Chapter 5

On Some Exotic Neutrino Phenomenology

In the previous chapter we have contemplated the changes that need to be made in the minimal standard model so as to incorporate a non-zero neutrino mass. We have also looked at one of the consequences of a possible breakdown of a global lepton number symmetry that a Majorana mass for the neutrino might induce. At this stage one question begs to be asked. Why need we consider a mass for the neutrino at all? Apart from the rhetorical answer "Well, why not? Nothing prevents it anyway.", there is the deeper and more practical reason of its potential to answer many ill-understood problems. Neutrinos being very light (?) and weakly interacting particles, do not manifest themselves too dramatically at ordinary interaction energies, but at the astrophysical scales, they are expected to play a very crucial role. Moreover, in view of the recent spate of results, both experimental and theoretical, in neutrino physics, it is quite conceivable that this field might afford the most accessible testing ground for new physics beyond the standard model.

In this chapter we look at different aspects of "non-standard" neutrino physics. To begin with, we examine the question of a sizable magnetic moment for a very light neutrino. We propose a new mechanism that decouples the question of neutrino masses and magnetic moments and based on this, develop a model which generates a large transition magnetic moment for an ultralight ν_e in a natural way. Next we take a look at the consequences of a non-zero neutrino mass in the context of gravitational interactions. We find that contrary to expectations, for a low-energy neutrino at the vicinity of a supernova the gravitational interaction could be the dominant one. We use this result to put very strong bounds on parity-violating effects in gravity. Finally we move on to present a phenomenologically consistent model for Simpson's $17keV$ neutrino that naturally accomodates a large magnetic moment for the ν_e. We also look at the grtavitational interaction of this neutrino as well as its effect on $(\beta\beta)_{0\nu}$ rates. It is here that the results of the previous exercises are used as inputs to achieve a coherent picture of the problem in its entirety.

5.1 Large Magnetic Moment for Nearly Massless Neutrinos

The question of the compatibility of a large magnetic moment and a very small mass for the neutrinos, apart from being very interesting in itself, is of much importance as a way out of the solar neutrino puzzle [67]. For, the neutrino spin rotation (flavour-changing or otherwise) in conjunction with the matter oscillation effects could lead to a substantial reduction in the ν_e-flux — irrespective of the validity of the adiabatic approximation — thus explaining the discrepancy between the standard solar model prediction [44] and the Davis and Kamiokande results [43]. Moreover a substantial neutrino magnetic moment could play a crucial role in supernova dynamics [68].

That the problem is a non–trivial one is not difficult to appreciate. The magnetic moment term being a non–renormalizable one, cannot occur in the bare Lagrangian and may appear only at the one–loop level or higher. But the very same diagram that gives rise to a non–zero μ_ν also, when the photon line is removed, gives a mass correction. This leads to a proportionality between μ_ν and m_ν with the result that normally one cannot be enhanced while the other is being suppressed. For example, in a minimal extension of the standard model, one gets

$$\mu_\nu \approx 10^{-19} \frac{m_\nu}{1\,eV} \mu_B,$$

and hence it is impossible to generate $\mu_\nu \gtrsim 10^{-12}\mu_B$ (needed for this mechanism to play any meaningful role in the solar context) without being saddled with an unaccepatably large mass for the ν_e.

It was first noticed by Voloshin [69] that if ν_e and ν_e^c transform as a doublet under some $SU(2)_\nu$ symmetry, then while the magnetic moment term is invariant, the mass term behaves as a triplet. This was incorporated in $SU(3)_L \otimes U(1)_Y$ electroweak models [70]. A variant in which $SU(2)_\nu$ was some kind of a horizontal symmetry with $(\nu_e \ \ \nu_\mu)$ as a doublet was also considered [71]. In the limit of exact $SU(2)_\nu$ symmetry then, there exists no mass term but only a nonzero magnetic (transition) moment. The breaking of this symmetry however genarates masses, the proportionality of which to the magnetic moments can be kept down only by imposing certain naturalness conditions.

In this section [1] we aim to generalize Voloshin's argument and see if we can have scenarios wherein the neutrino magnetic moment can exist independent of its mass even after the symmetry breaking, thus rendering the naturalness conditions redundant. We extend the standard model to include a horizontal symmetry that treats all fermions on an equal footing. The lepton number violating higgs (σ) is also responsible for breaking the horizontal symmetry. Thus in the exact symmetry limit, both the Majorana masses and the transition moments vanish. This is so because $\overline{\nu^c}_i\nu_j$ and $\overline{\nu^c}_i\sigma_{\mu\nu}\nu_j F^{\mu\nu}$ both violate lepton number. Since we do not have tree level Majorana or Dirac mass terms for the neutrinos, the origin of both the transition moments and the ν_c–mass lie in the radiative corrections. To one loop order they can be parametrized in terms of dimension five operators with the mass suppression scale being decided by the internal higgs particles in the relevant diagrams. Thus if the couplings and the *v.e.v.*s in the theory could be so chosen that only the antisymmetric terms get any contribution from the diagrams containing $\langle\sigma\rangle$, then the νs would acquire a transition moment while keeping the mass correction zero. In the case where one of the internal higgs flowing in the diagram happens to be a horizontal group singlet, this can be ensured simply by seeing to it that the effective *v.e.v.* structure couples only to antisymmetric combination of the fermions.

The gauge group we consider is $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes O(3)_H$ with the particle representations as under (for the scalars the super and subscripts denote the electric charge and the T_{3H} quantum numbers respectively):

[1] Based on the work in ref. [66]

Fermions

$$\begin{aligned}
\psi_L &\equiv \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{pmatrix}_L && (1,2,-1/2,3) \quad L=1 \\
\psi_L^c &\equiv (\, e_L^c \;\; \mu_L^c \;\; \tau_L^c \,) && (1,1,1,3) \quad L=-1 \\
Q_L &\equiv \begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L && (3,2,1/6,3) \quad L=0 \\
U_L^c &\equiv (\, u_L^c \;\; c_L^c \;\; t_L^c \,) && (\bar{3},1,-2/3,3) \quad L=0 \\
D_L^c &\equiv (\, d_L^c \;\; s_L^c \;\; b_L^c \,) && (\bar{3},1,-1/3,3) \quad L=0
\end{aligned} \tag{5.1.1}$$

Higgs:

$$\begin{aligned}
\sigma &\equiv (\, \sigma_1^0 \;\; \sigma_0^0 \;\; \sigma_{-1}^0 \,) && (1,1,0,3) \quad L=2 \\
\Sigma &\equiv (\, \Sigma_1^0 \;\; \Sigma_0^0 \;\; \Sigma_{-1}^0 \,) && (1,1,0,3) \quad L=0 \\
\phi_s &\equiv \begin{pmatrix} \phi_s^0 \\ \phi_s^- \end{pmatrix} && (1,2,-1/2,1) \quad L=0 \\
\Phi &\equiv \begin{pmatrix} \phi_1^0 & \phi_0^0 & \phi_{-1}^0 \\ \phi_1^- & \phi_0^- & \phi_{-1}^- \end{pmatrix} && (1,2,-1/2,3) \quad L=0 \\
H &\equiv \begin{pmatrix} h_2^0 & h_1^0 & h_0^0 & h_{-1}^0 & h_{-2}^0 \\ h_2^- & h_1^- & h_0^- & h_{-1}^- & h_{-2}^- \end{pmatrix} && (1,2,-1/2,5) \quad L=0 \\
\widetilde{H} &\equiv \begin{pmatrix} \tilde{h}_2^0 & \tilde{h}_1^0 & \tilde{h}_0^0 & \tilde{h}_{-1}^0 & \tilde{h}_{-2}^0 \\ \tilde{h}_2^- & \tilde{h}_1^- & \tilde{h}_0^- & \tilde{h}_{-1}^- & \tilde{h}_{-2}^- \end{pmatrix} && (1,2,-1/2,5) \quad L=0 \\
\eta &\equiv (\, \eta_1^{1/3} \;\; \eta_0^{1/3} \;\; \eta_{-1}^{1/3} \,) && (\bar{3},1,1/3,3) \quad L=-1 \\
\chi &\equiv \begin{pmatrix} \chi^{2/3} \\ \chi^{-1/3} \end{pmatrix} && (3,2,1/6,1) \quad L=-1
\end{aligned} \tag{5.1.2}$$

Here L denotes the lepton number.

On imposition of a further discrete symmetry

$$\begin{aligned}
\psi_L^c &\to -\psi_L^c, & U_L^c &\to -U_L^c \\
\Phi &\to -\Phi, & \widetilde{H} &\to -\widetilde{H}
\end{aligned} \tag{5.1.3}$$

the most general Yukawa term in the Lagrangian would look like

$$\begin{aligned}
\mathcal{L}_Y = \; & f\overline{\psi_L^c}Q_L\eta + f'D_L^c\psi_L\chi + \psi_L^c\psi_L(g_3^e\Phi + g_5^e\widetilde{H}) \\
+ \; & U_L^cQ_L(g_3^u\Phi + g_5^u\widetilde{H}) + D_L^cQ_L(g_3^d\phi_s + g_5^dH)
\end{aligned} \tag{5.1.4}$$

The global lepton number symmetry will also ensure baryon number conservation, so that even after σ acquires a *v.e.v.* , although lepton number is violated there is no proton decay. Since there is no direct coupling of the fermions with σ, the coupling of the Goldstone boson (Majoron) corresponding to the global lepton number symmetry breaking with the fermions is suppressed by the horizontal scale and thus remains invisible. The strictest bounds [72] on η and χ come from rare K–decay rates leading to

$$\frac{f^2}{m_\eta^2}, \frac{f'^2}{m_\chi^2} \lesssim 10^{-4} G_F \,. \tag{5.1.5}$$

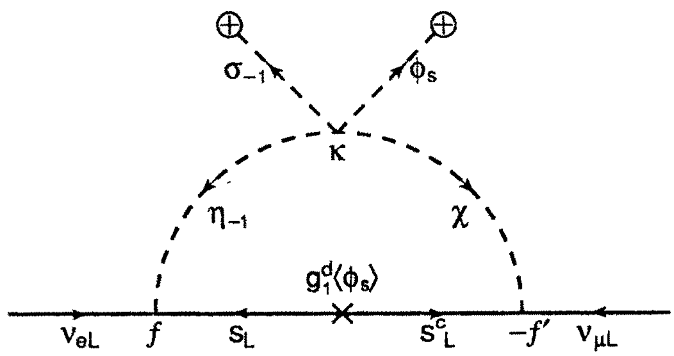
The $O(3)_H$ symmetry can be used to assure that of the three components of σ only σ_{-1} acquires a non-zero *v.e.v.* The assumption that of the five neutral h_i^0, only the $T_{3H} = -2$ component acquires a *v.e.v.* is consistent with this. To break the remaining $O(2)_H$ symmetry we choose $\langle \Sigma_0 \rangle \neq 0$. The scale for the horizontal symmetry breaking we choose to be of the order of $10^8 GeV$. The *v.e.v.*s are then

$$\begin{aligned}
\langle \sigma \rangle &= \begin{pmatrix} 0 & 0 & \langle \sigma_{-1}^0 \rangle \end{pmatrix} \\
\langle \Sigma \rangle &= \begin{pmatrix} 0 & \langle \Sigma_0^0 \rangle & 0 \end{pmatrix} \\
\langle \phi_s \rangle &= \begin{pmatrix} \langle \phi_s^0 \rangle \\ 0 \end{pmatrix} \\
\langle H \rangle &= \begin{pmatrix} 0 & 0 & 0 & 0 & \langle h_{-2}^0 \rangle \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} .
\end{aligned} \tag{5.1.6}$$

We have refrained from specifying $\langle \Phi \rangle$ and $\langle \widetilde{H} \rangle$ as these give masses only to the up-quark and the charged lepton sectors. Assuming the Yukawa couplings to be unity the most general form of mass matrices is then

$$\begin{pmatrix}
\langle \tilde{h}_2^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_1^0 \rangle - \langle \phi_1^0 \rangle & \frac{1}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle + \langle \phi_0^0 \rangle \\
-\frac{1}{\sqrt{2}} \langle \tilde{h}_1^0 \rangle + \langle \phi_1^0 \rangle & \frac{2}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_{-1}^0 \rangle - \langle \phi_{-1}^0 \rangle \\
\frac{1}{\sqrt{6}} \langle \tilde{h}_0^0 \rangle - \langle \phi_0^0 \rangle & -\frac{1}{\sqrt{2}} \langle \tilde{h}_{-1}^0 \rangle + \langle \phi_{-1}^0 \rangle & \langle \tilde{h}_{-2}^0 \rangle
\end{pmatrix} . \tag{5.1.7}$$

Thus there is a wide freedom to choose the Higgs couplings so as to obtain a *v.e.v.* structure amenable to giving phenomenologically consistent mass matrices in these sectors. We shall not discuss this sector any further.



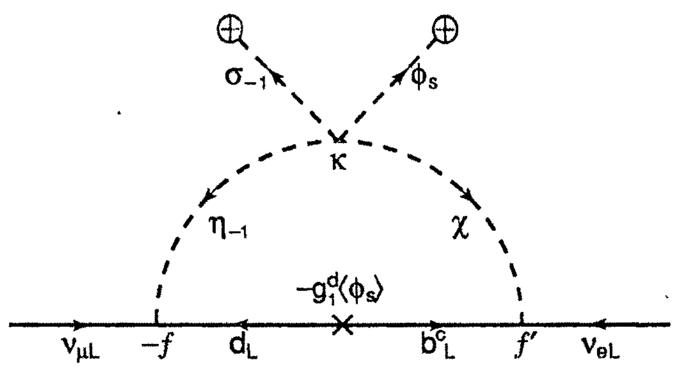


Figure 5.1: One–loop diagrams leading to possible neutrino mass corrections and transition magnetic moment $\mu_{\nu_e \nu_\mu}$.

The down-quark mass matrix reads

$$M_d = \begin{pmatrix} 0 & 0 & g_1^d \langle \phi_s^0 \rangle \\ 0 & -g_1^d \langle \phi_s^0 \rangle & 0 \\ g_1^d \langle \phi_s^0 \rangle & 0 & g_5^d \langle h_{-2}^0 \rangle \end{pmatrix} \tag{5.1.8}$$

For the eigenvalues one then gets the interesting hierarchy

$$m_s^2 = m_d m_b \tag{5.1.9}$$

a relation consistent with experimental data.

Of the multitude of terms in the Higgs potential, the one that interests us the most is $\kappa\phi_s\eta\chi\sigma$, where κ is a dimensionless constant. (Such a coupling for Σ is ruled out by lepton number conservation.) This gives rise to one loop diagrams as in Figure 5.1 that result in a non-zero transition moment of the form

$$\mu_{\nu_e\nu_\mu} = 2e\frac{ff'}{16\pi^2}m_s\frac{\kappa\langle\phi_s^0\rangle\langle\sigma_{-1}^0\rangle}{m_{\eta_{-1}}^2 m_{\chi^{1/3}}^2} \tag{5.1.10}$$

while the mass correction vanishes since the contribution from the two diagrams cancel exactly.

Assuming $\kappa \approx 1$, $\langle\phi_s^0\rangle \sim 100GeV$, $m_\chi \sim 50GeV$ and $m_\eta \sim 10^7 GeV$ and taking f^2/m_η^2, f'^2/m_χ^2 to be at the top of the allowed range one obtains a transition magnetic moment

$$\mu_{\nu_e\nu_\mu} = 10^{-12}\mu_B. \tag{5.1.11}$$

It should be noted that in the limit of exact $O(3)_H$ symmetry, $\mu_{\nu_e\nu_\mu}$ vanishes. But while it appears as a consequence of spontaneous breaking of $O(3)_H$, the mass term still remains zero on account of the absence of either a singlet or a 5-plet term in the effective *v.e.v.* structure. The naturalness condition required in refs.[70,71] to suppress the contribution of the Higgs mass splitting to the ν–masses, is redundant as because of χ being a $O(3)_H$ singlet only one set of scalars appear in the relevant diagrams.

It is easy to see that there are no diagrams giving rise to a mass to ν_τ or transition moments involving it. We have thus obtained a model in which the spontaneous breaking of the horizontal symmetry gives rise to a sole transition moment $\mu_{\nu_e\nu_\mu} \sim 10^{12}\mu_B$ while keeping all the neutrinos massless to 1–loop order without invoking any naturalness condition. In fact, the lepton number violating part of the relevant effective *v.e.v.* structure being a $O(3)_H$ triplet, there would be no radiative corrections to the neutrino mass. Thus the only source of a mass is the introduction of a tree level term as for example through a see–saw like mechanism induced by introduction of singlets. This analysis can easily be extended to the case of more than three genarations or higher symmetry groups. Care need only be taken that the effective lepton number violating higgs *v.e.v.* couples only to the antisymmetric combination(s) of the neutrinos.

Though such a small $\mu_{\nu_e\nu_\mu}$, while consistent with the bounds from supernova neutrino data might seem to be too uninteresting in the solar context, actually it is not so. For, coupled with a very small neutrino mass difference ($10^{-8}eV^2 < \Delta m^2 < 10^{-5}eV^2$) as is natural here, this could play a significant role in a moderately nonadiabatic evolution scenario [73].

Also one does not need to introduce extra higgs to suppress the influence of $\mu_{\nu_e \nu_\mu}$ during a supernova explosion.

5.2 Gravitational Helicity Flip of Neutrinos

If neutrinos did possess a small mass, they could, in principle, make a transition from a helicity state in which they would be dominantly participating in the electroweak process to one where they would have practically no interaction with matter. Such helicity flip mechanisms could drastically affect the evolution of astrophysical systems like neutron stars born in supernova explosions. Features most vulnerable to such helicity mechanisms are the cooling rate and the deleptonisation of the neutron star core. Normally one would expect the electromagnetic interactions of the neutrino (through μ_ν generated at the one-loop level) to give the dominant contribution to such effects. Gaemers *et al.* [75] have however argued that the Z-mediated process could be the more important one.

In this section [2] we wish to concentrate on a different mechanism for flipping the helicity of massive neutrinos that can be potentially important, namely that due to gravitational interactions. Neutron stars appear to be the most favourable candidates to look for such effects. The value of GM/Rc^2 for a typical neutron star of mass M and radius R, is around 0.1 and this is an important quantity which will lead to a sizeable helicity flip due to gravity as we shall see. In what follows, we shall treat the gravitational field in the so-called weak-field limit (also called the linearized approximation). It would however be desirable to formulate the contents of this section within the framework of general relativity.

The coupling of neutrinos to the gravitational field, strictly speaking, requires the introduction of tetrads (vierbeins). However in the weak-field limit the coupling can be described by an external field metric of the form

$$G\overline{\Psi}(P_2)[\gamma_\mu P_\nu + \gamma_\nu P_\mu]\Psi(P_1)h^{\mu\nu}(P_2 - P_1) \tag{5.2.1}$$

$$\text{where} \qquad P = \frac{1}{2}(P_2 + P_1) \qquad g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}. \tag{5.2.2}$$

Here P_1 and P_2 denote the initial and final neutrino momenta and G is the gravitational coupling strength.

[2] Based on the work in ref. [74]

For the moment, we shall assume that only the diagonal components of the stress tensor of the neutron star are important. This yields

$$h^{00} = GM/q^2 \quad \text{and} \quad h^{ij} = \delta^{ij} h^{00}\,, \tag{5.2.3}$$

where $q = |\mathbf{P}_2 - \mathbf{P}_1|$. At first sight this appears to be different from the Schwarzschild metric

$$\mathrm{d}s^2 = \left[1 - \frac{r_g}{r}\right] \mathrm{d}t^2 - \frac{\mathrm{d}r^2}{(1 - r_g/r)} - r^2[\sin^2\theta\, \mathrm{d}\phi^2 + \mathrm{d}\theta^2]\,, \tag{5.2.4}$$

(where r_g is the Schwarzschild radius) but the metric (5.2.3) is indeed the Schwarzschild metric expressed in the isotropic spherical coordinates [76]:

$$\mathrm{d}s^2 = \frac{(1 - r_g/4r)^2}{(1 + r_g/4r)^2} \mathrm{d}t^2 - \left[1 + \frac{r_g}{4r}\right]^4 [\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta\, \mathrm{d}\phi^2]. \tag{5.2.5}$$

The S-matrix element for the scattering of a massive neutrino of mass m is then given by

$$S_{fi} = -\frac{i}{V} \frac{m^2}{E_1 E_2} 2\bar{u}(\mathbf{P}_2, \lambda_2) P_\mu \gamma_\nu u(\mathbf{P}_1, \lambda_1) 2\pi\delta(E_2 - E_1) h^{\mu\nu}(q). \tag{5.2.6}$$

The differential cross section for helicity flip is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\pi^2} \frac{(GMm)^2}{q^2} 2EP(1 - \cos\theta)\,, \tag{5.2.7}$$

where $E(P)$ denote the initial neutrino energy (momentum). We have removed the subscripts since in the absence of recoil $E_1 = E_2 = E$ and $|\mathbf{P}_1| = |\mathbf{P}_2| = P$. The angle of scattering is denoted by θ . The total cross section for helicity flip is then given by

$$\sigma = \frac{(GMm)^2}{\pi} 2\frac{E^2}{P^4} \ln \frac{q_{max}}{q_{min}} \tag{5.2.8}$$

where q_{max} is equal to $2P$. q_{min}^{-1} determines in some sense the maximum impact parameter of the neutrinos. Since we want to determine what happens to the cooling rate and other aspects of the neutron star, we must restrict $q_{min}^{-1} \leq R$. Thus the relevant total cross section is

$$\sigma = \frac{2}{\pi}(GMm)^2 \frac{E^2}{P^4} \ln(2PR) \simeq 0.7 \times 10^{32} \ln(2PR) m^2(keV) \frac{E^2(MeV)}{P^4(MeV)}\, GeV^{-2}. \tag{5.2.9}$$

This is a very large number compared to the typical cross sections one encounters, namely the total cross section for helicity flip due to neutral current interactions [75]:

$$\sigma_z^{flip} \simeq 1.6 \times 10^{-23} m^2(keV)\, GeV^{-2}. \tag{5.2.10}$$

However what is physically relevant is not the cross section itself but the product $n_{\rm scat}\sigma$, where $n_{\rm scat}$ is the number density of scatterers. For the gravitational scattering this is (volume of the star)$^{-1}$. Thus the interesting ratio is

$$\frac{\sigma n_{\rm scat}}{\sigma_z n^z_{\rm scat}} = \frac{\sigma}{N\sigma_z} = \xi \tag{5.2.11}$$

where N is the total number of nucleons $\simeq 10^{57}\times 1.4$ (for a 1.4 solar mass neutron star). Then,

$$\xi = \frac{1}{4}\times 10^{-2}\times\frac{E^2(MeV)}{P^4(MeV)}[\ln 10^{16}P(MeV)R(km)] \simeq 0.1\frac{E^2(MeV)}{P^4(MeV)}. \tag{5.2.12}$$

Thus for low-energy neutrinos ($< O(1\,MeV)$) this can indeed become larger than the standard model effect. Although this mechanism is not of much interest for the cooling rate of newly born neutron stars where the average neutrino energy is $\sim 10\,MeV$ or more, it could be of potential interest when the neutrino temperature drops to 0.5 MeV or so. It should of course be kept in mind that at such temperatures the opacity due to weak interactions is also low. Only detailed investigations can tell whether the gravitational mechanisms play any observable role in late-time neutron star cooling.

By virtue of the fact that

$$u^\dagger_{\lambda'}(\mathbf{P}_2)\gamma_0\gamma_i u_\lambda(\mathbf{P}_1) = 0 \quad \text{for } \lambda\neq\lambda', \tag{5.2.13}$$

one sees that the rotation of the neutron star, manifested as a non-vanishing g^{0i} in the leading approximation, does not contribute to the effect.

In connection with the phenomenon of helicity flip due to magnetic moment in a magnetic field, Voloshin [79] has proposed the novel idea of a resonant helicity flip. We now demonstrate a similar effect for gravitational helicity flip. In the weak-field approximation the coupling of the neutrino spin $\mathbf{S}$ to the gravitational field is given by

$$\frac{3GM}{2R^3}(\mathbf{R}\times\mathbf{v})\cdot\mathbf{S}. \tag{5.2.14}$$

This can be thought of as if a magnetic moment μ were interacting with a magnetic field H, such that

$$\mu H = \frac{3GM}{2R^2}. \tag{5.2.15}$$

Thus the requirement for an adiabatic resonant helicity flip due to gravity becomes [79]

$$2\frac{GM}{R}\frac{R}{R_f}\frac{1}{R_f} > 2\times 10^{-2}\,cm^{-1}[\rho/(10^{12}\,g\,cm^{-3})]^{1/2}[80\,km/R_f]^{1/2}\,, \tag{5.2.16}$$

where R_f is the resonant radius at which the density of nucleons is ρ.

Interestingly, gravitational helicity flip mechanisms combined with our understanding of the cooling rates of newly born neutron stars can severely constrain the possible discrete symmetry violations in gravitation. The implications of the gravitational interactions not conserving discrete symmetries has been studied by Hari Dass [77]. He had also proposed a laboratory experiment based on ultra-cold neutron spin precession that could probe for such effects [78]. In a non-relativistic system interacting with a static non-rotating gravitational object of mass M such discrete symmetry breaking effects can be parametrised by the potential

$$V(r) = \alpha_1 \frac{GM}{r^3} \mathbf{S} \cdot \mathbf{r} + \alpha_2 \frac{GM}{r^2} \mathbf{S} \cdot \mathbf{v} \,. \tag{5.2.17}$$

The first term violates parity and time reversal while the second term violates parity and, through the CPT theorem (the status of this theorem in the context of gravitational interactions is not understood very well), charge conjugation invariance. The existing limits on the parameters are $\alpha_2 < 10^{-6}$ but $\alpha_1 < 10^4$. The experiment proposed by Hari Dass[77] is capable of probing $\alpha_1 \sim 1$ but technically is very hard to perform.

Before calculating the cross sections for helicity flip due to these interactions we present a (special) relativistic generalisation of the above potential. There are many such choices that reduce to the above form in the non-relativistic limit, but the choice is restricted if we demand smoothness in the zero fermion mass limit. Then the only possible term is

$$\alpha_1 \bar{u}(\mathbf{P}_2, \lambda_2) \gamma_5 [\gamma_\mu \sigma_{\nu\alpha} q^\alpha + \gamma_\nu \sigma_{\mu\alpha} q^\alpha] u(\mathbf{P}_1, \lambda_1) h^{\mu\nu}(P_2 - P_1). \tag{5.2.18}$$

In fact it is not possible to write a C violating term that contributes to helicity flip scattering. One need not be alarmed that the stress tensor in equation (5.2.18) does not appear to be conserved. It has been argued on general grounds that discrete symmetry violations in gravitation imply the breakdown of local Lorentz invariance [78] and hence the stress tensor is no longer symmetric. Even though the stress tensor is conserved, its symmetric part is not. It should be stressed that the asymmetry vanishes in the classical (as opposed to the quantum mechanical) limit so there is no conflict with the classical tests of general relativity. The total cross section for helicity flip due to the parity violating interaction is given by

$$\sigma^{\mathrm{pv}} = \alpha_1^2 \frac{8}{\pi} (GMm)^2 \frac{1}{PE} \ln(2PR). \tag{5.2.19}$$

Numerically the total contribution to $n_{scat}\sigma$ from the standard model effect as well as from gravitational interactions (both parity conserving and parity violating) is

$$10^{34} m^2(keV)\left[2.7 + \frac{1+4\alpha_1^2}{3E^2(MeV)}\right] = \sigma_0^{\text{hel.flip}} \quad \text{in}\, GeV^{-2} \tag{5.2.20}$$

assuming $E \simeq P \gg m$. The cooling rates of newly born neutron stars appear to limit this quantity by $< 3.2 \times 10^{37}\, GeV^{-2}$, leading to the constraint

$$m^2(keV)\left[1 + \frac{1+4\alpha_1^2}{8E^2(MeV)}\right] \leq 1.2 \times 10^3. \tag{5.2.21}$$

As noted by Gaemers *et. al.* [75], a limit of 40 keV is obtained for the neutrino mass, independent of the considerations of this section. If the actual mass of the tau neutrino saturates this bound, there will be no room for any parity violations in gravitation. Even if the tau neutrino mass turns out to be as small as as 1 keV, α_1 will be constrained to be smaller than 300 which is already two orders of magnitude better than the existing limits. We have seen that cosmological bounds on the stable neutrino masses require that those neutrinos with masses in the keV range be unstable on a cosmological scale [80]. If, however, the neutrino is stable with a mass of approximately $50\,eV$, the limit imposed on the parity violating parameter α_1 will be similar to the existing limit. On the other hand if the neutrinos are found to be massless no limit on the parity violating gravitational interaction will obtain.

5.3 Model for the 17 keV Neutrino

Signatures of a 17 keV neutrino that mixes with roughly 1% strength with the electron neutrino have recently been reported [62,63]. The observations, if corroborated, seem to call for the tau neutrino to be a 17 keV Dirac one (unless, of course, the new particle is an exotic one altogether) as an identification of the new neutrino with ν_μ is ruled out by the present experimental limits on mixings and the Majorana option negated by the non–observation of neutrinoless double beta decay. But even with this, further problems like the cosmological limit [80] of 100 eV for the masses of stable neutrino species and the aesthetic one of such a bizarre mass hierarchy persist. Various models to accomodate the new find in a phenomenologically viable way have been proposed [81,82] with different degrees of success but none of these address the issue of the reported anticorrelation of the solar neutrino flux with the sunspot activity [47]. As we have seen in the section 5.1, perhaps the only natural way to explain such a phenomenon is the assumption of a non–zero magnetic moment for the neutrino [67].

Thus the problem on hand is to realize a scheme that not only produces the required hierarchy for the neutrino masses consistent with the mixing and decay constraints, but can also account for a substantial (transition) magnetic moment for the nearly massless ν_e. While doing so, care must be taken to ensure that the result is not dependent on a severe fine–tuning of parameters. A particularly appealing solution for the first part of the problem has been proposed by Glashow [81]. The idea is to extend the SM fermion spectrum to include three gauge singlet right handed neutrinos and employ the singlet Majoron scheme [83] to break the global $B-L$ symmetry. The neutrino mass matrix, in the $(\nu_L\ \nu_R)$ basis, then reads

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}, \qquad (5.3.1)$$

where m gives the Dirac masses and $M = M^T$ is the Majorana mass term. Assuming that M is of rank two (*i.e.* it has one zero eigenvalue), the see–saw mechanism [55] generates the lighter masses to give a spectrum comprising of four Majorana neutrinos, two heavy ones of masses $\sim O(M)$, two light ones of masses $\sim O(m^2/M)$ and a nearly Dirac one of intermediate mass $\sim O(m)$. Taking $m \sim 17\,keV$ and $M \sim 300\,GeV$, one then identifies the Simpson neutrino with the pseudo–Dirac particle — comprised mainly of $\nu_{\tau L}$ and the massless ν_R — and has $m(\nu_e), m(\nu_\mu) \sim O(10^{-3}\,eV)$, values that can explain the solar neutrino puzzle via the MSW mechanism [46]. There however is one catch to this beautiful ansatz, for obtaining a 17 keV Dirac mass term necessitates Yukawa couplings of the order of 10^{-7}, a none–too–pleasing choice.

In this section [3] we marry the concepts outlined above and in Section 5.1 to construct a model with all the required features namely that the neutrino mass matrix should be such that it should accomodate a 17 keV Dirac ν_τ with a 1% mixing with ν_e and be consistent with the $(\beta\beta)_{0\nu}$ and neutrino oscillation experiments. Moreover a relatively large $\mu_{\nu_e\nu_\mu}$ should be present. However, unlike in Glashow's case [81], the neutrino mass matrix is a 5×5 one with M now a 2×2 matrix of rank one. This results in one of the neutrino being exactly massless at the tree level. Further, the Dirac terms for the neutrinos arise as

[3] Based on the work in ref. [84]

a consequence of radiative corrections and are hence naturally kept small.

The gauge group we choose is a slight modification of that in section 5.1 *viz.* $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)_H$ with the SM fermions ψ_L, ψ_L^c, Q_L, U_L^c and D_L^c transforming as triplets under the horizontal group. In addition, we include a singlet neutrino field N_L^c $[(1,1,0,1)_1]$ with a non-conventional lepton number denoted by the subscript. The scalar sector consists of the SM higgs ϕ_s and in addition the fields Φ $[(1,2,-1/2,3)_0]$, $H, \widetilde{H}$ $[(1,2,-1/2,5)_0]$ to give masses to the charged fermions; σ $[(1,1,0,3)_2]$, Σ $[(1,1,0,2)_8]$ to break the $SU(2)_H$ and lepton number. The neutrino Dirac masses $[\sim O(10keV)]$ are generated by $\widetilde{\phi}[(1,2,-1/2,2)_2]$, which acquires *v.e.v.* only through radiative correction through a diagram involving the fields ξ_1 $[(1,2,-3/2,1)_{-2}]$, ξ_2 $[(1,1,-1,3)_0]$, ξ_3 $[(1,2,-3/2,1)_2]$ and ξ_4 $[(1,1,-1,1)_4]$. We also need two color triplet fields η $[(\bar{3},1,1/3,3)_{-1}]$, χ $[(3,2,1/6,1)_{-1}]$ to radiatively generate the magnetic moment. The model thus offers charge quantization as there is no gaugeable (*i.e.* anomaly–free) global $U(1)$ symmetry [85].

We impose a further discrete symmetry (to be broken softly in the higgs sector), under which $(\psi_L^c, U_L^c, \Phi, \widetilde{H}, \xi_3) \longrightarrow -(\psi_L^c, U_L^c, \Phi, \widetilde{H}, \xi_3)$. The most general Yukawa term is then

$$\begin{aligned} \mathcal{L}_Y &= f\psi_L^c Q_L \eta + f' D_L^c \psi_L \chi + \psi_L^c \psi_L (g_3^e \Phi + g_5^e \widetilde{H}) + U_L^c Q_L (g_3^u \Phi + g_5^u \widetilde{H}) \\ &\quad + D_L^c Q_L (g_3^d \phi_s + g_5^d H) + g_D N_L^c \psi_L \widetilde{\phi} + g_M N_L^c N_L^c \sigma^\dagger . \end{aligned} \tag{5.3.2}$$

Instead of writing down the full scalar potential, we rather focus on the terms that are responsible for the physics we seek, namely a radiative *v.e.v.* generation and a one–loop magnetic moment. These are $\lambda_1 \widetilde{\phi} \Sigma^\dagger \xi_4 \xi_1^\dagger$, $\sigma \xi_1 \xi_2^\dagger (\lambda_2 H^\dagger + \lambda_3 \phi_s^\dagger)$, $\sigma \xi_2 \xi_3^\dagger (\lambda_4 \widetilde{H} + \lambda_5 \Phi)$, $\sigma \xi_3 \xi_4^\dagger (\lambda_6 \widetilde{H} + \lambda_7 \Phi)$ and $\eta \chi \sigma (\kappa \phi_s + \kappa' H)$.

A proper choice (always possible) of the higgs couplings along with the $SU(2)_H$ symmetry can be exploited to ensure that only the $T_{3H} = -1$ component of σ and $T_{3H} = -2$ component of H acquire *v.e.v.* and $\widetilde{\phi}$ does not acquire any tree level vacuum expectation values. The rest of the scalars may assume any *v.e.v.* consistent with charge conservation. For phenomenological consistency we demand that $\langle \sigma \rangle$, $\langle \Sigma \rangle \sim O(10^6\ GeV)$ and that any other *v.e.v.* be of the order of the electroweak scale or less. The *v.e.v.* of $\widetilde{\phi}$ is however not protected by any symmetry and at one–loop level the diagram in Figure 5.2 contributes.

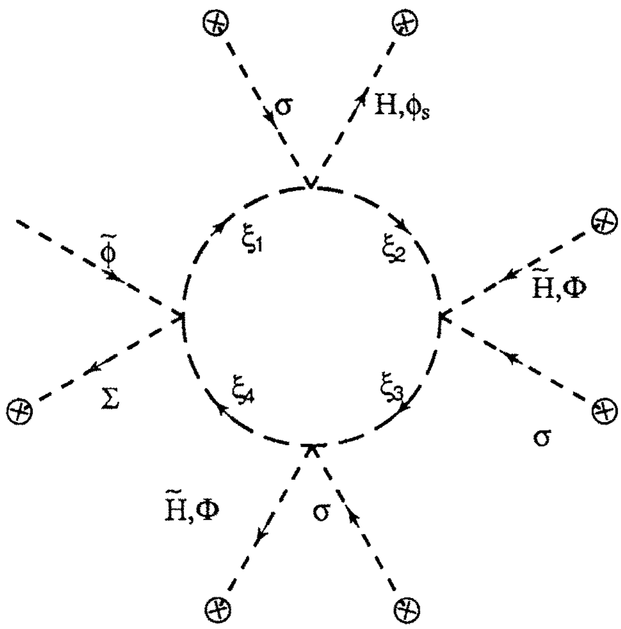


Figure 5.2: One–loop diagram responsible for radiative generation of $\langle\widetilde{\phi}\rangle$

Written symbolically,

$$\langle\widetilde{\phi}\rangle \sim \frac{\lambda^4}{16\pi^2}\frac{m_{Horiz}^4 m_{wk}^3}{m_\xi^4 m_{\widetilde{\phi}}^2} \approx 100\, keV - 1\, MeV, \tag{5.3.3}$$

on assuming that $m_\xi \sim m_{Horiz}$, $m_{\widetilde{\phi}} \sim m_{wk}$ and $\lambda_i \sim O(10^{-1})$.

The neutrino mass matrix is then a 5×5 one of a form similar to that in eqn.(5.3.1) (with M now a 2×2 matrix of rank one) and can be written as

$$\mathcal{M} = \begin{pmatrix} M_1 & M_2 \\ M_2^T & A \end{pmatrix} \tag{5.3.4}$$

where

$$M_1 = \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ a_1 & a_2 & a_3 & 0 \end{pmatrix} \quad \text{and} \quad M_2 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \tag{5.3.5}$$

Here $A = g_M\langle\sigma_{-1}^0\rangle$ and the elements a_i, b_i are of the order of $g_D\langle\widetilde{\phi}\rangle$ (the differences arising on account of the Clebsch–Gordon coefficients). As $\mathcal{M}$ is of rank 4, we have one exactly massless neutrino. $\mathcal{M}$ can be approximately block–diagonalized (for details, see Section 4.2) to the form $\begin{pmatrix} \widetilde{m} & 0 \\ 0 & \widetilde{A} \end{pmatrix} + O(\rho^2 M_1)$, where $\rho = A^{-1}M_2^T$, $\widetilde{m} = M_1 - M_2\rho$ and $\widetilde{A} = A + \frac{1}{2}(\rho M_2 + M_2^T\rho^T)$.

The rest of the spectrum then consists of a light Majorana neutrino of the order of $b_i^2/A \sim O(10^{-5}\ GeV)$, a pseudo–Dirac particle with mass $\sim \sqrt{\sum_i a_i^2} \equiv 17\ keV$ (the mass splitting $\sim b_i^2/A$) and a superheavy Majorana one with mass $\approx A \sim 10^4\ GeV$ (we have assumed Yukawa couplings g_D, $g_M \sim O(10^{-2})$).

The neutrino mixing angles are given essentially by the ratios of a_i and b_i apart from the contribution from the electron mixing matrix and can easily be chosen to satisfy the experimental constraints. The contribution to neutrinoless double beta decay amplitude is very small and assuming a diagonal form for the charged lepton mass matrix, is given by [51,53] $(\widetilde{m})_{11}$. However the ν_e and ν_μ masses are very small.

The Majoron (ϑ) in our model is mainly comprised of $Im(\sigma)$ and $Im(\Sigma)$ with a small admixture of $Im(\widetilde{\phi})$ of the order of $\langle\widetilde{\phi}\rangle/M_{Horiz}$ and contributions from other $SU(2)_L$ doublets further suppressed by a factor of $(\langle\widetilde{\phi}\rangle/M_{wk})^2$. The coupling of the charged fermions with ϑ is then very small and hence consistent with all astrophysical constraints [86]. Looking at the Dirac terms in the fourth row and column, we see that these arise due to the *v.e.v.* s of different components of $\widetilde{\phi}$. As these scalars do not have identical contributions to ϑ even to the leading order, the neutrino mass and the Majoron coupling matrices are not diagonalized simultaneously. This results in the light neutrinos having a substantial non–diagonal coupling with ϑ (of the order of m_ν/M_{Horiz}) and hence affords a decay channel to the tau neutrino of the form $\nu_\tau \to \nu_{\mu,e} + \vartheta$. The ν_τ is then comparatively short–lived, with a lifetime $\sim 10^5\ sec$ and cosmological requirements are easily satisfied[80]. As is easily recognised, this feature is a consequence of the Majoron having contributions both from $SU(2)_L$ doublets and singlets and is absent for the usual singlet Majoron models.

It is curious to note that the results of Section 5.2 can be sharpened in the context of the 17 keV neutrino. For, if it is actually comprised mainly of the $SU(2)_L$ doublet ν_τ, as is being hypothesised, then it satisfies all the criteria to be considered as a probe for the physics in the interior of a neutron star. This would result in the very strong bound (see equations 5.2.17 and 5.2.21) of $\alpha_1 \lesssim O(10)$ on parity violating effects in gravity.

The first set of criteria having been satisfied, we now turn to the problem of the magnetic moments. In fact, it is easy to see that the results of Section 5.1 are carried through without any modifications. We have thus obtained a model that naturally incorporates a 17 keV ν_τ

as well as a sole (neglecting lepton mixing) transition magnetic moment $\mu_{\nu_e \nu_\mu} \sim 10^{-12}\mu_B$ for nearly massless neutrinos. The latter effect obviously vanishes exactly in the limit of $SU(2)_\nu$ symmetry only to appear when the symmetry is broken. Yet the mass term remains identically zero, for the effective *v.e.v.* is antisymmetric in the family space. This effect allows us, unlike many earlier models [70,71], to totally dispense with any naturalness condition to suppress mass generation. Thus to this order, the only contribution to the light neutrino masses come from the see–saw terms which are very small anyway. However, as there is no conserved lepton number, one expects there to be radiative mass generation and the two–loop contribution would typically be of the order of 10^{-4}–10^{-5} eV. That this is so is easily seen from the fact that diagrams similar to those in Fig. 5.1 but with the coupling $\kappa' H\eta\chi\sigma$, contributes to the Majorana mass once the $T_{3H} = 0, -1$ components of H get *v.e.v.*s radiatively. The 17 keV ν_τ in the present model decays dominantly into a ν_μ and a doublet–singlet Majoron.