Chapter 4

Neutrino Masses and some Consequences

As opposed to the last two chapters, we shall concentrate here solely on certain aspects of neutrino physics. To begin with, we discuss the different types of mass terms possible for neutrinos and go on to give an outline of the most general case. A short discussion on the non-trivial consequences of neutrino mixing and oscillations follows next. The question of distinguishability of Dirac and Majorana particles leads us to the feature of neutrinoless double beta decay.

In the second part of the current chapter we present a new discussion on the connection between the Majorana mass of the neutrino and the neutrinoless double beta decay $[(\beta\beta)_{0\nu}]$ rate. It is argued that contrary to conventional wisdom, the latter does not distinguish between the Dirac and Majorana mass of the physical electron neutrino (ν_e). Building on this observation, we also identify scenarios where ν_e can naturally be a light Majorana neutrino with no $(\beta\beta)_{0\nu}$, and construct supersymmetric grand unified models that admit such possibilities.

4.1 Neutrino Masses

The question of neutrino masses is on a somewhat different footing than that of quark or charged lepton masses. For one, in the standard model the neutrinos are assumed to be strictly massless. This is forced upon us not by some theoretical constraint but rather by our failure so far to conclusively detect any non-zero mass for the neutrinos. However any negative experimental result is only as good as the resolution limits of the apparatus and in this case a lot of room is still left.

To achieve a symmetry between the leptonic and the hadronic sectors, one would like to consider the case where the neutrino does have a small (but non–zero) mass and look for consequences thereof. In this chapter we would venture to do the same.

As soon as one postulates a non-zero m_ν, one has to go beyond the minimal standard model as the lack of both ν_R's as well as triplet scalars in the SM prevents such a mass term. The simplest way then is to introduce one or more ν_R and as these are gauge singlets, the anomaly cancellation is not affected. One can then have m_ν through the usual method of Yukawa couplings and spontaneous symmetry breaking. However there is more to neutrino mass than just this and rather than duplicate the analysis in Chapter 2, we would take a different track.

To begin with, we digress somewhat to have a quick look at the charge conjugation properties of a spinor field:

$$C : \psi \to \psi^c \equiv C\overline{\psi}^T , \tag{4.1.1}$$

where C is a matrix in the Dirac space satisfying

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu, \quad C^\dagger C = 1, \quad C^T = -C. \tag{4.1.2}$$

A look at the Dirac equation then shows that

$$\begin{aligned} C : \quad \psi_L \to \quad (\psi_R)^c = C\overline{\psi_R}^T = P_L\psi^c \\ C : \quad \psi_R \to \quad (\psi_L)^c = C\overline{\psi_L}^T = P_R\psi^c , \end{aligned} \tag{4.1.3}$$

where $P_{L,R}$ are the left– and right–projection operators respectively. This is to say that the charge conjugation operator takes the state vector of a given particle to that of its antiparticle while preserving the momentum and helicity.

For a neutral left-handed particle we can then write a mass term of the form

$$m\overline{(\psi_L)^c}\psi_L ,$$

where m is either a bare mass term or arises from a $v.e.v.$ of some scalar as the case may be. Such a mass term is different from the usual Dirac term as it involves a field of only one helicity and is obviously absent for charged particles as that would violate charge conservation.

In the general case where one has n ν_L fields and m ν_R fields, the most general neutrino mass term in the Lagrangian would read

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{\nu_L} M_L \nu_L^c + \frac{1}{2}\overline{\nu_R} M_R \nu_R^c + \overline{\nu_L} M_D \nu_R + H.c.\,, \tag{4.1.4}$$

where M_L, M_R and M_D are matrices of dimension $n \times n$, $m \times m$ and $n \times m$ respectively. Now given any two fields ψ and χ,

$$\overline{\psi} M \chi^c = \overline{\psi} M C \overline{\chi}^T = \overline{\chi} M \psi^c. \tag{4.1.5}$$

Hence M_L and M_R are symmetric matrices. Writing

$$n_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} \quad \text{and} \quad \mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R^\dagger \end{pmatrix}, \tag{4.1.6}$$

we have

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{n_L} \mathcal{M} n_L^c.$$

Like M_L and M_R, $\mathcal{M}$ is also a complex symmetric matrix and hence can be "diagonalized" by an unitary matrix U such that

$$U^* \mathcal{M} U = \mathcal{M}_{diag}\,,$$

where U diagonalizes $\mathcal{M}^\dagger \mathcal{M}$ (See section 2.1). Defining

$$\chi = U^T n_L + U^\dagger {n_L}^c, \tag{4.1.7}$$

we have

$$-\mathcal{L}_{\nu\,\mathrm{mass}} = \frac{1}{2}\overline{\chi} \mathcal{M}_{diag} \chi\,. \tag{4.1.8}$$

Obviously $\chi_k^c = \chi_k$ and hence these are Majorana particles [40].

If $M_L = 0 = M_R$, then the eigenvalues of M are either zero ($|m-n|$ in number) or resolve into $min(m,n)$ pairs of the form $\pm m_i$ (m_i are complex). In the second case,

$$m(\overline{\chi_+}\chi_+ - \overline{\chi_-}\chi_-) = m(\overline{\chi_D}\chi_D + H.c.)\,, \tag{4.1.9}$$

where

$$\chi_D \equiv \frac{1}{\sqrt{2}}(\chi_+ + \gamma_5 \chi_-). \tag{4.1.10}$$

Thus two Majorana neutrinos with equal and opposite masses but the same CP properties (or equivalently degenerate neutrinos with opposite CP phases) combine to give a Dirac neutrino of the same mass. The number of degrees of freedom obviously remains the same.

At this stage one might ask whether the differences between a Majorana and a Dirac neutrino are limited only to their abstract vector space properties or if there exists any measurable quantity that distinguishes them. A more general question is that regarding the observational consequences of the neutrino mass matrix. We attempt to answer the last question first, not only because it closely parallels the discussion in Chapter 2, but also because the first problem, in a sense, is only a subset of the second.

4.1.1 Neutrino Mixing and Oscillations

Proceeding in a fashion analogous to that for the quarks, we define the charged lepton mass basis by

$$l_L = L_L l'_L \quad \text{and} \quad l_R = L_R l'_R, \tag{4.1.11}$$

where $L_{L,R}$ diagonalize the lepton mass matrix M^l through the biunitary transformation

$$L_L^\dagger M_l L_R = \widehat{M}_l. \tag{4.1.12}$$

Assuming now that all the left handed neutrinos are part of $SU(2)_L$ doublets with hypercharge $Y = -\frac{1}{2}$, and all right handed neutrinos are gauge singlets, we have then, for the relevant charged current

$$J_\mu^+ = \sum_{i,j=1}^{n} \sum_{\alpha=1}^{n+m} \overline{l_{Li}} \gamma_\mu \chi_{L\alpha} (L_L^\dagger)_{ij} (U^*)_{j\alpha} \tag{4.1.13}$$

leading to an effective neutrino mixing matrix (analogous to the CKM matrix) K^ν given by

$$(K^\nu)_{i\alpha} = \sum_{j=1}^{n} (L_L^\dagger)_{ij} (U^*)_{j\alpha}. \tag{4.1.14}$$

Notice that unlike in the hadronic case, we not only have the neutrino–CKM to be non-unitary, but it is rectangular $[n \times (n+m)]$ to boot. One has

$$(K^\nu K^{\nu\dagger})_{ik} = \delta_{ik} \quad \text{but} \quad (K^{\nu\dagger} K^\nu)_{\alpha\beta} = \sum_{k=1}^{n} U^T_{\alpha k} U^*_{k\beta}.$$

The non–orthogonality also manifests itself in the neutral current interactions, the relevant isotriplet part of which is given by

$$J^3_\mu = \sum_{i=1}^{n} \overline{\nu_{iL}}\gamma_\mu \nu_{iL} = \sum_{\alpha,\beta=1}^{n+m} (K^{\nu\dagger} K^\nu)_{\alpha\beta} \overline{\chi_{\alpha L}} \chi_{\beta L}.$$

Parameter counting in this case is slightly different from that in the hadronic sector. K^ν is best recognized as being a rectangular part of a $(n+m) \times (n+m)$ unitary matrix and hence, in the most general case is given by ${}^{n+m}C_2$ angles and ${}^{n+m+1}C_2$ phases. However, we can't proceed as for the quarks and eliminate $2(n+m) - 1$ phases by redefinition of wavefunctions, for the Majorana neutrinos obviously cannot absorb phase transformations. At most n phases can be eliminated by redefining only the charged lepton wavefunctions and thus we are left with ${}^nC_2 + \frac{m(2n+m+1)}{2}$ CP violating phases. It seems quite logical then that this difference can be exploited to distinguish a Majorana neutrino from a Dirac one, but Schechter and Valle [41] have shown that these extra CP violating effects are always suppressed by an additional factor of $(m_\nu/E_\nu)^2$, where m_ν and E_ν respectively are the mass and energy of the Majorana neutrino taking part in the process. The suppression is easily understood by appreciating that a process dependent on the Majorana mass must have an amplitude proportional to the latter and hence for dimensional reasons there has to be a suppression factor given by the relevant energy scale in the problem.

As in the case of the K^0–$\overline{K^0}$ system, we have, in the general case, a number of neutrinos with possibly all different masses mixing with each other. While the interaction terms in the Lagrangian conserve the individual lepton numbers (for a definition of lepton numbers, see section 4.1.2), the mass terms do not, and in the case of Majorana neutrinos even the total lepton number is not preserved. As a neutrino with definite interaction properties evolves in time, each of its massive modes propagates differently resulting in a periodic variation in their relative proportions in the generic neutrino 'beam'. Analogous to strangeness oscillations for the neutral kaons, we have then the possibility of lepton number oscillations [42].

To start with, we take a quick look at the oscillation of neutrinos in vacuum. In this section we shall adopt a slightly different and unorthodox notation. We extend the definition of flavour eigenstates to include the right–handed neutrinos as well, and shall denote them by $|\nu_i\rangle$ (where $i = 1...N(= n+m)$). Identifying the mass eigenstates as $|\chi_k\rangle$ as before, we have

$$|\nu_i(t=0)\rangle = \sum_{k=1}^{N} U_{ik}|\chi_k(t=0)\rangle \quad \text{and} \quad |\chi_k(t=0)\rangle = \sum_{i=1}^{N} U^*_{ik}|\nu_i(t=0)\rangle \tag{4.1.15}$$

where U is a $N \times N$ unitary matrix for N neutrino species. Then

$$\begin{aligned} |\nu_i(t)\rangle &= \sum_{k=1}^{N} U_{ik} e^{-iE_k t}|\chi_k(t=0)\rangle \\ &= \sum_{k=1}^{N} U_{ik} e^{-iE_k t} \sum_{j=1}^{N} U^*_{jk}|\nu_j(t=0)\rangle \end{aligned} \tag{4.1.16}$$

and thus

$$P_{\nu_i \to \nu_j}(t) \equiv |\langle \nu_j(t=0)|\nu_i(t)\rangle|^2 = \left|\sum_{k=1}^{N} U_{ik} U^*_{jk} e^{-iE_k t}\right|^2 . \tag{4.1.17}$$

Assuming the neutrinos do not decay,

$$\sum_{j=1}^{N} P_{\nu_i \to \nu_j}(t) = 1. \tag{4.1.18}$$

Now

$$CPT \text{ theorem} \implies P_{\bar{\nu}_j \to \bar{\nu}_i}(t) = P_{\nu_i \to \nu_j}(t) \tag{4.1.19}$$

$$\text{while} \quad CP \text{ conservation} \implies P_{\bar{\nu}_i \to \bar{\nu}_j}(t) = P_{\nu_i \to \nu_j}(t). \tag{4.1.20}$$

It is easy to see that in the case of two neutrino species, (4.1.18) and (4.1.19) together imply (4.1.20) and thus to detect CP violation in neutrino oscillations, one requires at least three neutrinos to mix (a not unexpected conclusion). However, henceforth we shall, for the sake of simplicity, assume that leptonic CP violation is absent and hence the matrix U shall be treated to be an orthogonal one.

Although a general study of the neutrino oscillation problem is quite a straightforward one, the physics issues involved are more transparent if one restricts oneself to the simplest possible case, namely that of only two neutrinos, say ν_e and ν_μ. The mixing matrix U then simplifies to

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} .$$

If we further assume that each neutrino is light enough so that we can write for its energy, $E \simeq p + \frac{m^2}{2p}$, where p is the momentum, we have

$$P_{\nu_e \to \nu_\mu}(R) = \frac{1}{2}\sin^2 2\theta \left[1 - \cos\frac{2\pi R}{L}\right] , \tag{4.1.21}$$

where R is the detector distance and

$$L \equiv \frac{4\pi p}{\Delta m^2} = 2.5 \frac{p\,(MeV)}{\Delta m^2\,(eV^2)} \text{ meters} \tag{4.1.22}$$

is the oscillation length. Here $\Delta m^2 = m_1^2 - m_2^2$ is the difference in neutrino mass squares. Thus for oscillations to be visible, one not only needs a non-zero mixing angle θ, but also $R \gtrsim L$. In practice however, it is not easy to recognize oscillation employing a single detector as one must average over the uncertainties in the zone of beam formation and detection *etc.*, leading to

$$\langle P_{\nu_e \to \nu_\mu} \rangle \approx \frac{1}{2} \sin^2 2\theta \quad \text{and} \quad \langle P_{\nu_e \to \nu_e} \rangle \approx 1 - \frac{1}{2} \sin^2 2\theta.$$

One of the prime motives behind the study of neutrino oscillations was the possibility of a resolution of the solar neutrino problem. The problem (real or imaginary, depending on the prejudices of the person concerned) lies in the low solar neutrino count in the Davis experiment [43] as compared to the predictions of the standard solar model [44]. An interesting solution would be to invoke transformation of the solar ν_e to ν_μ or ν_τ (which the Davis experiment cannot detect) while traversing the distance to earth. But the restrictions imposed by the terrestrial experiments on the $\sin 2\theta$–Δm^2 plane rules out a dominant role for vacuum oscillations in this context. A more practical solution lay in considering the effect of matter on neutrino oscillations [45]. While ν_e travelling in matter suffers both charged current (*c.c.*) and neutral current (*n.c.*) interactions, the other species have only the *n.c.* interactions. This induces an additional potential term proportional to the electron density for the ν_e or, equivalently, an extra term in the $(mass)^2$ matrix. With a matter density gradient, as is there in the Sun, this results in an quantum mechanical eigenvalue cross–over problem and consequently, in the adiabatic approximation, in a resonant conversion of ν_e to say, ν_μ [46]. This mechanism could magnify the oscillation effects due to even a small vacuum mixing angle sufficiently enough to explain the rather large discrepancy. But even this mechanism cannot explain the reported anticorrelation [47] between the solar magnetic activity and the observed neutrino flux. An explanation for such a behaviour is found if one ascribes a non–zero magnetic dipole moment to the neutrino thus enabling the solar magnetic field to rotate ν_e to some sterile (in the Davis context) species.

4.1.2 Neutrinoless Double Beta Decay

The most distinguishing feature of a Majorana mass term is the explicit breaking of a symmetry of the Lagrangian that its existence implies. In the absence of such terms, the Lagrangian is invariant under the global transformation

$$l'_{iL} \to e^{i\theta} l'_{iL}, \quad e'_{iR} \to e^{i\theta} e'_{iR}, \quad \nu'_{iR} \to e^{i\theta} \nu'_{iR}. \tag{4.1.23}$$

This obviously leads to an exactly conserved charge L (the lepton number, with values ± 1 for (anti–)leptons and zero for all other particles) with the consequence that the electroweak interactions (and trivially the strong interactions too) preserve the relative abundance of leptons over antileptons. However the individual flavour numbers are not conserved, leading to possible decays like

$$\mu \to e + \gamma, \quad \mu \to 3e, \quad K \to \pi \mu e. \tag{4.1.24}$$

On the other hand, if both the neutrino and the electron mass matrices be simultaneously diagonalizable, or in other words, if the neutrino mixing matrix is but a phase matrix, then the Lagrangian is invariant under independent global transformations

$$l'_{iL} \to e^{i\theta_i} l'_{iL}, \quad e'_{iR} \to e^{i\theta_i} e'_{iR}, \quad \nu'_{iR} \to e^{i\theta_i} \nu'_{iR}. \tag{4.1.25}$$

This leads to individually conserved lepton flavour numbers L_i (the corresponding invariances in the hadronic case are explicitly broken down to a conserved total baryon number by the non–zero quark mixings). In such a case, the interactions as in (4.1.24) are obviously absent.

With the introduction of the Majorana mass term (either M_L or M_R), even the total lepton number no longer remains a symmetry. In fact, a non–zero Majorana mass implies the existence of a propagator of the form $\langle \chi \chi^T \rangle$, leading to L violation by two units. This effect manifests itself most dramatically in neutrinoless double beta decay. It is to be noted, however, that existence of Majorana mass terms need not mean absence of any conserved lepton charge. For it might so happen that a certain (or more) combination(s) of L_i may still be a good symmetry. A particularly simple case is that of two left–handed neutrinos ν_{eL} and $\nu_{\mu L}$ such that the mass matrix reads

$$M_\nu = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} .$$

The Lagrangian then has a $U(1)$ invariance under which the two lepton generations being considered have opposite charges, *i.e.*,

$$
\begin{array}{ll}
e'_{L,R} \rightarrow e^{i\theta} e'_{L,R} & \nu'_{eL} \rightarrow e^{i\theta} \nu'_{eL} \\
\mu'_{L,R} \rightarrow e^{-i\theta} \mu'_{L,R} & \nu'_{\mu L} \rightarrow e^{-i\theta} \nu'_{\mu L}
\end{array}
$$

We have then a Dirac neutrino mass term of the form $m\bar{\nu}\nu$ where $\nu \equiv \nu'_{eL} + (\nu'_{\mu L})^c$. The conserved charge in this case is given by $L = L_e - L_\mu$ and was introduced by Zeldovich, Konopinsky and Mahmoud [48].

In 1937, Racah [49] pointed out that if the neutrino emitted by a neutron is a Majorana particle, then it can stimulate the decay of a second neutron. Furry [50] then pointed out that this neutrino can be a virtual one thus inventing the process of neutrinoless double beta decay [$(\beta\beta)_{0\nu}$]. We now expect that the $(\beta\beta)_{0\nu}$ experiment will give us the physical Majorana mass of the neutrino. Since a Dirac particle can be thought of as two Majorana particles with opposite CP properties, their contributions to $(\beta\beta)_{0\nu}$ cancel [51,52]. Thus we also expect that $(\beta\beta)_{0\nu}$ experiments will allow us to distinguish between a Dirac and a Majorana particle.

4.2 Naturally Light Majorana Neutrinos with no Neutrinoless Double Beta Decay

In this section [1], from a general analysis of the neutrino mass matrix, we argue that the $(\beta\beta)_{0\nu}$ amplitude does not depend on the physical Dirac or Majorana mass of the electron neutrino. We discuss the situation in which ν_e is a Majorana neutrino (and may even be the only light one) and yet there is no $(\beta\beta)_{0\nu}$. Conversely we also know of situations wherein there is $(\beta\beta)_{0\nu}$ inspite of the ν_e being a Dirac particle. We then proceed to construct certain supersymmetric grand unified theories that naturally have a light Majorana ν_e with no $(\beta\beta)_{0\nu}$ or, on the contrary, a massless ν_e with considerable $(\beta\beta)_{0\nu}$. Thus experimental signature or otherwise of $(\beta\beta)_{0\nu}$ gives very little information about the neutrino masses.

As is evident from the discussion in the last section, a mass matrix of the form in eqn.(4.1.6) in general induces $(\beta\beta)_{0\nu}$. It has been shown [54] that the amplitude for this

[1] Based on the work in ref. [53]

event goes as

$$A\left((\beta\beta)_{0\nu}\right) \propto \langle m \rangle \equiv \sum_{k=1}^{n} \left(U_{ek}\right)^2 m_k F(m_k, N) \tag{4.2.1}$$

where $F(m_k, N) = \langle e^{-m_k r}/r \rangle \langle 1/r \rangle^{-1}$, the average being done over the nucleus N in question. (In this section we would, for the sake of simplicity, assume that the charged lepton mass matrix is diagonal and hence L_L is the identity matrix.) For neutrinos lighter than a few MeV the suppression factor F is nearly one and then one has

$$\langle m \rangle \approx \sum_{k=1}^{n+m} \left(U_{ek}\right)^2 m_k = \mathcal{M}_{ee} \tag{4.2.2}$$

the last equality following from the definition of U. Thus for light neutrinos, $(\beta\beta)_{0\nu}$ level depends only on $\mathcal{M}_{ee}$. (Although this result was obtained by Wolfenstein [51] in 1981, its significance was not quite appreciated.) It is quite independent of whether ν_e is massless or if massive, whether it is of the Dirac or Majorana type.

However if one or more of the ν–species are too heavy to be kinematically produced inside the nucleus, then the effective mass $\langle m \rangle$ gets modified to

$$\langle m \rangle = \sum_{\text{light } \nu} \left(U_{ek}\right)^2 m_k F(m_k, N) \approx \sum_{\text{light } \nu} \left(U_{ek}\right)^2 m_k. \tag{4.2.3}$$

The last approximate equality follows under the assumption that the neutrinos are either too heavy to be of kinematic importance or quite light, *i.e.* their masses do not lie in the MeV region. In this case $\mathcal{M}_{ee}$ is no longer a measure of $(\beta\beta)_{0\nu}$.

To consider a concrete case, assume the mass matrix to be of the form

$$M = \begin{pmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{pmatrix} \tag{4.2.4}$$

where $M_{1,3}$ are real symmetric matrices and one has a hierarchy $M_1 \ll M_2 \ll M_3$ such that $M_1 \sim O(M_2 M_3^{-1} M_2^T)$. M can then be approximately block diagonalized by an orthogonal matrix

$$V = \begin{pmatrix} 1 - \frac{1}{2}\rho^T \rho & \rho^T \\ -\rho & 1 - \frac{1}{2}\rho\rho^T \end{pmatrix} \tag{4.2.5}$$

where $\rho = M_3^{-1} M_2^T$. Then one has

$$M_{BD} \equiv V^T M V = \begin{pmatrix} \widetilde{m} & 0 \\ 0 & \widetilde{M} \end{pmatrix} + O(\rho^2 M_1) \tag{4.2.6}$$

where

$$\widetilde{m} = M_1 - M_2 \rho \quad \text{and} \quad \widetilde{M} = M_3 + \frac{1}{2}(\rho M_2 + M_2^T \rho^T). \tag{4.2.7}$$

Let $K_{1,3}$ be orthogonal matrices such that $K = diag(K_1, K_3)$ diagonalizes M_{BD}. Then

$$U \equiv VK = \begin{pmatrix} (1-\frac{1}{2}\rho^T\rho)K_1 & \rho^T K_3 \\ -\rho K_1 & (1-\frac{1}{2}\rho\rho^T)K_3 \end{pmatrix} \tag{4.2.8}$$

diagonalizes the mass matrix M. If we assume the eigenvalues of M to be very large, then the effective mass $\langle m \rangle$ for $(\beta\beta)_{0\nu}$ is given by

$$\langle m \rangle = \sum_{k \in \tilde{m}} (U_{ek})^2 m_k = \tilde{m}_{11} \tag{4.2.9}$$

Thus if we block diagonalize the mass matrix M into two blocks $\tilde{m}$ and $\widetilde{M}$, such that the eigenvalues of $\widetilde{M}$ are much larger than $1MeV$ while those of $\tilde{m}$ are not, then the $(\beta\beta)_{0\nu}$ amplitude depends only upon the element $(\tilde{m})_{11}$ and not on the actual eigenvalues of $\tilde{m}$. We are here working in the basis $(\nu_e \nu_a \nu_b \cdots)$ where ν_a, $\nu_b \ldots$ can either be of different generations or sterile.

Let us now consider a few special cases. If $M = \begin{pmatrix} 0 & m \\ m & m' \end{pmatrix}$ where $m' \gg 1MeV \gg m$, then we get a light Majorana neutrino of mass m^2/m'. In this case $M_{11} = 0$, yet we do get a nonzero contribution to $(\beta\beta)_{0\nu}$. This is the well-known see-saw mechanism [55]. On the other hand, if in the same mass matrix we have $m, m' \lesssim 1MeV$, then M itself describes low energy ν–phenomena and we do not have to take recourse to constructing $\tilde{m}$. In this case though we have two Majorana particles of masses $\left(m' \pm \sqrt{m'^2 + 4m^2}\right)/2$, yet $A\left((\beta\beta)_{0\nu}\right) = 0$ as $M_{11} = 0$.

We have demonstrated two situations. In both the cases $M_{11} = 0$, but while $(\beta\beta)_{0\nu}$ is present in one, it is absent in the other. In either case the physical neutrino is a Majorana particle. Let us now consider the case when it is a Dirac particle, at least at the tree level. Since our analysis does not depend on the radiative corrections, we shall not talk about loop effects. Consider the mass matrix $M = \begin{pmatrix} m' & m \\ m & -m' \end{pmatrix}$. This corresponds to two Majorana particles of equal masses $\sqrt{m'^2 + m^2}$ with opposite CP properties, and hence they combine to give a two helicity state Dirac neutrino. Both the eigenvalues being equal we can ignore the factor F and write $\langle m \rangle \propto m'$. Now we can have two scenarios : $m = 0$ or $m \neq 0$. Each will predict a Dirac neutrino [56] but in the first there isn't any $(\beta\beta)_{0\nu}$ whereas in the second it does appear.

Although in the simplest example cited above, if the physical neutrino is a Dirac particle, then the contribution to $(\beta\beta)_{0\nu}$ is given by $\langle m\rangle = M_{11}$, this is not the case in general. Consider for example

$$M = \begin{pmatrix} 0 & 0 & 0 & m \\ 0 & 0 & m & 0 \\ 0 & m & m' & 0 \\ m & 0 & 0 & -m' \end{pmatrix}$$

where $m' \gg m$. This mass matrix predicts one light Dirac neutrino of mass m^2/m' and also gives nonzero $(\beta\beta)_{0\nu}$ ($\langle m\rangle = m^2/m'$) although $M_{11} = 0$.

The question we attempt to answer next is the one regarding the naturalness of the above arguments. We have demonstrated many scenarios which, in principle, can exist. But if we cannot get them naturally from any realistic theory, then it does not make much sense.

Models [57,58] were constructed to predict light Dirac neutrinos naturally, which give no $(\beta\beta)_{0\nu}$. The most popular versions start with three additional sterile neutrinos per generation. Then using some symmetry of the theory one gets a mass matrix in the $(\nu_e\,\nu_a\,\nu_b\,\nu_c)$ basis of the form [57]

$$M = \begin{pmatrix} 0 & 0 & A & 0 \\ 0 & 0 & B & C \\ A & B & 0 & 0 \\ 0 & C & 0 & 0 \end{pmatrix}$$

where $B \gg A, C$. This predicts a light Dirac neutrino of mass $AC/B \sim$ a few eV, so that this can explain the ITEP result [59] of $m_\nu \sim 20eV$, as well as the absence of $(\beta\beta)_{0\nu}$ [60].

We shall proceed in a similar fashion to demonstrate a scenario where we have a light Majorana neutrino with $m_\nu \sim 20eV$ but no $(\beta\beta)_{0\nu}$. The model can also accomodate a 17 keV Majorana ν with a small mixing with ν_e (similar to that seen by Simpson [62,63] albeit with a smaller mixing). The numbers are not very special to the model. What we would like to emphasize is that one can obtain light Majorana neutrinos from realistic GUTs naturally which do not admit $(\beta\beta)_{0\nu}$. We also start with three sterile neutrinos alongwith ν_e and seek to get in the $(\nu_e\,\nu_a\,\nu_b\,\nu_c)$ basis a mass matrix of the form

$$M = \begin{pmatrix} \alpha & 0 & 0 & a \\ 0 & 0 & k & 0 \\ 0 & k & 0 & G \\ a & 0 & G & B \end{pmatrix} \tag{4.2.10}$$

where $G \gg k, B \gg a \gg \alpha$. In fact α can even be zero. This mass matrix can be block diagonalized to $M_{BD} = diag(\tilde{m}, \widetilde{M})$ with

$$\tilde{m} = \begin{pmatrix} \alpha & -ak/G \\ -ak/G & k^2B/G^2 \end{pmatrix} \quad \text{and} \quad \widetilde{M} \approx \begin{pmatrix} 0 & G \\ G & B \end{pmatrix}. \tag{4.2.11}$$

Then if $aG \ll kB$, we have four Majorana neutrinos with masses

$$m_1 = \alpha - a^2/B, \quad m_2 = k^2B/G^2, \quad m_{3,4} = B \pm G/2 \tag{4.2.12}$$

With a suitable choice for the 5 parameters appearing in M we can obtain two light neutrinos and two superheavy ones. The ν–less double beta decay amplitude is proportional to $(\tilde{m})_{11} = \alpha$ as two of the neutrinos are too heavy to be kinematically produced at the ordinary decay energies.

The most interesting aspect of this exercise is the relation between α and m_1 . As has been mentioned earlier, α is a parameter in the mass matrix much smaller than the others. In the explicit models to be considered later, it turns out to be of the order of a^2/B or smaller. Thus we have three possibilities:

a) α of the same order as m_1 : This gives the usual picture of $(\beta\beta)_{0\nu}$ being proportional to the Majorana mass of ν_e;

b) $\alpha \approx 0$: then we get a light Majorana ν without appreciable $(\beta\beta)_{0\nu}$.

c) $\alpha \approx a^2/B$: this leads to a very small Majorana mass but a rather large amount of $(\beta\beta)_{0\nu}$.

We now proceed to present a model based on a supersymmetric $SO(10)$ grand unified theory in which the hierarchy of the parameters that we require appears naturally. We do not aim to construct a complete gauge theory; rather we give an illustration of how we can get light Majorana neutrinos, with no $(\beta\beta)_{0\nu}$. This model is on the same footing as those which predict light Dirac neutrinos. In particular, we require one $U(1)$ global symmetry to get the required form of the mass matrix as compared to the three $U(1)$ symmetries required for a light Dirac neutrino in a similar model.

We shall focus our attention on a single family assuming intergeneration mixing to be small. We start with a $SO(10)$ model with two fermion singlet superfields S and S' in addition to the usual 16-plet matter superfield χ. The symmetry breaking chain being considered is

$$\begin{aligned} SO(10) &\xrightarrow{M_{GUT}} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\ &\xrightarrow{M_{LR}} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ &\xrightarrow{M_{wk}} SU(3)_c \otimes U(1)_Q \end{aligned}$$

To give masses to the fermion fields as well as to prompt the last two stages of the above symmetry breaking chain, we introduce four Higgs superfields $\chi(\overline{16})$, $\Phi(10)$, $\Delta(126)$ and $\sigma(1)$ where the numbers in parentheses denote their transformation properties under $SO(10)$. The most general Yukawa coupling allowed then is

$$\mathcal{L}_Y = \overline{\psi^c}\psi(f_1\Phi + f_2\Delta) + \overline{\psi^c}\chi(f_3S + f_4S') + (f_5SS' + f_6SS + f_7S'S')\sigma.$$

We impose an additional U(1) global symmetry, the non-trivial transformations under it being

$$\chi \to e^{i\theta}\chi \qquad \sigma \to e^{-i\theta}\sigma \qquad S \to e^{-i\theta}S \qquad S' \to e^{2i\theta}S' \tag{4.2.13}$$

This eliminates the Yukawa couplings given by f_4, f_6 and f_7 as well the bare mass terms for S and S'. Then in the basis $(\nu\ S\ S\ \nu^c)$, the mass matrix reads as in eqn(4.2.10), and with a particular hierarchy of $v.e.v.$s [64]:

$$\begin{array}{llllllll} G &=& f_3\langle\chi\rangle & \sim O(10^9\,GeV), & B &=& f_2\langle\Delta\rangle & \sim O(10\,TeV), \\ k &=& f_5\langle\sigma\rangle & \sim O(10\,TeV), & a &=& f_1\langle\Phi\rangle & \sim O(10\,MeV) \text{ and} \\ \alpha &\sim& O(10\,eV). & & & & & \end{array}$$

While the value of G is self-evident, the others need some explanation. The scales of k and B, proportional to M_{SUSY}, appear naturally in a certain class of supersymmetric models where the corresponding scalars remain massless at the tree level, only to gain mass through radiative corrections. a reflects the electroweak breaking scale, assuming a value comparable to the light quark masses. A small value of α (proportional to $\langle\Phi\rangle^2/\langle\Delta\rangle$) is generated due to the features of potential minimization in a left-right symmetric model.

An $SU(5)$ analog of this model can easily be constructed using three singlet fermions $(S_{1,2,3})$ apart from the usual 10 (χ) and 5 (ψ) superfields. The Higgs sector is enlarged to accomodate two singlets (σ_1 and σ_2) and a 5–plet $\tilde{\Phi}$ alongwith the usual 24–plet Σ and the 5–plet Φ. Then the imposition of an $U(1)$ symmetry:

$$\begin{array}{lll} S_1 \to e^{i\theta}S_1 & S_2 \to e^{3i\theta}S_2 & S_3 \to e^{2i\theta}S_3 \\ \sigma_1 \to e^{-4i\theta}\sigma_1 & \sigma_2 \to e^{-5i\theta}\sigma_2 & \tilde{\Phi} \to e^{-2i\theta}\tilde{\Phi} \end{array} \tag{4.2.14}$$

will give us the Yukawa coupling

$$\mathcal{L}_Y = (f_1\psi\chi + f_2\chi\chi)\Phi + g_1\psi S_3\tilde{\bar{\Phi}} + g_4 S_2 S_3\sigma_2 + (g_2 S_1 S_2 + g_3 S_3 S_3)\sigma_1$$

This gives a neutrino mass matrix of the form of eqn.(4.2.10) with

$$\begin{array}{rclcrcl} G &=& g_4\langle\sigma_2\rangle, & & k &=& g_2\langle\sigma_1\rangle, \\ B &=& g_3\langle\sigma_1\rangle, & & a &=& g_1\langle\tilde{\bar{\Phi}}\rangle \end{array}$$

with $a \ll k, B \ll G$. It is to be noted that in this case $\alpha = 0$. With a suitable choice (depending on the details of the model concerned) of a, k, B and G we obtain naturally light Majorana neutrinos with no $(\beta\beta)_{0\nu}$.

As an aside we point out that in these scenarios with a careful, but not too unnatural, choice of the Yukawa couplings one could simultaneously accomodate a 25 eV Majorana neutrino with very low $(\beta\beta)_{0\nu}$ rate alongwith a 17 keV neutrino with a small mixing. Also the cosmological constraint on the masses of stable light neutrinos would not pose much of a problem as the keV mass neutrino could be made to decay through Majorons [65].

In summary we argue that the widely held belief that the neutrinoless double beta decay experiments would give us the Majorana mass of the physical electron neutrino is tenable if and only if there is just a single species of ultralight neutrino per generation (as for example in a minimal extension of the standard model with the inclusion of right–handed neutrino singlets) and if the inter-family mixing is non-existent. But in a generic grand unified theory, where there are more than one type of neutrino per generation, it fails to go through. While the absence of $(\beta\beta)_{0\nu}$ cannot say anything about the mass matrix except that $\tilde{m}_{11} = 0$, its presence only confirms the existence of lepton number violation in nature and hence the presence of a Majorana mass term but does not distinguish between a pseudo-Dirac and a Majorana particle. This is exploited in the construction of a supersymmetric grand unified theory in which a Majorana particle that can explain the ITEP results while not succumbing to the $(\beta\beta)_{0\nu}$ constraints is naturally generated..