CHAPTER II

ESTIMATION TECHNIQUES : PAST & PRESENT

2.1 General

The art of automatic control is very old. In fact, it originated with life itself as all living organisms, including human beings, are themselves wonderful models of automatic control systems adjusting to the environments. The automatic control in its early stages was developed in an empirical and a trial-and-error fashion as an engineering solution to the problems of industry. The principle of feedback did not follow from the sophisticated mathematical philosophy but evolved later as an analytical interpretation to help understand the operation and analysis of automatic control systems. The terms automatic control and feedback control are therefore used interchangeably. One of the earliest application of the principle of feedback in every day life is a thermostat controlling temperature in an electric iron, in an oven or inside the room.

The world war II gave an impetus, by sheer necessity, to the development of the theory and practice of automatic control. Systematic mathematical procedures for analysis and design were developed and standardized to meet the urgent and constant military needs. Here, the terms 'analysis' means the investigation of performance of a system which is already designed. The term 'design' is used in the sense that the system is to be found which will satisfy the required specifications of performance. In general, design is a trial-and-error procedure. However, if there is a clear-cut mathematical procedure for going from the given performance specifications to the corresponding system, it is called synthesis.

The problem of analysis and design of control systems can be tackled in three different ways. The first is the most common approach based upon the methods such as Nyquist plot, Bode diagram, Nichol's chart and the root-locus method of Evans. This is often referred to as the trial-and-error procedure. The control engineer is given a set of some of the specifications like gain margin, phase margin, M_{peak}, output impedance, rise time, settling time and peak overshoots. The system configuration including some standard power actuating devices and transducers is also more or less fixed by the general requirements. The designer's task then is to provide proper gain adjustment or equaliser compensation.

Another approach is the analytical design in which some criterion based on integrated history of the response of the entire system is used as a measure of performance. There are several performance criteria of this kind. For example, either the minimization of Mean-Square-Error(MSE) or the Integral-Square-Error (ISE) may be used as suggested by Wiener[6] and Hall[7]. The detailed discussion on such criteria is presented by Rideout[8], et.al. Use of classical methods of calculus of variations is made to minimize the criterion and to obtain consequently the compensating network. The system that minimizes the performance index is then said to be the "best" or "optimal". A typical classical feedback control system is illustrated in Fig. 2.1. The compensating network is introduced to obtain the desired output response. The function of the transducer is to transform the output into the same form as that of the input. Applicationsof these classical methods are limited to idealized and relatively simple feedback control systems.

During the last decade, the control theory has been involved in a period of evolutionary development in which significant advances have been made. The introduction of high speed digital computers has revolutionalized the philosophy of analysis and design of control systems. With the introduction of digital computers for system design and analysis, the older methodology and tools gave way to better ones. Also, the classical approach was confronted by severe limitations to the design of more complex, multivariable and time-varying systems.

Modern trend is in the direction of optimum control. The limited supply of natural resources, raw materials and energy and the immense pressure of business competition for producing better and cheaper products have forced all types of industry — chemical, steel, automobile, food processing, aircraft, textile, machine tools, etc. to seek greater and greater efficiency through optimum control. The businessman strives to get the maximum out of his investment. The plant manager tries to maximize production and to minimize cost. The rocket expert attempts to sent the rocket to the maximum height with minimum fuel. The design and operation of power plant by an engineer is also aimed at producing electric power with minimum cost. The weapons engineer attempts to design weapon-systems with maximum destructive power. During the last decade, the need for better controls in industrial, military and space applications has stimulated a great deal of interest in problems of optimum control and system optimization.

Most of these complex systems fall under the scope of system engineering — which involves a co-ordinated research and development of a certain system or a setup concerning several branches of engineering and science. Optimum design of such complex systems demands a computer program or an algorithm which could be processed on a digital computer.

The third approach to the design and synthesis of control systems evolved to meet the aforesaid requirements. It is a broad generalization of the second and has developed in different ways. The problem of optimum design of a control system may be roughly stated as follows. Given a plant or a process to be controlled, to determine the control law or an optimum control policy so that a set of specified performance criteria is minimized or maximized. The control law is an expression of the control variables as functions of plant variables, i.e. a feedback system results. The optimum control law is to be generated by the optimum controller or by the digital computer incorporated in the control system. This concept of optimum design has developed during the last decade and as such it is of ten referred to as the Modern Control Theory.

2.2 Modern Design Approach

The advent of high speed digital computers has considerably influenced the trend in modern design methods. The optimization techniques are now evaluated not only with respect to their mathmatical elegance, but also in relation to their computational feasibility. The state space approach has both these advantages and as such is given an increased emphasis.

The modern design method begins with the characterization of the system by state variables followed by its design employing state-space techniques. In a general formulation, the design of optimum control is usually viewed as a variational problem. There are several possible variational methods for minimizing or maximizing a functional over a function space. The range is from classical methods in the calculus of variations to numerical and successive approximation techniques of experimental or model systems. The methods[1] most commonly used are :

1) The Calculus of Variations — related to Euler — Lagrange equations,
2) The Pontryagin's Maximum Principle — related to Hamilton principle,
3) The Dynamic Programming — related to Hamilton - Jacobi theory.

Whatever the method, the object is to find the control law that seeks extremum of the given fuctional of the performance indices.

The state variables describing the dynamics of a system are sometimes all accessible for measurement and observation. For linear systems with this feature, the determination of the optimal control law as a function of state variables can be worked out even in the presence of measurement noise. However, it happens quite frequently in engineering systems that the state variables are not all accessible for measurement and observation. The optimal control law is then determined as a function of the best possible estimates of the state variables computed in an optimum fashion from the measurable noisy output signals. Consequently, the more general case involves both optimum estimation and optimum control.

Since the modern control theory is evolved to deal with more general, complex and multivariable systems, the basic control system configuration is markedly complicated from that of a classical feedback control system shown in Fig. 2.1. For example, in the automatic control of a boiler for an electric generating station, the outputs(or variables which can be measured) include the steam temperature, the rate of steam flow, the water level, etc. There are likewise several inputs(or variables which can be adjusted in order to realize the optimum operating conditions) including primary fuel input, water input, etc. Thus, the process is decribed by a diagram similar to that shown in Fig.2.2. In such a system, the task of the controller is typically to adjust automatically each of the input variables in order to realize the optimum economy of operation while simultaneously maintaining each of the output variables within the limits prescribed by safety considerations.

The matrix of transfer functions, relating the outputs with inputs, varies with the loading conditions of the power plant. If the matrix were to remain constant irrespective of the load, the optimum performance could be obtained by designing the controller, once for all, whatever may be the load condition. The optimum control is thus accomplished in an open-loop manner, having identified earlier the constant matrix of transfer functions. However, in order to realize optimum dynamic performance of the boiler, as the load varies over a range from the maximum value to a small fraction of the maximum, the transfer function matrix must be determined at each operating point. Once the process is characterized or identified off-line, the design of controller system, although complicated, can be realized with a digital computer implementation. If the process-matrix variation with plant loading is thus known at the time of system design, the controller can consist simply of a pre-programmed controller(shown by dotted line in Fig. 2.2) characteristic as a function of the single variable, the load. The controller would then modify itself according to load variations giving optimum performance under any load condition. Here again, off-line identification is necessary. This type of design is possible especially when the variations in the process characteristics are predictable and follow a definite pattern. It can be classified as a well-designed optimum or adaptable control system but it does not meet the definition of a self-optimizing[9] or adaptive control system. A detailed bibliography on adaptive control system is available in Reference (10). The need for an adaptive control system arises when the dynamic characteristics of a controlled system change very widely in a manner which is difficult or impossible to predict. As for example, in coal-fired boilers,the process matrix changes markedly and unpredictably with time and hence the designer finds it impracticable to determine a priori the process matrix for all loading conditions. Under such circumstances, the controller has to be adaptive, that is, it must during normal operation evaluate the dynamic characteristics of the system from measurable outputs, decide the control law and then generate an actuating signal which results in satisfactory overall performance. Thus, the adaptive control system involves three stages[11]: Identification, Decision and Actuation as shown in Fig. 2.3. This type of scheme necessitates on-line identification of the transfer function matrices which are to be computed from measurable inputs and outputs of the process without disturbing the normal operation. Both the inputs and outputs usually are corrupted with noise and as such the transfer function matrices are determined optimally to fit the given input-output data satisfying a certain performance criterion. Thus, this problem includes the optimum control preceded by optimum identification.

2.3 Estimation

The problem of identifying an unknown process or a black box is known under different guises such as Identification[12], Estimation, Characterization, Evaluation or Measurement. The word " Process" is more general and may include an engineering system, a biological system, an economical system or a sociological system. In the literature on circuit Theory and Communication Theory, the terms "Identification" and "Estimation" had been familiar for a long time. The recent trend of research towards adaptive control has brought an added significance to it. On many occassions, these two terms have been used synonymously. However, depending on the a priori and the desired knowledge of the process, the distinction can be made[13]. Identification really means the determination of the topology or structure of the process. Considering it as an absolute "Black box". On the other hand, estimation involves the determination of the parameter values of the process, assuming the topology to be known. The identification is therefore of a more general nature and hence the estimation can be considered to be a subclass of identification. For engineering purposes, the estimation is more realistic since some a priori information regarding the process is always available. For such cases, it may be possible to derive an incomplete mathematical structural model from observation and understanding of the physical process. The missing details like numerical values of parameters can then be determined by a suitable estimation technique. In view of this, the word "Estimation" will be used in context to differential equations(or transfer function) model of a process. Since no structural information need be known, the word "Identification" is more appropriate in context to impulse response.

Since estimation involves collecting normal operating input-output data and then computing the transfer function(or the weighting function) by some technique, it does take some time. When the process characteristics are changing faster, the estimation time must be shorter, if the estimation is to be of any subsequent use in decision and actuation. It is therefore essential that the estimation time be comparable to the time-constants of the system[14]. Moreover, the normal operation of the plant should not be unduly disturbed during identification.

The plant identification is not always necessary[15] for adaptive control as some criterion based on the integrated history of the response(like ITAE, ISE, etc.)[8] of the system can be used to determine whether the controller adjustment is optimum or whether changes are required.

Most of the research work done so far in the area assumes the plant to have a single input and single output with either complete, partial or no a priori knowledge. In the early stages of development, the idea of adaptive control aroused so much of attraction, interest and curiosity that people working in the area did not realize the gravity of sophisticated identification procedures but hastily resorted to the use of simple techniques employing crude gadgeteering. In the beginning, research workers were more inclined towards identification of impulse response, rather than transfer function, restricted to linear systems only. Thereafter much work has been done employing variety of techniques with increasing emphasis on estimation of a differential equation(or transfer function) model of the plant. The identification technique must be in the time domain if it is to be realistic from the adaptive control point of view. The techniques found in the literature are so diverse in principle and application that a control enginer facing the task of identifying a process is in a state of confusion as to what method is best suited for his problem. An overall review of literature and an investigation regarding the merits and demerits of different methods is very much desirable. An attempt is made in the succeeding pages to review different methods of identification from the available literature. Several types of classifications are possible depending upon

i) Whether the system to be identified is linear or non-linear,

ii) Whether the weighting function or the transfer function is desired. This depends upon whether the adaptive scheme is based on the former or the latter,

iii) Whether the presence of noise is considered or not. Consideration of noise is a more realistic case.

iv) Whether complete, partial or no a priori information regarding the noise and dynamics of the plant is available. It is not always possible, except in rare cases, that nothing is known of the plant. Some a priori information is always availble due to the familiarity with the plant,

v) Whether normal input-output record or external testing signals are used,

vi) Whether the identification scheme is based on the continous or sampled record of input-output,

vii) The classification can also be based on the analytical techniques employed. Various kinds of analytical and experimental methods are available.

Relatively a few articles are published on impulse response identification. All such papers are grouped together. The rest of the papers on estimation of a differential equation(or transfer function) model are discussed under different groups depending upon the techniques used. The next section deals with the review of methods of impulse response or weighting function identification.

2.4 Impulse Response Identification

The dynamic characteristics of any linear system can be completely represented by its impulse response. This is true because the characteristics of a linear system does not depend on the form or the magnitude of the system input. Unknown linear systems are therefore often identified in terms of impulse response. Moreover, the performance of a linear system can be evaluated[16] in terms of the impulse response and to accomplish self-adaption, controllable parameters can be adjusted until the identified impulse response takes the desired form. In practice, the output of the system is corrupted with noise and as such all realistic identification procedures aim at reaching the "best possible" or "optimum" estimate of impulse response of the system from normal noisy input-output data. The available methods can be subdivided into three groups:

A. Cross-correlation Identification,

B. Identification from sampled Input-Output Data, and

C. Matched Filter Identification.

2.4A Cross-correlation Identification

Most of the techniques employed so far use correlation functions. The cross-correlation of the input with output of the system is related to the auto-correlation of input through a convolution integral involving the systems impulse response. This relation is well known as the Wiener-Hopf equation[6]. As early as 1950, Wiener[17] and Lee[17,18] pointed out that when the input of a system is white noise, its auto-correlation function is an impulse function and as such the cross-correlation function in the Wiener-Hopf equation represents the impulse response. The same method was employed later by Anderson, et.al.[16] in their identification scheme for an adaptive control of an aircraft pitch damper.

2.4B Identification from Sampled Input-Output Data

Goodman and Reswick[5] developed in 1956 an experimental device, namely Delay Line Synthesizer (DLS), to obtain the impulse response at discrete instants by feeding to the device auto- and cross-correlations computed beforehand from the normal noisy input-output record of the system. In essence, the DLS unit performs "deconvolution" on the Wiener-Hopf equation to recover the weighting function. This is discussed in greater details in Chapter VII.

Levin[19] used the least squares method for identifying the impulse response at discrete instants from sampled input-output record observed over a limited period. The measured output is viewed as the sum of ideal output(not observable) and random noise as depicted in Fig. 2.4. It is assumed that the noise has zero mean. The optimum impulse response is sought as that which minimizes the sum of the squares of errors between the observed output and the computed output at sampling instants. The computed output is obtained from the expression of discrete version of the convolution integral relating the observed input and the system's impulse response to be estimated. The minimization procedure gives rise to a set of linear equations from which the estimate of impulse response at discrete instants is obtained. These equations are in the form of sampled-data analog of the Wiener-Hopf equation, thereby showing that the method of "deconvolution" by Goodman and Reswick and the least squares method would give almost the same results. It is further shown that (a) if the noise is white, the least squares estimates and Markov estimates coincide and are the same as the minimum variance unbiased estimates and (b) if the noise is white and also gaussian, then the least squares and Markov estimates are the same as the maximum likelihood estimates and are efficient(i.e. they have minimum variance among all unbiased estimates).

Kerr and Surber[20] used the same approach as suggested by Levin but they went further to provide a test of the reliability of identification by introducing a "sufficient record length" criterion. Emphasis is laid on the fact that the estimation scheme of time-varying systems must be based on as short an operating record as possible, consistant with the desired degree of accuracy. For short duration records, a strictly statistical description of the input and output signals is precluded. A conflict of requirement arises, in fact, since it is desirable to use as long a record as possible for noise smoothing, but as short a record as possible, so that the system may be assumed to be time-invariant over the estimation interval. Hence, for a given rate of parameter variation, a given noise level and a given type of control signal variation, it is shown that there does exist an optimum record length of input and output data.

It is assumed that the system has a finite settling time so as to represent the system's impulse response by a finite number of parameters. The precision with which the set of parameters can be estimated increases with the increase in the length of the observed record. This type of noise smoothing is a function of the degree of redundancy in the data, i.e. the number of independent output data samples relative to the number of parameters to be estimated. If the assumed settling time is too long, the apparent number of parameters to be estimated will be increased. This also increases the effective noise smoothing and results in poorer precision. However, if the settling time is too short a systematic error will be introduced into the estimates of the retained parameters. The best choice of settling time for a fixed observed record is obtained when the systematic error became of the same order of magnitude as the expected noise-induced estimation error.

If the system is assumed to have an effective upper cut-off frequency f_c, then no significant information is lost in setting the sampling interval $\Delta t = (1/2f_c)$. If Δt is chosen to be smaller then this, then a larger number of parameters will be required for the same settling time. In addition to requiring a faster sampling rate and a greater computer capacity, this increases the expected error in the parameter estimates. This provides less filtering of the noise in the output, thus increasing the effect of the noise energy relative to the signal energy[21]. However, if Δt were chosen higher than $(1/2f_c)$, some information in the high frequency region would be lost thereby introducing a systematic structural error.

The estimation procedure and the conclusions for the given statistical description of the noise are generally the same as those suggested by Levin. The reliability of estimate is shown to depend critically upon the nature of the input and noise. An expression for the expected integrated-squared-error between the actual and the estimated impulse responses is derived to indicate the degree of reliability. A "sufficient signal" is defined as the one for which the expected integral-squared-error does not exceed some specified value. It is shown that if the statistics of the input signal and noise are known a priori, "sufficient test signal" would be obtained with a record of certain length. Thus the "sufficient test signal" criterion is transformed in to a "sufficient record length" criterion.

2.4C <u>Matched Filter Identification</u>

A different approach developed by Turin[22] uses an estimating filter, as depicted in Fig. 2-5, at the system output to make a linear minimum mean-square-error estimate of the impulse response of the system. Such a procedure requires no multiplier, and the output of the filter is the impulse response as a continuous function of real time. The system in this case is a transmission medium such as ionosphere. The transmission medium characteristics does vary with time but it is assumed that it varies slowly and consequently it remains unchanged during the estimation period. Moreover, to make the problem realistic, the signal after transmission through the medium is considered to be perturbed by stationary random noise. The mean-square of the error(considering the statistical average over the ensemble of possible noises and impulse responses of the medium), between the output of the estimating filter and the impulse response of the medium, is minimized by adjusting the estimating filter impulse response and the nature of input. Making use of the Fourier transforms, this gives optimum transfer function of the estimating filter which gives at its output the optimum impulse response of the medium.

Although Turin's problem arose in the field of communications and radar, the idea was applied to process identification for adaptive control by Lichtenburger[23]. The method consists of injecting a special test signal at the input along with the regular actuating signal and then passing the output through a correlating filter whose output gives the estimate of impulse response of the time-invariant system. The amplitude of the test signal must be small with respect to the actuating signal for the practical reason that the process output is not appreciably disturbed. In this case, the actuating signal is treated as noise and as such the noise power will be relatively greater then the test signal power. The corresponding noise in the output will also be quite high. The effect of output noise is reduced by increasing the effective duration of the test signal so as to increase the energy of the test signal without increasing its average amplitude. This is accomplished in the following manner.

A train of a finite number of test pulses(as against only one testing pulse used by Turin) is added to the normal input of the linear process as shown in Fig. 2.6. The output of the process during each test pulse is added to the sum of all the previous tests in the measurement at each instant of time. This is accomplished by the use of delay or storage and an adder. This process is called coherent summation. After the last test pulse, the result of the summation, still a function of time, is passed into an estimating filter. The expected mean-square-error is then minimized by the choice of estimating filter, test signal, and number of testing signals to get the estimating filter output as the best estimate of process impulse response. This method does give better results but takes a longer time. This is unfortunate because one would like to have the results available as soon as possible from the point of view of control system performance and because even for slowly varying processes, error builds up seriously for sufficiently long measurements.

As pointed out by Lindenlaub and Cooper[24], the mathematical similarity of the above three methods is provided by Wiener-Hopf equation, the solution of which becomes simpler by considering the input to be white noise, However, since the external noise enters the problem differently in each method, different techniques are used to reduce the variance of the impulse response estimate. It is further shown that the identification time in each case is the same as the product of gain and bandwidth of the system to be identified divided by the product of variance of the impulse response estimate and the output SNR(Signal to Noise Ratio). All the three methods give a minimum variance estimate of the unknown impulse response, as is true for an ideal identifier, assuming that no a priori information of the unknown system is available.

2.5 Estimation of a Differential Equation or a Transfer Function Model

A transfer function is the frequency domain representation of a stationary linear system showing the relation between its input and output. It is derived from the differential equation description of the linear system by assuming the initial conditions to be zero. The estimation of a transfer function really means the determination of the constant coefficients of differential equation or constant parameters of the system. For a sampled-data case, the difference equation model is sought. The transfer function concept is not valid, in general, for a time varying system but from the adaptive control point of view, the system can be represented by a transfer function which changes from instant to instant with the variation in parameters which can be estimated from time to time. However, for a nonlinear system, only the differential equation (nonlinear) representation is possible and the estimation of such a system is aimed at determining its coefficients. With the increasing use of state-space concepts and the advent of the modern high-speed digital computer, the differential (or difference) equation description of the system is found more favourable.

In the early stages of development of adaptive control, the estimation of processes evolved in the form of parameter correction or parameter tracking by employing tracking servo loops as an engineering solution to the overall adaptive process. Most of these schemes employ physical models. The estimation is not given a separate and distinct identity in the overall adaptive scheme. Such schemes have been used for small size processes. Parameter tracking schemes are discussed in section 2.5A.

The estimation schemes based on explicit mathematical relations(i.e. mathematical models) and giving results in numerical quantities from the input-output record with the aid of digital computer, began to develop a little leter. The methods are employed especially for large size, multivariable and complex systems which can afford or justify a digital computer. The solution to the estimation problems of this class is obtained by means of variety of analytical techniques as will be seen in section 2.5B.

2.5A Estimation as Parameter Tracking Employing a Physical Model

The methods of this class are discussed under three separate groups: (i) Parameter perturbation, (ii) Input signal perturbation, (iii) Parameter tracking using Normal input-output record.

(i) Parameter Perturbation

In 1951, Draper and Li[25] presented a parameter perturbation scheme for optimizing the performance of an internal combustion engine. Mcgrath and Rideout[26] suggested that this techniques can be used for self-optimization of feedback control systems by adjusting the paramters so as to minimize the mean-square-error criterion. A similar system was developed independently by Nightingale[27] and Taylor[28]. The scheme had aroused a great deal of interest even in Britain and Russia as is evident from the publications of Douce and King[29], Feldbaum[30], Kazakov[31] and

Varygin[32]: The technique is relatively simple and has good noise immunity.

In this scheme, the controllable parameter is perturbed sinusoidally to test whether or not the system performance is optimum. The cross-correlation of the perturbation signal with the square of the error, between the outputs of the process and the model is then used to adjust the parameters of the process. As many perturbation signals as the number of parameters are required. Mcgrath, et. al.[33] pointed out the versatility of this technique by citing the number of control situations in which it can be profitably applied.

Folloing their remarks, Rajaraman[34] described a multiple-model system for simultaneous adjustment of two parameters of the process along the steepest descent to obtain a self-adaptive system which, according to Aseltine[35], is both input signal and process adaptive. It consists of two models receiving the same input as the process. Model I is an ideal version of the process and will be in general of an order different from that of the process. Model II is chosen to be of the same order as the process. The parameter of model II is perturbed by a low frequency sinusoidal signal. The square of the error, between the outputs of Model I and II, after multiplication (with the perturbation signal) and integration yields information to adjust the parameter. This ensures adaptation against input signal variations. The controller for the process is of feedback type and its parameter is adjusted (by a separate adaptive loop) in a similar manner, based on the error resulting from the comparasion of outputs of process and

Model II. The parameter of Model II is perturbed with a different frequency for this adaptive loop. The adaptive loop performs simultaneously system parameter tracking(estimation) and its correction(adaptation). When adjustment of two parameters is involved, another set of two adaptive loops is added.

(ii) Input Signal Perturbation

Besides parameter perturbation, sinusoidal perturbation of the input signal was also common in early stages of adaptive control. This method is also simple and has the advantage of constant amplitude of test signal and negligible noise effect. In addition, it can provide in general two identification signals for each sinusoidal frequency.

One of the earliest papers on input signal perturbation is by Weygandt and Puri[36]. It describes a system for determing the parameters of a transfer function of the form - one divided by a polynomial in terms of the Laplacian variable 's'. The method is shown to work for a polynomial of second order which involves the tracking of two coefficients in the denominator polynomial. A sinusoidal perturbing signal is injected in the normal input. The cross-correlations of the error(between the input and output of the same frequency) with (a) the output and (b) the output shifted in phase by 90^{o}, are then used to track the two parameters.

Eykhoff and Smith[37] used a dynamic model which is made to follow the process by adjusting its parameters by cross-correlation of model output with process output. The process and the model are fed with the same perturbation signal.

Smith[38] devised an adaptive scheme for automatic gain control of an Autopilot. Any change in phase shift or amplitude of the process output caused by environmental variations is detected by feeding model output and process output to an adaptive computer which then adjusts the gain of the Autopilot(controller) to hold the measured amplitude or phase shift constant. The adaptive computer is either a phase discriminator or an amplitude measuring device, tuned to the test signal frequency.

Another paper by Perlis[39] describes a technique based upon the use of existing external signals and claims that the overall system's figure of merit can be improved with such a scheme.

Smyth and Nahi[40] developed a technique to track the two parameters in their adaptive scheme, based on the variations in amplitude and phase of the output corresponding to the dither (perturbation) signal at the input. The scheme is an extension to the siggal-parameter amplitude dither adaptive system by Smith[38].

It is important to note that in both the perturbation techniques, the amplitude of the test signal should be neither too large to avoid undue distrubance to the normal working nor too small to keep the SNR sufficiently high.

(iii) Parameter Tracking Using Normal Input-Output Record

The techniques discussed in the previous sections employing perturbation becomes cumbersome when adjustment of many parameters is involved. However, the general method of computing the partial derivatives(i.e.gradients) of the performance criterion with respect to each parameter has the advantage of being applicable to many problems and independent of specific system configuration.

Margolis and Leondes[41,42] used only the normal input-output record(rather than any kind of perturbation) for parameter tracking of a physical process employing a dynamic model in their dynamic scheme. The learning model and the physical process are subjected to the same normal input signals. Their outputs are compared and the resultant error is fed to the adjusting mechanisms which operates on an approximation to steepest descent and adjusts the parameters of the learning model until the square of the error reaches minimum. Thus the dynamic model behaves as much like the process as possible. This information is then used in programming the controller. The method has been tried out successfully on both the first order and the second order processes with some restriction on the input for the latter. The input must be present all the time otherwise the error becomes zero(i.e.minimum) and the adjusting mechanism does not operate.

Narendra and McBride[43] suggested the use of correlation techniques to compute the partial derivatives to adjust the parameters along the path of steepest descent in a parameter space to the minimum error criterion.

2.5B Estimation Employing Mathematical Models

A number of methods which employ dynamic mathematical models for estimation of transfer functions or differential equation parameters are available to-day. They are classified in the following groups according to the analytical techniques employed.

(i) Statistical Methods,

(ii) Quasilinearization Technique,

(iii) Dynamic Programming,

(iv) Estimation as a TPBV Problem, and

(v) Miscelleneous Methods.

(i) Statistical Methods

Wiener-Kalman Filter

Wiener[44] pointed out that the natural setting of the estimation problems in communication and control belongs to the realm of probability theory and statistics. The estimation in general covers (i) data-smoothing or interpolation(estimation of the past state), (ii) filtering(estimation of the current state) and (iii) prediction(estimation of the future state). The solution of filtering or prediction problems leads to the well known Wiener-Hopf integral equation which can be solved by spectral factorization method. Many extensions and generalizations[45] followed Wiener's basic work. In all these works, the objective had been to obtain the model for Wiener filter which could accomplish prediction, separation or detection of a random signal. These methods are subject to a number of limitations[45] which seriously curtail their usefulness to practical problems.

Kalman[45] introduced a novel approach to solve the Wiener problem and overcame the difficulties by using the Bode-Shannon[46] representation of random processes and the "state-transition" method of analysis of dynamic systems. Linear filtering is regarded as orthogonal projection in Hilbert space. With the state transition method, a single derivation covers a large variety of problems: growing and infinite memory filters, stationary and nonstationary statistics, etc. The approach gives a nonlinear difference(or differential) equation for the covariance matrix of the optimal estimation error. This is vaguely analogous to the Wiener-Hopf equation. The solution of the equation begins with the first observation taken at time t_o. At each later time t, the solution of the equation represents the covariance of the optimal prediction error, given the observations in the interval (t_o , t). Use is made of conditional probability distributions and expectations. The coefficients(in general, time-varying) characterizing the optimal linear filter is obtained at once from the covariance matrix at time t , without any further calculations. The new formulation of the Wiener problem turns out to be the dual of the noise-free optimal regulator problem[45,47]. The power of the method is most apparent in theoretical investigations and in numerical answers to complex practical problems, with the aid of digital computers. The Kalman's solution to the Wiener problem is popularly known as Wiener-Kalman filter.

In working out an analogy for a continuous-time case, Kalman and Bucy[48] showed that the nonlinear differential equation for the covariance matrix of the optimal filtering error is of Riccati type which occurs in the calculus of variations and is closely related to the canonical form of Hamilton differential equation. The relationship gives a clue to the solution of the Riccati equation. The solution of this Riccati type equation completely specifies the optimal filter for either finite or infinite smoothing intervals and stationary and non-stationary statistics. It is concluded that this approach is better rather than attacking the Wiener-Hopf integral equation directly. The principle of duality relating stochastic estimation and deterministic control is used to the advantage in the proof of theoretical results.

Florentin[49] used a technique conceptially much simpler than the idea of orthogonal projection employed by Kalman and arrived at the same recursive relations for the estimation of a state vector when only a part of it corrupted by noise is observable. However, this does not directly demonstrate the filtering process to be interpreted as a linear dynamical system. Mayne[50], in his estimation procedure, considered all the components of the state vector as perfectly measurable. The forgoing ideas were unified and a procedure was devised by Kumar and Sridhar[51] for estimating the entire state vector and the coefficients of the differential equation from measurements on the system inputs and the observable outputs. Making use of some statistical concepts, the estimate of the current state is obtained by updating the immediate past estimate, as new observations are made. This sequential scheme can be very easily implemented on a digital computer. It is proposed that the method could be used for on-line identification since the identification time is not excessively long compared to the system time constants.

O'Donnell[52] presented a mathematically less sophisticated derivation of the one dimensional filtering problem. The other advantage is that the covariance matrix of the optimal estimation error is obtained in the closed form, in contrast to that in Kalman's paper. It is shown that, after some finite number of measurements, the above covariance matrix approaches zero and therefore no correction is provided to the estimate even if the actual values may vary. This is undersirable and can be remedied by including a disturbance term(with known statistics) in the assumed dynamic model of the system.

The design of the optimal filter by Kalman and Bucy[48] considers white noise in measurements. Many practical systems exist in which some of the measurements are corrupted with coloured noise, some with white noise and the rest with no noise. Such cases are singular problems within the framework of the Kalman-Bucy theory. The solution to this problem is provided by Bryson and Johansen[53]. The coloured noise is considered as the output of an auxiliary linear dynamic system(called a "shaping filter") with white noise inputs. With this approach the coloured noise vector becomes a part of an augmented state variable vector and the corresponding measurements now contain only linear combinationsof the augmented state variables without noise term. Thus the shaping filter approach makes the augmented system appear as a system in which the measurements are partly perfect and partly corrupted with white noise. The estimation of the states measured perfectly is not required. The rest of the states containing white noise are estimated using the Kalman-Bucy approach.

The solution of the Riccati type equation[48] for the design of optimal filter to give a conditional expectation of the state is not easy. A paper by Park[54] presents the derivation of a minimum variance filter which yields an approximation to the conditional expectation of the state. The filter is a model, of the plant and controller, which is reset after every independent observation by a device which averages the current observation with the past observations. After the filter is reset to this new average, it tracks with the plant until the next observation when the process is repeated. The filter tracks the state so that the statistical problem is reduced to that of determining a constant in additive noise.

After Kalman and Bucy[48] worked out the linear filtering theory, suggestions were made to use this theory to find an approximate solution to the nonlinear filtering problem by linearizing the nonlinear dynamics of the process and observation function. Bucy[55] showed that linearizing the optimum nonlinear problem leads to quite a different and probably more useful approximate solution than the above procedure. This is accomplished by representing the conditional density in terms of a functional of the stochastic integral of various functions with respect to the observed random process. A random partial differential equation for the conditional density is then obtained by using Ito's random calculus. Similar work is done earlier by Wonham[56] claiming that the performance of the optimal nonlinear filter is substantially better than that of the simple Wiener filter.

Ho[57] proved that with little manipulation, the recursive relations derived by Bryson and Frazier[58] for the estimation of a state in the presence of gaussian noise, can be transformed to those of Kalman[45]. Some connections are then established among the maximum likelihood estimate, the optimal filtering and the stochastic approximation of the estimate. The conditions for system's identifiability are given by Ho and Whalen[59].

Smoothing Estimate

Whereas Kalman gave solutions to filtering and prediction problems, some others[58,60,61] extended the results to the smoothing problem. Realizing that a smoothing solution would contain filtering subroutines, Weaver[60] gave a modified solution to the filtering problem. He made use of the fact that, in the gaussian case, if the loss function of a Bayes estimate is proportional to the square of the magnitude of the error vector, then the optimum estimate is also the maximum likelihood estimate and is the conditional mean of the Quantity to be estimated. Rauch[61] attacked a practical smoothing problem wherein the instantaneous position and velocity of a satellite(after its injection into orbit) are estimated in real time as observations are received while the smoothed estimates of the initial conditions(position and velocity immediately after the termination of thrust) are required for the evaluation of accuracy of the guidance system used during injection. For this class of problems, a solution is found which directly relates the smoothed estimate of the state at the particular time to the new observations. This form is more feasible computationally because it elimitates the need for storing observations and because it allows the smoothed estimate to be updated immediately as more observations are made. The solution is obtained using the state transition matrix and the covariance matrix of the estimate.

Bayesian Approach

Another statistical approach to the problem of estimation is based on the assumption that the system under consideration is Bayesian. A system is considered Bayesian if the general structure of the plant is known and some a priori distributions for the unknown parameters are available. Bayesian systems may be rare in general statistical work but are common in control field. According to Bayes rule, the probability(a posteriori) density of x given y is given as the product of the density function of y given x and the density function of x divided by the density function of x. Ho and Lee[62] demonstrated the use of Bayes rule for linear estimation and arrived at the closed form Wiener-Kalman solution in gaussian noise. Basically, the approach consists in proceeding step by step from the availble a priori probability density functions to the a posteriori conditional density function. Florentin[63] also assumed a Bayesian system to estimate the gain in the control path of a simple regulator, using the control as the probe. The approach becomes formidable in the light of computer time when the system is multidimensional and the observation is a nonlinear function of state variables. However, it is felt that the Bayesian approach[62] offers a unified and intuitive viewpoint for the general problems of estimation and control.

Regression Technique

Regression Analysis[64] is a powerful statistical tool for the determination of dynamic relationships among veriables. It is in no way restricted to any class of functions or, except for statistical tests of hypothesis, to any particular form of random distribution. It involves in general the simultaneous solution of m linear equations to determine n unknown coefficients($n < m$)

attached to n variables. These variables need not be mutually independent. In the estimation problem, the situation is similar since the output variable(observable) is represented as a linear combination of n controllable variables involving n unknown constant coefficients(parameters). For a noisefree case, n observations are sufficient to determine accurately the n parameters. This is a simple algebraic problem. But, in practice, the observations are noisy and the observations required are more, i.e. $m > n$, to smoothen the effect of noise. The solution is then obtained through the use of variance of controllable variables and covariance between observations and the controllable variables. Thus the regression analysis is a statistical problem. The least squares treatment to the problem would also give the same results. Since regression analysis involves dynamic relationships, it can, ideally at least proceed during the natural operation of the process without the necessity of special inputs for performance measuring purposes.

Bishop and Chope[65] employed this techniques to obtain information about constant parameters for optimal control of a multivariate nonstationary process citing an example of paper manufacturing. He considers a general case including nonlinearity.The adjustment of controllers is made periodically at the end of every calculation interval. As such the process operates open-loop during the calculation period. Elkind, et.al.[66] applied regression scheme to measurements of human pilot control(time-varying) characteristics with digital computer simulation. They considered a model consisting of parallel connections of filters whose impulse responses are orthogonalized exponential functions. The model is fed with the system input. The coefficients of linear regression of the system output on each of the filter outputs are determined. It is shown that the length of input-output record to determine the coefficients with given variance can be estimated with the known statistics about regression coefficients. Another paper[67] describes the technique for estimation of a state vector (for space vehicle orientation) which is augmented to include space vehicle paremeters. Giese and MeGhee[68] discussed the unifying ideas of least squares, regression, maximum likelihood and Bayesian estimate and tackled the practical problems like nonlinear pendulum equation and Ballistic Vehicle atmospheric reentry equation. The time taken by the computer is short enough to make the methods practicable.

Levin[69] used the generalized least squares theory for the estimation of pulse transfer function of a linear system from normal operating records. The estimates of the coefficients are obtained as the components of the eigen-vector corresponding to the smallest eigen-value of a matrix equation involving the sample auto- and cross-correlation functions of the input and output records and the covariance matrix of the corresponding noise components.

Steiglitz and McBride[70] proposed an iterative technique to estimate a linear system from noisy input-output samples by minimizing the mean square error between system and model outputs. The model chosen has a transfer function which is a ratio of polynomials in z^{-1} . Although the regression equations for the optimal set of coefficients are highly nonlinear and difficult to solve, it is shown that the problem can be reduced to the recursive solution of a related linear problem. The technique gave significant improvement over the Kalman's[71] linear regression estimate for a number of computer-simulated systems. Convergence is obtained successfully within 10-20 iterations thus saving considerable computer time. The procedure is found to be effective for SNR less than unity, and with as few as 200 samples of the input and output records.

Another filtering scheme of iterative nature for nonlinear time varying process, due to Mowery[72], is based on optimally weighted, linear differential corrections obtained by local linearization through Taylor series expansions. The set of normal equations obtained by minimizing the Quadratic function of errors are nonlinear whenever the observations are nonlinear functions of the state vector. These equations are then solved using the linear approximation obtained by truncating the Taylor series expansion about the state variables. The filter equations thus obtained are of a general nature and, with slight manipulation, can be reduced to Wiener-Kalman filter. The necessary and sufficient conditions which permit the use of technique for linear system are determined by Fisher[73].

Smith[94] made an attempt of reviewing the techniques of maximum likelihood, Bayesian decision-making, regression analysis, least squares and statistical filtering to show that the differences in various methods are due to differences in basic hypotheses. Kerr[95] suggested that the identification situations may be classified according to three basic "system signal" configurations, which the author has termed "statistical", "regression" and "structural" models explaing the distinguishing features.

(ii) <u>Quasilinearization Technique</u>

Quasilinearization[74,75] is the name given by Kalaba and Bellman for a generalized Newton-Raphson method of root-finding of a differential equation and is sometimes called the Newton-Raphson-Kantarovich method. The quasilinearization procedure is a general computational method for solving Multi-Point Boundary Value Problems (MPBVP). The method has been employed[76,77,78] to identify the differential equation, linear or nonlinear, describing the dynamics of a process. The quasilinearization technique as applied to identification problem is briefly discussed below.

Let the process be discribed by a vector differential equation

$$\dot{x} = f(x\ ,\ t) \qquad t_o \leq t \leq t_T \qquad (2.1)$$

where x is a n-vector which also includes constant parameters of the process to be estimated. Let the vector x , in a noise-free case, be observed as

$$y(t_i) = H\ x(t_i) \qquad i = 1,\ 2,\ .\ ,\ ,n \qquad (2.2)$$

Also

$$t_o \leq t_1 \leq t_2 \leq\ .\ \ .\ \ .\ \leq t_n \leq t_T$$

where y is a n-vector and H is a n x n constant matrix.

The identification problem is then to choose initial state $x(t_o)$ such that the solution $x(t)$ of equation (2.1) satisfies the observed values of equation (2.2). Let $x_o(t)$ be an aribitrary initial guess to the solution of equation (2.1). The (k+1)st approximation is then obtained from the k^{th} via

$$\dot{x}_{k+1} = f(x_k , t) + J\left[f(x_k , t)\right](x_{k+1} - x_k) \qquad (2.3)$$

where J is the Jacobian matrix whose $(i,j)^{th}$ element is the partial derivative of the i^{th} component of f with respect to the j^{th} component of x. The solution of equation (2.3) is written as

$$x_{k+1} = \Phi q + p \qquad (2.4)$$

where

Φ is the fundamental matrix solution of equation (2.3),

p is a particular solution vector of equation (2.3) and

q is a constant vector to be determined from equation (2.2) by substituting for x_{k+1} obtained from equation (2.4). The calculations are generally carried out on a digital computer and the sequence of vectors x_{k+1} will converge quardratically[74] to the disired solution.

For a noise-free case as discussed above, Prasannakumar and Sridhar assumed that for the n^{th} order system, exact fit through n arbitrarily selected observation points along the desired trajectory would give a unique solution to the identification problem. Lavi and Strauss[79] however disproved the presence of uniqueness by citing a simple example. Generally, the observations are corrupted with noise and the estimation of initial state is obtained in the least squares sense so as to minimize the performance index,

$$I = \sum_{i=1}^{N} \left[y(t_i) - H\,x(t_i)\right]' \left[y(t_i) - H\,x(t_i)\right] \qquad (2.5)$$

It is suggested[79] that the regression solution (i.e. when the number of observations is more than the order of the system) reduces the uniqueness difficulties and smoothens the noise effect giving an unbiased estimate. One major drawback is that the identification procedure often converges to the local minimum of the performance index.

Ohap and Stubberud[80] employed this technique for the estmation of the current position and velocity of a low earth-orbit satellite using range, elevation and azimuth observations from a ground tracking station. Another application due to Bellman, et. al.[81],is to the "Unscrambling Problem" which seeks to determine the correspondence between every stimulus and every source in a multivariable biological system.

(iii) Dynamic Programming

One of the most powerful tools being applied in the field of optimum control is the Ballman's[75,82] Dynamic programming. The fundamental idea underlying the method is the principle of invariant imbedding which reduces a very difficult or unsolvable multistage decision problem into a class of simpler, solvable problems. The multistage decision problem of optimizing is converted to a problem of determining the solution of a recursive functional equation by invoking the principle of optimality, which states:

> An optimal policy has the property that whatever the initial state and the initial decisions are, the

remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

This concept implies that, to solve a specific optimum decision problem, the original problem is embedded within a family of similar problems which are easier to solve. The dynamic programming approach is, in principle, illustrated below.

Let us consider a N-stage decision process. Let x be a n-dimensional state vector characterizing a physical system at any time. The transition of the state is described by the relation

$$x^{i+1} = g(x^i, m_i) \qquad i = 1, 2, \; . \; . \; . \; , N \tag{2.6}$$

This operation yields the total output (or return) for N stages

$$R_N = \sum_{j=1}^{N} r(x^j, m_j) \tag{2.7}$$

The problem is to choose an N-stage policy, i.e. the sequence $\{m_1, m_2, \ldots, m_N\}$, so as to maximize (or minimize) the return R_N for the given initial state (or allocation) x^1. The maximum return of the N stages is given by

$$f_N(x^1) = \max_{\{m_j\}} \left\{ \sum_{j=1}^{N} r(x^j, m_j) \right\} \tag{2.8}$$

The solution of this N-stage problem by simple calculus is formidable. However, invoking the principle of optimality, the maximum return is readily written as

$$f_N(x^1) = \max_{m_1} \left\{ r(x^1, m_1) + f_{N-1}\left[g(x^1, m_1)\right] \right\} \tag{2.9}$$

Thus,by applying the principle of optimality, the N-stage decision process is reduced to a sequence of N single-stage decision processes, thereby making it convenient to solve the optimization problem in a systematic, iterative manner.

Henry Cox[2,83] used the dynamic programming in conjuction with maximum likelihood approach to develop a sequential scheme for the estimation of state variables and parameters of a nonlinear discrete-time system in the presence of gaussian dynamic and measurement noise. The state vector is augmented to include constant parameters. The vector equation of linear system then contains certain terms as product of two or more state variables making it nonlinear-like. The estimation scheme discussed by Cox is in general valid for nonlinear systems including the case just discussed. It is possible to proceed sequentially by Bayes rule and after each measurement obtain the a posteriori probability density function for the current state variable. This task is easy only for linear and simple systems. The problem of estimating the state trajectory, given the observed sequence and the a priori distribution of the initial state, is reduced to the problem of minimizing a functional of the least-squares-error type. The multistage minimization problem is then solved through dynamic programming to give a sequential procedure for estimation. The sequential procedure allows the processing of each new observation as it occurs and gives the estimate of the present state by modifying the extrapolated value of the previous estimate on the basis of current observation. The estimation can be performed in real time on a digital computer. The results obtained are valid for smoothing, filtering or prediction. The filtering or smoothing solution does not need to store old values. The results obtained for a linear system are found to be similar to those of Kalman[84]. An intuitively appealing approximation technique based on Taylor series expansion is developed for producing estimates of state variables of a nonlinear system. Using the same approach, Cox[85] obtained a general solution in continuous-time for the linear problem, similar to the one developed earlier by Kalman and Bucy[48]. The dynamic programming formulation leads naturally to an approximation technique for the nonlinear problem.

(iv) Estimation as a TPBV Problem

Besides dynamic programming, an alternative approach suggested by Cox[2] is to introduce Lagrange multipliers in the minimizing functional to incorporate the constraints imposed by the dynamic equations of the system. Setting the differential of the modified functional to zero for minima and rearranging yields the necessary conditions (Euler-Lagrange equations) in the form of a TPBV problem. This is found to be the discrete analogue of the TPBV obtained earlier by Bryson and Frazier[58]. The solution of the TPBV problem for a linear system gives recurrence equations (dependent on covariance matrix) to produce an up-to-date estimate. Using some matrix algebra, the results obtained by Cox may be shown to be equivalent to those of Kalman[84]. The results are then extended for a nonlinear problem using an approximation technique.

Sridhar and Detchmandy[3] developed a sequential estimation scheme based on least squares criterion by solving the TPBV problem using invariant imbedding. No statistical assumptions are made concerning the nature of the input disturbances or of the measurement errors. This is of practical importance since in most of the practical problems, statistical description of disturbances is absent or difficult to determine. The filter equations obtained consider both the input and measurement disturbances. Bellman, et. al.[86] obtained similar equations considering only the measurement error but their method of derivation is not appticable to this case. The filter equations[3] are somewhat similar to those obtained by Cox[2]. Pearson[4] obtained the discrete version of these filter equations[3] using the same principle but the logic is conceptually much different. The details of this method are given in Appendix A.

(v) Miscelleneous Methods

Kohr[87] described a method for the determination of a nonlinear differential equation model for a physical time-invariant system. The system is represented by a nonlinear differential equation containing a single-value nonlinear function of a single variable(output) and other linear derivatives of output on left hand side and input function on the other sides. The nonlinear function can then be interpreted as the input function minus all the linear terms of the differential equations. Thus, if the input, the output and derivatives of all required order, and the linear coefficients are all available, the nonlinear function can be evaluated. Analog computer simulation may be used for this purpose. The method is then extended to systems containing several nonlinear elements. The system is assumed to have a specified periodic input and insignificant noise level in input and output.

In most of the published work on identification, the consideration of a priori knowledge about the system structure, measurement noise and time-varying system parameters is not given sufficient attention. It is shown by Perlis[88] that a desirable identification scheme under these conditions is a perturbation-correlation process. The idea was earlier[25,36] used but not for time-varying processes. Moreover, the approach adopted here differs from the recently published research in that the estimation procedure is based on spectral analysis rather than on time-domain methods. A perturbation sinusoidal signal is used along with the normal input. An expression for mean-square identification in terms of spectral densities of parameter variations and the measurement noise at the output is developed to give an optimum filter transfer function. Optimization procedures bring out the conflicting requirements of filter bandwidth. That is, the bandwidth must be widened to accomodate increased parameter variations and should be as small as possible to avoid high frequency measurement noise.

Stanton[89,90] extended the power spectral analysis for multivariate systems and used the technique to estimate quasilinear transfer functions, from normal operating data, for turboalternators operating in parallel with an interconnected power system. The present work uses the same data for the same objective but employing an entirely different method.

Bellman, et. al.[91] introduced a new method for identifying linear systems based on numerical inversion of Laplace transforms.

This is obtained by reducing the interval (0 - ∞) of the in- gral in the definition of Laplace transform to the interval (0 - 1) by proper substitution. The resulting integral is then approximated by using a Gaussian quadrature formula. The method employs, as required for Gaussian quadrature, irregularly spaced observations (assumed to be noisefree) and is extremely fast.

Most of the research work is focussed on the éstimation of lumped-parameter systems describable by ordinary differential equations. Relatively, a little has appeared in the literature on estimation of distributed parameter systems represented by partial differntial equations. The extension of methods for estmating lumped parameter systems to the case of distributed parameter systems was found cumbersome. Goodson and Perdreauville[93] solved the problem by reducing the partial differential equation to algebraic equations whose solution yield the unknown distributed parameters. It is assumed that the form of the system equations is known.

In a steady-state operation, the plant can be characterised by a unique steady-state operator[96] which maps the proper periodic input and output. The estimation of the steady-state operator is obtained by sarachik by employing the steepest descent technique for ultimate optimum control.

2.5C Estimation Schemes : Sequential Versus Nonsequential

The usual classical approach to least squares leads to nonsequential regressional estimation schemes. The disadvantage of the nonsequential scheme is that, each time when additional output observations are to be included, the entire least squares calculation has to be repeated. Obviously, the time required to perform calculation even for a single interation is considerable. Moreover, it can not be implemented in real time.

In contrast, one major advantage of sequential estimators is that they[2,3,4] show excellent and quick convergence making it suitable for on-line identification. It is the author's experience with references (3) and (4) that this is especially true only when the variance of noise is small as compared to the value of parameters. In short, the sequential estimators which do not account for the statistics of noise in their formulation are very sensitive to noise. However, if the variance of the noise is large compared to the magnitude of parameters to be estimated, experience (Appendix A) shows that the parameter being estimated does not converge at all but keeps on roaming around its expected value with the deviation (vaguely) proportional to the variance of noise. In sequal, the idea of using a sequential type of scheme for estimating the parameters of turbo-alternator transfer functions from the normal operating input-output corrupted by heavy noice had to be abandoned. The present work shows that better results are obtained by solving progressively the TPBV problem nonsequentially using the steepest descent technique. Since, in a nonsequential scheme, the entire span of assumed model output is compared again and again with the observed output data of the same span until the best fit is obtained in the least squares sense, this method takes considerable time. But this seems to be the prise one must pay when the reliability of results is more important than the cost.

The next Chapter deals with the formulation of the estimation problem as a TPBV problem.

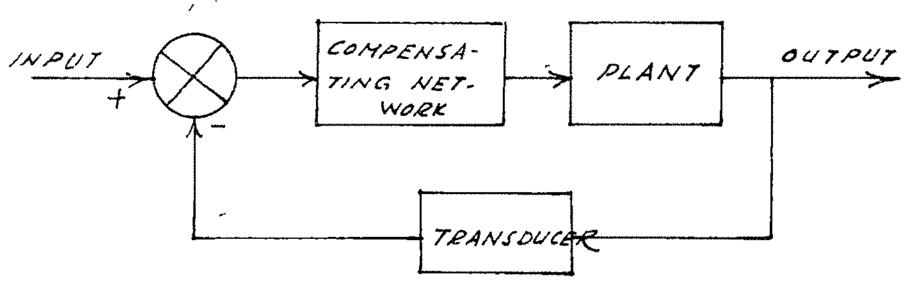

FIG. 2·1 CLASSICAL FEEDBACK CONTROL SYSTEM.

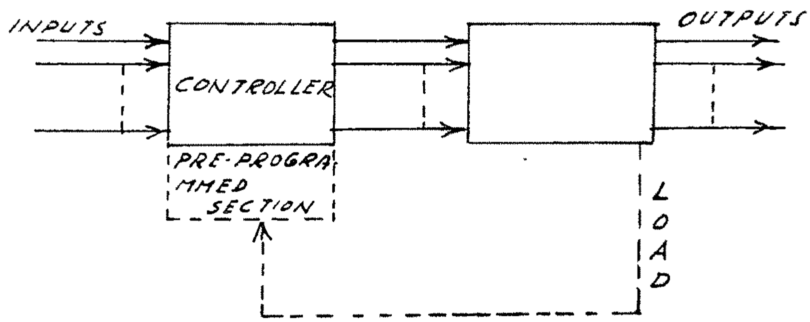

FIG. 2·2 OPTIMUM CONTROL OF A MULTIVARIATE SYSTEM IN AN OPEN-LOOP MANNER FOR CONSTANT PROCESS MATRIX. SCHEME FOR OPTIMUM DYNAMIC PERFORMANCE IS SHOWN BY DOTTED LINE WHEN THE PROCESS MATRIX VARIATION WITH PLANT LOADING IS KNOWN.

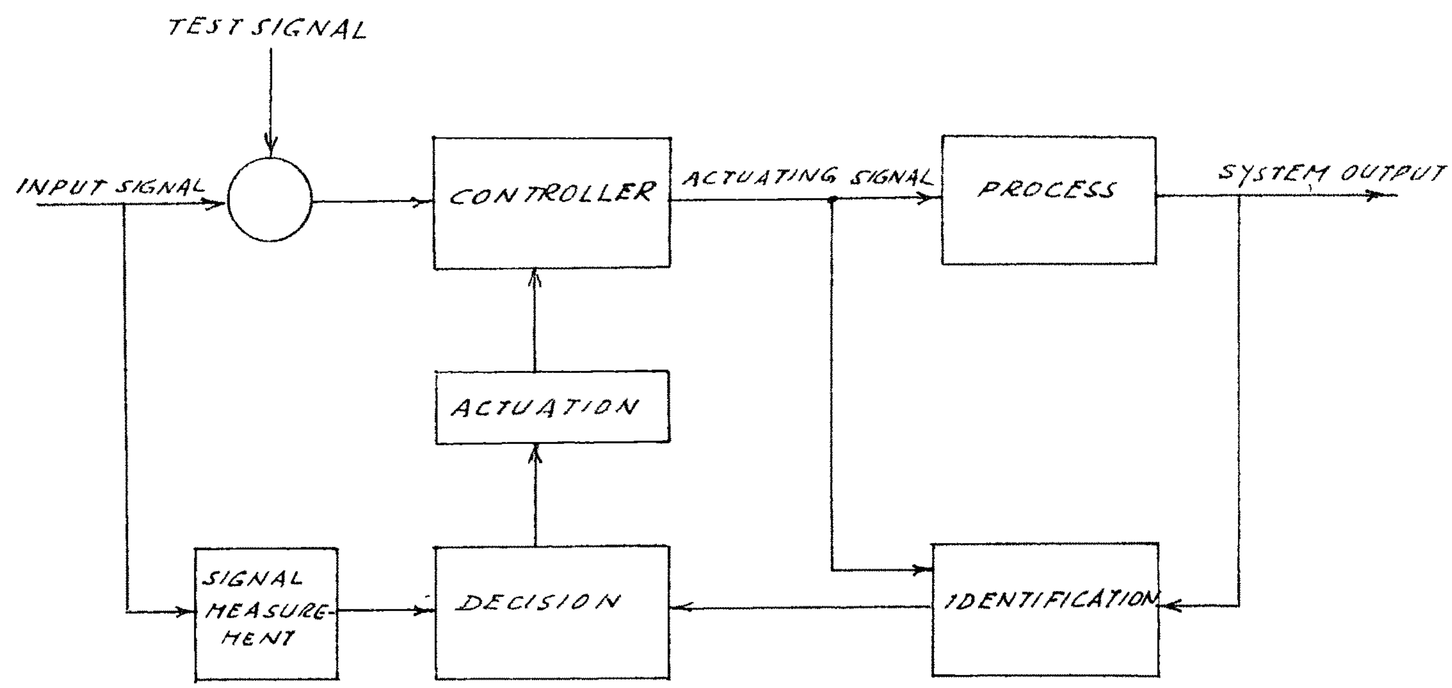

FIG. 2·3 A GENERAL ADAPTIVE CONTROL SYSTEM.

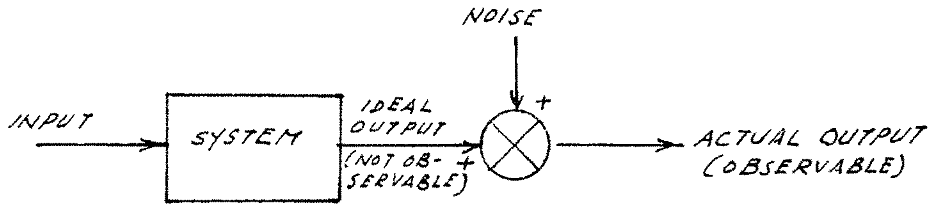

FIG. 2·4 THE MODEL FOR LEVIN'S IMPULSE RESPONSE IDENTIFICATION SCHEME

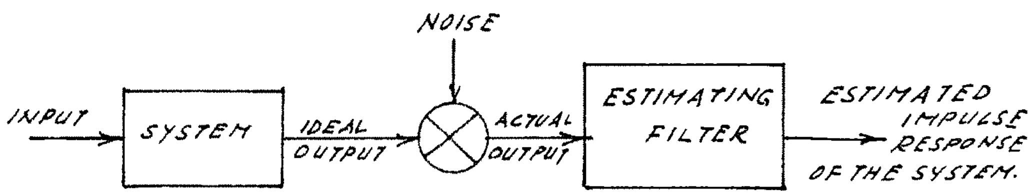

FIG. 2·5 TURIN'S IDENTIFICATION PROBLEM

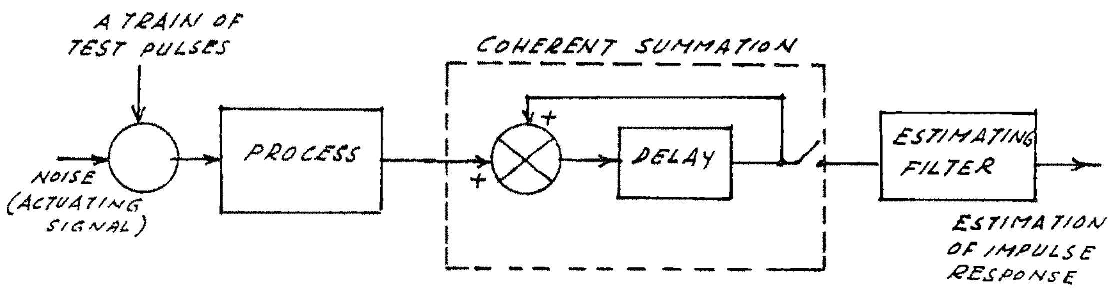

FIG. 2·6 EXTENSION OF TURIN'S IDENTIFICATION PROBLEM BY LITCHENBURGER.