Chapter 2

The Standard Model: Some Issues Revisited

In this chapter we first present a short discussion of the standard model — rather its electroweak part only — and some of its prime features. We discuss at some length the problem of the fermion masses in general and subsequently the quark masses in particular. A general account of quark mixing is given and is followed by the specific form for the case of three fermion generations. The experimental limits on these mixings are also presented.

Since color confinement makes it impossible to observe free quarks, a direct measurement of their masses is not possible. However estimates can be made using different and rather involved techniques and the result of such computations are presented without attempting any kind of discussion of the methods.

Finally we move on to a discussion of the neutral meson mixings and CP violation. The general form of the two independent parameters (*viz.* ϵ_K and ϵ'_K) giving a measure of CP violation in the K^0–$\overline{K^0}$ system are derived and the expression for these in the standard model calculated. A similar exercise is performed for the extent of mixing in the B_d^0–$\overline{B_d^0}$ system.

2.1 The Standard Model

As has been pointed out in the last section, the electroweak gauge group to be considered is $SU(2)_L \otimes U(1)_Y$, such that the left handed fermions transform as doublets of the $SU(2)_L$

and that the generator for $U(1)_{em}$ be given as a combination of the two diagonal generators. Hence we have for the charge operator,

$$Q = aT_{3L} + Y. \tag{2.1.1}$$

Considering either the lepton doublet $\binom{\nu}{e}_L$ or the quark doublet $\binom{u}{d}_L$, one immediately has $a = 1$. Keeping in mind the fact that the right handed fermions do not participate in the weak interactions and hence should be singlets under $SU(2)_L$, one gets for the fermions' transformation under the full symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ to be

$$\begin{array}{lll} l'_{iL} & \equiv \begin{pmatrix} \nu' \\ e' \end{pmatrix}_{iL} & (1,2,-1/2) \\ e'_{iR} & & (1,1,-1) \\ q'_{iL} & \equiv \begin{pmatrix} u' \\ d' \end{pmatrix}_{iL} & (3,2,1/6) \\ u'_{iR} & & (3,1,2/3) \\ d'_{iR} & & (3,1,-1/3) \end{array} \tag{2.1.2}$$

where i denotes the fermion generation.

As the low energy symmetry evinced in nature is only $SU(3)_c \otimes U(1)_{em}$, we must break $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{em}$. The simplest way to do this is to take recourse to spontaneous symmetry breaking involving a complex scalar field ϕ which transforms as $(1, 2, 1/2)$. Then the Lagrangian piece involving ϕ is

$$\mathcal{L}_\phi = (\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) - V(\phi) \tag{2.1.3}$$

where

$$\begin{array}{rrcl} & \mathcal{D}_\mu\phi & = & (\partial_\mu - igW_\mu - ig'B_\mu)\phi, \\ & V(\phi) & = & \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \\ \text{and} & W_\mu & = & W^a_\mu \tau^a \end{array} \tag{2.1.4}$$

W^a_μ and B_μ being the gauge fields corresponding to $SU(2)_L$ and $U(1)_Y$ and g and g' the respective couplings. The only restrictions imposed on $V(\phi)$ are the requirements of renormalizability and gauge invariance.

For $\mu^2 < 0$, $V(\phi)$ is minimized at $|\phi^\dagger\phi| = -\frac{\mu^2}{2\lambda} \equiv v^2$. Such a non-zero vacuum expectation value ($v.e.v.$) $\langle\phi\rangle = \binom{0}{v/\sqrt{2}}$ leads to a spontaneous breaking of both the $SU(2)_L$

and the $U(1)_Y$ symmetry. However a different $U(1)$ symmetry — which we shall identify as electromagnetism — generated by the combination of the diagonal generators

$$Q = T_{3L} + Y \tag{2.1.5}$$

is still preserved. The three Goldstone bosons due to symmetry breaking [6] are absorbed by three of the massless gauge bosons to appear as their longitudinal component thus rendering the latter massive. The essence of this Higgs [7] mechanism is encapsulated in the following brief discussion.

We reparametrize the scalar field ϕ, writing its four real components in terms of four new ones ξ_i and η by

$$\phi = U(\xi)\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

where $U(\xi) = e^{-i\xi\cdot T/2v}$ with T_i being any three independent generators of the gauge group that do not annihilate the vacuum.

Now taking advantage of the local gauge invariance of the theory one might as well work with the gauge transformed field

$$\begin{array}{lcll} \phi & \to & \phi' & = U^{-1}(\xi)\phi = \binom{0}{(v+\eta)/\sqrt{2}}, \\ W_\mu & \to & W'_\mu & = U^{-1}W_\mu U + \dfrac{i}{g}U^{-1}\partial_\mu U, \\ B_\mu & \to & B'_\mu & = B_\mu + \dfrac{i}{g}U^{-1}\partial_\mu U, \end{array}$$

and similarly transformed fermion fields.

Then (dropping the primes on the fields),

$$\mathcal{L}_\phi \to \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{4}\phi^T(g'B_\mu + gW_\mu)^2\phi - V\left(\frac{(v+\eta)^2}{2}\right),$$

and the gauge boson mass term reads

$$\frac{1}{8}v^2\left[(-g'B_\mu + gW_\mu^3)^2 + g^2\{(W_\mu^1)^2 + (W_\mu^2)^2\}\right].$$

Defining

$$\begin{array}{lcl} W_\mu^\pm & = & \frac{1}{\sqrt{2}}\left[W_\mu^1 \mp iW_\mu^2\right], \\ Z_\mu & = & W_\mu^3\cos\theta_W - B_\mu\sin\theta_W, \\ A_\mu & = & W_\mu^3\sin\theta_W + B_\mu\cos\theta_W, \end{array} \tag{2.1.6}$$

where

$$\theta_W = \tan^{-1}(g'/g), \tag{2.1.7}$$

we get

$$\begin{aligned} m_W &= \tfrac{1}{2} g v, \\ m_Z &= \tfrac{1}{2} g v \sec\theta_W \quad \text{and} \\ m_A &= 0. \end{aligned} \tag{2.1.8}$$

Thus with this special gauge choice (known as the unitary gauge), the bosonic spectrum of the theory consists of a massless and three massive gauge bosons and the single neutral scalar η. The other three degrees of freedom of the ϕ–field have been absorbed by the vector bosons only to appear as the corresponding longitudinal polarizations.

Writing the quark (and similarly for the other fermions) gauge boson coupling term in the new fields, we have

$$\begin{aligned} & g\overline{q'_L}\gamma^\mu(\sqrt{2}W^+_\mu\tau^- + \sqrt{2}W^-_\mu\tau^+ + W^3_\mu\tau^3)q'_L + g'(\overline{q'_L}\gamma^\mu Y q'_L + \overline{q'_R}\gamma^\mu Y q'_R)B_\mu \\ &= \frac{g}{\sqrt{2}}(\overline{u'_L}\gamma^\mu d'_L W^+_\mu + \overline{d'_L}\gamma^\mu u'_L W^-_\mu) + g\cos\theta_W Z_\mu \overline{q'_L}\gamma^\mu(\tan^2\theta_W Y - \tau^3)q'_L \\ &\quad + g\sin\theta_W A_\mu \overline{q'_L}\gamma^\mu Q q'_L. \end{aligned} \tag{2.1.9}$$

We then see that the massless vector field A couples vectorially with the fermion current and hence can be identified with the photon leading to the identification $g\sin\theta_W = e$. The massive gauge bosons, on the other hand, couple only to the chiral currents leading to the left handed weak interactions.

Looking now at the fermion masses, it is immediately apparent that we cannot write bare terms as

$$\overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

is not $SU(2)_L \otimes U(1)_Y$ invariant on account of ψ_L being a SU(2) doublet and ψ_R a singlet. This is not a problem though as we can use the same mechanism to generate fermion masses as for the gauge bosons *i.e.* spontaneous symmetry breaking. Recognising that the Yukawa term $\overline{q'_L}d'_R\phi$ is gauge invariant and of dimension $(mass)^4$, we have

$$\mathcal{L}_{Yuk} = f^{ij}_d \overline{q'_{Li}} d'_{Rj}\phi + f^{ij}_u \overline{q'_{Li}} u'_{Rj}\tilde{\phi} + f^{ij}_e \overline{l'_{Li}} e'_{Rj}\phi, \tag{2.1.10}$$

where $\tilde{\phi} \equiv \tau_2 \phi^*$ and i, j run over the fermion generations *i.e.* $d_1 \equiv d$, $d_2 \equiv s$, $d_3 \equiv b$ *etc.* In the unitary gauge we then have

$$\mathcal{L}_{Yuk} = \left(M_u^{ij} \overline{u'_{Li}} u'_{Rj} + \frac{1}{v} M_u^{ij} \overline{u'_{Li}} u'_{Rj} \eta + H.c. \right) + (u' \to d') + (u' \to e'), \tag{2.1.11}$$

where $M_u^{ij} = v f_u^{ij}$ is the mass matrix for the up-quarks and similarly for the others.

The theory does not specify f_u^{ij} and hence the mass matrices in any way. All structures for f_u^{ij} satisfy the symmetry requirements and these have to be determined only from experiments. In fact, the matrices do not even need to be hermitian and hence cannot be diagonalized by a unitary transformation. All is not lost though. As the left– and right–handed fermions have different $SU(2)_L$ and $U(1)_Y$ quantum numbers, one can define distinct unitary transformation for each. This is equivalent to treating the mass term as an hermitian operator in a $2n$ dimensional space (for n fermion generations) with a block off–diagonal representation and defining a unitary transformation in $U(2n)$ that is block diagonal in the left– and right–handed subspaces.

Now using a result in elementary linear algebra that any nonsingular matrix can be polar decomposed into a product of a positive definite Hermitian matrix and an isometric matrix, we can define

$$u_L \equiv U_L u'_L \quad \text{and} \quad u_R \equiv U_R u'_R, \tag{2.1.12}$$

such that

$$U_L^\dagger M_u U_R = \widehat{M_u} = \text{diagonal and positive definite.} \tag{2.1.13}$$

Thus U_L and U_R diagonalize the hermitian matrices $M_u M_u^\dagger$ and $M_u^\dagger M_u$ respectively. Defining similar transformations $D_{L,R}$ for the d–type quarks, we get

$$\mathcal{L}_{mass} = \overline{u_L} \widehat{M_u} u_R + \overline{d_L} \widehat{M_d} d_R + \overline{e_L} \widehat{M_e} e_R + H.c., \tag{2.1.14}$$

and

$$J_\mu^+ = \overline{u_L} \gamma_\mu K d_L + \overline{e_L} \gamma_\mu \nu_L, \tag{2.1.15}$$

where

$$K \equiv U_L^\dagger D_L \tag{2.1.16}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [8].

At this stage an interpretation of the results is called for. The original primed fields were the eigenstates of the weak hamiltonian (H_{wk}) but not of the full hamiltonian as H_{wk} does not commute with $H_s + H_{em}$. The unprimed states are the eigenstates of the total hamiltonian and hence have well-defined masses. The CKM matrix then represents the modification of the charged current vertices for the physical quarks induced by quark mixing. It should be noted that the corresponding matrix for the leptonic sector is but unity. This is due to the absence of the ν_R and hence a mass term for the neutrinos, as a consequence of which the diagonalization matrix for M_e can be absorbed into the definition of the ν_L.

The mixing matrix K lies in $U(n)$ and hence is described by n^2 real parameters. Recognizing that ${}^nC_2$ of these are nothing but the Euler angles for a real rotation, we find that the complexity is due to the rest of the ${}^{n+1}C_2$ parameters. But of these, $(2n-1)$ are of no physical significance as they can be absorbed by redefining the relative phases of the quark wavefunctions. So at the end of the day we are left with $2n$ quark masses, ${}^nC_2$ real rotations and ${}^{n-1}C_2$ phases in the mixing matrix.

Henceforth we shall specialize to three generations (unless otherwise stated) as most current experimental results favour such a scenario. Then the CKM matrix is 3×3 and parametrized by three angles θ_{ij} and a phase δ which, in this model, is responsible for CP violation. For explicit calculations involving the CKM matrix, we choose the parametrization due to Maiani [9]:

$$K = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23}e^{-i\delta} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{-i\delta} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{2.1.17}$$

where $c_{ij} = \cos\theta_{ij}$; $s_{ij} = \sin\theta_{ij}$.

While θ_{12} is very accurately determined from K_{e3} and hyperon decays [10]

$$s_{12} = 0.221 \pm 0.002, \tag{2.1.18}$$

θ_{23} and θ_{13} are rather poorly determined. The value of s_{23} may be extracted from a determination of V_{cb} (since $s_{23} \approx |V_{cb}|$ to a very good approximation) from the semileptonic B-meson partial width, under the assumption that it is given by the W-mediated process to be

$$\Gamma(b \to c l \overline{\nu}_l) = \frac{G_F^2 m_b^5}{192\pi^3} F(m_c^2/m_b^2)|V_{cb}|^2, \tag{2.1.19}$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$ is a phase space factor. Thus

$$s_{23}^2 = \frac{192\pi^3}{G_F^2} \frac{\mathrm{Br}(b \to c l \overline{\nu}_l)}{\tau_b m_b^5 F(m_c^2/m_b^2)} . \tag{2.1.20}$$

Using the experimental results for the branching ratio and the B-meson lifetime [11]

$$\mathrm{Br}(b \to c l \overline{\nu}_l) = 0.121 \pm 0.008 \qquad \tau_B = (1.16 \pm 0.16) \times 10^{-12}\, sec, \tag{2.1.21}$$

and the estimates for the quark masses (see section 2.2):

$$m_c = (1.35 \pm 0.05)\; GeV \qquad m_b = (5.3 \pm 0.1)\; GeV,$$

we get [12][1]

$$0.035 \leq s_{23} \leq 0.07 \tag{2.1.22}$$

The charmless B–meson decay width puts a limit [14][2]

$$0.07 \leq s_{13}/s_{23} \leq 0.22. \tag{2.1.23}$$

The CP–violating phase δ is allowed to adopt any value in the range $[0, \pi]$ by the current experimental results.

2.2 Quark Masses

In a renormalizable field theoretic treatment, the coupling constant and masses lose their absolute meaning and become dependent on the momentum scale one is addressing the problem at. This dependence arises from two sources, though the two cannot be demarcated easily.

In quantum field theoretical calculations infinities creep up quite often and are taken care of by what is called a 'regularization' prescription. Though there is *nothing ad hoc* about this program, there do exist different inequivalent schema for this procedure, the

[1]The experimental numbers quoted in this chapter are those used in [35,38]. Since then many of these have been revised. For example, now one has $s_{23} = 0.043^{+0.007}_{-0.009}$ [13].

[2]Current limits [13] are $0.05 \leq s_{13}/s_{23} \leq 0.13$.

difference lying in the amount of the finite part to be subtracted alongwith the divergent piece. Thus it creates a dependence on the momentum scale introduced, that is different in different schemes.

However this implies that the physical quantities would depend on the renormalization scheme adopted and the scale at which it is being performed, a situation clearly unacceptable as starting from a unique Lagrangian, all measurables ought to have a unique value. This then leads to the requirement that under a finite renormalization transformation, physical predictions be invariant. Expressed in a different language, this implies that all renormalized quantities should change with a change of the scale (equivalent to a finite renormalization transformation) in a well-defined fashion and that the functional dependence of measurables on these should change such that their (measurables') value remains the same. These finite renormalizations form a transformation group and the functional relations determining the changes can be expressed as differential equations of evolution known as the Renormalization Group (RG) equations.

When talking of quarks, the relevant theory of course is QCD and the RG equations important in our study are those governing the evolution of the quark masses and the strong coupling constant with the renormalization scale μ:

$$\begin{aligned} \mu\frac{\mathrm{d}g}{\mathrm{d}\mu} &= \beta(g), \\ \mu\frac{\mathrm{d}m_i}{\mathrm{d}\mu} &= -\gamma_{m_i}(g)m_i. \end{aligned} \tag{2.2.1}$$

In the modified minimal subtraction ($\overline{MS}$) scheme, the beta function and the anomalous dimension are respectively given by [15]

$$\beta(g) = -\frac{\beta_0}{(4\pi)^2}g^3 - \frac{\beta_1}{(4\pi)^4}g^5 + O(g^7), \tag{2.2.2}$$

and

$$\gamma_m(g) = \frac{\gamma_0}{(4\pi)^2}g^2 + \frac{\gamma_1}{(4\pi)^4}g^4 + O(g^6), \tag{2.2.3}$$

with

$$\begin{aligned} \beta_0 &= (11C_G - 4T_RN_f)/3, \\ \beta_1 &= \left[34C_G^2 - 45C_G + 3C_F)T_RN_f)\right]/3, \\ \gamma_0 &= 6C_F, \\ \gamma_1 &= C_F[9C_F + 97C_G - 20T_RN_f]/3, \end{aligned} \tag{2.2.4}$$

where N_f = number of quark flavours, T_R is given by the normalization of the generators $\left[Tr(T^a T^b) = N_f T_R\right]$ and C_G and C_F are the values of the quadratic Casimir operator on the gluons and the quarks respectively. For $SU(3)$, by convention, $T_R = 1/2$, and $C_G = 3$, $C_F = 4/3$ and hence

$$\begin{aligned} \beta_0 &= 11 - \tfrac{2}{3}N_f, \\ \beta_1 &= 102 - \tfrac{38}{3}N_f, \\ \gamma_0 &= 8, \\ \gamma_1 &= \tfrac{4}{3}\left(101 - \tfrac{10}{3}N_f\right). \end{aligned}$$

The solutions to the differential equations are

$$\alpha_s(\mu) \equiv \frac{g^2(\mu)}{4\pi} = \frac{4\pi}{\beta_0 L}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L}{L} + O\left(\left(\frac{\ln L}{L}\right)^2\right)\right], \tag{2.2.5}$$

and

$$m_i(\mu) = \overline{m}_i\left(\frac{L}{2}\right)^{-\gamma_0/2\beta_0}\left[1 - \frac{\beta_1\gamma_0}{2\beta_0^3}\frac{1+\ln L}{L} + \frac{\gamma_1}{2\beta_0^2 L} + O\left(\left(\frac{\ln L}{L}\right)^2\right)\right], \tag{2.2.6}$$

where $L = \ln(\mu^2/\Lambda^2)$. Here Λ and $\overline{m}_i$ are the RG–invariant scale parameter and masses, respectively defined through

$$\begin{aligned} e^{-\beta_0 g^2(0)} &= \tfrac{\lambda^2}{\Lambda^2}\left(\ln\tfrac{\lambda^2}{\Lambda^2}\right)^{\beta_1/\beta_0^2} \\ \text{and}\quad m_i(0) &= \overline{m}_i\left(\ln\tfrac{\lambda^2}{\Lambda^2}\right)^{-\gamma_0/2\beta_0}, \end{aligned}$$

λ being the momentum cutoff. This then takes care of the perturbation theory induced cutoff–dependence of the bare couplings. The arbitrary coupling constant $g(0)$ is thus replaced through 'dimensional transmutation' by a dimensionful parameter Λ, which along with the quark masses are the only arbitrary parameters in QCD and would be fixed by experimental data.

In all the above formulae the value of N_f to be used is determined by the energy scale of the problem at hand, with the assumption that all heavier degrees of freedom can be taken to be frozen. The physical mass of a quark is then its value calculated at the same scale. Thus to one loop order, the physical mass of the i'th quark is given by

$$m_i^{\text{phys}} = m_i(m_i)\left[1 + \frac{4}{3\pi}\alpha_s(m_i)\right]. \tag{2.2.7}$$

While non-observation of the top-quark puts a lower limit [16] to its mass [3]

$$m_t^{\text{phys}} \gtrsim 45\ GeV, \tag{2.2.8}$$

[3] Current bound [17] is $m_t^{\text{phys}} \gtrsim 89\ GeV$.

experimental consistency with the radiative corrections in the standard model requires [18]

$$m_t^{\rm phys} \lesssim 180\ GeV. \tag{2.2.9}$$

Substituting $N_f = 6$ and $\Lambda_{QCD} = 100MeV$, we have for the above range of interest

$$m_t^{\rm phys} \approx 0.6 m_t(1\ GeV),$$

which gives

$$75\ GeV \lesssim m_t(1\ GeV) \lesssim 300\ GeV. \tag{2.2.10}$$

The physical masses of the charm and bottom quarks can be calculated to a great degree of accuracy from e^+e^- data by using QCD sum rules for the vacuum polarization amplitude. We have then

$$\begin{array}{rcl} m_c(1\ GeV) & = & (1.35 \pm 0.05)\ GeV \\ m_b(1\ GeV) & = & (5.3 \pm 0.1)\ GeV. \end{array} \tag{2.2.11}$$

The determination of the lighter quark masses involves larger errors. These are best evaluated using chiral perturbation theory and meson and baryon spectroscopy. Though the individual errors are large, restrictions on the ratio of the masses reduce the indeterminacy somewhat:

$$\begin{array}{rcl} m_u & = & (5.1 \pm 1.5)MeV \\ m_d & = & (8.9 \pm 2.6)MeV \\ m_s & = & (175 \pm 55)MeV \\ m_s/m_d & = & (19.6 \pm 1.6) \\ m_d/m_u & = & (1.76 \pm 0.13) \\ m_s/m_u & = & (34.5 \pm 5.1) \\ \dfrac{m_u - m_d}{m_u + m_d} & = & (-0.28 \pm 0.03). \end{array} \tag{2.2.12}$$

2.3 *CP* Violation and Neutral Meson Systems

Apart from the usual continuous (gauge) symmetries that lead to conserved Nöther's currents, physical theories most often respect certain discrete symmetries as well. The most common of these are:-

Parity (P): this implies an equivalence of 'left' and 'right' *i.e.* a mirror image of an experiment would yield the same result in the reflected frame of reference as the original would in the initial frame.

Charge conjugation (C): implies invariance under replacing each particle by its antiparticle (*i.e.* reversing all additive quantum numbers).

Time reversal(T): referring to a formal reversal of time flow, this implies invariance under reversal of all momenta, angular momenta *etc.*

Though a theorem due to Lüders and Pauli [19] guarantees that any Lorentz-invariant unitary local field theory is invariant under the combined action CPT (in any order), the individual symmetries are not assured by any deep principle. In fact though gravitational, electromagnetic and the strong interaction seem to respect each of these to a very great degree (for a discussion of possible discrete symmetry violations in gravity, see section 5.2), it was established quite early on that the weak interactions violated both C and P conservation almost maximally. However even they seemed to respect CP and consequently T symmetry. In fact till date the only evidence of CP violation has been seen in the neutral kaon system and there too to a very small extent only. Thus any study of CP violation would demand as a prerequisite a thorough understanding of the K^0–$\overline{K^0}$ system. Also the heavier meson systems are exactly similar and most of the results obtained for the kaon system can easily be extended in a straightforward manner. As for the leptonic sector, CP violation is identically zero in the minimal standard model, but could arise if one were to include massive neutrinos (Chapter 4). Though the issues involved are somewhat different, most of the analysis here trivially follows through.

2.3.1 The K^0–$\overline{K^0}$ system:

The neutral K–mesons K^0 and $\overline{K^0}$ are characterised by definite strangeness values $S = 1$ and -1 respectively and hence are good basis states when one is talking about either the strong or the electromagnetic interactions. This is so because both these interactions do respect strangeness conservation and hence

$$\langle K^0 | H_s + H_{em} | \overline{K^0} \rangle = 0. \tag{2.3.1}$$

However weak interactions do not preserve strangeness and thus can mix K^0 and $\overline{K^0}$. This results in these particles not having well defined masses or weak decay rates. Rather there exist two independent linear combinations of these states namely K_L and K_S that do have precise masses and decay rates. These new states are characterized by the difference in their decay modes and hence their lifetimes. While K_S decays primarily through the 2π mode (a state with CP eigenvalue $+1$), K_L has many decay channels mostly going to final states with $CP = -1$ *e.g.* 3π or $\pi^{\pm} l^{\mp} \overset{(-)}{\nu}$. Obviously the two new states do not have well defined strangeness.

Working with a choice of basis such that

$$CP|K^0\rangle = -|\overline{K^0}\rangle \quad \text{and} \quad CP|\overline{K^0}\rangle = -|K^0\rangle, \tag{2.3.2}$$

if we define two new states as

$$|K^0_{1,2}\rangle \equiv \frac{1}{\sqrt{2}}\left[|K^0\rangle \pm |\overline{K^0}\rangle\right], \tag{2.3.3}$$

then obviously

$$CP|K^0_1\rangle = -|K^0_1\rangle \quad \text{and} \quad CP|K^0_2\rangle = |K^0_2\rangle. \tag{2.3.4}$$

So if CP were an exact symmetry, this would imply that

$$|K_L\rangle = |K^0_1\rangle \quad \text{and} \quad |K_S\rangle = |K^0_2\rangle. \tag{2.3.5}$$

However in 1964, Cronin *et al.* [20] observed that K_L does decay into the $\pi^+\pi^-$ channel (*i.e.* a $CP = +1$ state) with a branching ratio of 2×10^{-3}. Hence the identification in eqn.(2.3.5) is wrong and we should rather have

$$|K_{L,S}\rangle = N_{L,S}\left[|K^0\rangle \pm e^{i\xi_{L,S}}|\overline{K^0}\rangle\right], \tag{2.3.6}$$

where $\xi_{L,S}$ are complex numbers and $N_{L,S}$ the wavefunction normalizations. Since K^0 and $\overline{K^0}$ both mix and decay, their time evolution is governed by an effective hamiltonian $H = H_s + H_{em} + H_{wk}$ such that

$$i\frac{\mathrm{d}}{\mathrm{dt}}|\psi\rangle = H|\psi\rangle, \tag{2.3.7}$$

where

$$\begin{aligned} |\psi\rangle &= \begin{pmatrix} |K^0\rangle \\ |\overline{K^0}\rangle \end{pmatrix} \\ \text{and } H &= M - \tfrac{i}{2}\Gamma, \end{aligned} \tag{2.3.8}$$

with M and Γ being 2×2 hermitian matrices called the mass and decay matrices respectively. Now CPT invariance demands that

$$\begin{aligned} H_{11} &\equiv \langle K^0|H|K^0\rangle = \langle K^0|(CPT)^{-1}H(CPT)|K^0\rangle \\ &= \langle\overline{K^0}|H|\overline{K^0}\rangle \equiv H_{22} \end{aligned} \tag{2.3.9}$$

or $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. On the other hand CP conservation would require

$$\begin{aligned} H_{12} &\equiv \langle K^0 | H | \overline{K^0} \rangle &= \langle K^0 | (CP)^{-1} H (CP) | \overline{K^0} \rangle \\ &= \langle \overline{K^0} | H | K^0 \rangle &\equiv H_{21} \end{aligned} \tag{2.3.10}$$

and hence $M_{12} = M_{21}$ and $\Gamma_{12} = \Gamma_{21}$. Now, the eigenvalues of H are

$$E_{L,S} \equiv m_{L,S} - \frac{i}{2}\gamma_{L,S} = \frac{1}{2}\left[H_{11} + H_{22} \pm \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}\right] \tag{2.3.11}$$

and the difference is given by

$$E_L - E_S \equiv \Delta m - \frac{i}{2}\Delta\gamma = \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}. \tag{2.3.12}$$

If $K_{L,S}$ are to be the eigenvectors then we must have

$$e^{i\xi_L} = \frac{E_L - H_{11}}{H_{12}} \quad \text{and} \quad e^{i\xi_S} = \frac{H_{21}}{E_S - H_{22}}.$$

Invariance under CPT then demands that

$$\xi_L = \xi_S \equiv \xi = -\frac{i}{2}\ln\left(\frac{H_{21}}{H_{12}}\right),$$

and CP invariance would guarantee that

$$e^{i\xi} = 1. \tag{2.3.13}$$

However the last relation is phase convention dependent as can be seen by redefining the meson wavefunctions by

$$\begin{aligned} |K^0\rangle &\rightarrow |K^0\rangle' &= e^{i\alpha}|K^0\rangle, \\ |\overline{K^0}\rangle &\rightarrow |\overline{K^0}\rangle' &= e^{-i\alpha}|\overline{K^0}\rangle. \end{aligned} \tag{2.3.14}$$

Under this change of basis the diagonal matrix elements of any operator $\mathcal{O}$ remain invariant whereas the off-diagonal elements pick up phases

$$\mathcal{O}_{12} \rightarrow \mathcal{O}'_{12} = e^{-2i\alpha}\mathcal{O}_{12} \quad \text{and} \quad \mathcal{O}_{21} \rightarrow \mathcal{O}'_{21} = e^{2i\alpha}\mathcal{O}_{21}$$

and hence

$$\xi \rightarrow \xi' = \xi + 2\alpha. \tag{2.3.15}$$

Thus the basis invariant condition for CP conservation is that ξ be real [21]. An often used measure of CP violation is given by

$$\epsilon \equiv \frac{1 - e^{i\xi}}{1 + e^{i\xi}} \tag{2.3.16}$$

but this clearly is a phase convention dependent quantity.

We turn next to a discussion of the two pion decays of the neutral K-mesons. Bose statistics demands that the 2π state be in either total isospin $I = 0$ or $I = 2$ state. Hence defining the amplitudes

$$\langle n|H_{wk}|\overset{(-)}{K^0}\rangle = \overset{(-)}{a}_n \, e^{i\delta_n}, \qquad n = 0, 2 \tag{2.3.17}$$

where $|n\rangle \equiv |2\pi; I = n\rangle$ and δ_n is the 2π s-wave phase shift for the $I = n$ state, we have

$$\begin{array}{rl} CPT \text{ invariance} & \Longrightarrow \; \bar{a}_n = -a_n^* \\ \text{and} \quad CP \text{ invariance} & \Longrightarrow \; a_2/a_0 \text{ is real.} \end{array} \tag{2.3.18}$$

Under the phase rotation as described by equation (2.3.14),

$$a_n \to a_n' = a_n e^{i\alpha} \tag{2.3.19}$$

and hence the following combinations are phase choice independent

$$\begin{aligned} \epsilon_0 &\equiv \frac{\langle 0|H_{wk}|K_L\rangle}{\langle 0|H_{wk}|K_S\rangle} = \frac{a_0 - a_0^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} \\ \epsilon_2 &\equiv \frac{1}{\sqrt{2}}\frac{\langle 2|H_{wk}|K_L\rangle}{\langle 0|H_{wk}|K_S\rangle} = \frac{1}{\sqrt{2}}\frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}} e^{i(\delta_2 - \delta_0)} \\ \omega &\equiv \frac{\langle 2|H_{wk}|K_S\rangle}{\langle 0|H_{wk}|K_S\rangle} = \frac{a_2 - a_2^* e^{i\xi}}{a_0 + a_0^* e^{i\xi}}. \end{aligned} \tag{2.3.20}$$

The experimentally measurable quantities are

$$\begin{aligned} \eta^{+-} &\equiv \frac{\langle \pi^+\pi^-|H_{wk}|K_L\rangle}{\langle \pi^+\pi^-|H_{wk}|K_S\rangle} = \frac{\epsilon_0 + \epsilon_2}{1 + \omega/\sqrt{2}} = \epsilon_0 + \frac{\epsilon'}{1 + \omega/\sqrt{2}} \\ \text{and} \quad \eta^{00} &\equiv \frac{\langle \pi^0\pi^0|H_{wk}|K_L\rangle}{\langle \pi^0\pi^0|H_{wk}|K_S\rangle} = \frac{\epsilon_0 - 2\epsilon_2}{1 - \sqrt{2}\omega} = \epsilon_0 - \frac{2\epsilon'}{1 - \sqrt{2}\omega} \end{aligned} \tag{2.3.21}$$

where

$$\epsilon' \equiv \epsilon_2 - \frac{\omega\epsilon_0}{\sqrt{2}}. \tag{2.3.22}$$

In terms of the matrix elements of M and Γ then

$$\epsilon_0 = i\frac{Im(M_{12}a_0^2) - iIm(\Gamma_{12}a_0^2)}{Re(a_0^2 M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0|^2}{2}(\Delta m - \frac{i}{2}\Delta\gamma)} \tag{2.3.23}$$

$$\epsilon' = \frac{i}{\sqrt{2}}\frac{Im(a_2 a_0^*)(\Delta m - \frac{i}{2}\Delta\gamma)e^{i(\delta_2 - \delta_0)}}{Re(a_0^2 M_{12}) - \frac{i}{2}Re(a_0^2\Gamma_{12}) + \frac{|a_0|^2}{2}(\Delta m - \frac{i}{2}\Delta\gamma)} \tag{2.3.24}$$

All analysis till now has been the most general possible. No particular reference to the kaons have been made and all the results would hold equally well for any other neutral meson system. At this stage we would like to specialise to the K^0–$\overline{K^0}$ system and use some experimental results to obtain some approximate but easy to handle relations.

Now experimentlly we have [13]

$$\begin{aligned} m_K &= 0.498\,GeV, \qquad\qquad \Delta m_K = 3.5\times10^{-15}\,GeV, \\ \Delta\gamma_K &\approx -\gamma_{K_S} = -7.3\times10^{-15}\,GeV. \end{aligned} \tag{2.3.25}$$

The $\Delta I = 1/2$ rule for K–decays manifests itself in the form of a small suppression factor [22]

$$\omega \approx 0.045. \tag{2.3.26}$$

The dominant contribution to Γ_{12} comes from the 2π intermediate states and more specifically the $I = 0$ state. Thus

$$\Gamma_{12} \sim \langle K^0|H_{wk}^{\Delta S=1}|0\rangle\langle 0|H_{wk}^{\Delta S=1}|\overline{K^0}\rangle \tag{2.3.27}$$

and hence

$$\frac{Im\Gamma_{12}}{Re\Gamma_{12}} \simeq \frac{Im(a_0^*)^2}{Re(a_0^*)^2}. \tag{2.3.28}$$

Using the experimental values of η^{+-} and η^{00}, alongwith (2.3.26) we then get [13]

$$|\epsilon_0| = 2.3\times10^{-3}, \tag{2.3.29}$$

and the phase of ϵ_0 is nearly $\pi/4$. Such a small value of the CP violating effect can be best understood as resulting from

$$Im\Gamma_{12} \ll Re\Gamma_{12} \quad \text{and} \quad ImM_{12} \ll ReM_{12}.$$

Under this approximation eqn.(2.3.12) reduces to

$$\Delta m_K \approx 2ReM_{12} \quad \text{and} \quad \Delta\gamma \approx 2Re\Gamma_{12}. \tag{2.3.30}$$

In the SM, $K^0 \to 2\pi$ decays proceed through the "box diagram" (see section 2.3.2) and with a certain phase choice known as the 'quark phase convention', one can rotate away the phase of a_2 to have

$$a_0 = |a_0|e^{i\theta_0} \quad \text{and} \quad a_2 = \pm|a_2|. \tag{2.3.31}$$

Then using (2.3.25 – 2.3.31) in (2.3.23, 2.3.24) we get

$$\epsilon_K \equiv \epsilon_0 \approx \frac{e^{i\pi/4}}{\sqrt{2}}\left[\frac{ImM_{12}}{\Delta m_K} + \tan\theta_0\right] \tag{2.3.32}$$

$$\epsilon'_K \equiv \epsilon' \approx \mp\frac{1}{\sqrt{2}}\frac{|a_2|}{|a_0|}\sin\theta_0 e^{i(\delta_2-\delta_0+\pi/2)}. \tag{2.3.33}$$

To determine the parameter ϵ'_K one needs to measure η^{+-} and η^{00} to a great degree of accuracy, a task of considerable difficulty. However recently such measurements have been made to yield [23][4]

$$|\epsilon'_K/\epsilon_K| = (3.3 \pm 1.1) \times 10^{-3}. \tag{2.3.34}$$

2.3.2 Sources of CP Violation

The main thrust of the current chapter and the next is to establish a link between the CP violation in the K^0–$\overline{K^0}$ system and the quark mass matrices. But before jumping onto any conclusion, we would rather like to have a quick look at the various possible sources and only then point out the essential simplicity of the CKM picture.

CP violation in a theory satisfying the Lüders–Pauli criteria [19] can be categorised as those

a) violating each of C, P and T;

b) violating P and T but conserving C;

c) violating C and T but conserving P.

As parity violating effects in strong and electromagnetic interactions have been experimentally constrained to less than $O(10^{-5})$ [25,26], such theories obviously cannot explain ϵ_K. Thus if CP violation were to come from these sectors, then they must be of category (c). On the other hand, CP violating effects in the weak interactions are most likely to be of type (a), though H_{wk} might as well have small admixtures of categories (b) and (c). Keeping such considerations in mind, the candidate theories can be classified into four types. Of these, the millistrong and the electromagnetic models require an adequately small part of the corresponding hadronic interaction to be of type (c). The CP violation in $K \to 2\pi$

[4] It must be remembered though that a later experiment [24] gives a value $(-0.5 \pm 1.4) \times 10^{-3}$ *i.e.* consistent with zero.

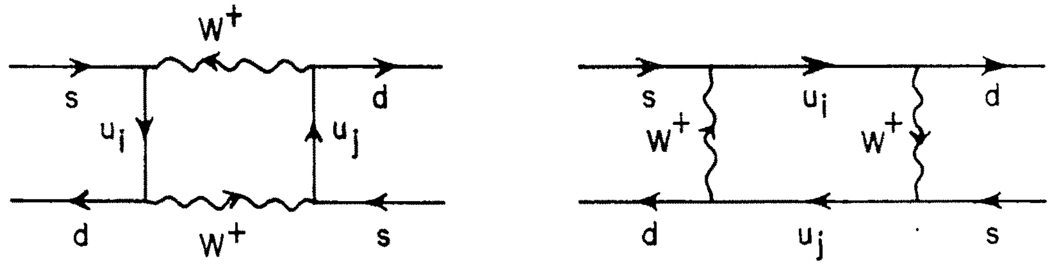

Figure 2.1: "Box"-diagram generating K^0–$\overline{K^0}$ mixing and ϵ_K in the SM

(which is supposed to occur through an intermediate state with one of the decays being driven by the CP conserving H_{wk}) then arises as a result of an interference of amplitudes. However, experimentally such models are not favoured [26].

Milliweak models require a part ($\sim O(10^{-3})$) of H_{wk} be CP violating, resulting in single–shot $K_L \to 2\pi$, and hence similar effects should be observable elsewhere, say in the B–decays. On the other hand superweak models predict a CP violating $\Delta S = 2$ piece in H_{wk} with $K_L \to 2\pi$ occuring through an intermediate K_S state. In such a case CP violation occurs only in the K^0–$\overline{K^0}$ system. Consistency with the observed value of Δm_K, which arises now as a first order effect requires $g_{sw} \sim 10^{-8}$ and hence the name. The distinguishing feature of this model is that ϵ'_K is identically zero.

In the 3–generation SM, which, for a complex CKM matrix, is a milliweak theory, K^0–$\overline{K^0}$ mixing and $K_L \to 2\pi$ come about because of the 1–loop Feynman diagrams in Figures (2.1) and (2.2) respectively, giving rise to

$$ImM_{12} = \frac{G_F^2}{12\pi^2} f_K^2 m_K m_W^2 B_K \left[\lambda_c^2 \eta_1 S(y_c) + \lambda_t^2 \eta_2 S(y_t) + \lambda_c \lambda_t \eta_3 S(y_c, y_t)\right], \qquad (2.3.35)$$

and

$$\tan\theta_0 = \frac{s_{13}s_{23}}{s_{12}} \sin\delta \left[\frac{150MeV}{m_s(1\,GeV)}\right]^2 \bar{H}, \qquad (2.3.36)$$

where

$$\begin{aligned} \lambda_i &\equiv K_{id}^* K_{is} & y_i &\equiv m_i^2/m_W^2 \\ f_K &= 0.16\,GeV & m_W &= 81.8\,GeV. \end{aligned} \qquad (2.3.37)$$

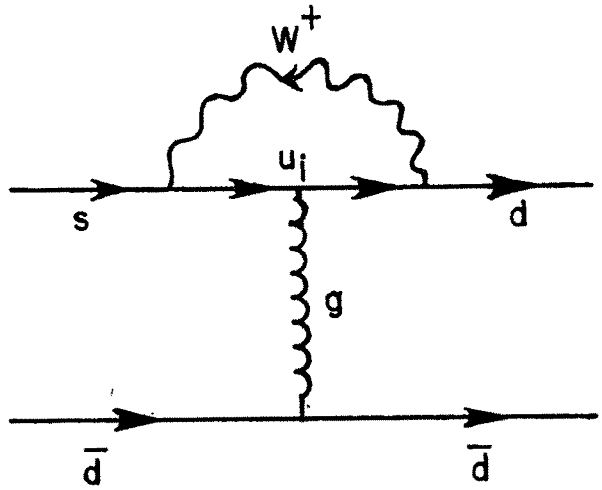

Figure 2.2: "Penguin"–diagram responsible for ϵ'_K in the SM

Whereas f_K is the pion decay constant, the bag parameter B_K reflects our ignorance of the hadronic matrix elements. If vacuum saturation approximation were correct then one would have $B_K = 1$, but theoretical estimates only put the rather loose bound of $1/3 \leq B_K \leq 1$. The functions $S(x)$ and $S(x,y)$ arise from the loop integral and are given by

$$
\begin{aligned}
S(x) &= x\left[\tfrac{1}{4} + \tfrac{9}{4(1-x)} - \tfrac{3}{2(1-x)^2}\right] + \tfrac{3}{2}\left[\tfrac{x}{x-1}\right]^3 \ln x \\
S(x,y) &= xy\left[\left\{\tfrac{1}{4} + \tfrac{3}{4(1-x)} - \tfrac{3}{4(1-x)^2}\right\}\tfrac{\ln x}{x-y} - \tfrac{3}{8}\tfrac{1}{1-x}\tfrac{1}{1-y}\right] + (x \leftrightarrow y)
\end{aligned}
$$

The quantities η_i represent QCD corrections [27]. While η_1 does not depend on m_t and is evaluated to be 0.85, η_2 is essentially independent of m_t for $40\ GeV \lesssim m_t^{\rm phys} \lesssim 130\ GeV$ and $\eta_2 = 0.61$. η_3 and $\bar{H}$ are slowly varying functions of m_t and are approximately 0.25 and 0.37 respectively [28]. However we shall allow for their full variation in our calculations.

2.3.3 The B_d^0–$\overline{B_d^0}$ system

The analysis for the B_d^0–$\overline{B_d^0}$ system proceeds exactly as for the K^0–$\overline{K^0}$ system. But unlike the latter, no trace of CP violation has yet been found here. Instead, we shall concentrate solely on the issue of particle–antiparticle mixing. Defining the time–integrated mixing parameters

$$
\begin{aligned}
r_d &\equiv \frac{\int_0^\infty |\langle \overline{B_d^0} | B_d^0 \rangle|^2 {\rm d}t}{\int_0^\infty |\langle B_d^0 | B_d^0 \rangle|^2 {\rm d}t} \\
&= \left|e^{i\xi_B}\right|^2 \frac{(\Delta m_B)^2 + (\Delta\Gamma_B)^2/4}{2\Gamma_B^2 + (\Delta m_B)^2 + (\Delta\Gamma_B)^2/4}
\end{aligned}
\qquad (2.3.38)
$$

(where in the second line we have dropped the subscript d) and similarly for $\bar{r}_d$, we see that CP conservation demands that $\bar{r}_d = r_d$. Experimentally however one cannot directly measure either r_d or $\bar{r}_d$ as generally the mesons are created as pairs. Rather one looks at the dilepton decay modes and defines parameters R_d and A_d which measure mixing and decay asymmetry (and thus CP violation) respectively:

$$R_d = \frac{N^{++} + N^{--}}{N^{+-} + N^{-+}} \qquad A_d = \frac{N^{++} - N^{--}}{N^{++} + N^{+-} + N^{-+} + N^{--}}, \tag{2.3.39}$$

where N's denote the number of dilepton pairs with the associated charges. For the $e^+e^- \to \Upsilon(4S) \to B_d^0\overline{B_d^0}$ process these relations reduce to

$$R_d = \frac{1}{2}(r_d + \bar{r}_d), \qquad A_d = \frac{r_d - \bar{r}_d}{2 + r_d + \bar{r}_d}.$$

For the 3-generation SM with a relatively heavy top, the dominant contribution to r_d comes from the corresponding box-diagram with the top flowing in it. With this simplifying assumption we have

$$x_d \equiv \frac{\Delta m_d}{\Gamma_d} = \frac{2G_F^2}{3\pi^2}\tau_B \eta B_B f_B^2 m_B m_W^2 S(y_t)|K_{td}^* K_{tb}|^2 \tag{2.3.40}$$

where τ_B is the B_d^0 lifetime, f_B the decay constant, B_B the bag parameter and η a QCD correction factor. Experimentally we have [11]

$$r_d = 0.21 \pm 0.08 \qquad \Longrightarrow \qquad x_d = 0.73 \pm 0.18 \tag{2.3.41}$$

and

$$\begin{array}{rclrcl} m_B & = & 5.28\, GeV & \tau_B & = & (1.16 \pm 0.16) \times 10^{-12} s \\ \eta & = & 0.85 & 0.1\, GeV \leq & f_B\sqrt{B_B} & \leq \; 0.2\, GeV. \end{array} \tag{2.3.42}$$

Armed with the resources of this chapter, we can now attack the problem of quark mass matrices and the various ansätze for them. The three experimental inputs discussed here *viz.* ϵ_K, ϵ'_K, and x_d shall be used in the next chapter to check for the phenomenological validity of various models for quark masses.