APPENDIX

The $d\underline{P}$, dp_z integrals occured in the second Born approximation of the present investigations (Chapter's III, IV and V), are evaluated using the standard integral techniques (Gradshiteyn and Ryzhik, 1965). The evaluation of those typical integrals is as follows :

A. 1 Evaluation of I_1 (......) integral :

$$I_1 (q^2, u^2, v^2) = \int_0^\infty \int_0^{2\pi} \frac{p \, dp \, d\phi}{(p^2 + u^2) (|\underline{q} - \underline{p} |^2 + v^2)}$$

$$= \int_0^\infty \frac{p \, dp \, 2\pi}{(p^2 + u^2) [(q^2 + p^2 + v^2)^2 - 4q^2 p^2]^{1/2}}$$

$$= \frac{\pi}{E} \log \left[\frac{(q^2 + v^2)(q^2 + v^2 + E) - u^2(v^2 - q^2)}{u^2 (E + v^2 - u^2 - q^2)} \right]$$

(where $E^2 = u^4 + (q^2 + v^2)^2 - 2 u^2 (v^2 - q^2)$)

$$= \frac{\pi}{(u^2 - v^2)} \log \frac{u^2}{v^2} \quad \text{when } q \longrightarrow o$$

$$= \frac{\pi}{y^2} \log \frac{\beta_i^2 + y^2}{\beta_i^2} \quad \text{when} \quad \begin{array}{l} u^2 = \beta_i^2 + y^2 \\ v^2 = B_i^2 \end{array}$$

These typical results are used in the present work to obtain the closed form of I_1 (.......) and I_1' s (.......) , which are occured in the imaginary part of the second Born approximation.

A. 2 Evaluation of I_2 (.......) integral :

$$I_2(q^2, \beta_i^2, y^2) = \int d\underline{p} \int_{-\infty}^{\infty} \frac{dp_z}{(p_z - \beta_i)(|\underline{q} - \underline{p}|^2 + p_z^2)(p^2 + p_z^2 + y^2)}$$

$$= \int \frac{d\underline{p}}{(p^2 - |\underline{q} - \underline{p}|^2 + y^2)} \mathcal{P}\int_{-\infty}^{\infty} \frac{dp_z}{(p_z - \beta_i)} \left[\frac{1}{p_z^2 - |\underline{q} - \underline{p}|^2} \right.$$

$$\left. - \frac{1}{p_z^2 + p^2 + y^2} \right] = -\pi\beta_i \left[\int \frac{d\underline{p}}{(|\underline{q} - \underline{p}|^2 - p^2 + y^2)p(p^2 + \beta_i^2)} \right.$$

$$\left. - \int \frac{d\underline{p}}{(p^2 - |\underline{q} - \underline{p}|^2 + y^2)(p^2 + \beta_i^2 + y^2)(p^2 + y^2)^{1/2}} \right]$$

$$= -\pi\beta_i [I_2' - I_2'']$$

where

$$I_2' = \int_0^{\infty} \frac{p\, dp}{p(p^2 + \beta_i^2)} \int_0^{2\pi} \frac{d\phi}{q^2 + y^2 - 2qp\cos\phi} ;$$

$$= 2\pi \int_0^A \frac{dp}{(p^2 + \beta_i^2)((q^2 + y^2)^2 - 4q^2p^2)^{1/2}} \; ; \qquad A = \frac{q^2 + p^2}{2q}$$

$$= \frac{2\pi}{2q} \int_0^A \frac{dp}{(p^2 + \beta_i^2)(A^2 - p^2)^{1/2}} = \frac{\pi^2}{\beta_i((q^2 + y^2)^2 + 4q^2\beta_i^2)^{1/2}}$$

and

$$I_2'' = \int_0^\infty \frac{p \cdot dp}{(p^2 + y^2)^{1/2}(p^2 + \beta_i^2 + y^2)} \; I_\phi$$

$$I_\phi = \int_0^{2\pi} \frac{d\phi}{(y^2 - q^2 + 2qp \cos \phi)} = \frac{2\pi}{((y^2 - q^2) - 4q^2p^2)^{1/2}} \; ;$$

$$\left(\frac{|y^2 - q^2|}{2q} > p, \; y^2 > q^2 \right) = -\int_0^{2\pi} \frac{d\phi}{q^2 - y^2 - 2qp \cos \phi}$$

$$= \frac{-2\pi}{((y^2 - q^2) - 4q^2p^2)^{1/2}} \; ; \qquad \frac{|y^2 - q^2|}{2q} > p, \; q^2 > y^2$$

$$= \frac{2\pi \,\mathrm{sgn}\,(y^2 - q^2)}{((y^2 - q^2)^2 - 4q^2p^2)^{1/2}}$$

$$\therefore \; I_2'' = \frac{\pi \,\mathrm{sgn}\,(y^2 - q^2)}{2q} \int_0^{A^2} \frac{dx}{(x + p^2 + y^2)(x + y^2)^{1/2}(A^2 - x)^{1/2}} \; ;$$

$$(A^2 = \frac{(y^2 - q^2)^2}{4q^2})$$

$$= \frac{\pi \operatorname{sgn} (y^2 - q^2)}{\beta_i[(y^2 + q^2)^2 + 4q^2\beta_i^2]^{1/2}} \left[\frac{\pi}{2} - \sin^{-1} A_1 \right]$$

Now combining I_2' and I_2'', we will get I_2 as

$$I_2 (q^2, \beta_i^2 , y^2) = \frac{- \pi^3}{((q^2 + y^2)^2 + 4q^2\beta_i^2)^{1/2}}$$

$$\left[1-\operatorname{sgn}(y^2-q^2) \left(\frac{1}{2} - \frac{\sin^{-1} A_1}{\pi} \right) \right]$$

$$= \frac{-\pi^3}{y^2}\left[\frac{1}{2} + \frac{\sin^{-1} A_1'}{\pi} \right] \quad \text{when} \quad q \longrightarrow o$$

where

$$A_1 = 1 - \frac{2\beta_i^2 (y^2 - q^2)}{(y^2 + q^2)^2 (y^2 + \beta_i^2)} \; ; \; A_1' = 1 - \frac{2\beta_i^2}{y^2(y^2+ \beta_i^2)}$$

A. 3 Evaluation of I_3 (.......) integral :

$$I_3 (\beta_i^2 , y) = \mathcal{P} \int \int_{-\infty}^{\infty} \frac{d\underline{P} \;\; dp_z}{(p_z - \beta_i)(p^2 + p_z^2 + y^2)}$$

$$= 2\pi \int_0^{\infty} p \; dp \, \mathcal{P}\int_{-\infty}^{\infty} \frac{dp_z}{(p_z - \beta_i)(p^2 + p_z^2 + y^2)}$$

$$= + 2\pi \int_0^{\infty} p \; dp \; \frac{-\pi\beta_i}{(p^2 + \beta_i^2 + y^2)(p^2 + y^2)^{1/2}}$$

$$= - 2\pi^2 \left[\frac{\pi}{2} - \tan^{-1} \frac{y}{\beta_i} \right]$$

A. 4 <u>Evaluation of I_4 (.......) integral</u> :

$$I_4 (q^2 , \beta_i^2 , y_1^2 , y_2^2) = \int d\underline{p} \, \mathcal{P}\int_{-\infty}^{\infty} \frac{dp_z \, (|\underline{q} - \underline{p}|^2 + y_2^2 + \beta_i^2)^{-1}}{(p_z - \beta_i)(p^2 + \beta_i^2 + y_1^2)}$$

$$= \int d\underline{p} \, \mathcal{P}\int_{-\infty}^{\infty} \frac{dp_z}{(p_z - \beta_i)} \left[\frac{1}{(p^2 - |\underline{q} - \underline{p}|^2 + y_1^2 - y_2^2)} \left(\frac{1}{|\underline{q} - \underline{p}|^2 + p_z^2 + y_2^2} - \frac{1}{p^2 + p_z^2 + y_1^2} \right) \right]$$

$$= \int d\underline{p} \, \mathcal{P}\int_{-\infty}^{\infty} \frac{dp_z}{(p_z - \beta_i)(p^2 - |\underline{q} - \underline{p}|^2 + y)} \left[\frac{1}{|\underline{q} - \underline{p}|^2 + p_z^2 + y_2^2} - \frac{1}{p^2 + p_z^2 + y_1^2} \right]$$

The evaluation of $d\underline{P}$, dp_z integrals is similar to that of I_2'' (A.2) evaluation. Using those results, the present I_4 (.......) can be obtained as

$$I_4 (q^2 , \beta_i^2 , y_1^2 , y_2^2) = - \pi^2 [\operatorname{sgn} (Y + q^2) \left(\frac{1}{2E_1} - \frac{\sin^{-1} A_1}{\pi E_1} \right) - \operatorname{sgn} (Y - q^2) \left(\frac{1}{2 E_2} - \frac{\sin^{-1} A_2}{\pi E_2} \right)]$$

where

$$E_1^2 = (Y + q^2)^2 + 4q^2 (\beta_i^2 + y_1^2)$$

$$E_2^2 = (Y - q^2)^2 + 4q^2 (\beta_i^2 + y_2^2)$$

$$A_1 = 1 - \frac{2\beta_i^2 (Y + q^2)^2}{[(Y + q^2)^2 + 4q^2 y_1^2] (\beta_i^2 + y_1^2)} \qquad ; Y = y_1^2 - y_2^2$$

$$A_2 = 1 - \frac{2\beta_i^2 (Y - q^2)^2}{[(Y - q^2)^2 + 4q^2 y_2^2] (\beta_i^2 + y_2^2)}$$

In the forward direction $q \longrightarrow o$, this I_4 (.......) tends to a finite value like I_2 (.......).

These typical results (A.2 , A.3 , A.4) are used to obtain the closed forms of I_2 (.......), I_2' (.......) and I_3 (.......) , I_3' (.......) and I_4 (.......) in the real parts of the second Born approximation.

(It is shown that all the integrals tends to a finite value in the forward direction ($q \longrightarrow o$) .