differentiable expressions (given under equation 4.17). Now the complete imaginary part (equation 4.50) can be obtained by substituting equation (4.46) in equation (4.51) and making use of the result (equation 4.52).

$$\mathrm{Im}\; F^{(2)}_{HEA} = \frac{2\, MN}{\pi\, k_i\, d'} \Big[\Big\{ \Big(- \frac{2 A^2}{y_2^3} - \frac{2\, A\, B}{y_3^3} \Big) D(y_1)$$

$$\frac{H(y_1)}{y_1^2} + \Big(- \frac{2 A^2}{y_1^3} - \frac{2\, A\, B}{y_4^3} \Big) D(y_2)$$

$$\frac{H(y_2)}{y_2^2} + \Big(- \frac{2\, A\, B}{y_1^3} - \frac{2 B^2}{y_4^3} \Big) D(y_3)$$

$$\frac{H(y_3)}{y_3^2} + \Big(- \frac{2 B^2}{y_3^3} - \frac{2\, A\, B}{y_2^3} \Big) D(y_4)$$

$$\frac{H(y_4)}{y_4^2} + 8 A^2 \; D(y_1)\; D(y_2)$$

$$\frac{I_4'(y_1^2, y_2^2)}{y_1^2 \quad y_2^2} + 8\,A\,B\;D(y_1)\;D(y_2)$$

$$\frac{I_4'(y_1^2, y_3^2)}{y_1^2 \quad y_3^2} + 8\,A\,B\;D(y_2)\;D(y_4)$$

$$\frac{I_4'(y_2^2, y_4^2)}{y_2^2 \quad y_4^2} + 8\,B^2\;D(y_3)\;D(y_4)$$

$$\frac{I_4'(y_3^2, y_4^2)}{y_4^2 \quad y_3^2}\;\Big)\Big) + y\;\Big(\Big(\;\Big(\frac{6\,A^2}{y_2'^4} + \frac{6\,A\,B}{y_3'^4}\Big)$$

$$D(y_1)\;\frac{H(y_1)}{y_1^2} + \Big(\frac{6\,AB}{y_2'^4} + \frac{B^2}{y_3'^4}\Big)\;D(y_4)$$

$$\frac{H(y_4)}{y_4^2} + \Big(-\frac{2\,A^2}{y_1^3} - \frac{2\,A\,B}{y_4^3}\Big)\;D^2(y_2')$$

$$\frac{H(y_2')}{y_2'^2} + \Big(-\frac{2\,A\,B}{y_1^3} - \frac{2\,B^2}{y_4^3}\Big)\;D^2(y_3')$$

$$\frac{H(y_3')}{y_3'^2} - 8\,A^2\;D(y_1)\;D^2(y_2')$$

$$\frac{I_4'(y_1^2, y_2'^2)}{y_1^2 \quad y_2'^2} - 8\,A\,B\;D(y_1)\;D^2(y_3')$$

$$\frac{I_4'(y_1^2, y_3'^2)}{y_1^2 \quad y_3'^2} - 8AB\ D(y_4)\ D^2(y_2')$$

$$\frac{I_4'(y_4^2, y_2'^2)}{y_4^2 \quad y_2'^2} - 8B^2\ D(y_4)\ D^2(y_3')$$

$$\left.\frac{I_4'(y_4^2, y_3'^2)}{y_4^2 \quad y_3'^2} \quad \right\} \quad]$$

The simplified form of this can be written as

$$= \frac{1}{\pi k_i} \Bigg[\sum_{k=1}^{4} \left(-A(k)\ D(y_k)\ \frac{H(y_k)}{y_k^2} \right)$$

$$+ \sum_{L=1}^{2} \left(B(L)\ D(y_L')\ \frac{H(y_L')}{y_L'^2} \right)$$

$$- \sum_{L=3}^{4} \left(B(L)\ D(y_L')\ \frac{H(y_L')}{y_L'^2} \right)$$

$$\sum_{\substack{M=1,1,2,3 \\ N=2,3,4,4}} C(M,N)\ D(y_M)\ D(y_N)$$

$$\frac{I_4'(y_M^2, y_N^2)}{y_M^2 \quad y_N^2}$$

$$- \sum_{\substack{m = 1,1,4,4 \\ n = 2,3,2,3}} E(m,n)\, D(y_m')\, D^2(y_n') \frac{I_4'(y_m'^2, y_n'^2)}{y_m'^2 \; y_n'^2} \Big] \qquad (4.53)$$

where $A(k)$'s, $B(L)$'s, $C(M,N)$'s and $E(m,n)$'s are constant which can be obtained from the equations (4.43 to 4.46), and y_k's and y_m''s are exponential parameters of the wave functions (equations 4.43, 4.44). All these constants can be obtained as

A (1)	= 6.97273 ,	y_1	= 3.41	y_1'	= 3.41
A (2)	= 2.23828 ,	y_2	= 2.275,	y_2'	= 1.932
A (3)	= 1.78839 ,	y_3	= 3.475,	y_3'	= 3.132
A (4)	= 5.57122,	y_4	= 4.61,	y_4'	= 4.61

B (1)	= 6.97270,	C (1,2)	= 4.45743,	E (1,2)	= 116.0981
B (2)	= 5.57120,	C (1,3)	= 1.52646,	E (1,3)	= 92.76262
B (3)	= 0.77399,	C (2,4)	= 1.94868,	E (4,2)	= 92.76262
B (4)	= 0.96869,	C (3,4)	= 0.66733,	E (4,3)	= 74.1174

Similarly real part equations (4.19, 4.22) can be obtained through the equations (4.19, 4.21, 4.47, 4.46). Like in the case of ESGHe process (equations 4.18, 4.21, 4.24), here also can write directly closed form of the real parts, by making use of the imaginary results (equation 4.53).

$$\mathrm{Rel}\; F^{(2)}_{HEA} = \frac{1}{\pi^2 k_i} \Big[\sum_{k=1}^{4} \Big(A(k)\, D(y_k) \frac{H'(y_k)}{y_k^2} \Big) -$$

$$\sum_{L=1}^{2} \Big(B(L)\, D(y_L') \frac{H'(y_L')}{y_L'^2} \Big) +$$

$$\sum_{L=3}^{4} \Big(B(L)\, D^2(y_L') \frac{H'(y_L')}{y_L'^2} \Big) -$$

$$\sum_{\substack{M=1,1,2,3\\ N=2,3,4,4}} C(M,N)\, D(y_M)\, D(y_N) \frac{I_4(y_M^2, y_N^2)}{y_M^2\; y_N^2} +$$

$$\sum_{\substack{m=1,1,4,4\\ n=2,3,2,3}} E(m,n)\, D(y'_m)\, D^2(y'_n) \frac{I_4(y'^2_m, y'^2_n)}{y'^2_m\, y'^2_n} \Big] \qquad (4.54)$$

$$\mathrm{Re}2\ F^{(2)}_{HEA} = \frac{1}{2\pi^2 k_i^2} \Big[\sum_{k=1}^{4} A(k)\, D(y_k)\, H''(y_k)$$

$$- \sum_{L=1}^{2} B(L)\, D(y_L'),H''(y_L')$$

$$+ \sum_{L=3}^{4} B(L)\, D^2(y_L')\, H''(y_L')$$

$$- \sum_{\substack{M=1,1,2,3\\ N=2,3,4,4}} \frac{C(M,N)}{2} D(y_M)\, D(y_N) \frac{I_5(y_M^2, y_N^2)}{y_M^2\, y_N^2}$$

$$+ \sum_{\substack{m=1,1,4,4\\ n=2,3,2,3}} \frac{E(m,n)}{2} D(y'_m)$$

$$D^2 (y_n') \frac{I_5 (y_m'^2 , y_n'^2)}{y_m'^2 \; y_n'^2}] \qquad (4.55)$$

All the functions $H'(y_k)$, $I_4 (y_m'^2 , y_n'^2)$ and $H''(y_k)$, $I_5 (y_m'^2 , y_n'^2)$ in the equations (4.54, 4.55) are similar to those in ESGHe process (given under equations 4.20, 4,23), and all the constants, in equations (4.54, 4.55) are same as given under equation (4.53).

4.3.2 Comparison of present ISHe results with the other data :

Using the ISHe (Sec. 4.3.1) process scattering amplitudes (equations 4.49, 4.53, 4.54, 4.55) we have performed the DCS calculations at incident energies E = 200 and 400 eV . We used an average excitation energy DE = 1.20 a.u (Byron and Joachain , 1977) in the calculation of second Born scattering amplitudes (equations 4.53, 4.54, 4.55). These DCS results and scattering amplitudes are listed in the Table (4.5). In Fig. (4.4) we have shown the DCS with and without real part (equation 4.55) in the DCS calculations . The present ISHe DCS results are found higher than the compared data. The discription of the Table and Fig. was as follows.

Fig. (4.4) shows present DCS (without exchange correction in equation 3.2) at incident energies 200 and 400 eV. Set A (E = 200 eV) shows present DCS (solid curves a' and b') at incident energy E = 200 eV. Set B (E = 400 eV) shows present DCS at E = 400 eV. It can be observed from these sets, that the variation of DCS due to real part (equation 4.55) was more at 200 eV than at 400 eV.

Table (4.5) shows the ISHe scattering amplitudes (equation 4.49, 4.53, 4.54, 4.55) and DCS at incident energies 200 and 400 eV along with the compared theoretical results of Singh and Tripathi (1980).

The present ISHe results are found to be very much higher than the GES results of Singh and Tripathi (1980).

Table – 4.1

Behaviour of the Born scattering amplitudes (equations 4.11 , 4.21 , 4.24, 4.31 , 4.18) in ESGHe process for Hartree – Fock orbitals at E = 200 eV.

Θ	$f^{(1)}_{i \to f}$	Re1 $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	$f^{(3)}_{GES}$ (–ve)	g_{Och} (–ve)	Im $f^{(2)}_{HEA}$	DCS*	(Present DCS with exchange) DCS
	1	2	3	4	5	6	7	8
10	0.7373	0.13590	0.0835	0.129	0.1248	0.6572	7.53 –1	1.3113
20	0.6103	0.03700	0.0513	0.126	0.9919 –1	0.4006	3.61 –1	6.0937 –1
30	0.4743	0.02180	0.0488	0.137	0.7246 –1	0.2818	1.70 –1	3.1266 –1
40	0.3622	0.01950	0.0503	0.117	0.5127 –1	0.2276	9.61 –1	1.8634 –1
50	0.2793	0.01850	0.0528	0.123	0.3626 –1	0.1955	4.63 –2	1.0794 –1
70	0.1775	0.01570	0.0563	0.102	0.1921 –1	0.1610	1.90 –2	5.3840 –2
90	0.1247	0.01280	0.0601	0.094	0.1135 –1	0.1376	8.68 –3	3.2170 –2
110	0.0958	0.01072	0.0625	0.086	0.7526 –2	0.1165	5.22 –3	2.1882 –2
130	0.0794	0.009364	0.0640	0.0802	0.5572 –2	0.1009	3.67 –3	1.6375 –2

...Contd...

Table - 4.1 Contd... at E = 400 eV

θ	1	2	3	4	5	6	7	8
10	0.6893	0.02739	0.0299	0.0678	0.5753 -1	0.3678	5.05 -1	6.7601 -1
20	0.4944	0.00931	0.0243	0.0689	0.3818 -1	0.2060	2.17 -1	2.899 -1
30	0.3350	0.00927	0.0281	0.0657	0.2313 -1	0.1522	9.08 -2	1.319 -1
40	0.2310	0.00857	0.0271	0.0580	0.1396 -1	0.1284	4.19 -2	6.605 -2
50	0.1660	0.00749	0.0287	0.0519	0.8711 -2	0.1113	2.15 -2	3.767 -2
70	0.0975	0.00545	0.0314	0.0432	0.3871 -2	0.0835	7.45 -3	1.602 -2
90	0.0660	0.00407	0.0329	0.0374	0.2052 -2	0.0602	3.47 -3	8.212 -3
110	0.0497	0.00322	0.0338	0.0334	0.1272 -2	0.0470	2.00 -3	5.175 -3
130	0.0409	0.00273	0.0343	0.0307	0.9042 -3	0.0390	1.36 -3	3.793 -3

DCS* Results of singh and Tripathi (1980).

Table - 4.2

TCS corresponding to Hartree - Fock and Hyllraas wave functions equations (4.2) and (4.25) respectively for incident energies E = 100 to 1000 eV in ESGHe process.

E	The theoretical and the experimental data						
	Ref(5)	(Present) Hartree Fock wave function	Ref(1)	(Present) Hyllraas wave function	Ref(2)	Ref(3)	Ref(4)
	1	2	3	4	5	6	7
100	3.97	4.94	4.05	4.30	4.68	6.16	4.154
200	2.58	2.93	2.68	2.56	2.92	3.37	2.732
300	1.98	2.14	2.03	1.86	2.15	2.38	2.021
400	1.64	1.69	1.66	1.48	1.71	1.86	1.611
500	1.35	1.42	1.39	1.24	1.43	1.56	1.346
600	1.19	1.22	-	1.07	-	-	1.161
700	1.04	1.08	1.06	0.94	1.09	1.16	1.011

....Contd...

Table - 4.2 Contd...

E	1	2	3	4	5	6	7
800	-	0.96	-	0.84	-	-	0.914
900	-	0.87	-	0.77	-	-	0.811
1000	-	0.80	-	0.70	-	-	0.739

Ref(1) de Heer and Jansen (1975)
Ref(2) Byron and Joachain (1975)
Ref(3) Byron and Joachain (1977)
Ref(4) Dalba et al (1979)
Ref(5) Blaauw et al. (1980)

Table - 4.3

The behaviour of the second Born amplitudes (equations 4.38, 4.39, 4.37) for different DE'S (= 1.20, = 1.10) in ESGHe process at E = 200 eV.

θ	Re1 $f^{(2)}_{HEA}$	Re1' $f^{(2)}_{HEA}$	Re2 $f^{(2)}_{HEA}$	Re2' $f^{(2)}_{HEA}$	Im $f^{(2)}_{HEA}$	Im' $f^{(2)}_{HEA}$
10	0.1221	0.1082	-0.4244 -1	-0.4267 -1	0.4385	0.4422
20	0.2477 -1	0.2016 -1	-0.1552 -1	-0.1607 -1	0.1869	0.1849
30	0.1049 -1	0.8638 -2	0.3786 -2	-0.3213 -2	0.8031 -1	0.7899 -1
40	0.9341 -2	0.8184 -2	0.1288 -1	0.1243 -1	0.3755 -1	0.3689 -1
50	0.1694 -1	0.1539 -1	-0.4644 -2	-0.5078 -2	0.1961 -1	0.1929 -1
70	0.1153 -1	0.1056 -1	0.9957 -2	0.9743 -2	0.7223 -2	0.7142 -2
90	0.1279 -1	0.1173 -1	-0.1969 -2	-0.2097 -2	0.3533 -2	0.3507 -2
110	0.1073 -1	0.9844 -2	-0.1473 -2	-0.1551 -2	0.2089 -2	0.2079 -2
130	0.9358 -2	0.8589 -2	-0.1211 -2	-0.1265 -2	0.1434 -2	0.1429 -2

Prime results corresponds to DE' = 1.10

Without Prime results corresponds to DE = 1.20

Table 4.4

ESGHe DCS at incident energies E = 100 to 800 eV , with DE = 1.20 and 1.10 in the Second Born amplitude (equations 4.37 , 4.38, 4.39).

E	θ = 10°	20°	30°	40°	50°	70°	90°	100°
	0.20212 +1	0.11267 +1	0.69937	0.46804	0.32448	0.15275	0.85271 -1	0.6692 -1
100	0.243 +1	0.129 +1	0.706	0.404	0.244	0.108	0.611 -1	0.498 -1
	0.20089 +1	0.11004 +1	0.68420	0.46003	0.31994	0.14967	0.83435 -1	0.6544 -1
	0.10522 +1	0.5403	0.3136	0.1858	0.10671	0.46121 -1	0.21232 -1	0.1596 -1
200								
	0.10313 +1	0.53303	0.31105	0.1846	0.1045	0.45680 -1	0.20911 -1	0.1570 -1
	0.74490	0.37460	0.1951	0.98134 -1	0.5373 -1	0.19706 -1	0.9228 -2	0.6831 -2
300								
	0.73307	0.3722	0.1943	0.97405 -1	0.5328 -1	0.1949 -1	0.9120 -2	0.6749 -2
	0.6076	0.28876	0.13505	0.6245 -1	0.3239 -1	0.11171 -1	0.5098 -2	0.3748 -2
400	0.664	0.280	0.121	0.560 -1	0.287 -1	0.100 -1	0.468 -2	0.347 -2
	0.6012	0.28773	0.13463	0.6206 -1	0.32156 -1	0.11073 -1	0.50494 -2	0.37115 -2

Contd...

Table 4.4 Contd...

E	$\theta = 10^o$	20^o	30^o	40^o	50^o	70^o	90^o	100^o
500	0.5293	0.23287	0.96106 -1	0.4325 -1	0.2157 -1	0.7158 -2	0.3218 -2	0.2358 -2
	0.5257	0.2323	0.9569 -1	0.4302 -1	0.2143 -1	0.7104 -2	0.3193 -2	0.2338 -2
600	0.4765	0.1928	0.7358 -1	0.31632 -1	0.1535 -1	0.4965 -2	0.2212 -2	0.1617 -2
	0.4743	0.1924	0.7329 -1	0.3147 -1	0.1526 -1	0.4932 -2	0.2199 -2	0.1605 -2
700	0.4365	0.1625	0.58224 -1	0.2411 -1	0.11459 -1	0.3641 -2	0.16122 -2	0.1177 -2
	0.450	0.154	0.545 -1	0.224 -1	0.107 -1	0.344 -2	0.154 -2	0.113 -2
	0.4352	0.1623	0.5802 -1	0.2399 -1	0.1140 -1	0.3619 -2	0.1602 -2	0.1169 -2
800	0.4043	0.1389	0.4717 -1	0.1895 -1	0.8869 -2	0.2781 -2	0.1226 -2	0.8939 -3
	0.4034	0.1388	0.4702 -1	0.1888 -1	0.8828 -2	0.2767 -2	0.12196 -2	0.8890 -3

For few incident energies there are three results upper one is with DE = 1.20 in the DCS calculations, middle results are taken from Byron and Joachain (1977), Lower one is with DE = 1.10 in the DCS calculations.

Table - 4.5

The behaviour of the Born seattering amplitudes (equations 4.49, 4.54, 4.55, 4.53) in ISHe process for the Hartree - Fock orbitals at E = 200 eV.

θ	$f^{(1)}_{i \to f}$ (-ve)	Re1 $F^{(2)}_{HEA}$ (-ve)	Re2 $F^{(2)}_{HEA}$ (-ve)	$f^{(3)}_{GES}$	Im $F^{(2)}_{HEA}$ (-ve)	(Present) DCS No exchange	DCS*
	1	2	3	4	5	6	7
10	3.30 -1	9.3132 -2	1.5331 -1	4.30 -2	1.4183 -1	2.4276 -1	1.30 -1
20	1.61 -1	7.5743 -2	6.1481 -2	3.43 -3	6.6524 -2	7.0631 -2	2.58 -2
30	6.73 -2	6.728 -2	9.302 -2	7.52 -3	5.3391 -2	2.8632 -2	5.37 -2
40	2.85 -2	5.7094 -2	1.7962 -1	5.70 -3	4.4489 -2	1.1749 -2	1.17 -3
50	1.30 -2	3.1161 -2	7.8223 -2	7.29 -3	3.5675 -2	4.521 -3	4.03 -4
70	3.52 -2	2.6374 -2	1.6246 -1	6.30 -3	2.2431 -2	2.2293 -3	8.64 -5
90	1.30 -3	2.3487 -2	1.6100 -1	5.71 -3	1.4969 -2	1.1083 -3	3.37 -5
110	6.17 -4	1.9258 -2	1.6463 -1	5.27 -3	1.1824 -2	8.2096 -4	1.77 -5
130	3.64 -4	1.6228 -2	1.6802 -1	4.96 -3	1.0201 -2	4.4437 -4	1.14 -5

....Contd...

Table - 4.5 Contd... at E = 400 eV.

θ	1	2	3	4	5	6	7
10	2.53 -1	4.3170 -2	1.2781 -2	1.14 -2	6.7240 -2	8.8068 -2	6.80 -2
20	7.51 -2	3.3226 -2	4.0755 -2	3.77 -3	4.0430 -2	1.7933 -2	6.11 -3
30	2.14 -2	3.0892 -2	3.9249 -2	6.65 -3	3.0750 -2	4.8095 -3	7.64 -4
40	7.08 -3	1.6905 -2	8.7245 -2	3.47 -3	3.1768 -2	2.1181 -3	1.21 -4
50	2.73 -3	1.3433 -2	9.4948 -2	3.10 -3	1.5021 -2	9.3659 -4	3.82 -5
70	6.05 -4	1.0383 -2	8.4648 -2	2.63 -3	8.5679 -3	2.780 -4	9.16 -6
90	2.00 -4	6.5389 -3	8.6841 -2	2.30 -3	6.5056 -3	1.1518 -4	3.62 -6
110	8.95 -5	5.9887 -3	8.8694 -2	2.06 -3	5.1727 -3	7.5058 -5	1.44 -6
130	5.10 -5	4.2842 -3	8.9485 -2	1.89 -3	4.1840 -3	4.287 -5	1.22 -6

DCS* Results taken from Singh and Tripathi (1980)

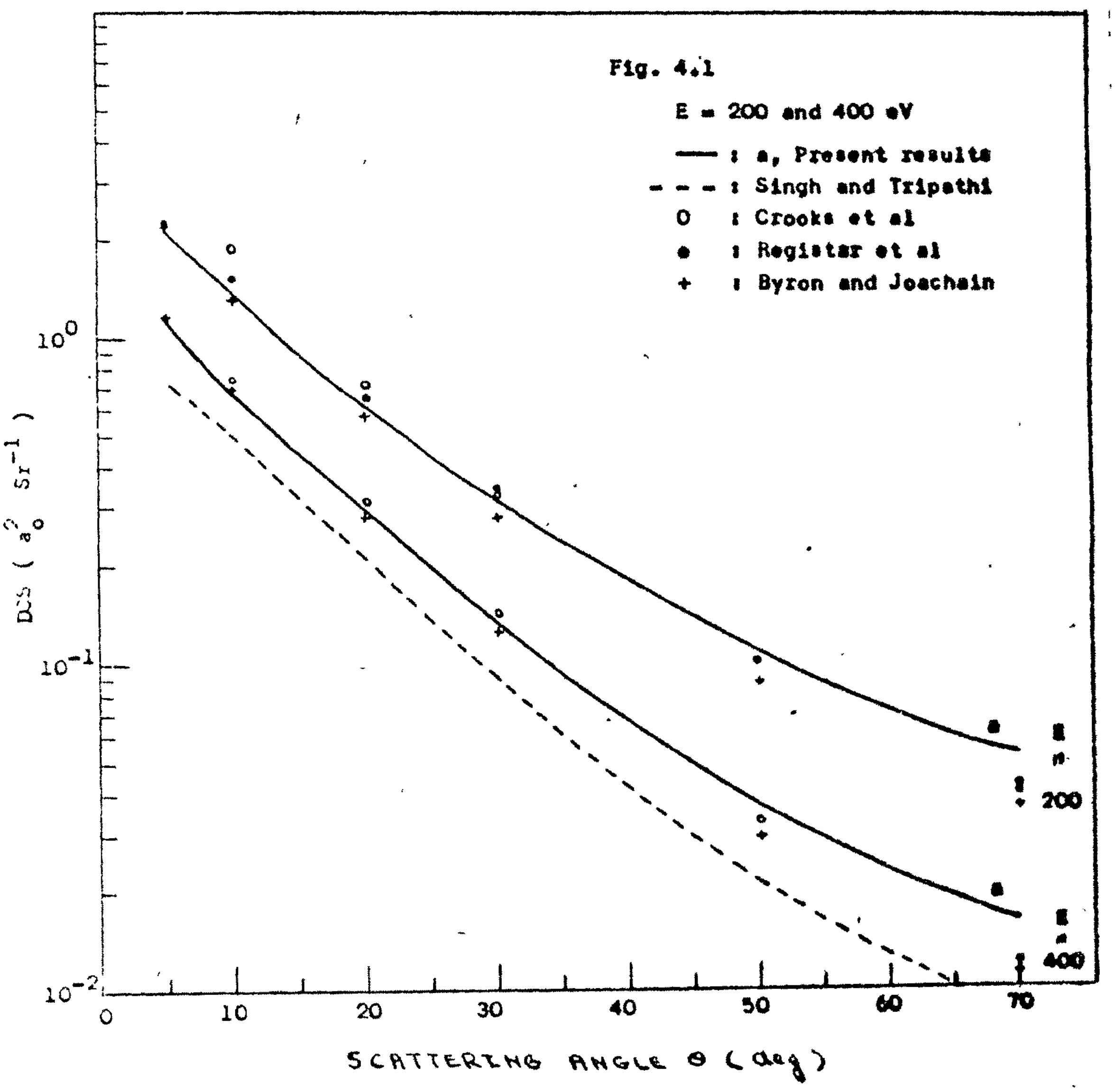


Fig. 4.1
E = 200 and 400 eV
: a, Present results
: Singh and Tripathi
O : Crooks et al
● : Registar et al
+ : Byron and Joachain
DCS (a_o^2 Sr^{-1})
10^0
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
200
400
SCATTERING ANGLE θ (deg)

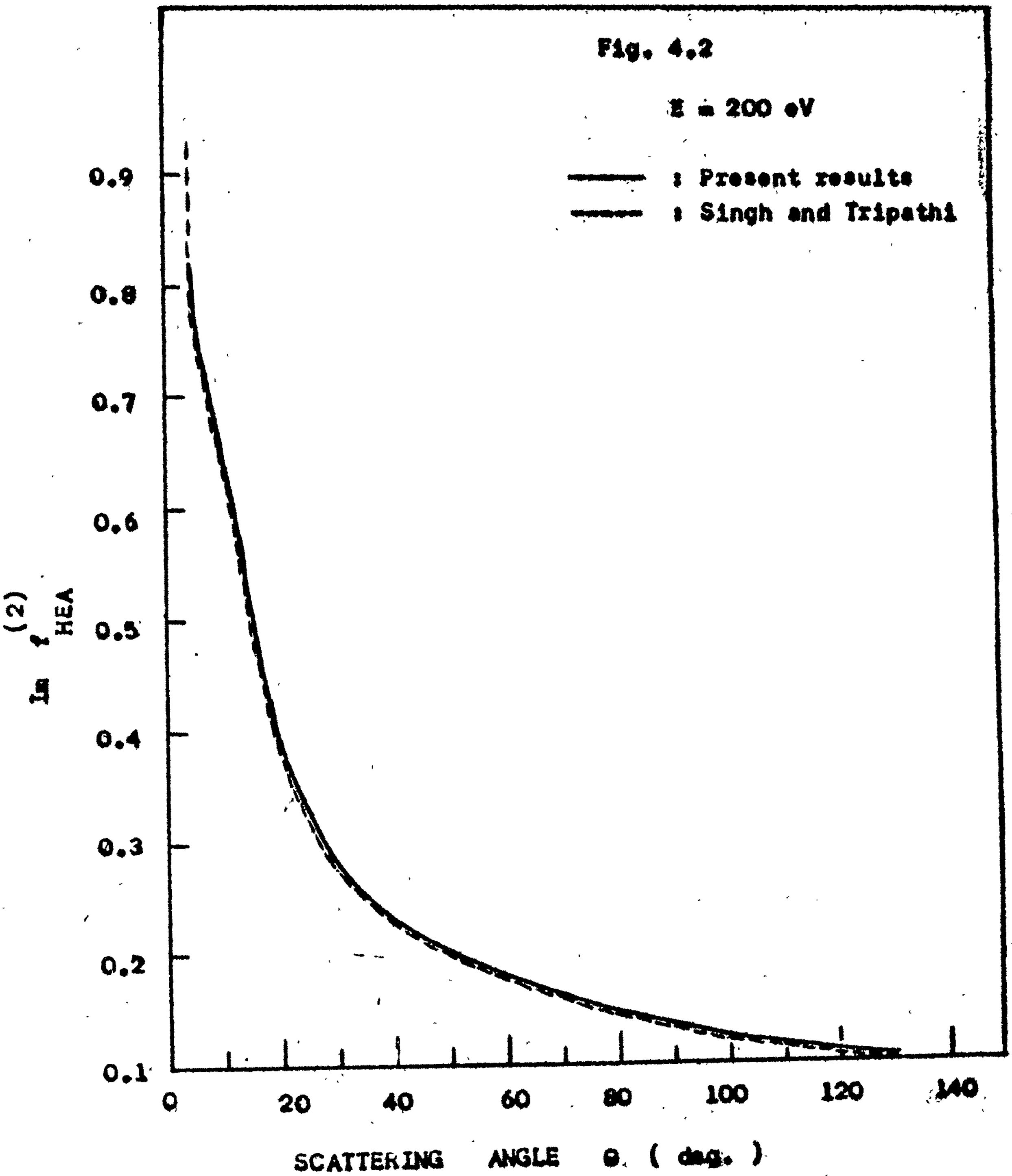
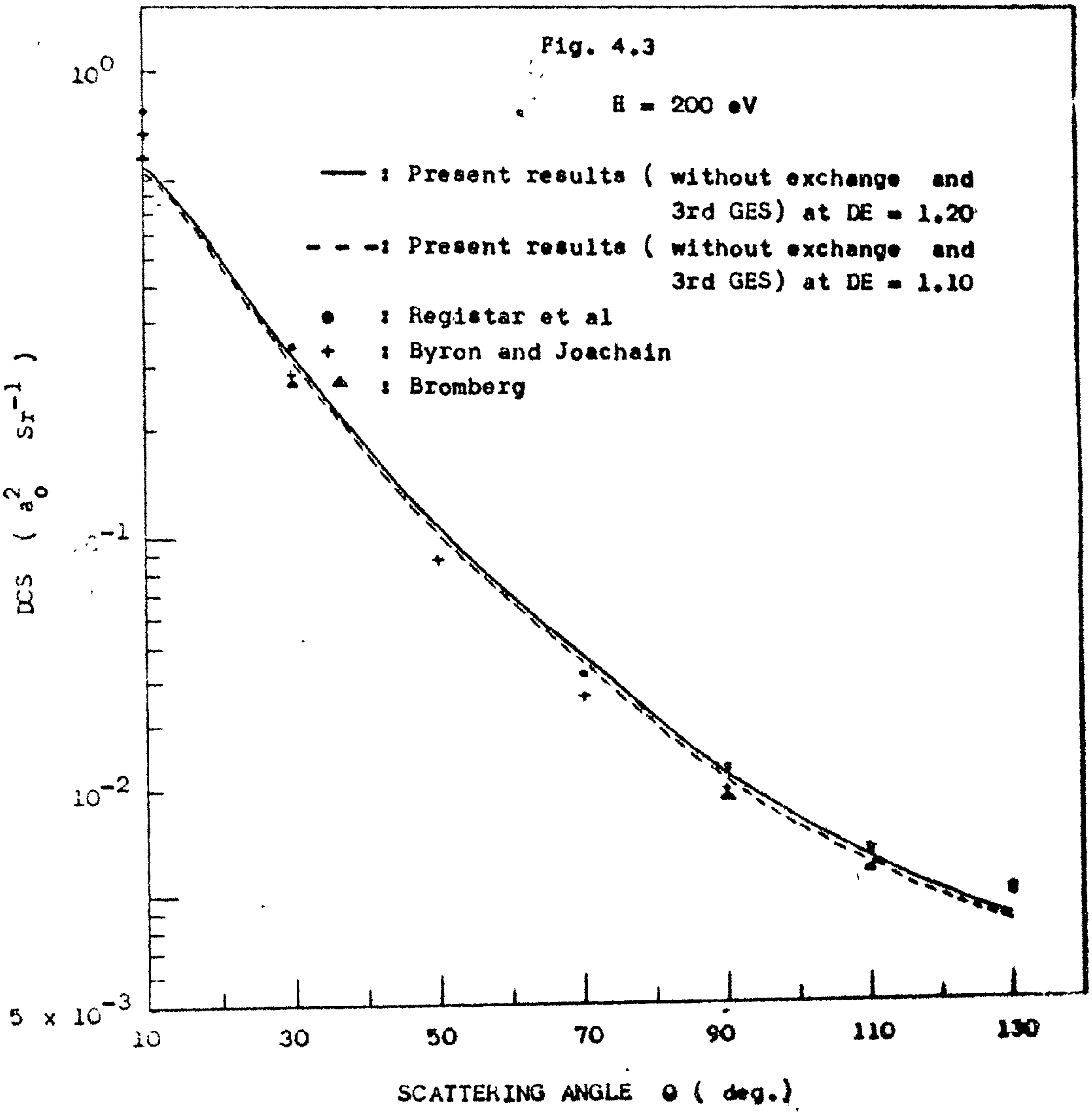


Fig. 4.2
E = 200 eV
: Present results
: Singh and Tripathi
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Im f(2) HEA
0
20
40
60
80
100
120
140
SCATTERING ANGLE θ (deg.)

Fig. 4.3


E = 200 eV
—— : Present results (without exchange and 3rd GES) at DE = 1.20
- - - : Present results (without exchange and 3rd GES) at DE = 1.10
● : Registar et al
+ : Byron and Joachain
▲ : Bromberg
DCS (a$_0^2$ Sr^{-1})
10^0
10^{-1}
10^{-2}
5 x 10^{-3}
10
30
50
70
90
110
130
SCATTERING ANGLE Θ (deg.)

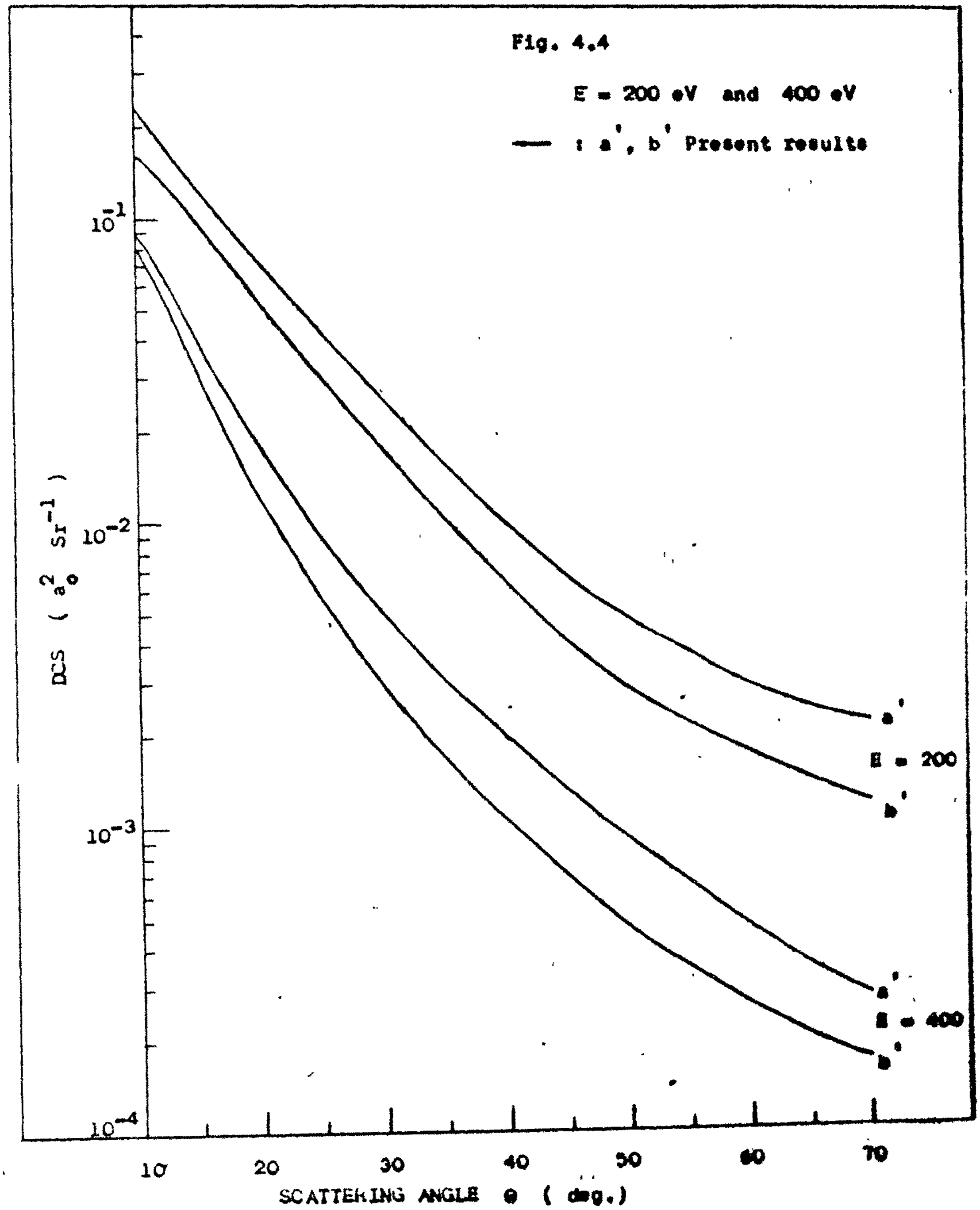


Fig. 4.4
E = 200 eV and 400 eV
: a', b' Present results
a'
E = 200
b'
a'
E = 400
b'
10^{-1}
10^{-2}
10^{-3}
10^{-4}
DCS (a_o^2 Sr^{-1})
10
20
30
40
50
60
70
SCATTERING ANGLE θ (deg.)