

# Exergoeconomic Optimization Methodology

This chapter is devoted to the description of the various steps involved in the development of a unified exergoeconomic optimization method to be adopted for the combination of heat energy source and the AAVAR system for minimum cooling cost. The methodology presented in this chapter is proposed to be applied to the existing large industrial AAVAR system dedicated for the brine chilling and compare the same with that applied to the other two options of heat energy source available with the fertilizer industry (as described in Chapter 3) to identify the best option with minimum cooling cost. In this context, a unified approach of combining the Thermo-economic Evaluation and Optimization (TEO) method by Tsatsaronis [1] along with the Entropy Generation Minimization (EGM) is proposed. The methodology for exergoeconomic optimization of thermal system using the unified approach can be divided in three steps viz. exergy analysis, exergoeconomic evaluation and exergoeconomic optimization. The following sections deal with them one by one.

### 4.1 Exergy Analysis

The exergy analysis gives the idea about the thermodynamic inefficiencies produced in a particular process quantitatively as well as qualitatively. This inefficiency increases the cost of the final product. When the system interacts with another system and is allowed to come to equilibrium, gives work as output. Exergy can be defined as the maximum amount of theoretical useful work obtainable when the state of a system comes to the state of the environment. Thus, exergy is a measure of the departure of the state of

a system from that of an environment (reference state). Pressure  $p_0$  and temperature  $T_0$  represent environment which is modelled as reference. In the present work, the value of  $p_0$  and  $T_0$  are taken as 1 atm and 25°C, respectively. Exergy is not a conserved property as some of it is destroyed due to irreversibilities. Hence it is reasonable to use exergy as a basis for thermodynamic analysis. In the absence of magnetic, electrical, nuclear and surface tension effect, the total exergy of the system is considered to be consists of four components, viz. physical exergy, kinetic exergy, potential exergy and chemical exergy.

$$\dot{E} = \dot{E}^{PH} + \dot{E}^{KN} + \dot{E}^{PT} + \dot{E}^{CH} \quad (4.1)$$

The sum of kinetic, potential and physical energies is also referred as thermo physical exergy. The physical exergy is given by the following equation.

$$\dot{E}^{PH} = \dot{m} \left[ (h - h_0) - T_0 (s - s_0) \right] \quad (4.2)$$

Considering the system at rest with respect to environment, total exergy becomes the sum of physical and chemical exergy.

$$\dot{E} = \dot{E}^{PH} + \dot{E}^{CH} \quad (4.3)$$

After calculating exergy at each station, exergy analysis is carried out either using exergy destruction method or entropy generation minimization method.

#### 4.1.1 Exergy Destruction Method (EDM)

The thermal system under consideration for the analysis is supplied with some exergy inputs (fuel exergy,  $\dot{E}_F$ ) derived from some energy source. In the process of conversion, these exergy inputs transform in to some exergy output (product exergy,  $\dot{E}_P$ ) and some exergy be destroyed (exergy destruction,  $\dot{E}_D$ ) and remaining is loss of exergy ( $\dot{E}_L$ ). For the exergy analysis, it is necessary to define product and fuel for each component and for the overall system. The product is defined according to the purpose of owning and operating the component under consideration and fuel represents the resources consumed in generating the product. Fuel and product are expressed in terms of exergy. Exergy destruction is the amount of exergy lost due to irreversibilities and can not be used anywhere. The exergy loss is the amount of exergy that is wasted from the system under

consideration, but can be useful to other system. The exergy destruction can be calculated by the exergy balance.

$$\dot{E}_{D,k} = \dot{E}_{F,k} - \dot{E}_{P,k} - \dot{E}_{L,k} \quad (4.4)$$

The exergetic efficiency of a component or system,  $\varepsilon$  is the percentage of the fuel exergy ( $\dot{E}_{F,k}$ ) found in the product exergy ( $\dot{E}_{P,k}$ ).

$$\varepsilon = \dot{E}_{P,k} / \dot{E}_{F,k} = 1 - [(\dot{E}_{D,k} + \dot{E}_{L,k}) / \dot{E}_{F,k}] \quad (4.5)$$

After calculating the exergy destruction and exergy losses for each component of the system, they are related to the fuel exergy of the component, total exergy supplied to the system and total exergy destruction in the system using exergy destruction and exergy loss ratios. The first exergy destruction ratio,  $Y_{D,k}$  compares the exergy destruction in the  $k^{\text{th}}$  component with total exergy supplied to the system using the following:

$$Y_{D,k} = \dot{E}_{D,k} / \dot{E}_{F,tot} \quad (4.6)$$

The second exergy destruction ratio,  $Y_{D,k}^*$  compares the exergy destruction in the  $k^{\text{th}}$  component with total exergy destruction in the system.

$$Y_{D,k}^* = \dot{E}_{D,k} / \dot{E}_{D,tot} \quad (4.7)$$

The two exergy destruction ratios are useful for comparisons among various components of the same system. The first exergy destruction ratio can also be invoked for comparisons among similar components of different systems using the same, or closely similar, fuels.

The exergy loss ratio,  $Y_{L,k}$  is defined as the ratio between the exergy loss in the  $k^{\text{th}}$  component and the total exergy supplied to the system.

$$Y_{L,k} = \dot{E}_{L,k} / \dot{E}_{F,tot} \quad (4.8)$$

The purpose of the exergy analysis is to identify the sources of the thermodynamic inefficiencies and to find the direction of improvement in the overall efficiency of the system through design changes.

### 4.1.2 Entropy Generation Minimization Method

The objective in the application of the entropy generation minimization (EGM) method is to find design in which the entropy generation is minimum. A minimum entropy generation design characterizes a system with minimum destruction of available work (exergy). In case of refrigeration plant, the minimum entropy generation rate is equivalent to maximum refrigeration load or minimum power input. This method consists of dividing the system in to sub systems those are in local (or internal) thermodynamic equilibrium. Entropy is generated at the boundaries between sub systems, as heat and mass flow through the boundaries. Using these flow rates, the total rate of entropy generation is calculated in relation to the physical characteristics of the systems. The total entropy generation is then monitored and minimized by properly varying the physical characteristics of the systems.

$$\dot{S}_g = m_e s_e - m_i s_i - \frac{Q_g}{T_{steam}} \quad (4.9)$$

$$I_g = T_0 \dot{S}_g \quad (4.10)$$

The EGM method is useful for the components like throttle valve and expansion valve where fuel and product can not be defined.

## 4.2 Exergoeconomic Analysis

Exergy analysis, in the previous section gives the quantitative values of the exergy destruction and the exergy loss in the transformation of fuel exergy to the product exergy in the system. The cost of the product depends upon the cost of fuel and cost of exergy destruction and losses. By reducing the exergy destruction and losses, the fuel requirement can be reduced and exergetic efficiency can be increased. As a result, the cost of exergy input and losses are decreased if the unit exergy cost is constant. This improvement of the system accompanies with additional investment cost. Thus, the main

objective of the design engineer is to get best possible configuration to have lowest product cost by optimizing the system using exergoeconomics.

The exergoeconomic concept based cost minimization methodology calculates the economic costs of all the internal flows and products of the system by formulating exergoeconomic cost balances. The system is then exergoeconomically evaluated to identify the effects of design variables on costs and thereby enables to suggest values of design variables that would make the overall system cost-effective. Based on these suggestions put forward by Bejan et al. [155], the optimization of the system is carried out through an iterative procedure. This information is made available through the formulation of cost balance equations. The cost rate associated with the product of the system,  $\dot{C}_{p,tot}$  is the total rate of expenditure made to generate it, i.e. the summation of the fuel cost rate  $\dot{C}_{F,tot}$  and the cost rate associated with the total capital investment  $\dot{Z}_{tot}^{TCI}$  and operation and maintenance  $\dot{Z}_{tot}^{OM}$  of the system.

$$\dot{C}_{p,tot} = \dot{C}_{F,tot} + \dot{Z}_{tot} \quad (4.11)$$

$$\dot{Z}_{tot} = \dot{Z}^{TCI} + \dot{Z}^{OM} \quad (4.12)$$

#### 4.2.1 Exergy Costing

In exergoeconomics, it is assumed that each exergy stream of the system is associated with the cost rate. Exergy costing involves cost balance formulations for each component separately. A cost balance applied to the  $k^{th}$  component of a system shows that the sum of the cost rates associated with all leaving exergy streams equals the sum of cost rates of all entering exergy streams, the appropriate charge due to total capital investment (TCI) and operation and maintenance (O&M) expenses. The cost balance equation for the component receiving heat and generating power would be

$$\sum_e \dot{C}_{e,k} + \dot{C}_{w,k} = \dot{C}_{q,k} + \sum_i \dot{C}_{i,k} + \dot{Z}_k \quad (4.13)$$

Expressing the costs in terms of cost per unit exergy ( $\dot{c} = \dot{C}/\dot{E}$ ),

$$\sum_e (c_{e,k} \dot{E}_e)_k + c_{w,k} \dot{W}_k = c_{q,k} \dot{E}_{q,k} + \sum_i (c_{i,k} \dot{E}_i)_k + \dot{Z}_k \quad (4.14)$$

For a system with 'n' number of components with 'm' exergy streams ( $n < m$ ), 'm' number of cost balance equations are required to calculate the cost flow rates of all the streams. For such a system, 'n' number of cost balance equations corresponding to the number of components can be developed by using Eqs. 4.13 and 4.14. The remaining '(m-n)' number of auxiliary equation can be developed through following principles.

1. When the product definition for a component involves a single exergy stream, the unit cost of this leaving stream can be calculated from the cost balance. The auxiliary relations are formulated for the remaining leaving exergy streams that are used in the definition of fuel or in the definition of exergy loss associated with the component being considered.
2. When the product definition for a component involves m leaving exergy streams, '(m-1)' auxiliary relations referring to these product streams must be formulated. In the absence of information about the production process of each of m streams, it may be assumed that each unit of exergy is supplied to each product stream at the same average cost.
3. When the fuel definition for a component involves the difference between the entering and leaving states of the same stream of matter the average cost per unit exergy remains constant for this stream. This cost changes only when exergy is supplied to the stream, which then becomes part of the product definition.

Once the cost rates ( $\dot{C}_i$ ) associated with each stream in a system are known, the cost of fuel ( $\dot{C}_F$ ) and the product ( $\dot{C}_P$ ) for each of the components are obtained by using exergetic fuel and product relationship.

The term  $\dot{Z}_k$  can be obtained by calculating the TCI cost and O&M costs associated with the  $k^{\text{th}}$  component and then computing the levelized values of these costs

per unit time (year, hour, second) of system operation. The variables ( $c_{e,k}$ ,  $c_{w,k}$ ,  $c_{q,k}$ ,  $c_{i,k}$ ) are the cost per unit exergy of the exergy streams associated with the  $k^{\text{th}}$  component. In analyzing the component, it is assumed that the cost per unit exergy of all entering streams is known as these are either the product streams of other components or the fuel streams of the overall system. The cost of fuel stream would be the purchase cost of that stream. The unknown variables can be calculated by solving the cost balance equations for all the components.

#### 4.2.2 Economic Analysis

Economic analysis of the system includes calculation of total capital investment which includes purchased equipment cost (PEC) of all the components, installation cost, material cost, instrumentation and control cost etc and O&M costs. For incorporating these costs in the exergoeconomic cost balance equations, they are converted to levelized cost.

##### 4.2.2.1 Estimation of TCI

The capital needed to purchase the land, build all necessary facilities and purchase and install the required machinery and equipment for a system is called fixed capital investment (FCI). The TCI is the sum of the FCI and other outlays as explained in Table 4.1 with detailed breakdown of TCI.

##### Purchased Equipment Cost (PEC)

The estimation of PEC can be obtained through vendor's quotation. In case of unavailability of these, PEC can be obtained from the cost estimating chart or mathematical correlations with respect to equipment size when all available cost data are plotted against equipment size. The plot will be the straight line which is represented by the following equation.

$$C_{PE,Y} = C_{PE,W} \left( \frac{X_Y}{X_W} \right)^\alpha \quad (4.15)$$

Eq. 4.15 allows the purchase cost of an equipment item ( $C_{PE,y}$ ) at a given capacity or size (as represented by the variable  $X_y$ ) to be calculated when the purchase cost of the same equipment at a different capacity or size (expressed by  $X_w$ ) is known. In the absence of other cost information, an exponent value of 0.6 may be used as suggested by Bejan et al. [155]

**Table 4.1: Break down of TCI**

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I	Fixed capital investment (FCI)
	A Direct cost (DC)
	1 Onsite costs (ONSC)
	<ul style="list-style-type: none"> <li>• Purchased equipment cost (PEC)</li> <li>• Purchased equipment installation (20-90% of PEC)</li> <li>• Piping (10-70% of PEC)</li> <li>• Instrumentation and control (6-40% of PEC)</li> <li>• Electrical equipment and material (10-15% of PEC)</li> </ul>
	2 Off-site costs (OFSC)
	<ul style="list-style-type: none"> <li>• Land (0-10% of PEC)</li> <li>• Civil, structural and architectural work (15-90% of PEC)</li> <li>• Service facilities (30-100 % of PEC)</li> </ul>
	B Indirect cost (IC)
	1 Engineering and supervision (25-75% of PEC)
	2 Construction cost with contractors profit (15% of DC)
	3 Contingencies (20% of FCI)
II	Other outlays
	A Startup cost (5-12% of FCI)
	B Working capital (10-20% of TCI)
	C Cost of licensing
	D Allowance for funds used during construction

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#### 4.2.2.2 Cost Index

Once the PECs are known from the above method, they must be brought to the reference year, i.e. the year used as the base for the cost calculations [156,157]. It is because all data are historical and costs do change with time. This is done with the aid of appropriate cost index using the following relation.

$$\text{Cost at the reference year} = \text{original cost} \times \left( \frac{\text{cost index for the reference year}}{\text{cost index for the year when the original cost was obtained}} \right) \quad (4.16)$$

Cost Index, an inflation indicator, is used to correct the cost of equipment, material, labour and supplies to the date of estimate. Existing cost indicators include the following: Chemical engineering plant cost index (based on construction costs for chemical plant listed in the Journal of Chemical Engineering), Marshal & Swift (M&S) equipment cost index (based on construction cost for various chemical process industries, listed in Journal of Chemical Engineering and in Oil and Gas Journal), Nelson Ferrar Refinery Cost Index (based on construction costs in the petroleum industry) and Engineering News Record (ENR) Construction Cost Index (based on general industrial construction, published in Engineering News Record). For thermal design project, the use of M&S cost index is recommended by Bejan et al. [155]. The capital recovery factor ( $\beta$ ) is defined by the following equation

$$\beta = \left( \frac{i_{\text{eff}}(1+i_{\text{eff}})^n}{(1+i_{\text{eff}})^n - 1} \right) \left( \frac{1}{\tau} \right) h^{-1} \quad (4.17)$$

Where 'n' is the plant life which is considered to be 30 years,  $\tau$  is the number of hours of operation per year which is taken as 8000 and  $i_{\text{eff}}$  effective annual rate of return which is taken as 10 % per year. Hence  $\beta$  is found to be 0.1061.

The Operation and maintenance (O&M) cost ( $\gamma$ ) excluding fuel is assumed to be 1.092 percent of the investment cost for each component as suggested by Tsatsaronis et al. [114]. Under these assumptions, the cost flow rate ( $\text{₹/hr}$ ) associated with levelized O&M cost ( $Z_k$ ) for the  $k^{\text{th}}$  component is calculated from the following relation

$$\dot{Z}_k = \frac{(\beta + \gamma) * TCI_k}{\tau} \quad (4.18)$$

Where  $TCI_k$  is the total capital investment for the  $k^{\text{th}}$  component.

#### 4.2.2.3 Fuel Cost

The fuel cost obtained from the vendor is to be updated to the processing year. It is done with the help of the economic term escalation rate ( $r_n$ ).

$$\text{Fuel cost at the reference year} = \left( \begin{array}{l} \text{Fuel cost at the original year} \\ \text{at which it is available} \end{array} \right) \times (1 + r_n)^n \quad (4.19)$$

Where ' $r_n$ ' is escalation rate and 'n' is the difference between year at which the cost is available and the processing year.

#### 4.2.3 Exergoeconomic Evaluation

After introducing the cost rates associated with the fuel ( $\dot{C}_F$ ) and product ( $\dot{C}_p$ ), one can define cost per unit exergy of fuel and product for the  $k^{\text{th}}$  component,  $c_{F,k}$  and  $c_{p,k}$ , respectively as follows:

$$c_{F,k} = \frac{\dot{C}_{F,k}}{\dot{E}_{F,k}} \quad (4.20)$$

$$c_{p,k} = \frac{\dot{C}_{p,k}}{\dot{E}_{p,k}} \quad (4.21)$$

In the cost balance equation for a component, there is no cost term directly associated with the exergy destruction and exergy loss. They are hidden cost and can be defined as

$$\dot{C}_{D,k} = c_{F,k} \dot{E}_{D,k} \quad (4.22)$$

$$\dot{C}_{L,k} = c_{F,k} \dot{E}_{L,k} \quad (4.23)$$

The relative cost difference ( $r_k$ ) for the  $k^{\text{th}}$  component is defined by

$$r_k = \frac{c_{p,k} - c_{F,k}}{c_{F,k}} \quad (4.24)$$

This variable expresses the relative increase in the cost per unit exergy between fuel and product of the component. In the iterative cost optimization of a system, if the cost of fuel of a major component changes from one iteration to the next, the objective of the cost optimization of the component should be to minimize the relative cost difference instead of minimizing the cost per unit exergy of the product for this component.

$$r_k = \frac{c_{F,k} (E_{D,k} + E_{L,k}) + (Z_k^{TCI} + Z_k^{OM})}{c_{F,k} E_{p,k}} \quad (4.25)$$

$$r_k = \frac{1 - \varepsilon_k}{\varepsilon_k} + \frac{(Z_k^{TCI} + Z_k^{OM})}{c_{F,k} E_{p,k}} \quad (4.26)$$

The cost sources in a component may be grouped in two categories, viz., non exergy related cost due to TCI and O&M and exergy related cost due to exergy destruction and exergy loss and the relative significance of each category can be determined by the exergoeconomic factor  $f_k$  defined for component k by

$$f_k = \frac{Z_k}{Z_k + (C_{D,k} + C_{L,k})} \quad (4.27)$$

$f_k$  is the ratio of non exergy related cost to the total cost. A low value of it for a major component suggests that cost saving in the entire system might be achieved by improving the component efficiency (reducing the exergy destruction) even if the capital investment for this component will increase. On the other hand, a high value of this factor suggests a decrease in the investment cost of the component at the expense of its exergetic efficiency. The system can be evaluated with the help of exergoeconomic variables given in Eqs. 4.20 to 4.27.

Bejan et al. [155] has suggested the following methodology for exergoeconomic evaluation:

1. Consider design changes initially for the components for which the value of the sum  $(\dot{Z}_k + \dot{C}_{D,k})$  is high.
2. Pay particular attention to the components with a high relative cost difference  $r_k$ , especially when the cost rates  $\dot{Z}_k$  and  $\dot{C}_{D,k}$  are high.
3. Use the exergoeconomic factor  $f_k$  to identify the major cost source (capital investment or exergy destruction).
  - a. If the  $f_k$  value is high, investigate whether it is cost effective to reduce the capital investment at the expense of component efficiency.
  - b. If the  $f_k$  value is low, try to improve the component efficiency by increasing the capital investment.
4. Eliminate any sub processes that increase the exergy destruction or exergy loss without contributing to the reduction of capital investment or fuel cost for other component.
5. Consider improving the exergetic efficiency of the component if it has low exergetic efficiency or large value of the rate of exergy destruction, the exergy destruction ratio or exergy loss ratio.

### 4.3 Exergoeconomic Optimization

Optimization means the modification of the structure and the design parameters of a system to minimize the total levelized cost of the system product under the given boundary conditions. The objective of the exergoeconomic optimization is to minimize costs including costs owing to thermodynamic inefficiencies. The objective function expresses the optimization criteria as a function of dependent and independent variables.

$$\text{Minimize } \dot{C}_{p,tot} = \dot{C}_{F,tot} + \overset{TCL}{\dot{Z}_{tot}} + \overset{OM}{\dot{Z}_{tot}} \quad (4.28)$$

$\dot{C}_{p,tot}$  is total cost rate associated with the product instead of the cost rate per unit exergy of product  $\dot{c}_p$ , since the exergy flow rate of the product  $\dot{E}_p$  is constant. In this approach the cost optimal exergetic efficiency can be obtained for a component isolated from the

remaining system components. The optimization approach is based on the following assumptions.

1. The exergy flow rate of the product  $\dot{E}_p$  and the unit cost of fuel  $c_p$  remain constant for the  $k^{\text{th}}$  component to be optimized.

$$\dot{E}_{p,k} = \text{constant}$$

$$c_{F,k} = \text{constant}$$

2. For every component, it is expected that the investment cost increases with increasing capacity and increasing exergetic efficiency of the component. Therefore  $TCI_k$  for the  $k^{\text{th}}$  component can be approximated by the following relation. [155]

$$TCI_k = B_k \left( \frac{\varepsilon_k}{1 - \varepsilon_k} \right)^{n_k} \dot{E}_{p,k}^{m_k} \quad (4.29)$$

Where  $\dot{E}_{p,k}$  is the exergy rate of the product for the  $k^{\text{th}}$  component and  $\varepsilon_k$  is the component's exergetic efficiency. The term  $[\varepsilon_k / (1 - \varepsilon_k)]$  expresses the effect of exergetic efficiency (thermodynamic performance) while the term  $\dot{E}_{p,k}^{m_k}$  expresses the effect of capacity (component size) on the value of  $TCI_k$ . Eq. 4.29 is valid within a certain range of design conditions for the  $k^{\text{th}}$  component. Within that range, the parameter  $B_k$  and the exponents  $n_k$  are constants, and can be calculated based on cost data through curve fitting technique. For simplicity, the value of  $m_k$  can be assumed equal to the scaling exponent  $\alpha$  for the respective equipment as explained in Eq. 4.15 and suggested by Bejan et al. [155].

3. Usually a part of the O&M cost depends on the total investment cost and another part on the actual production rate. Then the annual O&M cost for the  $k^{\text{th}}$  component can be represented by

$$Z_k^{OM} = \gamma_k (TCI_k) + \omega_k \tau \dot{E}_{p,k} + R_k \quad (4.30)$$

In this equation,  $\gamma_k$  is a coefficient that accounts for the part of the fixed O&M cost depending on the  $TCI_k$  associated with the  $k^{\text{th}}$  component,  $\omega_k$  is a constant

accounts for the variable O&M cost associated with the  $k^{\text{th}}$  component and denotes the O&M cost per unit of product exergy.  $\tau$  is the average annual time of plant operation at the nominal load and  $R_k$  includes all the remaining O&M cost that are independent of the TCI and exergy of the product.

4. The economic analysis of the system is simplified by neglecting the effect of financing, inflation, taxes, insurance and construction time and by considering the start-up cost, working capital and the cost of licensing, research and development together with the total capital investment. The annual carrying charge associated with the  $k^{\text{th}}$  component can then be obtained by multiplying the TCI for this component by the capital recovery factor  $\beta$ .

$$Z_k^{CI} = \beta(TCI_k) \quad (4.31)$$

The above assumptions form the cost model. The total annual costs, excluding fuel cost, associated with the  $k^{\text{th}}$  component are obtained by combining Eqs. 4.30 and 4.31.

$$Z_k = Z_k^{CI} + Z_k^{OM} = (\beta + \gamma_k)(TCI_k) + \omega_k \tau \dot{E}_{p,k} + R_k \quad (4.32)$$

The corresponding cost rate  $\dot{Z}_k$  is obtained by dividing Eq. 4.32 by annual hours of operation  $\tau$ .

$$\dot{Z}_k = \frac{(\beta + \gamma_k)}{\tau}(TCI_k) + \omega_k \dot{E}_{p,k} + \frac{R_k}{\tau} \quad (4.33)$$

Inserting the value of  $TCI_k$  from Eq. 4.29

$$\dot{Z}_k = \frac{(\beta + \gamma_k)B_k}{\tau} \left( \frac{\varepsilon_k}{1 - \varepsilon_k} \right)^{n_k} \dot{E}_{p,k}^{m_k} + \omega_k \dot{E}_{p,k} + \frac{R_k}{\tau} \quad (4.34)$$

The objective function to be minimized expresses the cost per exergy unit of product for the  $k^{\text{th}}$  component

$$\text{Minimize } c_{p,k} = \frac{c_{F,k} \dot{E}_{F,k} + \dot{Z}_k}{\dot{E}_{p,k}} \quad (4.35)$$

From Eqs. 4.5 and 4.34, this objective function may be expressed as

$$\text{Minimize } c_{p,k} = \frac{c_{f,k}}{\varepsilon_k} + \frac{(\beta + \gamma_k)B_k}{\tau \dot{E}_{p,k}^{1-m_k}} \left( \frac{\varepsilon_k}{1-\varepsilon_k} \right)^{n_k} + \omega_k + \frac{R_k}{\tau \dot{E}_{p,k}} \quad (4.36)$$

The values of parameters  $\beta$ ,  $\gamma_k$ ,  $B_k$ ,  $\tau$ ,  $\omega_k$  and  $R_k$  remain constant during optimization process and so  $c_{p,k}$  varies only with  $\varepsilon_k$ . Thus the optimization problem reduces to the minimization of Eq. 4.36 subject to constrain explained in assumption 1. The minimum cost per unit exergy of product can be obtained by differentiating Eq. 4.36 and setting the derivative to zero.

$$\frac{dc_{p,k}}{d\varepsilon_k} = 0$$

The resulting cost optimal exergetic efficiency is

$$\varepsilon_k^{OPT} = \frac{1}{1 + F_k} \quad (4.37)$$

Where

$$F_k = \left( \frac{(\beta + \gamma_k)B_k n_k}{\tau c_{f,k} \dot{E}_{p,k}^{1-m_k}} \right)^{\frac{1}{(n_k+1)}} \quad (4.38)$$

Eqs. 4.37 and 4.38 show that the cost optimal exergetic efficiency increases with increasing cost per unit exergy of fuel  $c_{f,k}$ , increasing annual number of hours of system operation  $\tau$ , decreasing capital recovery factor  $\beta$ , decreasing fixed O&M cost factor  $\gamma_k$  and decreasing cost exponent  $n_k$ . From Eq. 4.37

$$F_k = \frac{1 - \varepsilon_k^{OPT}}{\varepsilon_k^{OPT}} \quad (4.39)$$

From Eqs. 4.5 and 4.39

$$F_k = \left( \frac{\dot{E}_{D,k} + \dot{E}_{L,k}}{\dot{E}_{p,k}} \right)^{OPT} \quad (4.40)$$

Since the exergy rate of the product is assumed constant during optimization, the cost optimal value of the sum  $(\dot{E}_{D,k} + \dot{E}_{L,k})$  can be given by

$$(\dot{E}_{D,k} + \dot{E}_{L,k})^{OPT} = \dot{E}_{p,k} F_k = \dot{E}_{p,k} \left( \frac{1 - \varepsilon_k^{OPT}}{\varepsilon_k^{OPT}} \right) \quad (4.41)$$

Eq. 4.36 can be reduced by neglecting the last two terms as

$$\text{Minimize } c_{p,k} = c_{F,k} \left( 1 + \frac{\dot{E}_{D,k} + \dot{E}_{L,k}}{\dot{E}_{p,k}} \right) + \frac{(\beta + \gamma_k) B_k}{r \dot{E}_{p,k}^{1-n_k}} \left( \frac{\dot{E}_{p,k}}{\dot{E}_{D,k} + \dot{E}_{L,k}} \right)^{n_k} \quad (4.42)$$

By differentiating Eq. 4.42, with respect to  $(\dot{E}_{D,k} + \dot{E}_{L,k})$  and setting the derivative to zero, the relation between cost optimal values of the cost rates can be expressed by  $c_{F,k}(\dot{E}_{D,k} + \dot{E}_{L,k})$  and  $Z_k$

$$n_k = \frac{c_{F,k}(\dot{E}_{D,k} + \dot{E}_{L,k})^{OPT}}{Z_k} \quad (4.43)$$

From Eqs. 4.22 and 4.23

$$n_k = \frac{(C_{D,k} + C_{L,k})^{OPT}}{Z_k} \quad (4.44)$$

Thus, the cost exponent  $n_k$  expresses the ratio between the cost optimal rates associated with the exergy destruction and exergy loss and cost optimal rates associated with capital investment. From this equation, the expressions for cost optimal values of the non fuel related cost rate  $Z_k$ , the relative cost difference  $r_k$  and the exergoeconomic factor  $f_k$  can be obtained as shown below.

$$Z_k^{OPT} = c_{F,k} \dot{E}_{p,k} \frac{F_k}{n_k} \quad (4.45)$$

$$r_k^{OPT} = \left( \frac{n_k + 1}{n_k} \right) F_k \quad (4.46)$$

$$f_k^{OPT} = \frac{1}{1 + n_k} \quad (4.47)$$

In the present optimization problem, though the main goal is to obtain the optimum value of the product cost, the cost of exergy destruction and the cost of exergy



loss also have to be minimum. Therefore, the objective function for the overall system can be defined as

$$\text{Minimize OBF} = \dot{C}_{p,tot} + \dot{C}_{D,tot} + \dot{C}_{L,tot} \quad (4.48)$$

Therefore  $\dot{C}_{p,tot}$  is to be optimized. From Eq. 4.21

$$\dot{C}_{p,tot} = c_{p,tot} \dot{E}_{p,tot} \quad (4.49)$$

$c_{p,tot}$  can be optimized using Eq. 4.29. To solve this equation for the local optimum condition, the parameter  $B_k$  and the exponents  $n_k$  and  $m_k$  are to be evaluated. They are calculated based on the cost data for each component. The value of  $m_k$  can be assumed equal to the scaling exponent  $\alpha$  explained in Eq. 4.15 and can be referred from Bejan et al. [155].

Then, based on thermodynamic and the cost data, the variation of the exergetic product ( $\dot{E}_{p,k}$ ), the exergy destruction ( $\dot{E}_{D,k}$ ) and the  $TCl_k$  with respect to the exergetic efficiency corresponding to variation in local decision variable can be generated. From the generated data for each component,  $(TCl_k / \dot{E}_{p,k})^{m_k}$  can be plotted against  $(\dot{E}_{p,k} / \dot{E}_{D,k})$  which is equivalent to  $[\varepsilon_k / (1 - \varepsilon_k)]$ . By curve fitting technique, the equivalent power law can be found and the required value of  $B_k$  and  $n_k$  for each component can be determined.

After calculating the values of constants  $B_k$  and  $n_k$ , the cost optimal values of the exergetic efficiency  $\varepsilon_k^{OPT}$ , the relative cost difference  $r_k^{OPT}$ , total exergy loss  $(\dot{E}_{D,k} + \dot{E}_{L,k})^{OPT}$ , the capital investment  $Z_k^{OPT}$  and thermoeconomic factor  $f_k^{OPT}$  can be calculated from Eqs. 4.37 to 4.47.

The optimization procedure using the above approach is an iterative one that aims at finding out a better solution for the system, unlike conventional optimization procedure, where the aim is to calculate the global optimum. In the iterative optimization

procedure, the following thermoeconomic variables are defined to facilitate the decision-making:

$$\Delta \varepsilon_k = 100 \times \frac{\varepsilon_k - \varepsilon_k^{OPT}}{\varepsilon_k^{OPT}} \quad (4.50)$$

$$\Delta r_k = 100 \times \frac{r_k - r_k^{OPT}}{r_k^{OPT}} \quad (4.51)$$

The value of  $\Delta \varepsilon_k$  and  $\Delta r_k$  express the respective relative deviation of actual values from optimal values. In the iterative optimization procedure, engineering judgments and critical evaluations are used in deciding on the changes made to the decision variables from one iterative step to the next. Also, while taking the decision on the changes of the decision variables, the practical limitations of the system, mentioned earlier, are also considered. The criteria followed in decision-making on the changes of the decision variables from one iterative step to the next are as follows

- Calculation of  $\Delta \varepsilon_k, \Delta r_k, \Delta f_k, \dot{C}_{p,tot}$  and  $\dot{C}_{D+L}$  variables for a change in one decision variable in a certain step, while keeping other decision variables constant.
- Examination of its effects on the exergoeconomic variables.
- If the effect is positive, i.e.,  $\dot{C}_{p,tot}$  and  $\dot{C}_{D+L}$  has reducing trend, then in the next iterative step this variable becomes a candidate for a similar change, otherwise, this variable remains unchanged in the next iterative step.
- Repetition of the above three steps for the other decision variables.

#### 4.4 Unified Approach for Exergoeconomic Optimization

A unified approach of combining the TEO method by Tsatsaronis [1] along with the iterative procedure by Bejan et al.[155] is presented in the above sections dealing with the three steps proposed, viz. exergy analysis, exergoeconomic evaluation and exergoeconomic optimization. A computer code written in EES software based on the unified approach is developed. The flow chart of the same is given in Fig. 4.1.

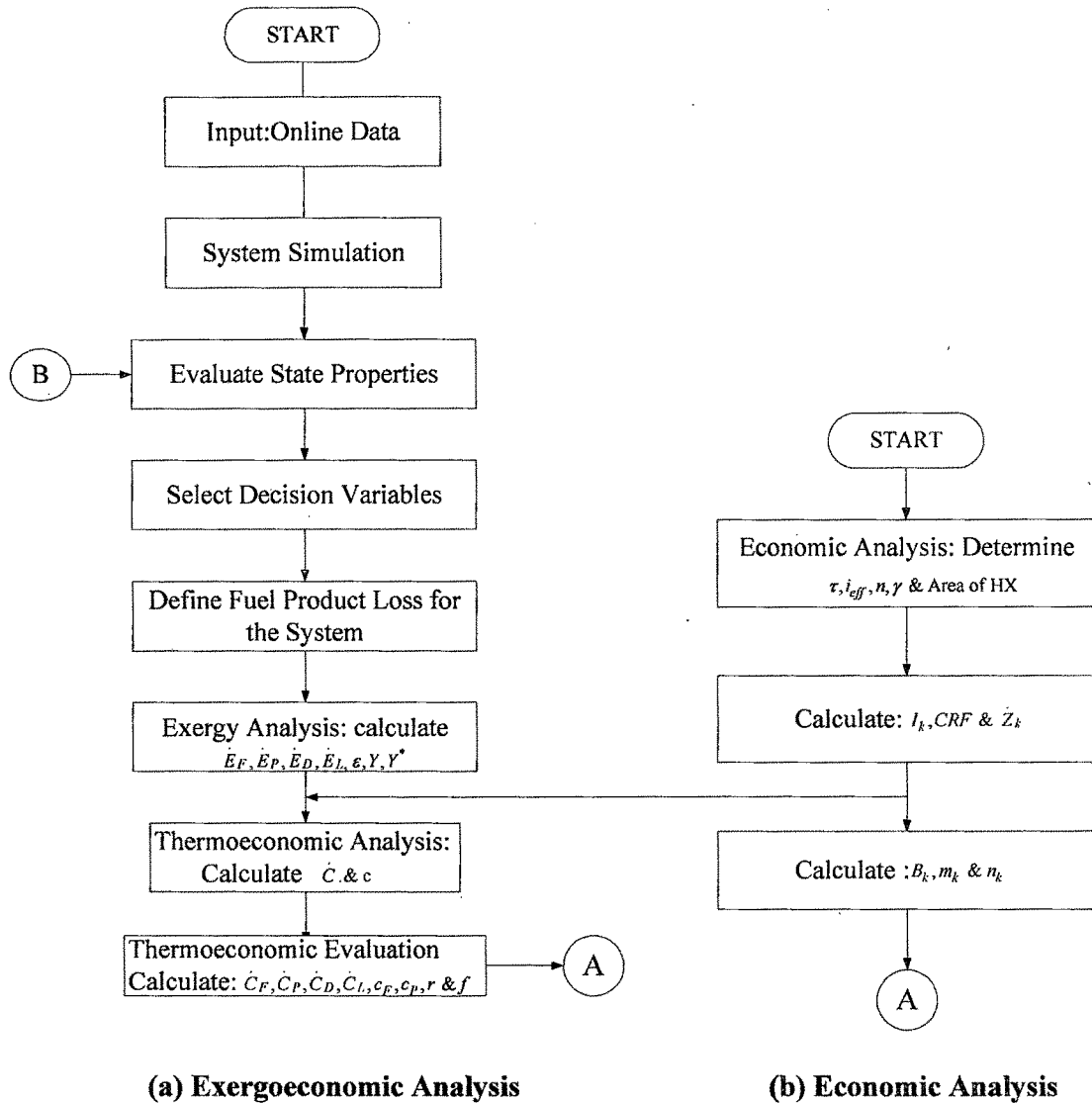
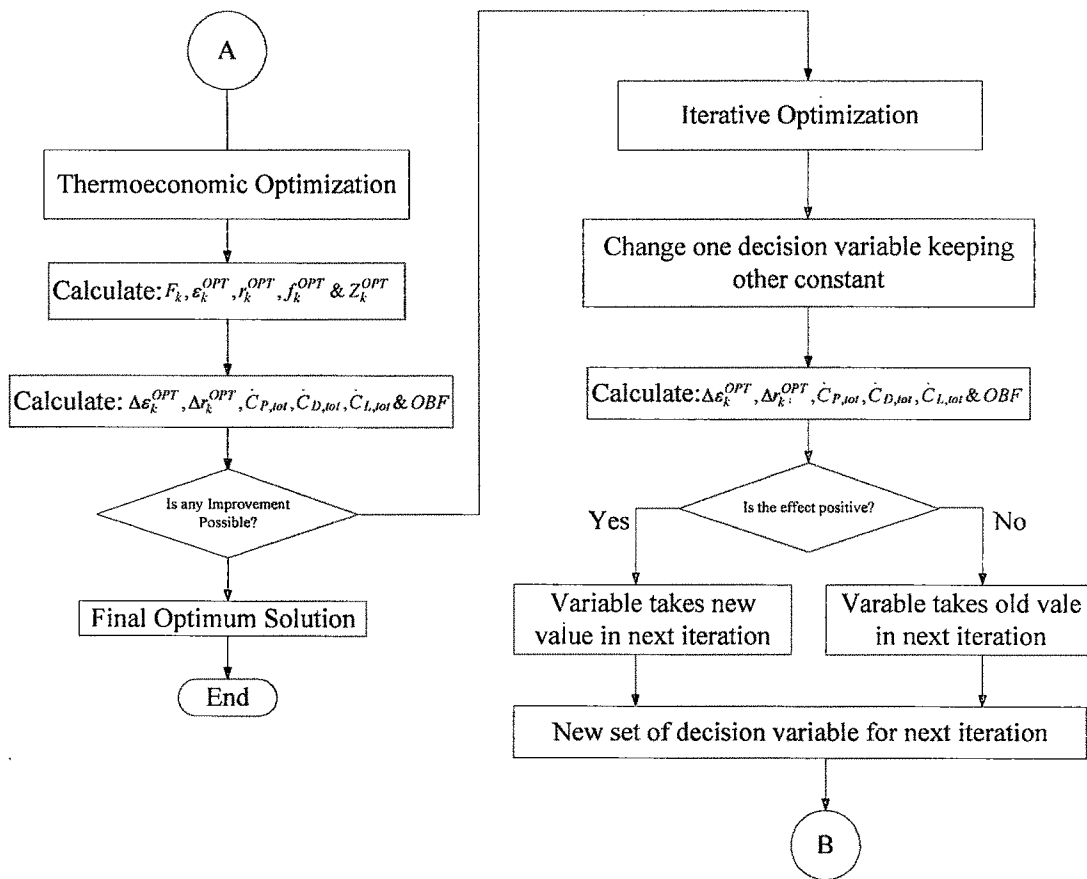


Fig. 4.1 Flow Chart for Unified Approach of Exergoeconomic Optimization  
(Continued)



(c) Thermoeconomic Optimization

Fig. 4.1 Flow Chart for Unified Approach of Exergoeconomic Optimization

The flow chart is described in the following section

**Module 1:**

- Step 1: Input the online data from control panel
- Step 2: Simulate the system through energy balance and mass balance
- Step 3: Evaluate the state properties and Exergy at each station (Eqs. 4.1 to 4.3)
- Step 4: Select decision variables
- Step 5: Define Fuel, Product and Loss for every component
- Step 6: Exergy analysis: Calculate  $\dot{E}_F, \dot{E}_P, \dot{E}_D, \dot{E}_L, \varepsilon, Y$  and  $Y^*$  using Eqs. 4.4 to 4.10

Step 7: Thermo-economic analysis: Calculate  $\dot{C}$  &  $c$  using the values of  $I_k, CRF$  &  $\dot{Z}_k$  from step 2 of module 2 and Eqs.4.13, 4.14 & 4.19

Step 8: Thermo-economic evaluation: Calculate  $\dot{C}_F, \dot{C}_P, \dot{C}_D, \dot{C}_L, c_F, c_P, r$  &  $f$  using Eqs. 4.20-4.27

## Module- 2

Step 1: Economic analysis: Consider the value of  $\tau, i_{eff}, n$  &  $\gamma$  as input and calculate area of heat exchanger using eq. 4.15 and PEC from cost model for other components

Step 2: Using step1, calculate  $I_k, CRF$  &  $\dot{Z}_k$  using Eqs 4.16- 4.18

Step 3: Calculate the values of  $B_k, m_k$  &  $n_k$  using Eq. 4.29

## Module 3

Step 1: Thermo-economic optimization: Calculate  $F_k, \varepsilon_k^{OPT}, r_k^{OPT}, f_k^{OPT}$  &  $Z_k^{OPT}$  using step 8 of module 1 and step 3 of module2 and Eqs.4.37 to 4.45

Step 2: Calculate  $\Delta\varepsilon_k^{OPT}, \Delta r_k^{OPT}$  &  $OBF$  using Eqs 4.48-4.51

Step 3: If any improvement possible? Go to iterative optimization

Step 4: Change one decision variable keeping other constant

Step 5: Calculate  $\dot{C}_{P,tot}, \dot{C}_{D,tot}, \dot{C}_{L,tot}$  &  $OBF$  for the new value of variable

Step 6: Calculate  $\Delta\varepsilon_k^{OPT}$  and  $\Delta r_k^{OPT}$  and check whether the effect is positive or negative

Step 7: If the effect is positive, the variable takes new value in next iteration and reach final optimum solution.

Step 8: If the effect is negative, the variable takes old value in the next iteration then go to Step 3 of module 1 and determine new set of decision variable.

Step 9: By more iteration, reach the final optimum solution.