CHAPTER VI

ESTIMATION OF THE THIRD ORDER CLOSED-LOOP
TRANSFER FUNCTION OF THE TURBO-ALTERNATOR

The estimation of second order closed-loop plant failed to give reliable results as discussed in previous chapter. An attempt was therefore made to try out a third order model where the governor action in the feedback loop is represented by two time constants. The representation of the governor by two time constants is more typical. It also involves a backlash nonlinearity but this was neglected.

After obtaining the state variable formulation of the transfer function, the same computational algorithm was used and the convergence of parameters was excellent when the open-loop plant parameters τ_m and D' were assumed to have been known. The experimental data used in this case was the same as that for the second order system considered earlier. The results obtained from the simulated as well as the actual data were quite satisfactory.

6.1 State Variable Formulation

Denoting the two time constants for the governor by τ'_g and τ''_g (in the feedback loop shown by dotted line in Fig. 4.1), the closed-loop transfer function is given by

$$\frac{\Delta \Omega(s)}{\Delta P'(s)} = \frac{\dfrac{1}{D(1+\tau_m s)}}{1 + \dfrac{1}{D(1+\tau_m s)} \dfrac{K}{(1+\tau'_g s)(1+\tau''_g s)}} \qquad (6.1)$$

$$= \frac{1 + (\tau'_g + \tau''_g)s + \tau'_g \tau''_g s^2}{D\left[1 + (\tau_m + \tau'_g + \tau''_g)s + (\tau_m \tau'_g + \tau'_g \tau''_g + \tau''_g \tau_m)s^2 + \tau_m \tau'_g \tau''_g s^3\right] + K} \qquad (6.2)$$

Using equation (4.8) in equation (6.2)

$$\frac{\Delta Q'(s)}{\Delta P'(s)} = \frac{1 + (\tau_g' + \tau_g'')s + \tau_g'\tau_g''s^2}{D'\left[1 + (\tau_m + \tau_g' + \tau_g'')s + (\tau_m\tau_g' + \tau_g'\tau_g'' + \tau_g''\tau_m)s^2 + \tau_m\tau_g'\tau_g''s^3\right] + K'}$$

(6.3)

where D' and K' are related to D and K respectively by
equations (5.4) and (5.5). According to Stanton's[90] estimate,
the values of D' , K' , τ_m are 0.458 p.u./ c/s , 2.91 p.u./ c/s ,
2.5 secs. respectively. No definite estimate of τ_g' and τ_g''
is available but they are expected to be around 0.5 secs. each.
The problem in this chapter is to estimate the parameters D' ,
K' , τ_m , τ_g' and τ_g'' using the same operating data as used by
Stanton and using the estimation scheme developed in Chapter III.
The estimation scheme requires the transfer function of equation
(6.3) to be transformed into state variable equations. This will
be obtained by using computer diagram. Equation (6.3) can be
written as

$$\frac{\Delta Q'(s)}{\Delta P'(s)} = \frac{a_1 s^2 + a_2 s + a_3}{b_1 s^3 + b_2 s^2 + b_3 s + b_4}$$

(6.4)

where

$a_1 = \tau_g'\tau_g''$; $a_2 = \tau_g' + \tau_g''$; $a_3 = 1$; $b_1 = D'\tau_m\tau_g'\tau_g''$;

$b_2 = D'(\tau_m\tau_g' + \tau_m\tau_g'' + \tau_g'\tau_g'')$; $b_3 = D'(\tau_g' + \tau_g'' + \tau_m)$; $b_4 = D' + K'$.

Dividing the numerator and denominator on the right hand side
of equation (6.4) by $b_1 s^3$, one obtains

$$\frac{\Delta Q'(s)}{\Delta P'(s)} = \frac{\frac{a_1}{b_1}s^{-1} + \frac{a_2}{b_1}s^{-2} + \frac{a_3}{b_1}s^{-3}}{1 + \frac{b_2}{b_1}s^{-1} + \frac{b_3}{b_1}s^{-2} + \frac{b_4}{b_1}s^{-3}}$$

(6.5)

Let

$$E(s) = \frac{\Delta P'(s)}{1 + \frac{b_2}{b_1} s^{-1} + \frac{b_3}{b_1} s^{-2} + \frac{b_4}{b_1} s^{-3}}$$ (6.6)

which is equivalent to

$$E(s) = \Delta P'(s) - \frac{b_2}{b_1} s^{-1} E(s) - \frac{b_3}{b_1} s^{-2} E(s) - \frac{b_4}{b_1} s^{-3} E(s)$$ (6.7)

Use of equation (6.6) in equation (6.5) yields

$$\Delta\Omega'(s) = (\frac{a_1}{b_1} s^{-1} + \frac{a_2}{b_1} s^{-2} + \frac{a_3}{b_1} s^{-3}) E(s)$$ (6.8)

Equations (6.7) and (6.8) can be interpreted to give the computer diagram shown in Fig. 6.1 . Let the outputs of integrators in Fig. 6.1 be denoted by state variables x_1 , x_2 , . . . starting with x_1 for the last integrator and proceeding backwards. Thus, one obtains

$$\dot{x}_1 = x_2$$ (6.9)

$$\dot{x}_2 = x_3$$ (6.10)

$$\dot{x}_3 = -\frac{b_4}{b_1} x_1 - \frac{b_3}{b_1} x_2 - \frac{b_4}{b_1} x_3 + u$$ (6.11)

and the ideal (without noise) output $\Delta\omega'$ is given by

$$\Delta\omega' = \frac{a_3}{b_1} x_1 + \frac{a_2}{b_1} x_2 + \frac{a_1}{b_1} x_3$$ (6.12)

where $u = \Delta p'$; b_1 , b_2 , b_3 , b_4 , a_1 , a_2 , a_3 are constants and x_1 , x_2 , u are functions of time. Representing the parameters as state variables given by

$$x_4 = 1/\tau_m$$ (6.13)

$$x_5 = 1/D'$$ (6.14)

$$x_6 = 1/\tau_g' \tag{6.15}$$

$$x_7 = 1/\tau_g'' \tag{6.16}$$

$$x_8 = K' \tag{6.17}$$

and substituting the relevant state variables for a_1, a_2, a_3, b_1, b_2, b_3 and b_4, equations (6.9), (6.10) and (6.11) become

$$\dot{x}_1 = x_2 \tag{6.18}$$

$$\dot{x}_2 = x_3 \tag{6.19}$$

$$\dot{x}_3 = -x_4 x_6 x_7 (1+x_5 x_8)x_1 - (x_4 x_6 + x_6 x_7 + x_7 x_4)x_2 -$$
$$(x_4 + x_6 + x_7)x_3 + u \tag{6.20}$$

Noting that x_4, x_5, x_6, x_7 and x_8 are constants, one obtains

$$\dot{x}_4 = 0 \tag{6.21}$$

$$\dot{x}_5 = 0 \tag{6.22}$$

$$\dot{x}_6 = 0 \tag{6.23}$$

$$\dot{x}_7 = 0 \tag{6.24}$$

$$\dot{x}_8 = 0 \tag{6.25}$$

The output $\Delta\omega'$ in equation (6.12) now becomes

$$\Delta\omega' = x_4 x_5 x_6 x_7 x_1 + x_4 x_5 (x_6 + x_7) x_2 + x_4 x_5 x_3 \tag{6.26}$$

Equation (6.26) represents the theoretical or ideal output since it does not consider the random diturbances. Considering the additive noise, the discretely measured output $y(i)$ is given by

$$y(i) = x_4(i) \, x_5(i) \, x_6(i) \, x_7(i) \, x_1(i) + x_4(i) \, x_5(i) \left[x_6(i) + \right.$$

$$\left. x_7(i) \right] x_2(i) + x_4(i) \, x_5(i) \, x_3(i) + n(i) \qquad (6.27)$$

Comparison of equation (6.27) with equation (3.3) gives the constant row vector H or $H(i)$ given by

$$H(i) = \left[x_4(i)x_5(i)x_6(i)x_7(i) \, , \, x_4(i)x_5(i)\{x_6(i)+x_7(i)\} \, , \right.$$

$$\left. x_4(i)x_5(i) \, , \, 0 \, , \, 0 \, , \, 0 \, , \, 0 \, , \, 0 \right] \qquad (6.28)$$

and the state vector $x(i)$ given by

$$x(i) = \text{col} \left[x_1(i) \, , \, x_2(i) \, , \, \ldots \, , \, x_8(i) \right] \qquad (6.29)$$

The aim is to estimate the initial states $x_1(0)$, $x_2(0)$, $\ldots$, $x_8(0)$ from observations of input $u(i)$ and output $y(i)$; $i = 0, 1, \ldots ,N$.

6.2 Estimation Scheme

The best estimate of the initial states $x_1(0)$, $x_2(0)$, $\ldots$, $x_8(0)$ will be obtained on the basis of minimization of the performance index I given by

$$I = \sum_{i=0}^{N} Q \left[y(i) - \bar{x}_4(i)\bar{x}_5(i)\bar{x}_6(i)\bar{x}_7(i)\bar{x}_1(i) - \bar{x}_4(i)\bar{x}_5(i). \right.$$

$$\left. \{\bar{x}_6(i)+\bar{x}_7(i)\}\bar{x}_2(i) - \bar{x}_4(i)\bar{x}_5(i)\bar{x}_3(i) \right]^2 \qquad (6.30)$$

This is obtained by substituting for H from equation (6.28) in the equation (3.11). Since the observation $y(i)$ is scalar, Q is also scalar. The expression $\left[\bar{x}_4(i)\bar{x}_5(i)\bar{x}_6(i)\bar{x}_7(i)\bar{x}_1(i) + \bar{x}_4(i)\bar{x}_5(i)\{\bar{x}_6(i)+\bar{x}_7(i)\}\bar{x}_2(i) + \bar{x}_4(i)\bar{x}_5(i)\bar{x}_3(i) \right]$ in equation (6.30) represents the output $\bar{y}(i)$ of the dynamic model simulated on the digital computer. Equation (6.30) imply that the model output $\bar{y}(i)$ is compared with the observed system output

in the least squares sense to obtain the best values of $\bar{x}_1(i)$, $\bar{x}_2(i)$, . . . , $\bar{x}_8(i)$ which then give the best estimates of $x_1(0)$, $\bar{x}_2(0)$, . . . , $\ddot{x}_8(0)$ respectively. The dynamic model is represented by the following equations.

$$\dot{\bar{x}}_1 = \bar{x}_2 \qquad\qquad (6.31)$$

$$\dot{\bar{x}}_2 = \bar{x}_3 \qquad\qquad (6.32)$$

$$\dot{\bar{x}}_3 = -\bar{x}_4\bar{x}_6\bar{x}_7(1 + \bar{x}_5\bar{x}_8)\bar{x}_1 - (\bar{x}_4\bar{x}_6 + \bar{x}_6\bar{x}_7 + \bar{x}_7\bar{x}_4)\bar{x}_2 -$$

$$(\bar{x}_4 + \bar{x}_6 + \bar{x}_7)\bar{x}_3 + u \qquad\qquad (6.33)$$

$$\dot{\bar{x}}_4 = 0 \qquad\qquad (6.34)$$

$$\dot{\bar{x}}_5 = 0 \qquad\qquad (6.35)$$

$$\dot{\bar{x}}_6 = 0 \qquad\qquad (6.36)$$

$$\dot{\bar{x}}_7 = 0 \qquad\qquad (6.37)$$

$$\dot{\bar{x}}_8 = 0 \qquad\qquad (6.38)$$

with some initial conditions $\bar{x}_1(0)$, $\bar{x}_2(0)$, ... , $\bar{x}_8(0)$ and using the same input as that of the system whose parameters are to be estimated. Equation (3.41) is in the general vector form representing the above 8 scalar equations. The Euler-Lagrange equations for minimization of I of equation (6.30) are given by

$$\bar{x}_k(i+1) = f_k\left[\bar{x}_1(i) , \bar{x}_2(i) , ... , \bar{x}_8(i)\right] \qquad (6.39)$$

$$k = 1, 2, . . . , 8$$

and since $\bar{x}_4$, $\bar{x}_5$, ... , $\bar{x}_8$ are constants ,

$$\bar{x}_j(i+1) = \bar{x}_j(i) \qquad ; \quad j = 4, 5, 6, 7, 8 . \qquad (6.40)$$

and

$$\lambda_1(i-1) = f^{11}_{\underset{\sim}{x}(i)}\lambda_1(i) + f^{21}_{\underset{\sim}{x}(i)}\lambda_2(i) + \ldots + f^{81}_{\underset{\sim}{x}(i)}\lambda_8(i) +$$

$$2Q\,\bar{x}_4(i)\bar{x}_5(i)\bar{x}_6(i)\bar{x}_7(i)\left[y(i) - \bar{x}_4(i)\bar{x}_5(i)\bar{x}_6(i)\bar{x}_7(i).\right.$$

$$\bar{x}_1(i) - \bar{x}_4(i)\bar{x}_5(i)\ \bar{x}_6(i)+\bar{x}_7(i)\ \bar{x}_2(i) -$$

$$\left.\bar{x}_4(i)\bar{x}_5(i)\bar{x}_3(i)\right] \tag{6.41}$$

$$\lambda_2(i-1) = f^{12}_{\underset{\sim}{x}(i)}\lambda_1(i) + f^{22}_{\underset{\sim}{x}(i)}\lambda_2(i) + \ldots + f^{82}_{\underset{\sim}{x}(i)}\lambda_8(i) +$$

$$2Q\,\bar{x}_4(i)\bar{x}_5(i)\{\bar{x}_6(i)+\bar{x}_7(i)\}\left[y(i) - \bar{x}_4(i)\bar{x}_5(i).\right.$$

$$\bar{x}_6(i)\bar{x}_7(i)\bar{x}_1(i) - \bar{x}_4(i)\bar{x}_5(i)\ \bar{x}_6(i)+\bar{x}_7(i)\ \bar{x}_2(i) -$$

$$\left.\bar{x}_4(i)\bar{x}_5(i)\bar{x}_3(i)\right] \tag{6.42}$$

$$\lambda_3(i-1) = f^{13}_{\underset{\sim}{x}(i)}\lambda_1(i) + f^{23}_{\underset{\sim}{x}(i)}\lambda_2(i) + \ldots + f^{83}_{\underset{\sim}{x}(i)}\lambda_8(i)+$$

$$2Q\,\bar{x}_4(i)\bar{x}_5(i)\left[y(i) - \bar{x}_4(i)\bar{x}_5(i)\bar{x}_6(i)\bar{x}_7(i)\bar{x}_1(i) -\right.$$

$$\left.\bar{x}_4(i)\bar{x}_5(i)\ \bar{x}_6(i) + \bar{x}_7(i)\ \bar{x}_2(i) - \bar{x}_4(i)\bar{x}_5(i)\bar{x}_3(i)\right] \tag{6.43}$$

$$\lambda_j(i-1) = f^{1j}_{\underset{\sim}{x}(i)}\lambda_1(i) + f^{2j}_{\underset{\sim}{x}(i)}\lambda_2(i) + \ldots \dotplus f^{8j}_{\underset{\sim}{x}(i)}\lambda_8(i)$$

$$j = 4,\ 5,\ 6,\ 7,\ 8\ . \tag{6.44}$$

$i = 0, 1, \ldots , N$ for all equations from (6.39) to (6.44). The boundary conditions to be satisfied are

$$\lambda_k(-1) = 0\quad ;\quad k = 1,\ 2,\ \ldots ,\ 8 \tag{6.45}$$

$$\lambda_k(N) = 0\quad ;\quad k = 1,\ 2,\ \ldots ,\ 8 \tag{6.46}$$

Equations (6.39) are discrete-time equivalent of equations (6.31) to (6.38). They need not be known in the closed form when the values $\bar{x}_1(i),\ \bar{x}_2(i),\ \ldots ,\ \bar{x}_8(i)$ for $i = 0, 1, \ldots ,N$ are

obtained by numerical integration of equations (6.31) to (6.38).

The derivation for obtaining difference equations from differe-

ntial equations (6.31) to (6.32) and then to find Jacobian matrix

$f_{\bar{x}(i)}$ were too lengthy, tedious and cumbersome. These were the-

refore obtained by numerical integration. The subroutine used

for this purpose was based on the first term in Taylor's expan-

sion. The results obtained by this subroutine were almost the

same as those obtained by using AMRK subroutine. Moreover, this

subroutine is four times faster than AMRK subroutine. The Jaco-

bian matrix elements are obtained by solving equation (3.46).

The matrix $g_{\bar{x}}$ needed in the equation (3.46) for the present

case is given by equation (6.47) on the next page. The state

transition matrix $\bar{\Phi}(i+1, i)$ is a 8×8 matrix and is equiva-

lent to the Jacobian matrix by equation (3.47). The computation-

al procedure followed here is the same as detailed in the pre-

vious chapter. The initial conditions $\bar{x}_1(0), \bar{x}_2(0), \ldots,$

$\bar{x}_8(0)$ are modified on every iteration as per the following eq-

uation.

$$\text{new } \bar{x}_k(0) \;=\; \text{old } \bar{x}_k(0) \;+\; \frac{\Delta A}{\sqrt{\displaystyle\sum_{j=1}^{8} \lambda_j^2(-1)}} \; \lambda_k(-1) \qquad (6.48)$$

$$k = 1, 2, \ldots, 8$$

The step-size ΔA is taken to be 0.1 to start with. The next

section deals with the estimation from the input-output data

obtained for a third order computer-simulated system.

$$
g_{\underline{x}} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
g_{\underline{x}}^{31} & g_{\underline{x}}^{32} & g_{\underline{x}}^{33} & g_{\underline{x}}^{34} & g_{\underline{x}}^{35} & g_{\underline{x}}^{36} & g_{\underline{x}}^{37} & g_{\underline{x}}^{38} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{6.47}
$$

where

$$g_{\underline{x}}^{31} = - \bar{x}_4 \bar{x}_6 \bar{x}_7 (1 + \bar{x}_5 \bar{x}_8) \quad ;$$

$$g_{\underline{x}}^{32} = - (\bar{x}_4 \bar{x}_6 + \bar{x}_6 \bar{x}_7 + \bar{x}_7 \bar{x}_4) \quad ;$$

$$g_{\underline{x}}^{33} = - (\bar{x}_4 + \bar{x}_6 + \bar{x}_7) \quad ;$$

$$g_{\underline{x}}^{34} = - \bar{x}_6 \bar{x}_7 (1 + \bar{x}_5 \bar{x}_8) \bar{x}_1 - (\bar{x}_6 + \bar{x}_7) \bar{x}_2 - \bar{x}_3 \quad ;$$

$$g_{\underline{x}}^{35} = - \bar{x}_4 \bar{x}_6 \bar{x}_7 \bar{x}_8 \bar{x}_1$$

$$g_{\underline{x}}^{36} = - \bar{x}_4 \bar{x}_7 (1 + \bar{x}_5 \bar{x}_8) \bar{x}_1 - (\bar{x}_7 + \bar{x}_4) \bar{x}_2 - \bar{x}_3 \quad ;$$

$$g_{\underline{x}}^{37} = - \bar{x}_4 \bar{x}_6 (1 + \bar{x}_5 \bar{x}_8) \bar{x}_1 - (\bar{x}_4 + \bar{x}_6) \bar{x}_2 - \bar{x}_3 \quad ;$$

$$g_{\underline{x}}^{38} = - \bar{x}_4 \bar{x}_5 \bar{x}_6 \bar{x}_7 \bar{x}_1 \quad .$$

6.3 Estimation from Input-Output Record of a Computer - Simulated System Similar to the Third Order Closed-loop Plant

The third order transfer function represented by differential equations (6.18) to (6.25) was simulated on the digital computer. The true output given by equation (6.27) excluding the noise term, for $i = 0$ to 99 (i.e. 100 measurements), was computed from equations (6.18) to (6.25) using sinusoidal input $u(i) = 1.0 \sin(0.125 \, i)$ and with initial conditions $x_1(0) = 1.0$, $x_2(0) = 0.5$, $x_3(0) = 0.125$, $x_4(0) = 0.5$, $x_5(0) = 2.0$, $x_6(0) = 2.0$, $x_7(0) = 2.0$ and $x_8(0) = 3.0$. Random numbers having their values within ± 0.05 were added to this output while considering the case of additive noise. The method described in case II(a) of chapter III was used to take care of mean of noise. Several experiments were made and their results are summarized below.

(1) The first trial for estimation was begun with initial guesses of initial conditions 10 % off from their true values and considering no noise with the output. In the process of convergence, it was observed that $\bar{x}_4(0)$ overshooted considerably and did not converge to its true value. This created difficulty for other initial states to converge to their true values. The performance index did not decrease further beyond this stage. The results are shown in Fig. 6.2 and Table 6.1 . The computer time for this program was about 5 minutes for 60 iterations.

(2) The difficulty in convergence was avoided by keeping $\bar{x}_4(0)$ and $\bar{x}_5(0)$ constants to their true values as if they were known, so that while working with the experimental data,

$\bar{x}_4(0)$ and $\bar{x}_5(0)$ may be assumed to have been known from the results of open-loop plant estimation. When $\bar{x}_4(0)$ and $\bar{x}_5(0)$ were kept constants to their true values 0.5 and 2.0 respectively, the problem reduced to the estimation of six initial states excluding $\bar{x}_4(0)$ and $\bar{x}_5(0)$. Instead of making an entirely new program for this new situation, the same program was used with computation of $\lambda_4(i)$ and $\lambda_5(i)$ neglected and by keeping $\lambda_4(i)$ and $\lambda_5(i)$ equal to zero throughout. The results of estimation without considering noise are shown in Fig. 6.3 and Table 6.2, Fig. 6.4 and Table 6.3, Fig. 6.5 and Table 6.4 for different initial guesses.

TABLE 6.1

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
True values*	1.00	0.50	0.125	0.50	2.00	2.00	2.00	3.00
Initial guess*	0.90	0.45	0.10	0.45	1.80	1.80	1.80	2.70
Final results*	0.958	0.517	0.115	0.663	1.772	1.933	1.933	2.621

* No Noise

TABLE 6.2

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
Without Noise:								
True values	1.00	0.50	0.125	0.50	2.00	2.00	2.00	3.00
Initial guess	0.90	0.45	0.10	0.50*	2.00*	1.80	1.80	2.70
Final results	1.053	0.445	0.088	0.50*	2.00*	2.016	2.016	2.931

* Kept constants

TABLE 6.3

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
Without Noise:								
True values	1.00	0.50	0.125	0.50	2.00	2.00	2.00	3.00
Initial guess	0.50	0.50	0.50	0.50*	2.00*	0.50	0.50	0.50
Final results	0.994	0.311	0.042	0.50*	2.00*	1.896	1.896	3.116

* Kept constants

TABLE 6.4

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
Without Noise:								
True values	1.0	0.5	0.125	0.50	2.00	2.00	2.00	3.00
Initial guess	1.0	1.0	1.0	0.50*	2.00*	1.0	1.0	1.0
Final results	1.092	0.259	0.771	0.50*	2.00*	2.05	2.05	2.888

* Kept constants

The computer time taken to obtain the results shown in Table 6.3 and Table 6.4 was about 3 minutes for about 100 iterations. Some experiments were done for estimation starting with arbitrary initial guesses for all initial states as done in experiment (1) of section 6.3 and then varying $\bar{x}_4(0)$ (when I did not decrease further and got stuck) alone towards minimizing the performance index I. and then going back to regular computational algorithm. This brought the initial state $\bar{x}_4(0)$ towards true value and overcame the trouble in convergence posed by over-shooting of $\bar{x}_4(0)$. Such experiments were also done for second

order system in Chapter IV . This method worked well but the computer time taken was enormous.

5.4 Estimation from Actual Input and Output Data

Before starting with the estimation from actual input and output, some experiments similar to those in section 6.3 were made for estimation using the actual input (instead of sinusoidal as was used in the previous section) and the corresponding output of the simulated system considering 100 measurements. The results regarding convergence were as good as those in section 6.3 , when the states $\bar{x}_4(0)$ and $\bar{x}_5(0)$ were kept constants to their true values. Having obtained confidence that the estimator works for both sinusoidal as well as actual input of random nature, the technique was tried using actual input and actual output data.

Both the input and output data were treated for h.f. filtering with $f_c = 1.0$ c/s, thus reducing the number of input and output measurements to 600. As discussed in Chapter IV, h.f. filtering with $f_c = 2.0$ c/s could have been better from accuracy point of view but this was deliberately avoided as this would increase the number of measurements to 1200 and hence the computer time. The initial states $x_4(0)$ and $x_5(0)$ representing the reciprocals of time constant τ_m and damping coefficient D' for the open loop plant were assumed to be constant at the values 0.25 and -2.158 (Table 4.3) respectively, obtained earlier for open-loop plant estimation. Three hundred measurements were used to estimate the rest of the initial states (i.e. $x_1(0)$, $x_2(0)$, $x_3(0)$, $x_6(0)$, $x_7(0)$ and $x_8(0)$). Since

the sampling interval for the actual data was 1.0 second, the integration subroutine having integration step of 0.01 sec. was required to be called 100 times in order to reach the next sampling instant. This involved a lot of computation. The convergence was observed to be excellent as was evident from smooth decrease in performance index I on every iteration but each iteration took about two minutes. It was observed that no significant difference in the results was found by taking the integration step of 0.1 sec. This required to CALL the integration sub-routine 10 times only to integrate over 1 second of time. This reduced the computer time considerably. The results obtained from the experimental input-output data of the turbo-alternator are shown in Table 6.5.

TABLE 6.5

	$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$	$x_5(0)$	$x_6(0)$	$x_7(0)$	$x_8(0)$
Initial guess	1.0	1.0	1.0	0.25*	-2.158*	1.0	1.0	1.0
Final results	-19.2	0.708	0.694	0.25*	-2.158*	2.373	2.373	-3.232

* Kept constant

Referring to equations (6.15), (6.16) and (6.17), the estimates of τ_g', τ_g'' and K' are given by

$$\tau_g' = 1/x_6(0) = 1/2.373 = 0.42 \tag{6.49}$$

$$\tau_g'' = 1/x_7(0) = 1/2.373 = 0.42 \tag{6.50}$$

$$K' = x_8(0) = -3.232 \tag{6.51}$$

Using equation (5.5), one obtains

$$K = 1200 \ K'/10^4$$

$$= - (1200 \times 3.232)/10^4$$

$$= - 0.388 \tag{6.52}$$

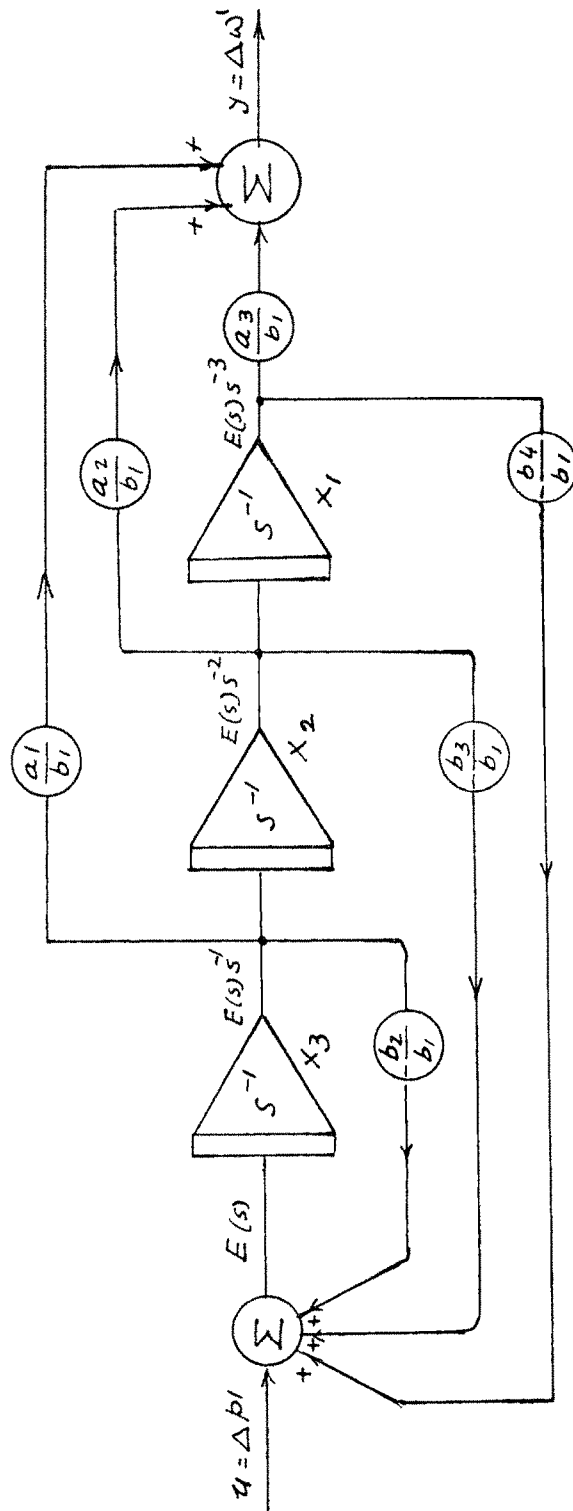The results of estimation are in close agreement with expectations.

FIG. 6.1 COMPUTER DIAGRAM TO OBTAIN STATE VARIABLE
EQUATIONS FOR THE CLOSED LOOP THIRD ORDER
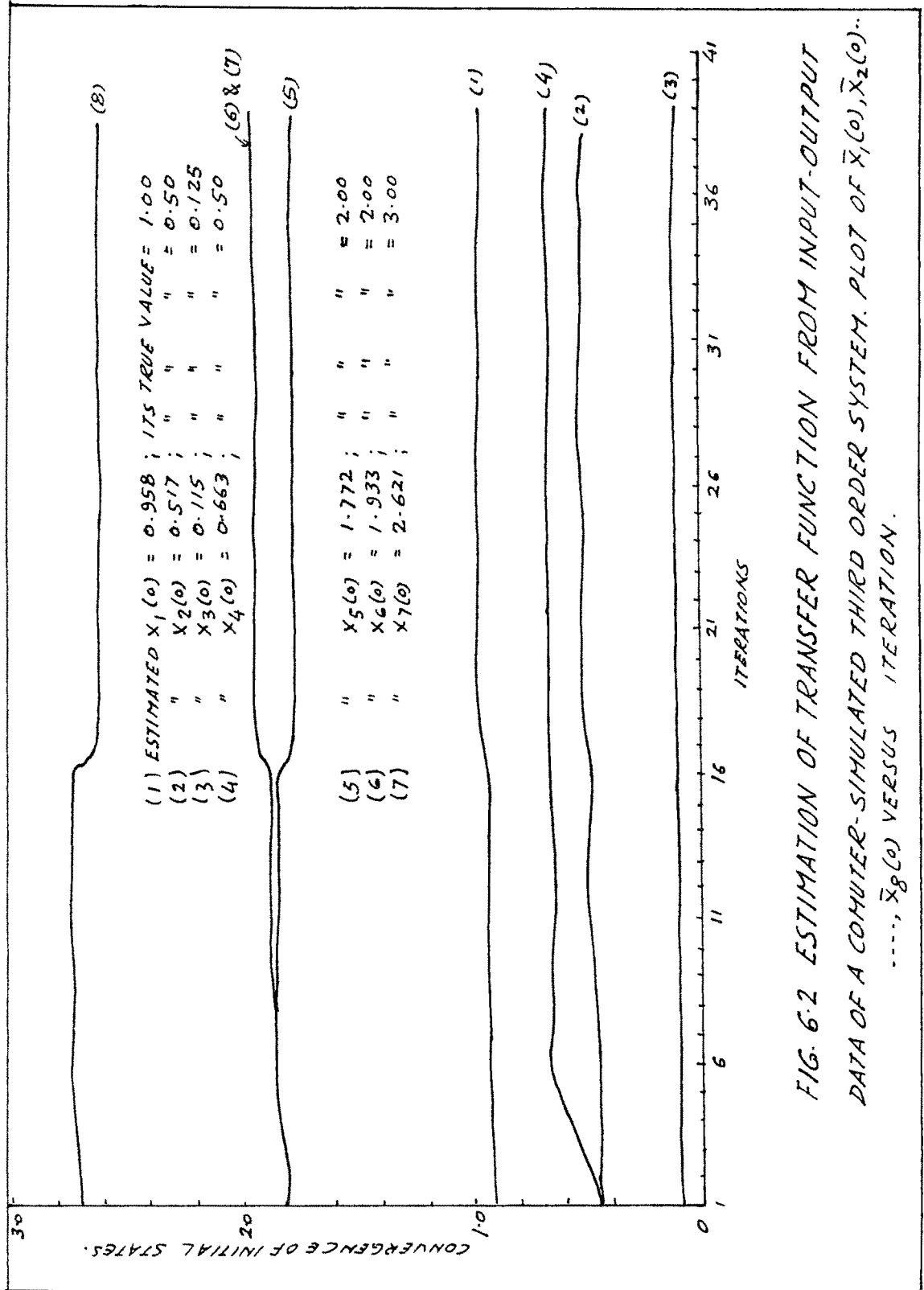TRANSFER FUNCTION

FIG. 6·2 ESTIMATION OF TRANSFER FUNCTION FROM INPUT-OUTPUT
DATA OF A COMPUTER-SIMULATED THIRD ORDER SYSTEM. PLOT OF $\bar{x}_1(o), \bar{x}_2(o)$..
----, $\bar{x}_8(o)$ VERSUS ITERATION.

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(1) ESTIMATED $X_1(0)$ = 1·053 ; ITS TRUE VALUE = 1·00
(2) " $X_2(0)$ = 0·443 ; " " " = 0·50
(3) " $X_3(0)$ = 0·088 ; " " " = 0·125
(4) $X_4(0)$ KEPT CONSTANT AT 0·50
(5) $X_5(0)$ " " 2·00
(6) ESTIMATED $X_6(0)$ = 2·017 ; " " " = 2·00

(7) " " $X_7(0)$ = 2·017 ; " " = 2·00
(8) " " $X_8(0)$ = 2·93 ; " " = 3·00

(8)
(6)&(7)
(5)
(1)
(4)
(2)
(3)

3·0
2·0
1·0
0

CONVERGENCE OF INITIAL STATES

1   6   11   16   21   26   31   36
INTERATIONS

FIG 6·3 ESTIMATION OF TRANSFER FUNCTION FROM INPUT-OUTPUT DATA OF A
COMPUTER-SIMULATED THIRD ORDER SYSTEM. PLOT OF $\tilde{X}_1(0)$ $\tilde{X}_2(0)$,.... $\tilde{X}_8(0)$ VERSUS
INTERATIONS, ASSUMING $X_4(0)$ AND $X_5(0)$ TO BE KNOWN. INITIAL GUESSES 10%.
OFF FROM THEIR TRUE VALUES.

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FIG.6.4. ESTIMATION OF TRANSFER FUNCTION FROM INPUT-OUTPUT DATA OF A COMPUTER-SIMULATED THIRD ORDER SYSTEM. PLOT OF $\bar{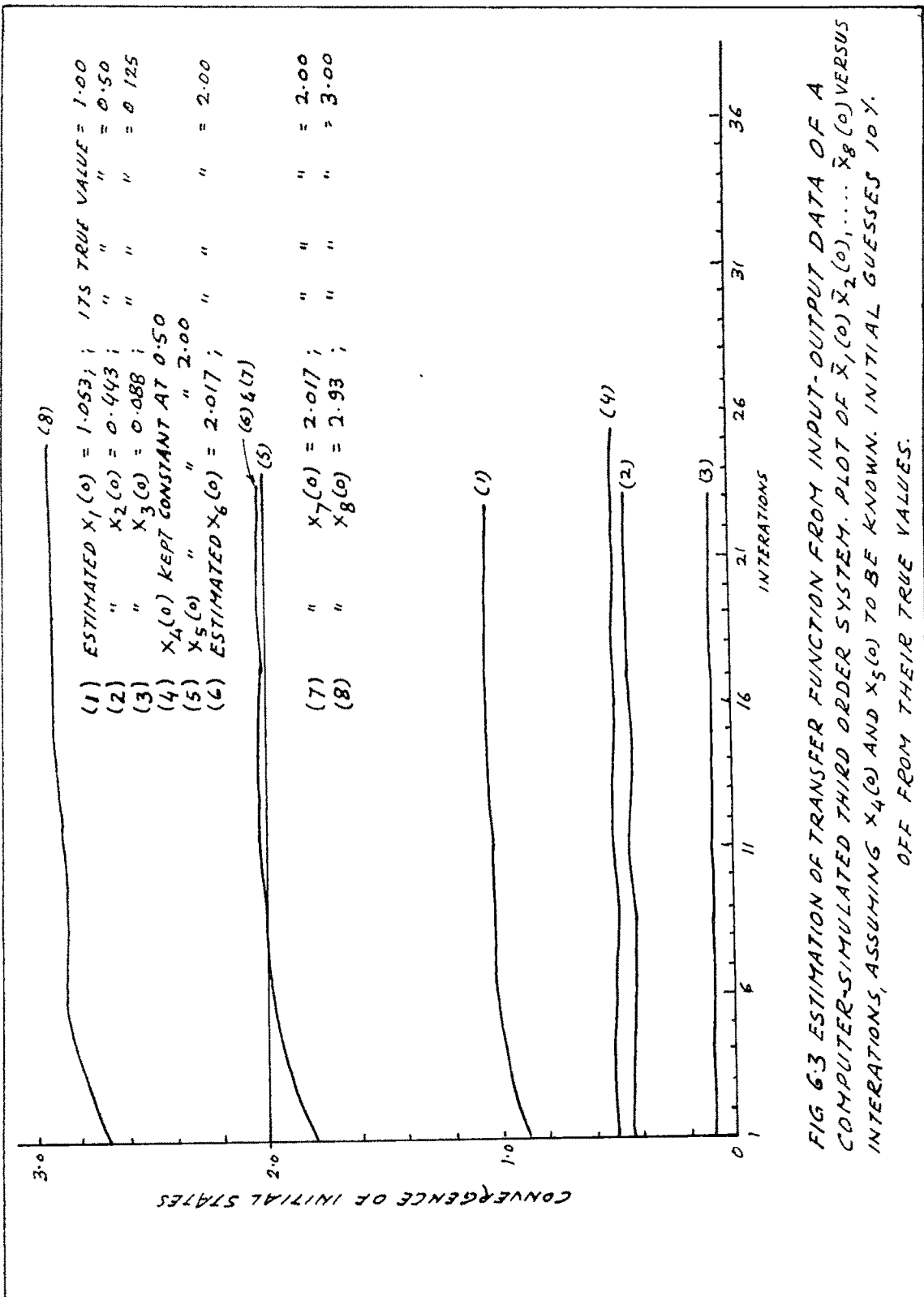x}_1(0), \bar{x}_2(0), \ldots \bar{x}_8(0)$ VERSUS ITERATIONS, ASSUMING $x_4(0)$ & $x_5(0)$ TO BE KNOWN. INITIAL GUESSES 0.5 FOR ALL OTHER INITIAL STATES.
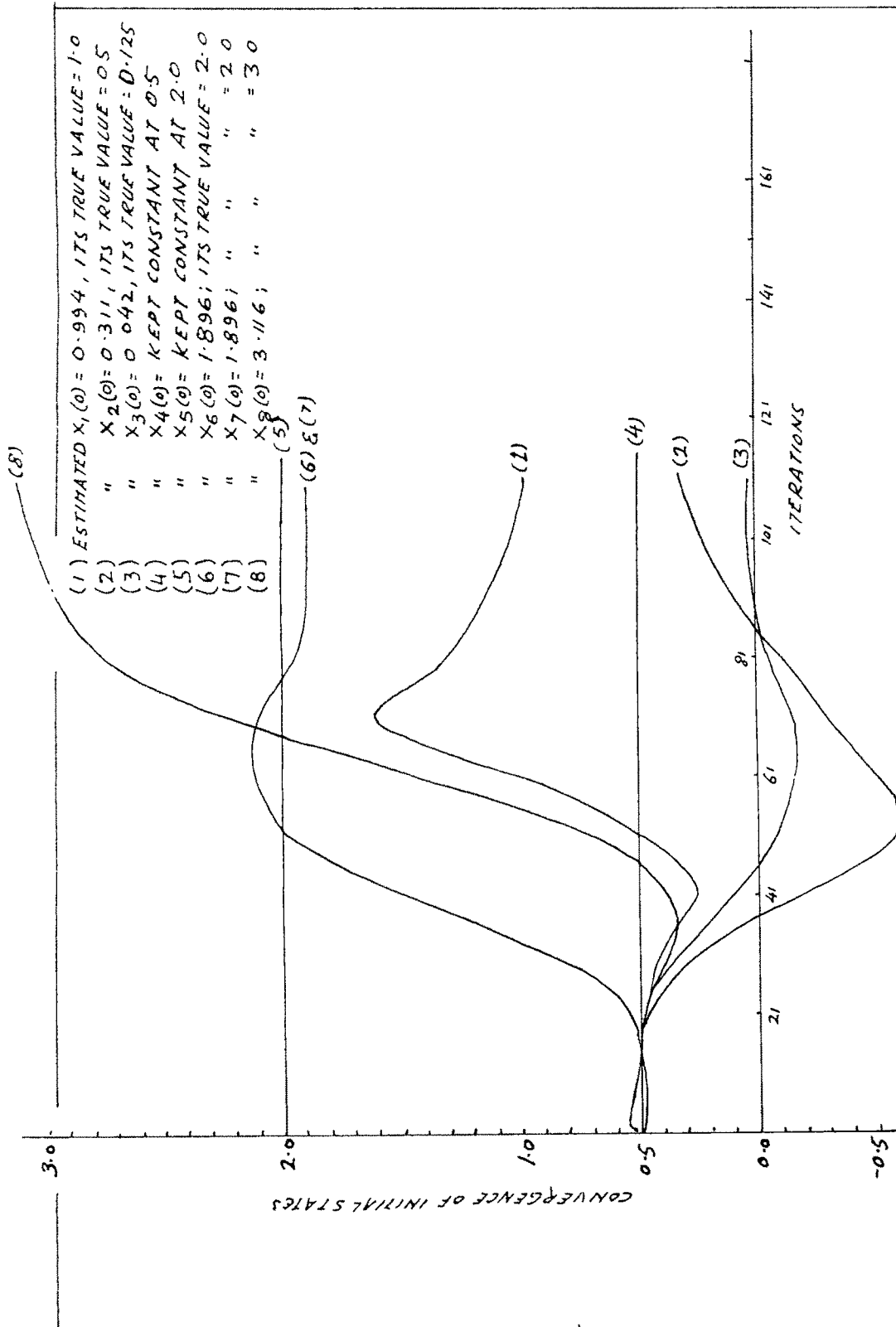
(1) ESTIMATED $x_1(0) = 0.994$, ITS TRUE VALUE = 1.0
(2)   " $x_2(0) = 0.311$, ITS TRUE VALUE = 0.5
(3)   " $x_3(0) = 0.042$, ITS TRUE VALUE = 0.125
(4)   " $x_4(0) = $ KEPT CONSTANT AT 0.5
(5)   " $x_5(0) = $ KEPT CONSTANT AT 2.0
(6)   " $x_6(0) = 1.896$; ITS TRUE VALUE = 2.0
(7)   " $x_7(0) = 1.896$;   "   "   " = 2.0
(8)   " $x_8(0) = 3.116$;   "   "   " = 3.0
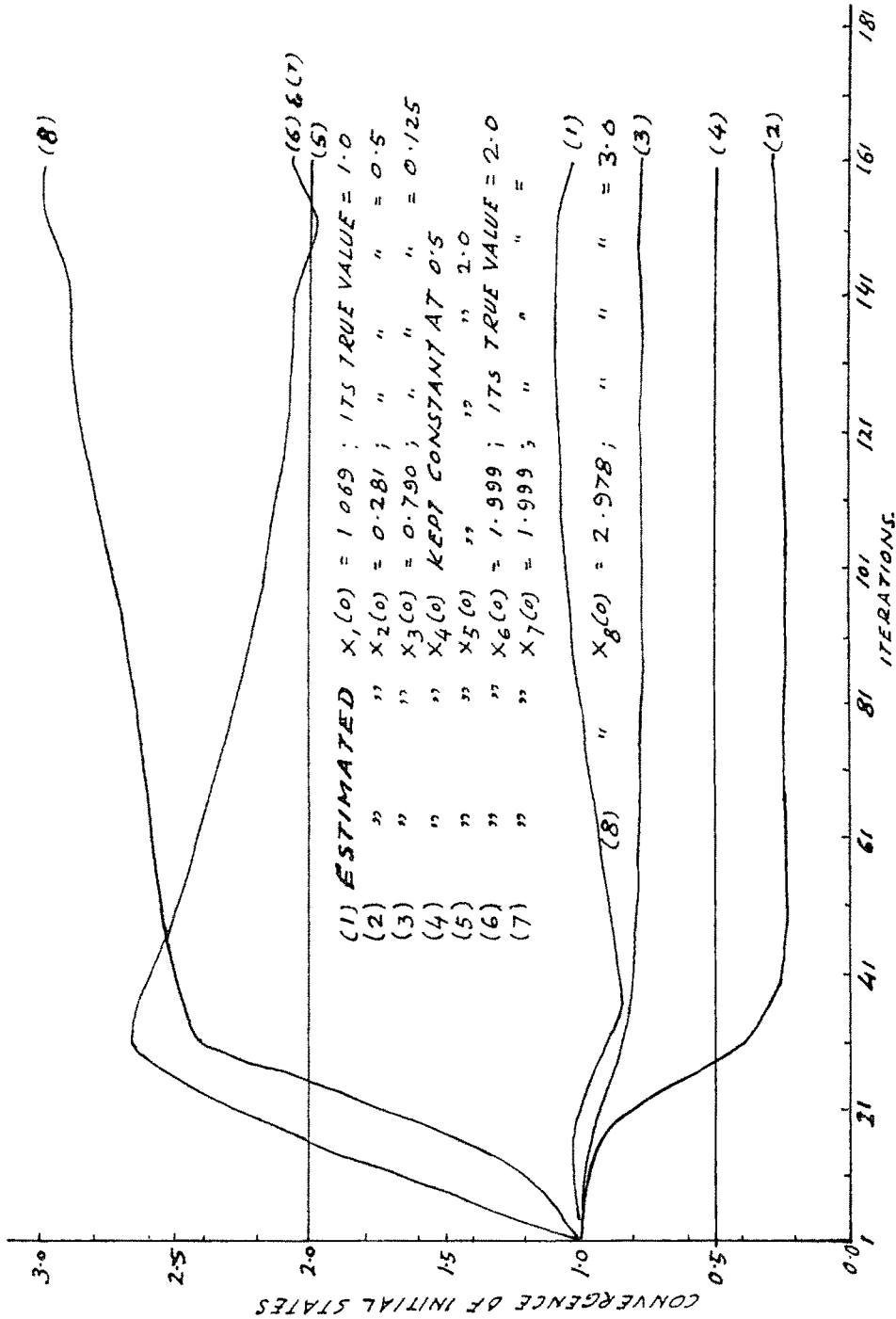
CONVERGENCE OF INITIAL STATES

FIG. 6.5   ESTIMATION OF TRANSFER FUNCTION FROM INPUT-OUTPUT DATA OF A
COMPUTER-SIMULATED THIRD ORDER SYSTEM. PLOT OF $\bar{x}_1(o), \bar{x}_2(o), \ldots, \bar{x}_8(o)$
VERSUS ITERATIONS, ASSUMING $x_4(o), x_5(o)$ TO BE KNOWN. INITIAL GUESSES
1·0 FOR ALL OTHER INITIAL STATES.

(1) ESTIMATED $x_1(o)$ = 1·069 ; ITS TRUE VALUE = 1·0
(2)   ,, $x_2(o)$ = 0·281 ;   ,,   ,,   ,, = 0·5
(3)   ,, $x_3(o)$ = 0·790 ;   ,,   ,,   ,, = 0·125
(4)   ,, $x_4(o)$ KEPT CONSTANT AT 0·5
(5)   ,, $x_5(o)$,,   ,,   ,,   = 2·0
(6)   ,, $x_6(o)$ = 1·999 ; ITS TRUE VALUE = 2·0
(7)   ,, $x_7(o)$ = 1·999 ;   ,,   ,,   ,,   ,,
(8)   ,, $x_8(o)$ = 2·978 ;   ,,   ,,   ,, = 3·0