CHAPTER VII

IDENTIFICATION OF SYSTEM CHARACTERISTICS FROM NORMAL OPERATING DATA USING THE DELAY LINE SYNTHESIZER PRINCIPLE

A computational technique is presented here for identifying the impulse response of a linear system from normal operating noisy data. No assumption, however, is made regarding the nature of the noise. The technique derives its idea from the Delay Line Synthesizer (DLS) though in this case the DLS coefficients which discretely represent the weighting function are computed automatically employing the steepest descent method. The method has been tried out on a first order as well as a second order system simulated on a digital computer and the estimated impulse response is found to be very close to the actual one.

7.1 Introduction

The objective of adaptive control is to enable the control system to operate while satisfying certain performance criteria under changing conditions, environmental or inherent. Identification of the transfer function, impulse response or equivalent characteristic of a system is necessary for self-adaptation. For a linear system the performance can be evaluated in terms of the inpulse response[16, 102] and to accomplish self-adaptation, controllable parameters can be adjusted until the identified impulse response takes the desired form.

Goodman and Reswick[5] developed an experimental device, namely, Delay Line Synthesizer (DLS), to obtain an impulse response by feeding to the device auto- and cross-correlations computed beforehand from the input-output record of the physical system. The auto-correlation $\Phi(t)$ of the input and the cross-correlation $\Psi(t)$ of the input with the output bear the same

convolution relation[5,103] as that between the input $r(t)$ and the output $c(t)$ at time t (Fig. 7.1) as shown by equations (7.1) and (7.2), were $h(t)$ is the impulse response.

$$c(\tau) = \int_0^{\infty} h(t)\, r(\tau-t)\, dt, \quad ; \quad h(t) = 0 \quad t<0 \qquad (7.1)$$

$$\Psi(\tau) = \int_0^{\infty} h(t)\, \Phi(\tau-t)\, dt \quad ; \quad h(t) = 0 \quad t<0 \qquad (7.2)$$

Equation (7.2) is well known in the literature as the Wiener-Hopf equation.

For a damped type-0 system[104], $h(t)$ tends to zero as $t \to \infty$ and becomes practically negligible beyond a certain instant $t = t_2$. Thus equation (7.2) becomes

$$\Psi(\tau) = \int_0^{t_2} h(t)\, \Phi(\tau-t)\, dt \qquad (7.3)$$

where $h(t) = 0$ for $t < 0$ and $t \geq t_2$. If the input to the system is a random noise with zero mean and no dominating periodic component, the auto-correlation $\Phi(t)$ of the input tends to zero[103] as $t \to \infty$. However, in a practical situation, $\Phi(t)$ becomes negligible beyond some finite value of time $t = t_1$ as can be seen in Fig. 7.2. Therefore, when $\tau \geq t_1 + t_2$, the argument $(\tau-t)$ of Φ in equation (7.3) is greater than t_1 for any value of t between 0 and t_2, whence $\Phi(\tau-t) = 0$. Therefore $\Psi(\tau) = 0$ for $\tau \geq t_3$ where $t_3 = t_1 + t_2$. Thus, the cross-correlation $\Psi(\tau)$ in equation (7.3) is always zero beyond the instand t_3 as can be seen in Fig. 7.3. Since t_2 is not known for an unknown system, it can be estimated as $t_2 = t_3 - t_1$, where t_1 and t_3 are known from the given data for auto- and cross-correlations.

Goodman and Reswick used the DLS to simulate the integral in equation (7.3) discretely by dividing the period 0 to t_2 into K equal segments each of duration T (Fig. 7.4) and then using the trapezoidal rule for integration, as given below

$$\Psi(\tau) \approx \tfrac{1}{2}\, h(0)\, \bar{\Phi}(\tau)\, T + \tfrac{1}{2}\, h(KT)\, \bar{\Phi}(\tau-KT)\, T + \sum_{n=1}^{K-1} h(nT)\bar{\Phi}(\tau-nT)\, T \qquad (7.4)$$

Here "$\approx$" means "is approximately". Equation (7.4) contains (K + 1) terms each being a product of an ordinate of the weighting function at an instant nT by the auto-correlation delayed by the time nT where n = 0, 1, . . . ,K. The device thus contains K delay lines (neglecting the zero delay in the first term) and (K+1) multipliers h(nT) for n = 0, 1, . . . ,K . The technique used by Goodman and Reswick to obtain h(nT)'s in their DLS is to feed the auto-correlation of the input data as input to DLS and adjust all h(nT)'s manually until the output, i.e. the cross-correlation, tallies with the given cross-correlation at all instants of time. The pulse-like shape of the auto-correlation makes this task easier.

The technique discussed here does not require the DLS as a unit as this is simulated on the digital computer and moreover, the DLS coefficients are computed automatically using the steepest descent method.

7.2 Computational Scheme for Cross-Correlation

In applying the trapezoidal rule in equation (7.4), it is assumed that the function $h(nT)\,\bar{\Phi}(\tau-nT)$ is linear between samples. But the auto-correlation is wavy and so, from an accuracy

point of view, this assumption is justified when the sampling interval T is very small. However, during the identification process it is desirable to have less samples, i.e. wider sampling interval, to save computational time without losing accuracy. This is accomplished by representing the auto-correlation by a polynomial of suitable degree higher than the first and the weighting function by a straight line (i.e. first degree polynomial) between widely spaced samples and then performing the exact integration. The auto-correlation is to be substituted by a higher degree polynomial rather than linear because it is more wavy as compared to the impulse response. Representing the intergral from 0 to t_2 in equation (7.3) by the sum of the integrals over each of the sampling intervals, equation (7.3) becomes

$$\Psi(\tau) = \sum_{n=0}^{K-1} \int_{nT}^{(n+1)T} h(t)\, \bar{\Phi}(\tau - t)\, dt \qquad (7.5)$$

The cross-correlation $\Psi(\tau)$ is computed at discrete instants mT, $m = 0, 1, 2, \ldots, M$ and so writing mT for τ, letting Ψ_m for $\Psi(\tau)$ at $\tau = mT$ and noting that $\bar{\Phi}(t) = \Phi(-t)$, equation (7.5) becomes

$$\begin{aligned} \Psi_m &= \sum_{n=0}^{K-1} \int_{nT}^{(n+1)T} h(t)\, \bar{\Phi}(t - mT)\, dt \\ &= \sum_{n=0}^{K-1} \int_{0}^{T} h(nT + t)\, \Phi(nT - mT + t)\, dt \qquad (7.6) \\ &\qquad\qquad m = 0, 1, \ldots, M \end{aligned}$$

Let h_n denote $h(nT)$ and suppose that the weighting function for the time interval nT to $(n+1)T$ is represented by the relation

$$h(nT + t) = h_n + \alpha_n t \qquad (7.7)$$

for $n = 0, 1, \ldots, K-1$ and $0 \leq t \leq T$, where α_n is the slope of the impulse response $h(t)$ at $t = nT$ and is given by

$$\alpha_n = \frac{d}{dt}[h(t)]\Big|_{t=nT} \approx \frac{h_{n+1} - h_n}{T}$$

Also writing Φ_n for the auto-correlation $\Phi(nT+t)$, $0 \leq t \leq T$, and representing Φ_n by a polynomial of N^{th} degree, one obtains

$$\Phi_n = \sum_{i=0}^{N} a_{n,i}\, t^i \quad ; \quad n = -K, -(K-1), \ldots, -1, 0, 1 \ldots, K-1 \qquad (7.8)$$

where the $a_{n,i}$'s are the coefficients of polynomial for the auto-correlation for the time interval nT to $(n+1)T$. Substituting for $\Phi(t)$ and $h(t)$ in equation (7.6), one obtains

$$\Psi_m = \sum_{n=0}^{K-1} \int_0^T [h_n + \alpha_n t]\left[\sum_{i=0}^{N} a_{n-m,i}\, t^i\right] dt \qquad (7.9)$$

Performing the integration, equation (7.9) simplifies to

$$\Psi_m = \sum_{n=0}^{K-1} \left[\sum_{i=0}^{N} a_{n-m,i}\left\{\frac{T^{i+1}}{i+1} h_n + \frac{T^{i+2}}{i+2}\alpha_n\right\}\right] \qquad (7.10)$$

$$m = 0, 1, \ldots, M$$

7.3 Identification Scheme

In the identification problem, the auto-correlation (and hence the $a_{n,i}$'s) and the cross-correlation are given and only the h_n's are to be evaluated. Let the given cross-correlation (computed from the given input-output data) be denoted by Ψ'_m for $m = 0, 1, \ldots, M$. If the impulse response (i.e. h_n's) were precisely known, the cross-correlation Ψ_m computed by using equation (7.10) would not be identical with Ψ'_m but

there would be some error. This error may be attributed to the presence of noise or imperfect measurements or approximations involved in the computation. Therefore, denoting the errors by E_m for $m = 0, 1, \ldots, M$, one obtains

$$\Psi'_m = \Psi_m + E_m \quad ; \quad m = 0, 1, \ldots, M \tag{7.11}$$

Substituting for Ψ_m and rearranging, equation (7.11) becomes

$$E_m = \Psi'_m - \sum_{n=0}^{K-1} \left[\sum_{i=0}^{N} \left\{ a_{n-m,i} \frac{T^{i+1}}{i+1} h_n + \frac{T^{i+2}}{i+2} \alpha_n \right\} \right] \tag{7.12}$$

$$m = 0, 1, \ldots, M$$

As noted earlier, even if the impulse response were precisely known, the Ψ_m and Ψ'_m would not be the same but close to each other and so the error E_m would be very small. Conversely, the h_n's in equation (7.12) can be adjusted in a scientific manner until the errors become negligibly small to arrive close to the system's true impulse response. Indeed, our identification procedure consists in assuming some arbitrary set of values for the h_n's in equation (7.12) and then modifying the h_n's successively using the steepest descent rule until the sum of the squares of the errors E_m, $m = 0, 1, \ldots, M$ reaches a minimum. The results thus obtained give the best possible impulse response of the linear system based on the minimum square error criterion. This criterion yields the same results as would be obtained by the minimum mean square error criterion, i.e. $\frac{1}{M+1} \sum_{m=0}^{M} E_m^2$, because the two criteria differ only by a constant. The best possible linear system thus obtained is also known as the optimum linear system[103].

In equation (7.12), It is required to determine K values of h_n's for $n = 0, 1, \ldots, K-1$ and so we should know at least K values of the errors E_m. Therefore let $M = K-1$. Making use of the minimum square error criterion, the performance index I to be minimized is given by

$$I = \sum_{m=0}^{K-1} E_m^2 \tag{7.13}$$

The magnitude of increments (or decrements) in h_n's needed to reach the minima are determined by the steepest descent rule Taking the differential of I with respect to the h_n's (noting that each of the errors E_m's depends on all h_n's), one obtains

$$I = 2 \sum_{m=0}^{K-1} E_m \left\{ \sum_{j=0}^{K-1} \frac{\delta E_m}{\delta h_j} \Delta h_j \right\} \tag{7.14}$$

where

$$\frac{\delta E_m}{\delta h_j} = - \sum_{i=0}^{N} a_{j-m,i} \left\{ \frac{T^{i+1}}{i+1} + \frac{T^{i+2}}{i+2} \frac{\delta \alpha_j}{\delta h_j} \right\} - \sum_{i=0}^{N} a_{j-1-m,i} \cdot \left\{ \frac{T^{i+1}}{i+2} \frac{\delta \alpha_{j-1}}{\delta h_j} \right\} \tag{7.15}$$

Substituting for

$$\frac{\delta \alpha_j}{\delta h_j} = \frac{\delta}{\delta h_j} \left\{ \frac{h_{j+1} - h_j}{T} \right\} = - \frac{1}{T} \tag{7.16}$$

and

$$\frac{\delta \alpha_{j-1}}{\delta h_j} = \frac{\delta}{\delta h_j} \left\{ \frac{h_j - h_{j-1}}{T} \right\} = \frac{1}{T} \qquad j = 0, 1, \ldots, K-1 \quad \text{and} \quad h_{-1} = 0 \tag{7.17}$$

in equation (7.15), one obtains

$$\frac{\delta E_m}{\delta h_j} = - \sum_{i=0}^{N} T^{i+1} \left\{ \frac{a_{j-m,i}}{i+1} + \frac{a_{j-1-m,i}}{i+2} - \frac{a_{j-m,i}}{i+2} \right\} \qquad (7.18)$$

and using this result in equation (7.14), one gets

$$I = -2 \sum_{m=0}^{K-1} E_m \left[\sum_{j=0}^{K-1} \left\{ - \sum_{i=0}^{N} T^{i+1} \left(\frac{a_{j-m,i}}{i+1} + \frac{a_{j-1-m,i}}{i+2} - \frac{a_{j-m,i}}{i+2} \right) \right\} \Delta h_j \right]$$

$$= \sum_{j=0}^{K-1} \left[-2 \sum_{m=0}^{K-1} E_m \left\{ \sum_{i=0}^{N} T^{i+1} \left(\frac{a_{j-m,i}}{i+1} + \frac{a_{j-1-m,i}}{i+2} - \frac{a_{j-m,i}}{i+2} \right) \right\} \right] \Delta h_j \qquad (7.19)$$

The expression within the outer brackets in equation (7.19) is the increment of I with respect to the increment of h_j. Let this increment be represented by $\delta I/\delta h_j$. Thus, equation (7.19) becomes

$$\Delta I = \sum_{j=0}^{K-1} \left[\frac{\delta I}{\delta h_j} \right] \Delta h_j \qquad (7.20)$$

The steepest descent rule[97], used to reach the minima of I requires that the increments in h_n's satisfy

$$\Delta h_n = -\alpha \frac{\delta I/\delta h_n}{\left[\sum_{j=0}^{K-1} \left(\frac{\delta I}{\delta h_j} \right)^2 \right]^{1/2}} \qquad n = 0, 1, \ldots, K-1 \qquad (7.21)$$

In equation (7.21), the coefficient α fixes the step-size[97] of the increment along the gradient and is to be chosen suitably so that on each successive iteration the performance index becomes smaller and smaller. Large values of α may lead to overcorrection of the h_n's and the performance index, instead of approaching the minima, may jump to and fro around the minima. It has been found from experience that, in the early stages, α

should be chosen as about 5 % of the peak value of the impulse response. However, since the peak value of the impulse response is not known, its peak value may be roughly estimated as the ratio of the peak value of the cross-correlation to that of the auto-correlation. In the later stages, when the performance index is getting closer to the minima, this value of α is found to be quite large and must be successively reduced whenever the current value of the performance index exceeds its previous value. In fact, the whole operation can be performed automatically by proper computer programming.

The entire computational procedure can be outlined as follows.

(1) It must be first assured that both the input and output data have zero means and no predominant periodic component. The statistical means for both should be computed and, if they are not zero they should be deduced respectively from the input and output records which, in turn, must be used to compute $\Phi(t)$ and $\Psi(t)$ (in a similar manner[3]) taking a sampling interval small enough to justify the trapezoidal rule for numerical integration. The values of t_1 and t_3 are to be fixed by inspecting the entire range of $\Phi(t)$ and $\Psi(t)$ such that, beyond t_1 and t_3, the function $\Phi(t)$ and $\Psi(t)$ are respectively less than 1% of their peak values. The value of t_1 can not be determined if the $\Phi(t)$ fails to satisfy this condition which means that either the input record contains a predominant periodic component or, in other words, both the input and the output data require similar pre-whitening[21] treatment.

The interval 0 to t_2, i.e. (t_3-t_1), should be divided into K equal parts with a sampling interval or delay time of t_2/K. For a first order system in which case the impulse response is not too wavy, (K=10) gives quite satisfactory results. However, considering the possibility of higher order damped-oscillatory systems where the samples should be relatively closer, (K=20) is, in general, a better choice. A polynomial of degree lowest enough to give the fit of desired accuracy should be fitted to all sections of $\Phi(t)$ to compute $a_{n,i}$'s in equation (7.8).

(2) Let the initial guess for the h_n's, $n = 0, 1, \ldots, K-1$ be all zeroes.

(3) The E_m's and I should be computed using equations (7.12) and (7.13).

(4) Having fixed α as discussed earlier the Δh_n's can be computed from equation (7.21). The h_n's are then to be modified as

$$\text{new } h_n = \text{old } h_n + \Delta h_n, \quad n = 0, 1, \ldots, K-1 \qquad (7.22)$$

where "new h_n" and "old h_n" denote the new and old values of h_n respectively.

(5) The sequence (7.3) to (7.4) is to be repeated until the performance index I reaches a minimum. If at any stage the quantity I becomes larger than its previous value, the value of α should be halved and the above sequence should be continued until further reduction is necessary.

7.4 Numerical Results

The method has been tried out on the following four systems

With different forms of weighting functions.

(1) $G(s) = 1/(s + 1)$; $h(t) = e^{-t}$

(2) $G(s) = 1/(s + 1)(s + 2)$; $h(t) = e^{-t} - e^{-2t}$

(3) $G(s) = 1/(s^2 + 2s + 5)$; $h(t) = e^{-t} \sin 2t$

(4) $G(s) = 1/(s^2 + 2s + 10)$; $h(t) = e^{-t} \sin 3t$

The output for each system was computed on a digital computer with a very small sampling interval for a random input with zero mean, using relation (1). The random data was generated by using the RRN (Rectangular Random Number generator) subroutine[101]. The correlations $\Phi(t)$ and $\Psi(t)$ were computed following the discussion given in reference (5). Figures 7.2 and 7.3, which depict respectively $\Phi(t)$ and $\Psi(t)$ for a first order system with the transfer function $G(s) = 1/(s+1)$, suggest $t_1 = 26$ secs. and $t_3 = 31$ secs. Hence $t_2 = t_3 - t_1 = 5$ secs. Taking $K = 20$, T becomes 0.25 secs. A fourth order was found to be the minimum order of the polynomial for all sections of $\Phi(t)$ satisfying the "F test"[105,106]. The criterion for good fit chosen for the "F test" is that the probability of hypothesis, that an extra coefficient, introduced due to the next higher order polynomial, is zero, is more than 95 % . Computer runs were made following the computational procedure outlined in the previous section with zero as the initial guess on h_n's. Similar procedure was followed for each of these systems. The results of the impulse response identified for these systems are shown in Figures 7.5, 7.6, 7.7 and 7.8. The time taken for identification was about 1.5 mins. on IBM 7094 after having computed a $a_{n,i}$'s and $\Psi(t)$. The time taken for computing $\Phi(t)$ and $\Psi(t)$

and then fitting a polynomial to $\tilde{\Phi}(t)$ to obtain $a_{n,i}$'s was about 4 to 5 mins.

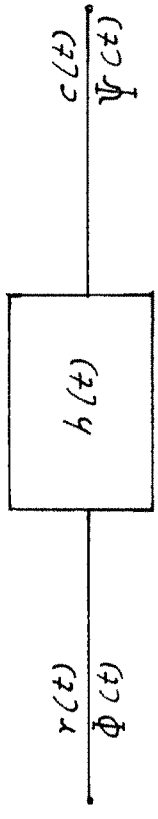

FIG. 7.1 A LINEAR SYSTEM WITH A WEIGHTING FUNCTION $h(t)$

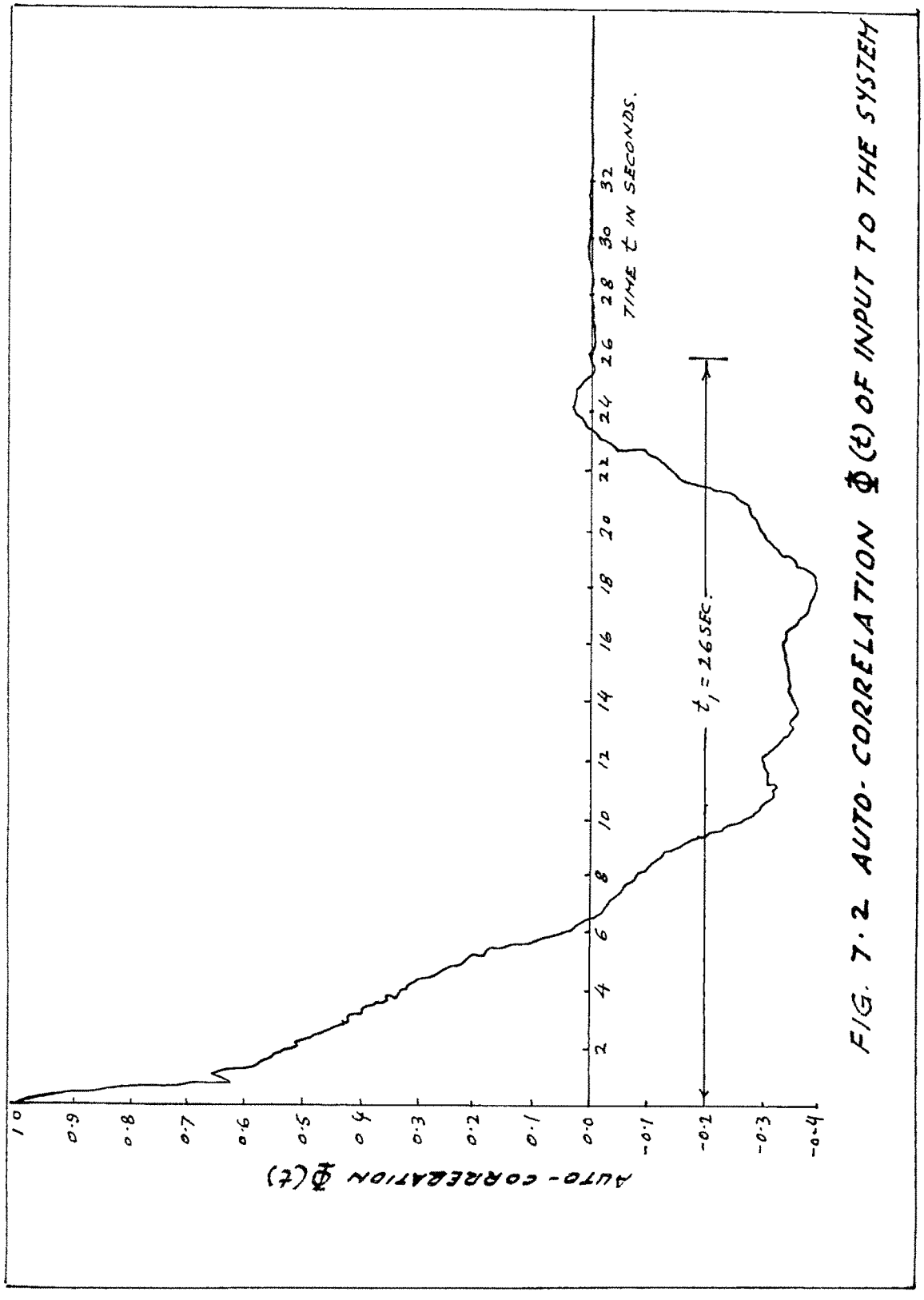

FIG. 7·2 AUTO-CORRELATION $\Phi(t)$ OF INPUT TO THE SYSTEM

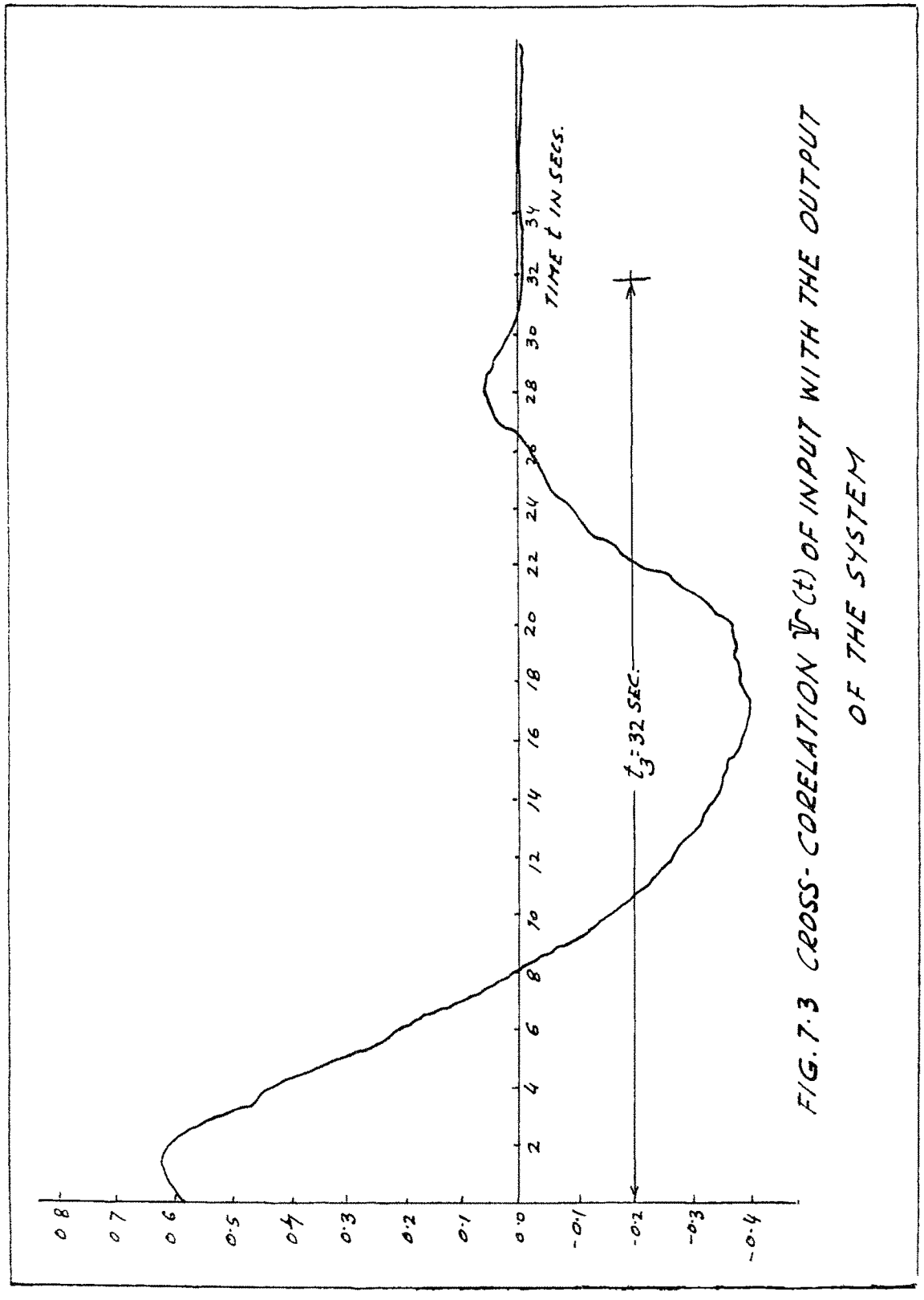

FIG. 7.3 CROSS-CORELATION $\Psi(t)$ OF INPUT WITH THE OUTPUT OF THE SYSTEM

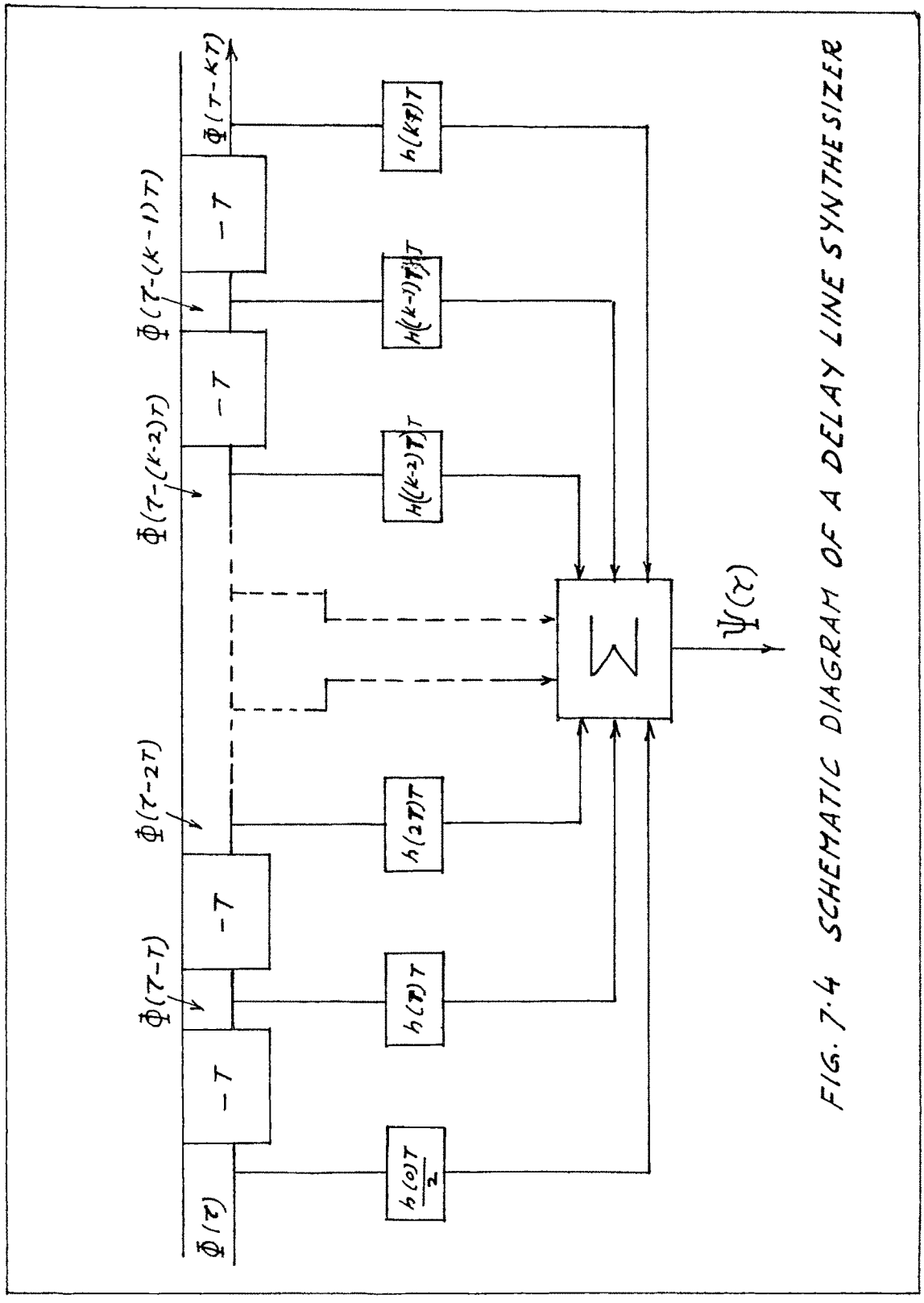

FIG. 7.4 SCHEMATIC DIAGRAM OF A DELAY LINE SYNTHESIZER

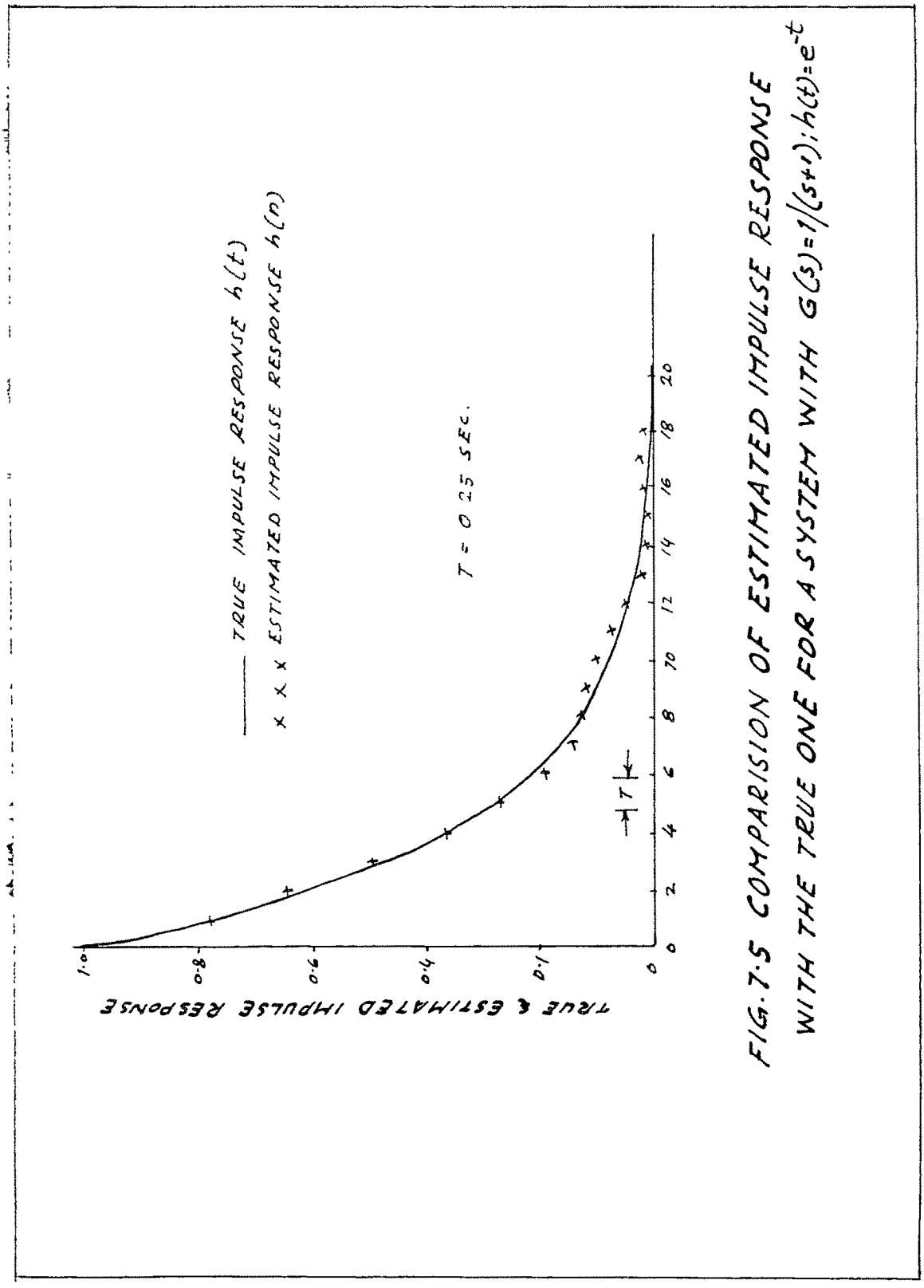

FIG. 7·5 COMPARISION OF ESTIMATED IMPULSE RESPONSE WITH THE TRUE ONE FOR A SYSTEM WITH $G(s)=1/(s+1)$; $h(t)=e^{-t}$

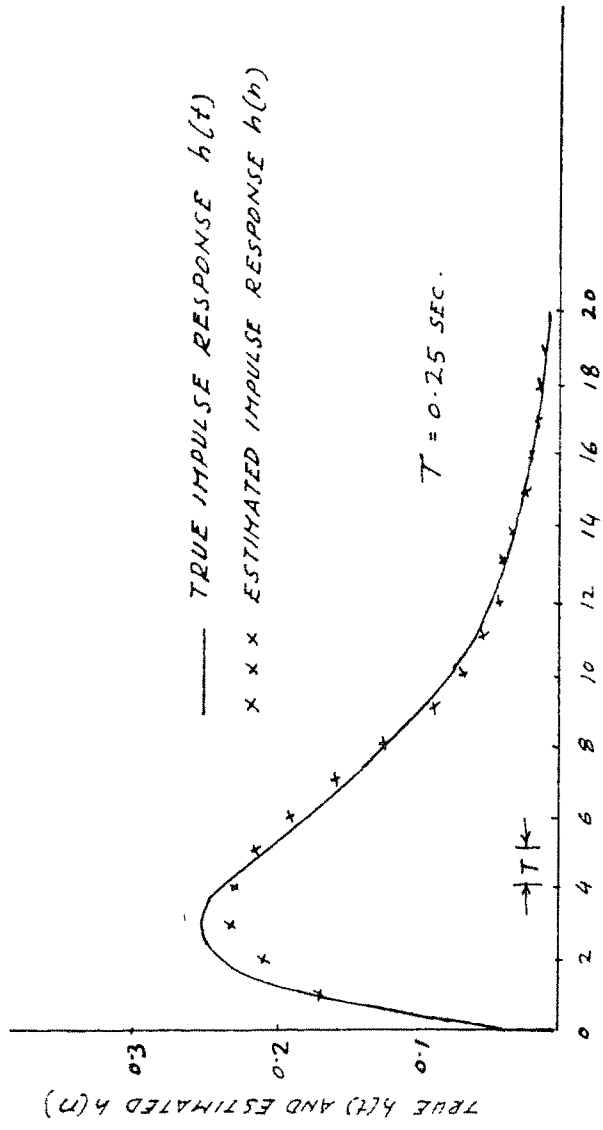

FIG. 7.6 COMPARISION OF ESTIMATED IMPULSE RESPONSE WITH THE TRUE ONE FOR A SYSTEM WITH $G(s) = 1/(s+1)(s+2)$; $h(t) = e^{-t} - e^{-2t}$

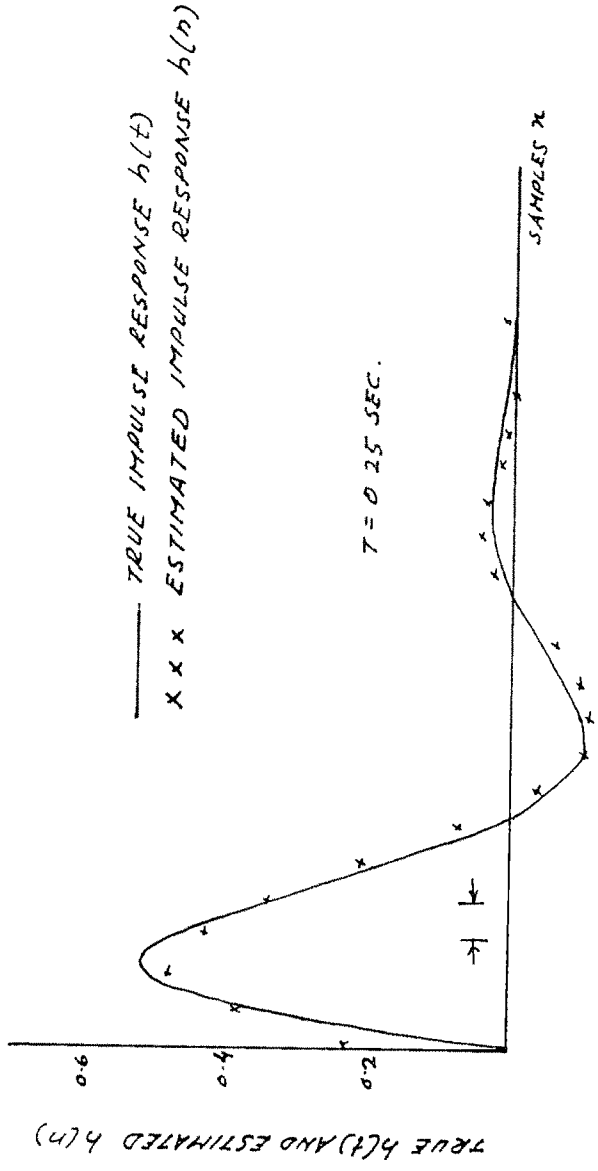

FIG. 7·7 COMPARISION OF ESTIMATED METHOD IMPULSE RESPONSE WITH THE TRUE ONE FOR A SYSTEM WITH $G(s) = 1/(s^2+2s+5)$; $h(t) = e^{-t} \sin 2t$

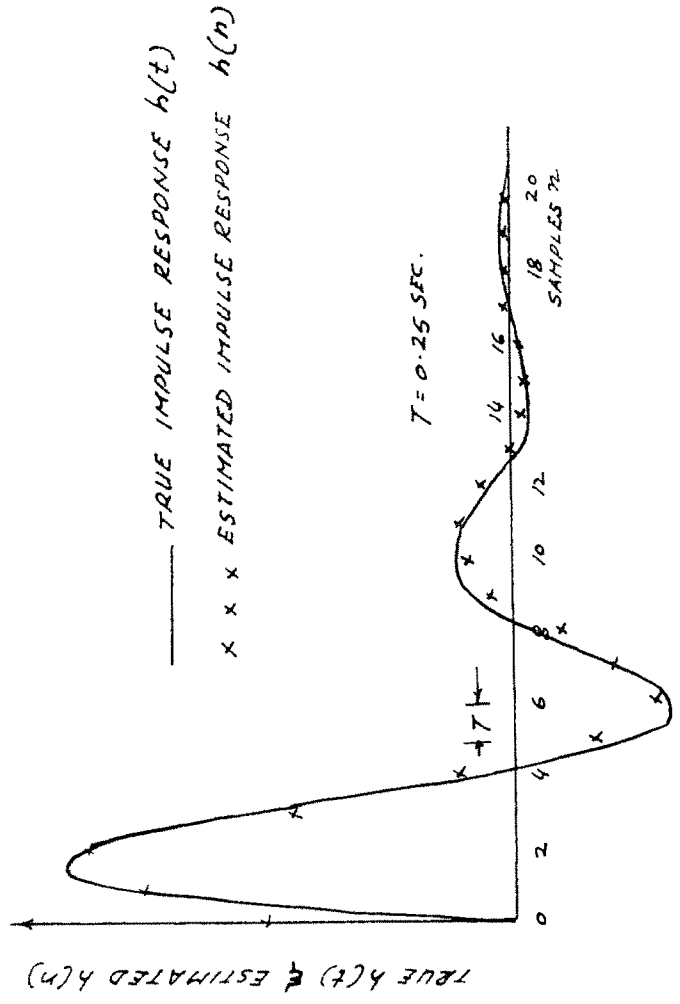

FIG. 7·8 COMPARISION OF ESTIMATED IMPULSE RESPONSE WITH THE TRUE ONE FOR A SYSTEM WITH $G(s) = 1/(s^2+2s+10)$;
$h(t) = e^{-t} \sin 3t$