Chapter 1

Introduction

While the $V - A$ [1] current-current form of the universal weak-interaction Lagrangian had successfully supplanted the earlier general 'four-fermi theory' [2], it still failed to answer many questions. The very 'weakness' of the interaction strength as reflected by $G_F = 1.16 \times 10^{-5} GeV^{-2}$ in

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} J^{\dagger}_{\mu 1} J^{\mu}_2$$

where

$$J_\mu = \overline{\psi}_p \gamma_\mu (1 - \gamma_5) \psi_n,\ \overline{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu \ \ etc.$$

is puzzling. The coupling constant has mass dimension -2 and closely parallels $\frac{e}{q^2}$ for the electromagnetic interactions (q_μ being the momentum of the internal photon). In fact, Fermi was tempted to carry on the analogy with QED and describe the weak interactions as being mediated by some vector boson, but he did not pursue the idea any further. However this very structure of the Lagrangian ensures bad high energy behaviour for the cross sections. Clearly for a point interaction as above, just from dimensional arguments one has for the cross section, $\sigma \sim G_F^2 s$ where s is the invariant energy. But this being the $\ell = 0$ mode of a partial wave expansion, one has $\sigma = const/s$. Thus for $s > G_F^{-1}$, one runs into contradiction with unitarity requirements. The cure for this problem has already been hinted at: to formulate the weak interactions so as to be mediated by Intermediate Vector Bosons (IVB). With $\mathcal{L} = g J^-_\mu W^{+\mu}$ and g a dimensionless coupling constant, one has for the amplitude $M \sim g^2 \frac{J^+_\mu J^{\mu -}}{M^2_W - q^2}$ and hence for a large M_W, the low energy interaction would adequately be parametrized by $G_F \sim \frac{g^2}{M^2_W}$, whereas at large energy transfers, the cross-section falls off as required.

But even this would fail to please the purist. For, like the original theory, the massive IVB theory too is non-renormalizable. In fact, the only renormalizable theories involving vector bosons as fundamental constituents are those with a local gauge symmetry. But then is there a gauge symmetry associated with the weak interactions, and even if there is, how does one get the IVB's to be massive? These questions were answered by Glashow, Salam and Weinberg [3] and while the crux of their arguments shall be presented in the next chapter, the rest of this chapter is concerned with motivating their solution.

The near equality of the muon decay constant and the vector coupling constants for neutron and pion β-decay, inspite of only the latter receiving strong interation corrections, gives an indication of the symmetry one is looking for. Drawing an analogy from electrodynamics where the equality of the proton and the positron charges is explained by an assumption of equal bare charges and the current conservation law $\partial_\mu A^\mu(x) = 0$, it was proposed that the $\Delta Y = 0$ (Y being the hypercharge) vector currents be part of the divergenceless isospin current of the strong interactions. According to this 'Conserved Vector Current' hypothesis then, the vector current strength would not be renormalized. This also implies that the $\Delta Y = 0$ semileptonic weak interactions and the electromagnetic interactions involving hadrons are related. This is because the electromagnetic current contains the third component of the isospin current. To wit,

$$\begin{aligned} V^\mu(x) &= J^\mu_{1+i2}(x) \\ J^\mu_{em}(x) &= J^\mu_3(x) + J^\mu_Y(x) \end{aligned}$$

where $J^\mu_Y(x)$ denotes the isoscalar hypercharge current [4].

Thus one is led to consider a $SU(2)$ group as the gauge symmetry for the weak interactions with the associated charged gauge bosons identified with the IVB's. Phenomenological reasons dictate that only the left-handed fermion fields transform nontrivially under this group. Hence any attempt to identify the neutral gauge boson with the photon is bound to fail as the electromagnetic interaction is vectorial. The next most economical way is to consider a direct product of the $SU(2)$ with a $U(1)$ and let the photon be a combination of the two neutral gauge bosons. This scheme has the added advantage of suggesting a common origin for the electromagnetic and the weak interactions and is the one favoured experimentally.

Thus was born the electroweak theory, a renormalizable gauge field theory chosen from amongst the many competing ones for better experimental agreement. Alongwith Quantum Chromodynamics (QCD), believed to be the theory explaining the strong nuclear force, this forms the so-called Standard Model (SM) of elementary particle physics, a model expected to explain very well physical processes that involve interaction energies upto at least $\sim O(100\, GeV)$.

However all activities (fortunately) do not cease with the choice and even establishment of a model. Apart from testing its consequences in hitherto uncharted areas, one must also identify its limitations and especially so in the context of delineating questions that it cannot presume to address. One such question of prime importance is that of fermion masses, the resolution to which promises insights into physics beyond the Standard Model. Apart from prescribing a method to obtain the masses, the model is absolutely silent about their relative magnitudes and the strength of their consequences. Although experiments do give you the numbers, yet one strives for a theoretical understanding, the first step to which is the act of model building with certain assumptions. Of course, before one takes any of these models seriously, their validity must be checked *vis. a. vis.* experimental agreement. The first part of this thesis aims to do just that for the case of the quarks.

The scenario for the leptons is even more challenging. Whereas Pauli [5] had proposed the neutrinos to be absolutely massless neutral particles reacting only to the weak nuclear force, the modern attitude is to view them not to be strictly massless but rather lacking any mass in the observationally discernible range. In fact, certain experimental puzzles can be resolved if one does postulate a very small but non-zero mass for the neutrinos. But doing this would open up a plethora of new interactions with perhaps rather startling consequences, and some of these issues we examine in the second part of the current work.

The plan of the rest of this document is as follows. In Chapter 2, we give a brief discussion of the essential features of the SM, followed by an account of the formal aspects of determination of quark masses, mixing in the neutral meson systems and CP violation. Chapter 3 deals with the examination of various ansätze for the quark mass matrices and their phenomenological implications. A model–independent analysis of the problem is also presented. The focus in Chapter 4 is on the question of neutrino masses and their experimental signatures. The extent of neutrinoless double beta decay is estimated for various configurations. It is established that contrary to expectations, this rate is not governed by the Majorana mass of ν_e and models are constructed to demonstrate the naturalness of such scenarios. Chapter 5 deals with some aspects of neutrino physics related to the neutrino mass namely a large magnetic moment on the one hand and the gravitational interactions of fermions on the other. To correlate these results, we propose a model for massive neutrinos to accomodate the new finding of a 17 keV neutrino as well as large transition magnetic moment for the ν_e. We also comment on the gravitational interaction of the new particle. Finally, in the concluding chapter, we summarise the results of the investigations conducted.