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CHAPTER - 2

ANALYSIS OF BASIC EQUATIONS

2.1 INTRODUCTION :

Partial differential equations governing fluid flow and heat transfer in free convection from rectangular enclosure of an arbitrary aspect ratio, oriented horizontally, are presented here.

2.2 ENCLOSURE CONFIGURATION :

A rectangular, horizontally oriented enclosure of width W and gap height H, having the bottom surface at a temperature T_s and the remaining surfaces at a temperature t_1, was considered for the analysis. A typical enclosure configuration is shown in Fig.2.1.

2.3 BOUSSINESQ APPROXIMATION :

Generalised partial differential equations governing fluid flow and heat transfer in free convection, appearing in standard textbooks[75,], are highly non-linear even after steady state assumption, due to an appearance of viscous dissipation function in energy conservation equation, and hence are not easily solvable, even after numerical simplifications. However, as was first pointed out by Boussinesq[9, 10], there are many practical applications in which these equations can be simplified considerably. This is possible when variability in density and other coefficients is due to variations in temperature of only moderate magnitude. This is due to smallness of the volumetric coefficient of thermal expansion of gases and liquids (10^{-3} to 10^{-4} 1/°C), in comparision to temperature variations (of

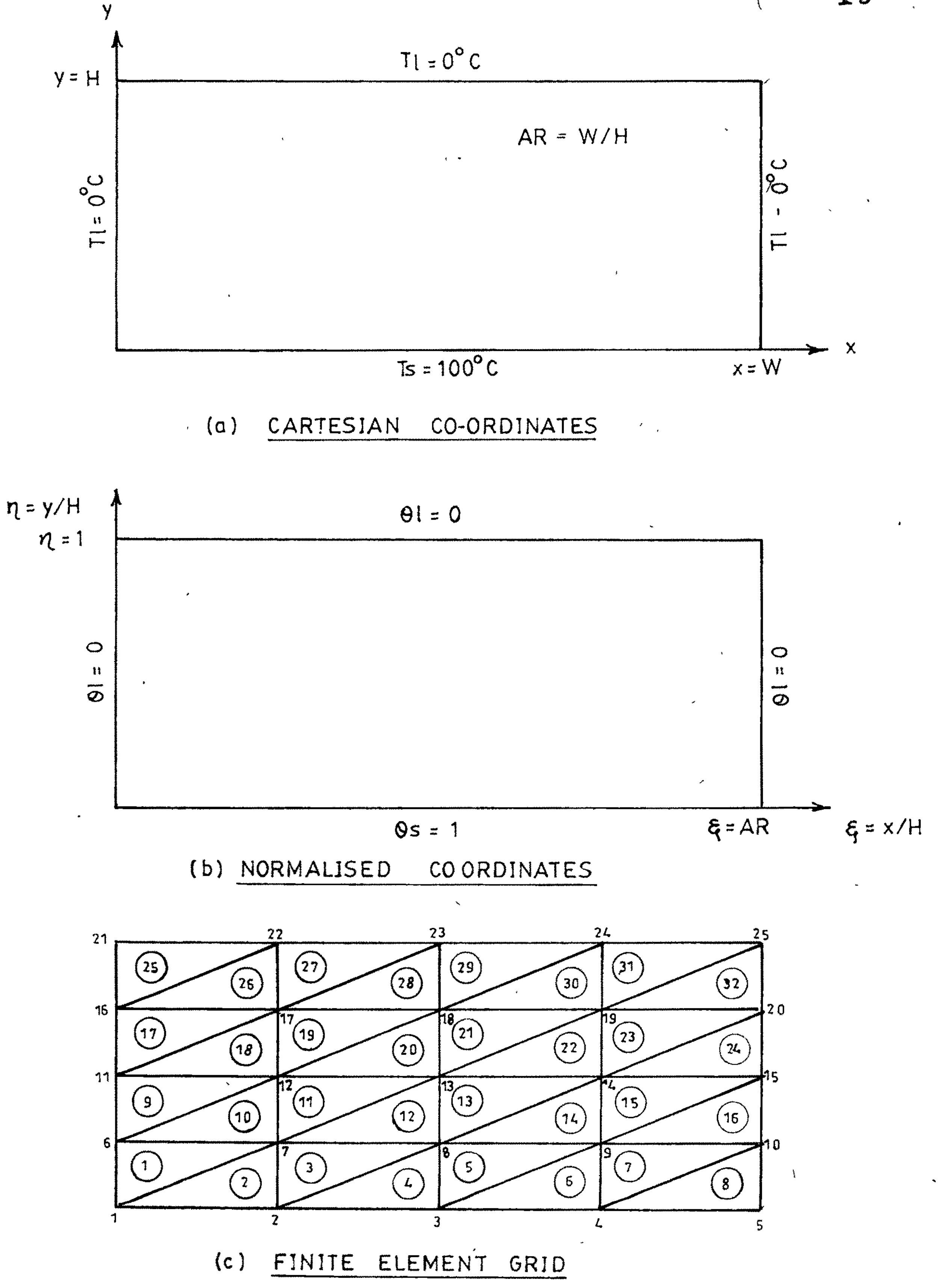
y
y = H
Tl = 0°C
AR = W/H
Tl = 0°C
Tl - 0°C
x
Ts = 100°C
x = W
(a) CARTESIAN CO-ORDINATES
η = y/H
η = 1
θl = 0
θl = 0
θl = 0
θs = 1
ξ = AR
ξ = x/H
(b) NORMALISED COORDINATES
(c) FINITE ELEMENT GRID

FIG: 2·1

the order of 10°C). Under above situation, variations in density and other coefficients shall not exceed 1% and hence may be ignored.

However, variability of density ρ in ρX_i terms in momentum conservation equations can not be ignored, because the acceleration resulting from $\delta \rho X_i = \beta \Delta T X_i$ can be quite large, compared to accleration due to inertial term $u_j \frac{\partial u_i}{\partial x_j}$. Thus, we may treat density as constant in all terms in the momentum equations except the one in the external force. This simplification is called Boussinesq approximation. Use of this Boussinesq approximation in the generalised governing equations, results in simplified governing equations of practical interest, which can be numerically solved, due to ellimination of highly complex viscous dissipation function from the energy conservation equation,,in addition to simplifications obtained in mass and momentum conservation equations.

2.4 GOVERNING EQUATIONS :

Simplification of the generalised partial differential equations, mentioned earlier[75], for the case of steady free convection in cartesian co-ordinate system, using Boussinesq approximation, resulted into :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad \ldots \qquad (2.1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\cdot\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad \ldots \qquad (2.2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\cdot\frac{\partial p}{\partial y} + \beta g(T - T_0) + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad \ldots \qquad (2.3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad \ldots \quad (2.4)$$

First of the above equations, is mass conservation equation, next two being x-momentum and y-momentum conservation equations while the last is energy conservation equation, for Newtonian, incompressible fluids, considering gravity vector in negative Y-direction. In the above, ρ_o is the density of the fluid at a reference temperature $T_o = (T_s + T_l)/2$.

Above equations were normalized i.e. non-dimensionalised to obtain :

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} = -\omega \quad \ldots \quad (2.5)$$

$$\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} = \frac{\partial \psi}{\partial \eta}\cdot\frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\cdot\frac{\partial \theta}{\partial \eta} \quad \ldots \quad (2.6)$$

$$\frac{\partial^2 \omega}{\partial \xi^2} + \frac{\partial^2 \omega}{\partial \eta^2} = \frac{1}{P_r}\left(\frac{\partial \psi}{\partial \eta}\cdot\frac{\partial \omega}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\cdot\frac{\partial \omega}{\partial \eta}\right) - Ra\cdot\frac{\partial \theta}{\partial \xi} \quad \ldots \quad (2.7)$$

Normalising parameters in the above equations are :

$\xi = x/H$, $\eta = y/H$, $u^* = uH/\alpha$, $v^* = vH/\alpha$

$p^* = pH^2/\rho_o\alpha^2$, $\theta = (T-T_l)/(T_s-T_l)$

Temperature function θ in the above equations is as defined above, while stream function ψ , vorticity function ω , Rayleigh number Ra and Prandtl number Pr are defined as

$$u^* = \frac{\partial \psi}{\partial \eta}, \quad v^* = -\frac{\partial \psi}{\partial \xi}, \quad \omega = \frac{\partial v^*}{\partial \xi} - \frac{\partial u^*}{\partial \eta}$$

$$Ra = \frac{\beta g H^3 (T_s - T_L)}{\alpha \nu}, \quad Pr = \frac{\nu}{\alpha}$$

Derivation of the above three normalised equations from the earlier four basic governing equations, using the normalising parameters defined above, is included in Appendix:A-3.

It is interesting to note that the normalisation of the governing equations results into ellimination of pressure gradient terms from momentum equations and reduction in number of equations to be solved.

2.5 BOUNDARY CONDITIONS :

Boundary conditions appropriate to the configuration of Fig.2.1 are :

$\psi = 0$ at $\xi = 0, 1$ and $\eta = 0, 1$ ... (2.8)

$\theta = 0$ at $\xi = 0, 1$ and $\eta = 1$ ... (2.9)

$\theta = 1$ at $\eta = 0$ ... (2.10)

$\omega = -2\Delta\psi/(\Delta n)^2$ at $\xi = 0, 1$ and $\eta = 0, 1$ ... (2.11)

First of the above boundary conditions (eqn. 2.8) shows that numerical value of stream function at the rigid boundaries of the enclosure is zero, which is obvious, while the next two boundary conditions (eqns. 2.9 and 2.10) represent dimensionless temperatures at the boundaries. Such a boundary condition where the side walls are isothermal (θ=0), is called Dirichlet boundary condition[76] and it differs from an alternative Neumann boundary condition[76] where the side walls are insulating ($\frac{\partial \theta}{\partial n}$ =0) .

The last boundary condition (eqn.2.11) is the most complex of all and is obtained by expanding stream function ψ in a Taylor series around a boundary point (second order approximation) and substituting ψ =0 and $\frac{\partial \psi}{\partial n}$ =0 at the surface (where ξ =0,1 and η =0,1), to obtain $\Delta\psi$ which is the value of ψ at a point Δn distance away from the surface, which is finally related to ω at the boundary using eqn.2.5, resulting in eqn.2.11. This is elaborately discussed by Shembharkar et al[42] in 1975 and somewhat later by Abdel-Khalik et al[53] .