CHAPTER V

ESTIMATION OF THE SECOND ORDER CLOSED-LOOP TRANSFER FUNCTION OF THE TURBO-ALTERNATOR

This chapter is concerned with the estimation of transfer function of the turbo-alternator when the feedback loop employing governor is closed as normal. The governor is represented by a first order transfer function (Fig. 4.1) like the open-loop transfer function considered in the previous chapter. The overall closed-loop transfer function is of the second order. The estimation procedure followed here is more or less the same as that discussed in chapter IV. The state varaible formulation of the second order plant is obtained in the beginning and a suitable algorithm for estimation developed under simulated conditions before trying with the actual data. The success in estimation was very limited due to difficulties in convergence of states towards their true values.

5.1 State Variable Formulation

Referring to Fig. 4.1, the closed-loop transfer function is given by

$$\frac{\Delta\Omega(s)}{\Delta P(s)} = \frac{\frac{1}{D(1+\tau_m s)}}{1 + \frac{1}{D(1+\tau_m s)}\cdot\frac{K}{(1+\tau_g s)}} \tag{5.1}$$

$$= \frac{(1 + \tau_g s)}{D[1 + (\tau_m + \tau_g)s + \tau_m \tau_g s^2] + K} \tag{5.2}$$

Recalling from chapter IV that the input Δp (power load fluctuations) and the output $\Delta\omega$ (frequency variations) were amplified before they were measured as $\Delta p'$ and $\Delta\omega'$ given by equations (4.6) and (4.7). Using equation (4.8) in equation (5.2), one obtains

$$\frac{\Delta\Omega'(s)}{\Delta P'(s)} = \frac{(1 + \tau_g s)}{D'\left[1 + (\tau_m+\tau_g)s + \tau_m\tau_g s^2\right] + K'} \qquad (5.3)$$

where

$$D' = \frac{10^4 D}{1200} \qquad (5.4)$$

and

$$K' = \frac{10^4 K}{1200} \qquad (5.5)$$

According to Stanton's[90] estimate, the values of D', K', τ_g, and τ_m respectively are 0.458 p.u. / c/s, 2.91 p.u. / c/s, 2.5 secs. and 2.5 secs. (refer equations (4.2) and (4.4)). The problem here is to obtain the estimates of D', K', τ_m and τ_g from the normal operating data for the closed-loop plant using the technique used in the previous chapter. The transfer function given by equation (5.3) has to be transformed into state variable differential equations.

The state variable formulation of the first order plant in chapter IV was comparatively easier. However, for a second order transfer function having a zero, it is not so simple. It is obtained by using computer diagrams[99]. Equation (5.3) can be written as

$$\frac{\Delta\Omega'(s)}{\Delta P'(s)} = \frac{a_1 s + a_2}{b_1 s^2 + b_2 s + b_3} \qquad (5.6)$$

where

$a_1 = \tau_g$; $a_2 = 1$; $b_1 = D'\tau_m\tau_g$; $b_2 = D'(\tau_m+\tau_g)$;
$b_3 = D' + K'$

Dividing both the numerator and demominator by $b_1 s^2$, equation

(5.6) becomes

$$\frac{\Delta\Omega'(s)}{\Delta P'(s)} = \frac{\frac{a_1}{b_1} s^{-1} + \frac{a_2}{b_1} s^{-2}}{1 + \frac{b_2}{b_1} s^{-1} + \frac{b_3}{b_1} s^{-2}} \qquad (5.7)$$

Let

$$E(s) = \frac{\Delta P'(s)}{1 + \frac{b_2}{b_1} s^{-1} + \frac{b_3}{b_1} s^{-2}} \qquad (5.8)$$

which is equivalent to

$$E(s) = \Delta P'(s) - \frac{b_2}{b_1} s^{-1} E(s) - \frac{b_3}{b_1} s^{-2} E(s) \qquad (5.9)$$

Use of equation (5.8) in equation (5.7) yields

$$\Delta\Omega'(s) = \left(\frac{a_1}{b_1} s^{-1} + \frac{a_2}{b_1} s^{-2}\right) E(s) \qquad (5.10)$$

Equation (5.9) and subsequently equation (5.10) can be interpreted to give the computer diagram depicted in Fig. 5.1. Denoting the outputs of the integrators in Fig. 5.1 by state variables x_1, x_2, . . . starting with x_1 for the last integrator and proceeding backwards, one obtains

$$\dot{x}_1 = x_2 \qquad (5.11)$$

$$\dot{x}_2 = -\frac{b_3}{b_1} x_1 - \frac{b_2}{b_1} x_2 + u \qquad (5.12)$$

and the ideal (without noise) output $\Delta\omega'$ is given by

$$\Delta\omega' = \frac{a_2}{b_1} x_1 + \frac{a_1}{b_1} x_2 \qquad (5.13)$$

where

$u = \Delta p'$; b_1, b_2, b_3, a_1, a_2 are constants and x_1, x_2, u are functions of time. Denoting the constant parameters by state variables as

$$x_3 = 1/\tau_m \qquad (5.14)$$

$$x_4 = 1/D' \qquad (5.15)$$

$$x_5 = 1/\tau_g \qquad (5.16)$$

$$x_6 = K' \qquad (5.17)$$

and representing a_1, a_2, b_1, b_2 and b_3 in terms of state variables, the equations (5.11) and (5.12) become

$$\dot{x}_1 = x_2 \qquad (5.18)$$

$$\dot{x}_2 = -x_3x_5(1 + x_4x_6)x_1 - (x_3 + x_5)x_2 + u \qquad (5.19)$$

and noting that x_3, x_4, x_5 and x_6 are constants, one gets

$$\dot{x}_3 = 0 \qquad (5.20)$$

$$\dot{x}_4 = 0 \qquad (5.21)$$

$$\dot{x}_5 = 0 \qquad (5.22)$$

$$\dot{x}_6 = 0 \qquad (5.23)$$

The output $\Delta\omega'$ in equation (5.13) now becomes

$$\Delta\omega' = x_3x_4(x_1x_5 + x_2) \qquad (5.24)$$

This output $\Delta\omega'$ ignores the random disturbances and represents the theretical or ideal output. However, the observed values of $\Delta\omega'$ are assumed to be corrupted with additive noise. Thus the discretely measured output is given by

$$\begin{aligned} y(i) &= x_3(i)x_4(i)\left[x_1(i)x_5(i) + x_2(i)\right] + n(i) \\ &= \overline{x_3(i)x_4(i)x_5(i)}\, x_1(i) + \overline{x_3(i)x_4(i)}\, x_2(i) + n(i) \end{aligned} \qquad (5.25)$$

Comparing equation (5.25) with equation (3.3), one obtains the constant vector H or H(i) given by

$$H(i) = [x_3(i)x_4(i)x_5(i),\ x_3(i)x_4(i),\ 0,\ 0,\ 0,\ 0] \tag{5.26}$$

which is a row vector, and

$$x(i) = \text{col}\,[x_1(i),\ x_2(i),\ x_3(i),\ x_4(i),\ x_5(i),\ x_6(i)] \tag{5.27}$$

which is the state vector representing the dynamic state of plant at an instant i.

The problem is to estimate the initial states $x_1(0)$, $x_2(0), \ldots, x_6(0)$ from the measurements of input $u(i)$ and output $y(i)$; $i = 0, 1, \ldots, N$.

5.2 Estimation Scheme

The best estimate of $x_1(0), x_2(0), \ldots, x_6(0)$ will be obtained by minimizing I given by

$$I = \sum_{i=0}^{N} Q\,[y(i) - \bar{x}_3(i)\bar{x}_4(i)\bar{x}_5(i)\bar{x}_1(i) - \bar{x}_3(i)\bar{x}_4(i)\bar{x}_2(i)]^2 \tag{5.28}$$

which is obtained by substituting for H from equation (5.26) in the equation (3.11). Here Q is scalar. The expression $[\bar{x}_3(i)\bar{x}_4(i)\bar{x}_5(i)\bar{x}_1(i) + \bar{x}_3(i)\bar{x}_4(i)\bar{x}_2(i)]$ in equation (5.28) is the output $\bar{y}(i)$ of the dynamic model simulated on the digital computer. Equation (5.28) means that the model output $\bar{y}(i)$ is compared with the observed system output in the least squares sense to obtain the best values of $\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i)$ which then become the best estimates of $x_1(0), x_2(0), \ldots, x_6(0)$ respectively. The dynamic model is represented by the following equations.

$$\dot{\bar{x}}_1 = \bar{x}_2 \tag{5.29}$$

$$\dot{\bar{x}}_2 = -\bar{x}_3\bar{x}_5(1 + \bar{x}_4\bar{x}_6)\bar{x}_1 - (\bar{x}_3 + \bar{x}_5)\bar{x}_2 + u \tag{5.30}$$

$$\dot{\bar{x}}_3 = 0 \tag{5.31}$$

$$\dot{\bar{x}}_4 = 0 \tag{5.32}$$

$$\dot{\bar{x}}_5 = 0 \tag{5.33}$$

$$\dot{\bar{x}}_6 = 0 \tag{5.34}$$

for some initial conditions $\bar{x}_1(0), \bar{x}_2(0), \ldots, \bar{x}_6(0)$ and the same input as that of the system whose parameters are to be estimated. Equations (5.29) to (5.34) together are equivalent to the vector differential equation (3.41). The minimization of I requires to satisfy equations (3.17) to (3.20). These equations, when interpreted in the present case, give

$$\bar{x}_1(i+1) = f_1[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] \tag{5.35}$$

$$\bar{x}_2(i+1) = f_2[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] \tag{5.36}$$

$$\bar{x}_3(i+1) = f_3[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] = \bar{x}_3(i) \tag{5.37}$$

$$\bar{x}_4(i+1) = f_4[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] = \bar{x}_4(i) \tag{5.38}$$

$$\bar{x}_5(i+1) = f_5[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] = \bar{x}_5(i) \tag{5.39}$$

$$\bar{x}_6(i+1) = f_6[\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i), i] = \bar{x}_6(i) \qquad i = 0, 1, \ldots, N \tag{5.40}$$

and

$$\lambda_1(i-1) = f^{11}_{\bar{x}(i)}\lambda_1(i) + f^{21}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{61}_{\bar{x}(i)}\lambda_6(i) + 2\Omega\bar{x}_3(i)\bar{x}_4(i)\bar{x}_5(i)[y(i) - \bar{x}_3(i)\bar{x}_4(i)\bar{x}_5(i)\bar{x}_1(i) - \bar{x}_3(i)\bar{x}_4(i)\bar{x}_2(i)] \tag{5.41}$$

$$\lambda_2(i-1) = f^{12}_{\bar{x}(i)}\lambda_1(i) + f^{22}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{62}_{\bar{x}(i)}\lambda_6(i) + 2Q\bar{x}_3(i)\bar{x}_4(i)[y(i)-\bar{x}_3(i)\bar{x}_4(i)\bar{x}_5(i)\bar{x}_1(i) - \bar{x}_3(i)\bar{x}_4(i)\bar{x}_2(i)] \quad (5.42)$$

$$\lambda_3(i-1) = f^{13}_{\bar{x}(i)}\lambda_1(i) + f^{23}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{63}_{\bar{x}(i)}\lambda_6(i) \quad (5.43)$$

$$\lambda_4(i-1) = f^{14}_{\bar{x}(i)}\lambda_1(i) + f^{24}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{64}_{\bar{x}(i)}\lambda_6(i) \quad (5.44)$$

$$\lambda_5(i-1) = f^{15}_{\bar{x}(i)}\lambda_1(i) + f^{25}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{65}_{\bar{x}(i)}\lambda_6(i) \quad (5.45)$$

$$\lambda_6(i-1) = f^{16}_{\bar{x}(i)}\lambda_1(i) + f^{26}_{\bar{x}(i)}\lambda_2(i) + \ldots + f^{66}_{\bar{x}(i)}\lambda_6(i) \quad (5.46)$$

with the following boundary conditions

$$\lambda_k(-1) = 0 \quad ; \quad k = 1, 2, \ldots, 6 \quad (5.47a)$$

$$\lambda_k(N) = 0 \quad ; \quad k = 1, 2, \ldots, 6 \quad (5.47b)$$

Equations (5.35) to (5.40) are discrete-time version of equations (5.29) to (5.34). They need not be known in the closed form when the values $\bar{x}_1(i), \bar{x}_2(i), \ldots, \bar{x}_6(i)$ for $i =$ 0 to N are obtained by numerical integration of equations (5.29) and (5.34). Equations (5.37) to (5.40) imply that $\bar{x}_3(i)$, $\bar{x}_4(i)$, $\bar{x}_5(i)$ and $\bar{x}_6(i)$ are constants. The elements of the Jacobian matrix required to solve equations (5.41) to (5.46) can be computed by solving equation (3.46). The matrix $g_{\bar{x}}$ needed in equation (3.46) for the present case is given by equation (5.48) on the next page. The state transition matrix $\Phi(i+1, i)$ is a 6×6 matrix in the present case and it represents the Jacobian matrix by virtue of equation (3.47). The numerical integration

$$g_{\bar{x}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{x}_3\bar{x}_5(1+\bar{x}_4\bar{x}_6) & -(\bar{x}_3+\bar{x}_5) & -\bar{x}_5(1+\bar{x}_4\bar{x}_6)\bar{x}_1-\bar{x}_2 & -\bar{x}_3\bar{x}_5\bar{x}_6\bar{x}_1 & -\bar{x}_3(1+\bar{x}_4\bar{x}_6)\bar{x}_1-\bar{x}_2 & -\bar{x}_3\bar{x}_5\bar{x}_4\bar{x}_1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.48}$$

method using AMRK or other simpler subroutine took considerable computer time to compute $\bar{x}(i)$ and $f_{\bar{x}(i)}$. It was therefore decided to obtain the same by transforming the differential equations (5.29) to (5.34) into difference equations of the form shown by equations (5.35) to (5.41). The values of parameters in the equations (5.29) to (5.34) change on every iteration of estimation procedure (until they finally converge to their true values) driving the second order system dynamics into either over- , under- or critically-damped condition. The subroutine involving difference equations must incorporate all the three possibilities. The difference equations for the differential equations are obtained considering linear interpolation between the successive samples of the input $u(i)$. These are given in Appendix B.

It is not easy to write down equations for certain elements (especially those involving partial differentiation of $\bar{x}_1(i+1)$ and $\bar{x}_2(i+1)$ with respect to the parameters appearing as exponents) of Jacobian matrix from the difference equations given in Appendix B. These were computed numerically by considering small variations in parameters. The computational procedure followed is the same as mentioned in Chapter IV with the only difference that the initial conditions $\bar{x}_1(0), \bar{x}_2(0), \ldots, \bar{x}_6(0)$ are changed on every iteration as per following equation.

$$\text{new } \bar{x}_k(0) = \text{old } \bar{x}_k(0) + \frac{\Delta A}{\sqrt{\sum_{j=1}^{6} \lambda_j(-1)^2}} \lambda_k(-1) \qquad (5.49)$$

$$k = 1, 2, \ldots, 6$$

Here too, the step-size ΔA is chosen to be 0.1 to start with. The estimation scheme detailed in this section for the second order system is first tried with the input-output data of a computer-simulated system as discussed in the next section.

5.3 Estimation from Input-Output Record of a Computer-Simulated System Similar to the Second Order Closed-loop Plant.

The second order transfer function of equation (5.3) represented by differential equations (5.18) to (5.23) was simulated on the digital computer. The true output $[x_3(i)x_4(i)x_5(i)x_1(i) + x_3(i)x_4(i)x_2(i)]$ for $i = 0$ to 299 (i.e. 300 measurements) was computed from equations (5.18) to (5.23) using sinusoidal input $u(i) = 1.0 \sin (0.125\, i)$ and with initial conditions $x_1(0) = 1.0$, $x_2(0) = 0.5$, $x_3(0) = 0.5$, $x_4(0) = 2.0$, $x_5(0) = 0.5$ and $x_6(0) = 3.0$. The sampled input and output were stored and used for estimation following the method described in case II(a) of chapter IV. The results of experiments are discussed below.

(1) The first trail for estimation was begun with initial guesses for initial conditions 10 % off from their true values and it was observed that $\bar{x}_3(0)$ and $\bar{x}_5(0)$ overshooted and did not converge back to their true values. The performance index I did not decrease further beyond a certain value. The results are shown in Table 5.1 on the next page.

(2) The second experiment was begun with an objective to improve convergence by keeping parameters $\bar{x}_3(0)$ and $\bar{x}_4(0)$ constants to their respective true values and estimating the remaining initial conditions $\bar{x}_1(0)$, $\bar{x}_2(0)$, $\bar{x}_5(0)$ and $\bar{x}_6(0)$. It was

thought that if it worked successfully this way, the scheme could be applied to the experimental data assuming the initial states $\bar{x}_3(0)$ and $\bar{x}_4(0)$ to be known from open-loop plant estimation of Chapter IV. The results are shown in Table 5.2 .

It may be observed that this experiment also failed to show any sign of reliable and satisfactory covergence. It was thought that if the coefficients of differential equations representing the second order plant could be changed, it might give better convergence. Three such possibilities were tried as discussed below in schemes (3), (4) and (5).

TABLE 5.1

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$
True values	1.0	0.5	0.5	2.0	0.5	3.0
Initial guess	0.9	0.45	0.45	1.80	0.45	2.70
Final results	0.919	0.454	0.588	1.833	0.588	2.722

TABLE 5.2

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$
True values	1.0	0.5	0.5	2.0	0.5	3.0
Initial guess	0.9	0.45	0.5*	2.0*	0.45	2.70
Final results	0.965	0.478	0.5*	2.0*	0.581	2.877

(3) All the relevant equations were modified by redefining parameters as follows:

$$x_3'(0) = \frac{1}{x_3(0)} = \tau_m \quad (5.50)$$

$$x_4'(0) = \frac{1}{x_4(0)} = D' \quad (5.51)$$

$$x_5'(0) = \frac{1}{x_5(0)} = \tau_g \quad (5.52)$$

$$x_6'(0) = x_6(0) = K' \quad (5.53)$$

This indeed changed the coefficients of dynamic equations but did not show convergence to obtain good estimate of $x_3'(0)$, $x_4'(0)$, $x_5'(0)$ and $x_6'(0)$. The difference equations given Appendix B are still valid. Only the expressions for α_1 and α_2 given by equations (B.2) and (B.3) have changed.

(4) The above experiment did not succeed and so another possible modification was tried. The parameters were redefined as shown below.

$$x_3'(0) = \frac{1}{x_3(0)} = \tau_m \quad (5.54)$$

$$x_4'(0) = x_4(0) = 1/D' \quad (5.55)$$

$$x_5'(0) = \frac{1}{x_5(0)} = 1/\tau_g \quad (5.56)$$

$$x_6'(0) = x_6(0) = K' \quad (5.57)$$

All other equations were modified in view of this and computer run for estimation were made but there was no improvement in covergence and another alternative was tried.

(5) The differential equations were modified by redefining the variables given by

$$x_3'(0) = x_3(0) = 1/\tau_m \quad (5.58)$$

$$x_4'(0) = x_4(0) = 1/D' \quad (5.59)$$

$$x_5'(0) = x_3(0)\, x_5(0) = 1/(\tau_m \gamma_g) \tag{5.60}$$

$$x_6'(0) = x_6(0) = K' \tag{5.61}$$

The convergence during estimation procedure in this case was better but $x_5'(0)$ was overshooting a little more and prevented convergence. The computational algorithm developed for first order system in Chapter IV was added one more step in the end.

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(4d) If large step search along the gradient fails to get I lower than previous minimum I, vary $\bar{x}_5(0)$, say by 2 to 3 % (plus or minus) of its current value. If this gives I lower than previous minimum I, continue variation in $\bar{x}_5(0)$ until stuck and then go back to step (2).

The results of estimation (with and without additive noise) for this case (with initial guesses 10 % off from their true values) are shown in Table 5.3 below.

TABLE 5.3

	$x_1'(0)$	$x_2'(0)$	$x_3'(0)$	$x_4'(0)$	$x_5'(0)$	$x_6'(0)$
True values	1.0	0.5	0.5	2.0	0.25	3.0
Initial guess	0.9	0.45	0.45	1.80	0.3	2.7
Final results :						
i) Without noise	0.994	0.510	0.489	2.025	0.247	3.038
ii) With 5% noise	0.975	0.487	0.488	1.952	0.262	2.923

However, this algorithm failed to work when initial guesses for initial conditions were far away from their true values. This was observed when the initial guess for all initial conditions was taken to be 0.2 or 0.5 or 1.0 successively. This frittered away the hopes of getting good convergence for estimation with actual operating data in which case the initial guess has to be arbitrary and not nearer to their expected values, if it is to be an estimation problem.

5.4 Estimation from Actual Operating Data of the Closed-loop Plant

In the beginning, some experiments for estimation were made by using the actual input (instead of sinusoidal) and the computed or simulated output using the values of parameters same as discussed in section 5.3 . The convergence was satisfactory when the initial guesses were made close to the true values. But it failed when the initial guesses were far away from their true values. No conclusion could be made regarding the convergence of parameters while using the actual input and actual output for estimation. Thus the estimation of second order plant from the experimental data almost failed and consequently the estimation of closed-loop plant will be tackled again considering two time-constants (Fig. 4.1) for the feedback loop.

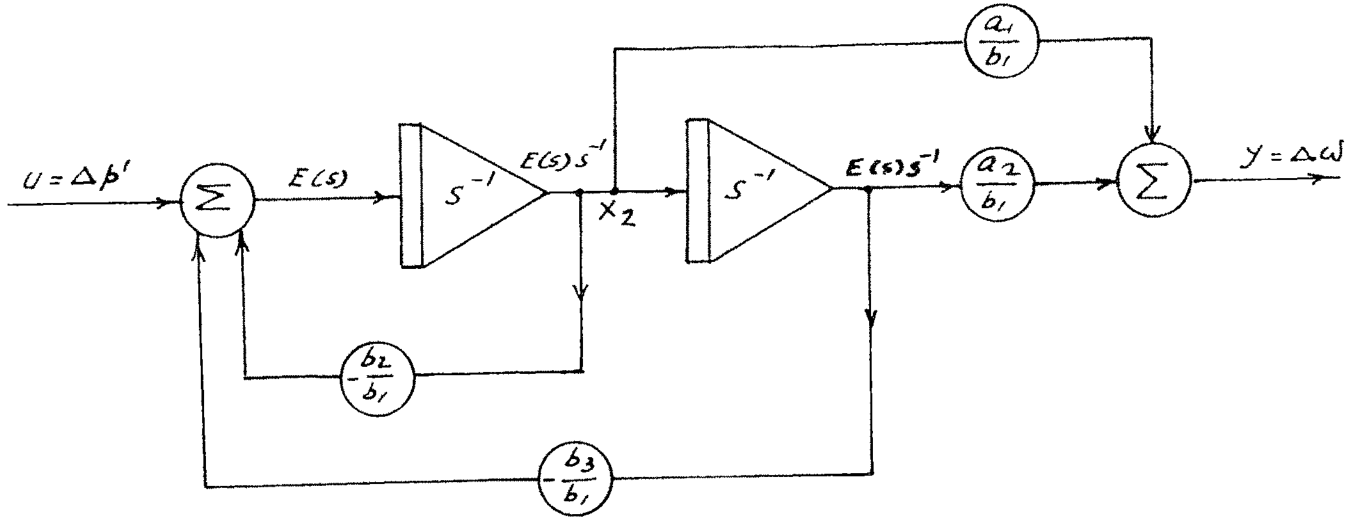

FIG. 5·1. COMPUTER DIAGRAM TO OBTAIN STATE VARIABLE EQUATIONS FOR THE CLOSED-LOOP SECOND ORDER TRANSFER FUNCTION.