# Chapter 6

# Summary

In this concluding chapter we take stock of the achievements catalogued in the earlier parts. We had started out with the objective of understanding the generation of fermion masses in the standard model and its extensions made popular by the possibility of answering questions that the former cannot even deign to ask. As with the case of the objects of our study, the thesis is also divided into two parts. The first half deals with the strongly interacting particles, namely the quarks, whereas the second half is devoted to the study of leptons, or without any loss of generality, aspects of neutrino physics.

To circumvent the the lack of any predictive power of the standard model as far as the quark sector parameters are concerned, various models that go beyond the $SM$ have been proposed in the literature. Based as these are on higher symmetries, they give specific forms of the quark mass matrices resulting in relations between the ten parameters *viz.* the six masses, the three mixing angles and the $CP$–violating phase. In the first part of this thesis we have looked at the predictions of these models and compared them with the results of some recent experiments. It was found that, contrary to previous claims, the $B_d^0$–$\overline{B_d^0}$ mixing extent $x_d$ and the $\Delta S = 2$ $CP$ violationparameter $\epsilon_K$ do not rule out either the Stech or the Fritzsch model or for that matter even their derivatives. Regions of validity, though narrow, could still be found. However once the $UA1$ result for the direct $CP$ violationparameter $\epsilon'_K$ came into the picture, the situation changed completely. The Stech model and all others incorporating the ansatz were ruled out emphatically. While the Fritzsch scheme did survive, the parameter space available to it was curtailed significantly with experimental agreement limited to the case of a none–too–heavy top quark ($m_t \lesssim 90\,GeV$). Later experiments have shown that such a top quark is not allowed in the context of the minimal standard model, thus ringing the death–knell for such models as candidates for low energy physics.

The failure of the existing ansätze for quark masses and mixings led to a model–independent analysis of the problem. Looking at the most general form for such matrices, we found that observational data do not constrain them to a great degree but rather allow them a continuous range that is divided into four disjoint sectors associated with the relative signs of the mass terms. The large width afforded to these parameters is traced to the indeterminacy in the masses of the light quarks and owes comparatively little to the lack of knowledge about the mixings involving the third generation. Surprisingly (?), all the models discussed in the literature lie in the same sector. This could provide a clue to the direction that efficient model–building could adopt in the future.

The first question that we address in our effort to understand the wide field that neutrino physics is, relates to neutrinoless double beta decay [$(\beta\beta)_{0\nu}$ ]. Conventional wisdom had it that the extent of $(\beta\beta)_{0\nu}$ is specified by the Majorana mass of the electron neutrino $\nu_e$. A reexamination of the problem showed us that this need not be the case and that many different possibilities do exist. In fact the $(\beta\beta)_{0\nu}$ amplitude is determined primarily by only one element of the effective low–energy neutrino mass matrix. This then means that one could have a scenario where the physical $\nu_e$ is a Dirac particle but the $(\beta\beta)_{0\nu}$ rate is quite high and conversely a case of a light Majorana $\nu_e$ with identically zero $(\beta\beta)_{0\nu}$ rate. Thus the numerous ongoing $(\beta\beta)_{0\nu}$ experiments cannot differentiate between a (pseudo–)Dirac and a Majorana $\nu_e$ except in the simplest of cases. The only comment they can make about is that regarding the magnitude of the particular term mentioned before and hence the extent of lepton number violation that could be visible at low energies. We further go on to construct supersymmetric grand unified theories (both $SO(10)$ and $SU(5)$ ) that naturally accomodate the scenarios that we talk about.

The longstanding discrepancy between the observed levels of solar neutrino flux and the theoretically calculated rates have been a source of embarassment. A possible explanation that has the added advantage of accounting for the reported anticorrelation of the observed flux with the solar magnetic activity is that of a non–zero neutrino magnetic moment ($\mu_\nu$). However it is very difficult to generate a large enough $\mu_\nu$ while keeping the neutrino mass within acceptable limits. Though some models had been proposed in the literature, they all suffered from one weakness or the other. We have proposed a novel mechanism to achieve the same goal. The cornerstone of the ansatz is that to a given order in perturbation theory, the only effective higgs *v.e.v.* term coupling to fermion currents violating lepton number by two units is antisymmetric in the generation space. This leads to a non-zero transition moment while keeping the mass term identically zero. The $O(3)_H$ model that we construct, unlike the others of its genre, neither needs extra fermions nor does it treat the standard model fermions unequally. Moreover, it has the added advantage of avoiding the pitfalls of earlier efforts, namely severe fine tuning of parameters or an unwanted Goldstone boson.

Neutrinos, though seemingly insignificant on account of their very weak interactions and very low mass, are actually of great importance, especially in the astrophysical and the cosmological context. A non-zero mass for the neutrino would lead to interactions flipping its helicity, and as a consequence allow neutron stars to lose energy at a very high rate. Normally one would expect the electroweak interactions to dominate, but we find that gravity could be a strong contender for supremacy in the case of low energy neutrino scattering! Though at the initial stages of stellar collapse the neutrino energy is probably too high for such an eventuality, at the late stages of supernova evolution these effects could be of great importance. Proceeding with the study, we also look at the possible discrete symmetry violations in gravity that a deviation from the geometric theory would allow. It is found that there is only one discrete symmetry (in this case, parity) violating term consistent with special relativity that could lead to helicity flip. Comparing the scattering cross-sections with the limits put by the observed neutrino bursts from the supernova $SN\ 1987A$, very strong bounds are put on such interactions.

The aspects of neutrino physics discussed hitherto might seem to be bit disjointed to the casual reader. That this is not so, is brought out by the last topic that we consider. A lot of interest has been generated recently by independent claims of the experimental signature of a 17 $keV$ neutrino that mixes with the $\nu_e$ to as great an extent as 1%. The study of $(\beta\beta)_{0\nu}$ clearly demonstrates that it cannot be a Majorana particle unless there are other such particles with exactly the right mixing with $\nu_e$ so that the individual contributions to $(\beta\beta)_{0\nu}$ cancel giving an acceptably low level. Accomodating such a (pseudo-)Dirac neutrino quite often leads to problems either with their decay or with unaesthetically small Yukawa couplings. We present a model that avoids these difficulties. The 100 $keV$ scale is generated radiatively and hence there is no need to suppress the Yukawa couplings. The see-saw mechanism assures a superheavy Majorana neutrino, a pseudo-Dirac 17 $keV$ $\nu_\tau$ and extremely light Majorana $\nu_e$ and $\nu_\mu$. The $\nu_\tau$ evades the cosmological problem by decaying very fast into a lighter neutrino and a singlet–doublet Majoron that is not constrained significantly by either the $Z$–decay width or the various astrophysical bounds. The resolution of the solar neutrino problem is achieved through the radiative generation of a large transition magnetic moment connecting the two ultralight neutrinos. As for the $\nu_\tau$, its relatively large mass can be used to better the previous bounds on parity violation in gravitational interactions by as much as three orders of magnitude.