CHAPTER VI

MULTIPLE THREE CLASS ATTRIBUTES SAMPLING PLAN - CURTAILED AS WELL AS UNCURTAILED

6.1 In the previous chapter we have introduced semi-curtailed and fully-curtailed forms of the single three class attributes sampling plan. In this chapter we introduce multiple three class attributes sampling plan (MTCAP) which is an extension of a single three class attributes sampling plan. Furthermore, we introduce three forms of MTCAP: Uncurtailed, Semi--curtailed, and Fully-curtailed. A particular case of MTCAP, namely, double three class attributes sampling plan (DTCAP) is studied extensively.The expressions for the average sample number (ASN), the maximum likelihood estimators of the proportion of bad units and that of marginal units when m lots have undergone the inspection, the relations between the asymptotic variances of estimators and the ASN are all obtained under the three different forms of DTCAP. These results, then, are generalized to MTCAP.

6.2 Notations and Assumptions :

The symbols, notations etc. introduced in section 5.3 of

Chapter V are modified to suit the developments of this chapter. These modifications, along with a few symbols remained unaltered are given below :

N = Number of units in a lot.

k = Maximum number of samples.

n_i = Size of the ith sample.

a_{1i} = Maximum allowable number of nongood units in a sequence of first i samples. Nongood units means bad unit or marginal unit.

a_{2i} = Maximum allowable number of bad units in a sequence of first i samples.

r_{1i} = Rejection number for nongood units in a sequence of first i samples.

r_{2i} = Rejection number for bad units in a sequence of first i samples.

$g_{oi} = \sum_{j=1}^{i} n_j - a_{1i}$ = Acceptance number for good units in a sequence of first i samples.

$g_{1i} = \sum_{j=1}^{i} n_j - a_{2i}$ = Acceptance number for nonbad units in a sequence of first i samples.

Y = Number of units inspected.

D_{1i} = Number of marginal units found in the ith sample.

D_{2i} = Number of bad units found in the ith sample.

D_{oi} = Number of good units found in the ith sample.

Z = Indicator variable.

p_2 = Proportion of bad units in the production run.

p_1 = Proportion of marginal units in the production run.

p_o = Proportion of good units in the production run.

$= 1-p_1-p_2$.

$p_1' = p_1/(p_1+p_o)$

$p_1'' = p_1/(p_1+p_2)$

$p_o' = p_o/(p_1+p_o)$

$p_2'' = p_2/(p_1+p_2)$.

m = Number of lots submitted for inspection.

(TB) = Total number of bad units observed in m lots.

(TM) = Total number of marginal units observed in m lots.

(TG) = Total number of good units observed in m lots.

(TU) = Total number of units inspected in m lots.

= (TB) + (TM) + (TG).

$$B(r;n,p) = \sum_{x=o}^{r} \binom{n}{x} p^x q^{n-x}$$

$$b(x;n,p) = \binom{n}{x} p^x q^{n-x} = B(x;n,p) - B(x-1;n,p).$$

The assumptions given at the end of Section 5.3 of Chapter V are also true for the development of the results in this chapter.

6.3 The Multiple Three Class Attributes Sampling Plan :

In this section we have given statements of the three forms of MTCAP.

6.3.1 Statement of the Uncurtailed MTCAP :

The concern here is with an uncurtailed MTCAP in which a sequence of upto k samples is taken from a lot of size N for inspection. The size of the ith sample is n_i (i=1,2,...k). The units in the sample are randomly selected. Then the inspection is stopped due to the occurrence of one of the following mutually exclusive 4k events $E_{u,ti}$ (t=1,2,3,4, i=1,2,...,k):

$E_{u,1i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that

$$\sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} \leq a_{1i} \text{ and } \sum_{j=1}^{i} D_{2j} \leq a_{2i},$$

$E_{u,2i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that $\sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} < r_{1i}$ and $\sum_{j=1}^{i} D_{2j} \geq r_{2i}$,

$E_{u,3i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that

$$\sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} \geq r_{1i} \text{ and } \sum_{j=1}^{i} D_{2j} < r_{2i},$$

$E_{u,4i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that

$$\sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} \geq r_{1i} \quad \text{and} \quad \sum_{j=1}^{i} D_{2j} \geq r_{2i}.$$

The lot is accepted if any one of the k events $E_{u,1i}$ (i=1,2,...,k) occurs and is rejected if any one of the 3k events $E_{u,ti}$ (t=2,3,4, i=1,2,3,...,k) occurs. It may be noted, that the occurrence of one of the following 3(k-1) events $E_{u,ti}$ (t=5,6,7) implies that the decision of acceptance or rejection of a lot is deferred until the next sample of size n_{i+1} (i=1,2,...,k-1) is inspected :

$E_{u,5i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that

$$\sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} \leq a_{1i} \quad \text{and} \quad a_{2i} < \sum_{j=1}^{i} D_{2j} < r_{2i},$$

$E_{u,6i}$: $\sum_{j=1}^{i} n_j$ units are inspected and its is found that

$$a_{1i} < \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} < r_{1i} \quad \text{and} \quad \sum_{j=1}^{i} D_{2j} \leq a_{2i},$$

$E_{u,7i}$: $\sum_{j=1}^{i} n_j$ units are inspected and it is found that

$$a_{1i} < \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} < r_{1i} \quad \text{and} \quad a_{2i} < \sum_{j=1}^{i} D_{2j} < r_{2i}.$$

The constants a_{vi} and r_{vi} (v=1,2,; i=1,2,...,k) be the predetermined integers satisfying the following conditions:

$$0 \leq a_{11} \leq a_{12} \leq \quad \cdots \quad \leq a_{1k} \qquad \ldots(6.3.1)$$

$$0 \leq a_{21} \leq a_{22} \leq \quad \cdots \quad \leq a_{2k} \qquad \ldots(6.3.2)$$

$$0 \leq r_{11} \leq r_{12} \leq \quad \cdots \quad \leq r_{1k} \qquad \ldots(6.3.3)$$

$$0 \leq r_{21} \leq r_{22} \quad \cdots \quad \leq r_{2k} \qquad \ldots(6.3.4)$$

$$a_{1i} + 1 \leq r_{1i} \qquad \ldots(6.3.5)$$

$$a_{2i} + 1 \leq r_{2i} \qquad \ldots(6.3.6)$$

$$a_{1k} + 1 = r_{1k} \qquad \ldots(6.3.7)$$

$$a_{2k} + 1 = r_{2k} \qquad \ldots(6.3.8)$$

$$a_{2i} < a_{1i} \quad i = 1,2,\ldots,k \qquad \ldots(6.3.9)$$

The conditions (6.3.7) and (6.3.8) ensure that not more than k samples are required to be inspected. For each $i=1,2,\ldots,k-1$, at least one of the expressions (6.3.5) and (6.3.6) is a strict inequality.

6.3.2 Statement of the Semi-Curtailed MTCAP :

Inspect randomly selected units of a lot one at a time until one of the following mutually exclusive 4k events $E_{s,ti}$ ($t=1,2,3,4$; $i=1,2,\ldots,k$) occurs :

$$E_{s,1i} : Y = \sum_{j=1}^{i} n_j, \ \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} \leq a_{1i}, \ \sum_{j=1}^{i} D_{2j} \leq a_{2i},$$

$$E_{s,2i} : \sum_{j=1}^{i} n_{j-1} < Y \leq \sum_{j=1}^{i} n_j, \ \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} < r_{1i},$$

$$\sum_{j=1}^{i} D_{2j} = r_{2i},$$

$$E_{s,3i} : \sum_{j=1}^{i} n_j - 1 < Y \leq \sum_{j=1}^{i} n_j, \; \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} = r_{1i},$$

$$\sum_{j=1}^{i} D_{2j} < r_{2i},$$

$$E_{s,4i} : \sum_{j=1}^{i} n_{j-1} < Y \leq \sum_{j=1}^{i} n_j, \; \sum_{v=1}^{2} \sum_{j=1}^{i} D_{vj} = r_{1i},$$

$$\sum_{j=1}^{i} D_{2j} = r_{2i},$$

where = n_o = 0 by convention. The decision rule is to accept the lot if one of the k events $E_{s,1i}$ (i=1,2,...k) occurs and to reject the lot if one of the 3k events $E_{s,ti}$ (t=2,3,4; i=1,2,...,k) occurs.

6.3.3 <u>Statement of the Fully-Curtailed MTCAP</u> :

Inspect randomly selected units of a lot one at a time until one of the following 6k mutually exclusive events $E_{f,ti}$ (t=1,2,...,6; i=1,2,...,k) occurs :

$$E_{f,1i} : \sum_{j=1}^{i} n_{j-1} < Y \leq \sum_{j=1}^{i} n_j, \; \sum_{v=o}^{1} \sum_{j=1}^{i} D_{vj} = g_{1i},$$

$$\sum_{j=1}^{i} D_{oj} > g_{oi},$$

$$E_{f,2i} : \sum_{j=1}^{i} n_{j-1} < Y \leq \sum_{j=1}^{i} n_j, \; \sum_{v=o}^{1} \sum_{j=1}^{i} D_{vj} > g_{1i},$$

$$\sum_{j=1}^{i} D_{oj} = g_{oi},$$

$E_{f,3i}$: $\sum_{j=1}^{i} n_{j-1} < Y \le \sum_{j=1}^{i} n_j$, $\sum_{v=o}^{1} \sum_{j=1}^{i} D_{vj} = g_{1i}$,

$\sum_{j=1}^{i} Do_j = g_{oi}$,

$E_{f,4i}$: Event $E_{s,2i}$ of semi-curtailed MTCAP,

$E_{f,5i}$: Event $E_{s,3i}$ of semi-curtailed MTCAP,

$E_{f,6i}$: Event $E_{s,4i}$ of semi-curtailed MTCAP,

where $n_o=0$ by convention. The decision rule is to accept the lot if one of the 3k events $E_{f,ti}$ (t=1,2,3,; i=1,2,...,k) occurs and to reject the lot if one of the 3k events $E_{f,ti}$ (t=4,5,6; i=1,2,...,k) occurs.

6.4 Double Three Class Attributes Sampling Plan (DTCAP) :

In this section we discuss, in detail, the double three class attributes sampling plan (DTCAP) which is a particular case of MTCAP for k=2. The description of DTCAP is given. Furthermore, we give the probability functions and the expressions for the ASN for all the cases-uncurtailed, semi-curtailed, and fully-curtailed. The maximum likelihood estimators of the proportion of bads and that of marginals and the asymptotic variances of the maximum likelihood estimators are obtained under semi-curtailed DTCAP.

6.4.1 Description of DTCAP and its Particular Case :

It follows from the definition of MTCAP together with the conditions (6.3.7) and (6.3.8) that one requires the following six numbers of acceptance and rejection :

$$a_{11} \quad r_{11} \quad a_{12} \quad (= r_{12}-1)$$

$$a_{21} \quad r_{21} \quad a_{22} \quad (= r_{22}-1)$$

to carry out the usual DTCAP. However, in case of double sampling plan for two class attributes, the practice is to have the common rejection number for both the samples. Then, following this practice in this case too, we take

$$r_{11} = r_{12} = r_1$$

$$r_{21} = r_{22} = r_2 \quad \text{say,} \qquad \ldots(6.4.1)$$

Furthermore, for the convenience, we take

$$a_{11} = a_{12} = a_1 \quad \text{say,} \qquad \ldots(6.4.2)$$

Due to the condition (6.4.2), the event $E_{u,61}$ and $E_{u,71}$ explained in Section 6.3.1, will not occur, but the event $E_{u,51}$ will occur. The occurrence of $E_{u,51}$ indicates when one has to draw a second sample. Thus, the DTCAP considered here needs the following three numbers only as acceptance and rejection numbers :

$a_1 \quad a_{21} \quad a_{22}$ ...(6.4.3)

where $a_1 = r_1-1$ and $a_{22} = r_2-1$. In this case DTCAP have only three numbers given by (6.4.3) in addition to n_1, n_2, and N. The subsections followed here onwards take this fact into consideration.

6.4.2 Probability Functions :

We determine now the probability functions of DTCAP :

For k=2, one can list down 8 events $E_{u,ti}$ (t=1,2,3,4; i=1,2) of the uncurtailed, 8 events $E_{s,ti}$ (t=1,2,3,4; i=1,2) of the semi-curtailed DTCAP and 12 events $E_{f,ti}$ (t=1,2,...,6; i=1,2) of the fully-curtailed DTCAP.The sets of the possible values attained by the random variables Y and D_{vj} (v=0,1,2; j=1,2) are listed in Table 6.1. The sets denoted by $B_{u,ti}$ (t=1,2,3,4; i=1,2), $B_{s,ti}$ (t=1,2,3,4; i=1,2), and $B_{f,ti}$ (t=1,2,...,6; i=1,2) correspond to $E_{u,ti}$ $E_{s,ti}$ and $E_{f,ti}$ respectively. Some events of one plan being identical with some events of the other plan, one would find actually 20 rows instead of 28 rows in Table 6.1.

Let ($Y=y$, $D_{01}=d_{01}$, $D_{11}=d_{11}$, $D_{21}=d_{21}$, $D_{02}=d_{02}$, $D_{12}=d_{12}$, $D_{22}=d_{22}$) be denoted by a vector representation $\underline{X} = \underline{x}$. Then considering the appropriate trinomial and inverse trinomial distributions, the probability functions for the uncurtailed, semi-curtailed and fully-curtailed DTCAP are given below :

Uncurtailed :

$$P(\underline{X}=\underline{x},\ Z=z) = \begin{cases} \delta_1(n_1,\ d_{o1},\ d_{11},\ d_{21};\ p_1,p_2) & \underline{x} \in B_{u,t1};\ t=1,2,3,4;\ z=t \\ \delta_1(n_1,\ d_{o1},\ d_{11},\ d_{21};\ p_1,p_2)\ \delta_1(n_2,\ d_{o2},\ d_{12},\ d_{22};\ p_1,\ p_2) & \underline{x} \in B_{u,t2};\ t=1,2,3,4;\ z=t+4 \end{cases} \quad \ldots(6.4.4)$$

Semi-curtailed :

$$P(\underline{X}=\underline{x},\ Z=z) = \begin{cases} \delta_1(n_1,\ d_{o1},\ d_{11},\ d_{21};\ p_1,\ p_2) & \underline{x} \in B_{s,11};\ z=1 \\ \delta_2(y,\ d_{o1},\ d_{11},\ r_2;\ p_1,p_2) & \underline{x} \in B_{s,21};\ z=2 \\ \delta_3(y,\ d_{o1},\ r_1-d_{21},\ d_{21};\ p_1,p_2) & \underline{x} \in B_{s,31};\ z=3 \\ \delta_4(y,\ d_{o1},\ r_1-r_2,\ r_2;\ p_1,p_2) & \underline{x} \in B_{s,41};\ z=4 \\ \delta_1(n_1,\ d_{o1},\ d_{11},\ d_{21};\ p_1,p_2)\ \delta_1(n_2,\ d_{o2},\ d_{12},\ d_{22};p_1,p_2) & \underline{x} \in B_{s,12};\ z=5 \end{cases}$$

cont...

$$\begin{cases} \delta_1(n_1,\ d_{01},\ d_{11},\ d_{21};\ p_1,\ p_2)\ \delta_2(y-n_1, d_{02}, d_{12}, r_2-d_{21}; p_1, p_2) \\ \qquad\qquad\qquad\qquad\qquad\qquad \underline{x} \in B_{s,22};\ z=6 \\ \delta_1(n_1,\ d_{01},\ d_{11},\ d_{21};\ p_1, p_2)\ \delta_3(y-n_1,\ d_{02},\ r_1-(d_{11}+d_{21}) \\ \qquad -d_{22},\ d_{22};\ p_1,\ p_2) \qquad\qquad \underline{x} \in B_{s,32};\ z=7 \\ \delta_1(n_1,\ d_{01},\ d_{11},\ d_{21};\ p_1, p_2)\ \delta_4(y-n_1,\ d_{02},\ r_1-r_2-d_{11}, \\ \qquad r_2-d_{21};\ p_1,\ p_2) \qquad\qquad \underline{x} \in B_{s,42};\ z=8 \end{cases}$$

...(6.4.5)

Fully-curtailed :

$P\ (\underline{X}=\underline{x},\ Z=z) =$

$$\begin{cases} \delta_5(y, d_{01},\ g_{11}-d_{01},\ d_{21};\ p_1, p_2) & \underline{x} \in B_{f,11};\ z=1 \\ \delta_6(y,\ g_{01},\ d_{11},\ d_{21};\ p_1,\ p_2) & \underline{x} \in B_{f,21};\ z=2 \\ \delta_7(y,\ g_{01},\ g_{11}-g_{01},\ d_{21};\ p_1, p_2) & \underline{x} \in B_{f,31};\ z=3 \\ \delta_2(y, d_{01}, d_{11}, r_2;\ p_1, p_2) & \underline{x} \in B_{f,41};\ z=4 \\ \delta_3(y, d_{01},\ r_1-d_{21}, d_{21};\ p_1, p_2) & \underline{x} \in B_{f,51};\ z=5 \\ \delta_4(\ y, d_{01}, r_1-r_2, r_2;\ p_1, p_2) & \underline{x} \in B_{f,61};\ z=6 \\ \delta_1(n_1, d_{01}, d_{11}, d_{21};\ p_1, p_2)\ \ \delta_5(y-n_1, d_{02}, g_{12}-(d_{01}+d_{11}) & \\ \qquad -d_{02}, d_{22};\ p_1, p_2) & \underline{x} \in B_{f,12};\ z=7 \\ \delta_1(n_1, d_{01}, d_{11}, d_{21}; p_1, p_2)\ \delta_6(y-n_1, g_{02}-d_{01}, d_{11}, d_{22}; p_1, p_2) & \\ & \underline{x}\ \in B_{f,22};\ z=8 \end{cases}$$

Cont...

$$\begin{cases} \delta_1(n_1, d_{01}, d_{11}, d_{21}; p_1, p_2)\ \delta_7(y-n_1,\ g_{02}-d_{01}, g_{12}-g_{02}-d_{11}, d_{22}; \\ \quad p_1, p_2) \qquad\qquad \underline{x} \in B_{f,32};\ z=9 \\ \delta_1(n_1, d_{01}, d_{11}, d_{21};\ p_1, p_2)\ \delta_2(y-n_1,\ d_{02}, d_{12}, r_2-d_{21}; p_1, p_2) \\ \qquad\qquad \underline{x} \in B_{f,42};\ z=10 \\ \delta_1(n_1, d_{01}, d_{11}, d_{21}; p_1, p_2)\ \delta_3(y-n_1, d_{02}, r_1-(d_{11}+d_{21}) \\ \quad -d_{22}, d_{22}; p_1, p_2) \qquad\qquad \underline{x} \in B_{f,52};\ z=11 \\ \delta_1(n_1, d_{01}, d_{11}, d_{21};\ p_1, p_2)\ \delta_4(y-n_1, d_{02}, r_1-r_2-d_{11}; \\ \quad r_2-d_{21}; p_1, p_2) \qquad\qquad \underline{x} \in B_{f,62};\ z=12 \end{cases} \qquad \ldots(6.4.6)$$

where

$$\delta_1(y, y-x_1-x_2,\ x_1, x_2;\ p_1, p_2) = \frac{y!\ p_0^{y-x_1-x_2}\ p_1^{x_1}\ p_2^{x_2}}{x_1!\ x_2!\ (y-x_1-x_2)!} \qquad \ldots(6.4.7)$$

$$\delta_2 = \frac{x_2}{y}\delta_1 \qquad \ldots(6.4.8)$$

$$\delta_3 = \frac{x_1 + x_2}{y}\ \delta_1 \qquad \ldots(6.4.9)$$

$$\delta_4 = \frac{x_2}{y}\ \delta_1 = \delta_2 \qquad \ldots(6.4.10)$$

$$\delta_5 = \frac{y-x_2}{y}\ \delta_1 \qquad \ldots(6.4.11)$$

$$\delta_6 = \frac{y-x_1-x_2}{y}\ \delta_1 \qquad ..(6.4.12)$$

$$\delta_7 = \frac{y-x_2}{y} \delta_1 = \delta_5 \qquad \ldots(6.4.13)$$

6.4.3 The average Sampl[redacted] Number :

Summing the probability functions (6.4.4), (6.4.5) and (6.4.6) over d_{vj} ($v=0,1,2$; $j=1,2$) one gets the joint probability functions of (Y,Z). Let these probability functions be denoted by $\beta_u(y,z;\ p_1,p_2)$, $\beta_s(y,z;\ p_1,p_2)$ and $\beta_f(y,z;\ p_1,p_2)$ respectively. Then the expressions of the ASN for these sampling plans are given below:

$$\text{ASN(uncu)} = \sum_y y \sum_z \beta_u (y,z;\ p_1,p_2)$$

$$\text{ASN(semi)} = \sum_y y \sum_z \beta_s (y,z;\ p_1,p_2)$$

$$\text{ASN(fully)} = \sum_y y \sum_z \beta_f (y,z;\ p_1,p_2)$$

With little hard algebra the expressions for the ASN are obtained as follows :

$$\text{ASN(uncu)} = n_1 \Big[1- \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2) B(r_1-1-d_{21};\ n_1-d_{21},p_1')\Big] + (n_1+n_2) \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};\ n_1,p_2)\ B(r_1-1-d_{21};\ n_1-d_{21},p_1')$$

$$= n_1+n_2 \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2)\ \ B(r_1-1-d_{21};\ n_1-d_{21},p_1') \qquad \ldots(6.4.14)$$

$$ASN(semi) = n_1 \sum_{d_{21}=o}^{a_{21}} b(d_{21};n_1,p_2)\, B(r_1-1-d_{21};\, n_1-d_{21},p_1')$$

$$+ \frac{r_2}{p_2} \sum_{y'=r_2+1}^{n_1+1} \{B(r_2;\, y'-1,p_2)-B(r_2;y',p_2)\}$$

$$\cdot B(r_1-r_2;y'-r_2-1,p_1')$$

$$+ \frac{r_1}{1-p_o} \{1-B(r_1;n_1+1,\, 1-p_o)\}\, B(r_2-1;r_1,p_2'')$$

$$+(n_1+n_2) \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2) \sum_{d_{11}=o}^{r_1-1-d_{21}}$$

$$b(d_{11};n_1-d_{21},p_1')$$

$$\cdot\left[\sum_{d_{22}=o}^{r_2-1-d_{21}} b(d_{22};n_2,p_2)\, B(r_1-1-(d_{11}+d_{21})\right.$$

$$\left.-d_{22};n_2-d_{22},p_1')\right] + \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2)$$

$$\sum_{d_{11}=o}^{r_1-1-d_{21}} b(d_{11};n_1-d_{21},p_1') \left[\frac{r_2-d_{21}}{p_2} \sum_{y'=r_2+1-d_{21}}^{n_2+1}\right.$$

$$\cdot\{B(r_2-d_{21};\, y'-1,p_2) - B(r_2-d_{21};y',p_2)\}$$

$$B(r_1-r_2-d_{11};\, y'-(r_1+1-d_{21}),p_1')$$

$$+ n_1 \sum_{y'=r_2-d_{21}}^{n_2} \{B(r_2-1-d_{21};y'-1,p_2) - B(r_2-1-d_{21};y',p_2)\}$$

$$\cdot B(r_1-r_2-d_{11};\, y'-(r_2-d_{21}),p_1')$$

cont...

$$+\frac{r_1-(d_{11}+d_{21})}{1-p_o}\{1-B(r_1-(d_{11}+d_{21});\ n_2+1,1-p_o)\}$$

$$\cdot B(r_2-1-d_{21};\ r_1-(d_{11}+d_{21}),\ p_2'')$$

$$+n_1\{1-B(r_1-1-(d_{11}+d_{21});\ n_2,1-p_o)\}$$

$$\cdot B(r_2-1-d_{21};\ r_1-(d_{11}+d_{21}),\ p_2'')\Big] \qquad \ldots(6.4.15)$$

$$ASN(fully) = \frac{g_{11}}{1-p_2}\{1-B(g_{11};\ n_1+1,\ 1-p_2)\}\{1-B(g_{o1}-1;\ g_{11},p_o')\}$$

$$+\frac{g_{o1}}{p_o}\sum_{y'=g_{11}+1}^{n_1+1}\{B(g_{o1};y'-1,p_o) - B(g_{o1};y',p_o)\}$$

$$\cdot\{B(n_1-g_{o1};y'-(g_{o1}+1),p_1'') - B(g_{11}-g_{o1};y'-(g_{o1}+1),p_1'')\}$$

$$+\frac{r_2}{p_2}\sum_{y'=r_2+1}^{n_1+1}\{B(r_2;y'-1,p_2) - B(r_2;y',p_2)\}$$

$$B(r_1-r_2;\ y'-r_2-1,p_1')$$

$$+\frac{r_1}{1-p_o}\{1-B(r_1;\ n_1+1,\ 1-p_o)\}B(r_2-1;\ r_1,p_2'')$$

$$+\sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2)\sum_{d_{11}=o}^{r_1-1-d_{21}} b(d_{11};n_1-d_{21},p_1')$$

$$\cdot\Big[\frac{g_{12}-(d_{o1}+d_{11})}{1-p_2}\{1-B(g_{12}-(d_{o1}+d_{11});\ n_2+1,1-p_2)\}$$

$$\{1-B(g_{o2}-d_{o1}-1;\ g_{12}-(d_{o1}+d_{11}),p_o')\}$$

$$+n_1\{1-B(g_{12}-(d_{o1}+d_{11})-1;n_2,1-p_2)\}$$

$$\cdot\{1-B(g_{o2}-d_{o1}-1;\ g_{12}-(d_{o1}+d_{11}),p_o')\}$$

cont...

$$+ \frac{g_{o2}-d_{o1}}{p_o} \sum_{y'=g_{12}-(d_{o1}+d_{11})+1}^{n_2+1} \{B(g_{o2}-d_{o1};y'-1,p_o)$$

$$-B(g_{o2}-d_{o1};y',p_o)\}$$

$$\cdot\{B(n_2-(g_{o2}-d_{o1});y'-(g_{o2}-d_{o1}+1),p_1'')$$

$$-B(g_{12}-g_{o2}-d_{11};\ y'-(g_{o2}-d_{o1}+1),p_1'')\}$$

$$+n_1 \sum_{y'=g_{12}-(d_{o1}+d_{11})}^{n_2} \{B(g_{o2}-d_{o1}-1;y'-1,p_o)$$

$$-B(g_{o2}-d_{o1}-1;\ y',p_o)\}$$

$$\{B(n_2-g_{o2}+d_{o1};y'-(g_{o2}-d_{o1}),p_1'')$$

$$-B(g_{12}-g_{o2}-d_{11};\ y'-(g_{o2}-d_{o1}),p_1'')\}\Big]$$

$$+ \sum_{d_{21}=a_{21}+1}^{r_2-1} b(d_{21};n_1,p_2) \sum_{d_{11}=o}^{r_1-1-d_{21}} b(d_{11};n_1-d_{21},p_1')$$

$$\cdot\Big[\frac{r_2-d_{21}}{p_2} \sum_{y'=r_2+1-d_{21}}^{n_2+1} \{B(r_2-d_{21};y'-1,p_2)$$

$$-B(r_2-d_{21};y',p_2)\}$$

$$\cdot B(r_1-r_2-d_{11};\ y'-(r_2+1-d_{21}),\ p_1')$$

$$+ n_1 \sum_{y'=r_2-d_{21}}^{n_2} \{\ B(r_2-1-d_{21};y'-1,p_2)-B(r_2-1-d_{21};y',p_2)\}$$

$$B(r_1-r_2-d_{11};\ y'-(r_2-d_{21}),p_1'\ )$$

$$+ \frac{r_1-(d_{11}+d_{21})}{1-p_o} \{\ 1-B(r_1-(d_{11}+d_{21});\ n_2+1,1-p_o)\}$$

cont...

$$\cdot B(r_2-1-d_{21};\ r_1-(d_{11}+d_{21}),p_2'')$$
$$+n_1\ \{1-B(r_1-1-(d_{11}+d_{21});\ n_2,1-p_o)\}$$
$$\cdot B(r_2-1-d_{21};\ r_1-(d_{11}+d_{21}),p_2'')\Big] \qquad \ldots(6.4.16)$$

These expressions can be calculated with the help of usual binomial tables such as [34] , [43] etc. The relation $B(r;n,p) = 1-B(n-r-1;n,1-p)$ may be found useful while using the binomial table [43].

6.4.4 The Maximum Likelihood Estimators and the Asymptotic Variances of the Maximum Likelihood Estimators Under Semi-Curtailed DTCAP :

The inspection of units of a lot according to a semi-curtailed DTCAP implies the occurrence of the events $E_{s,ti}$ $(t=1,2,3,4;\ i=1,2)$. Let the events $E_{s,ti}$ occur m_{ti} times when m lots have undergone the inspection such that $\sum_{t=1}^{4}\sum_{i=1}^{2} m_{ti} = m$. Then the m 8-tuple observations associated with the random variable Y, D_{vj} $(v= 0,1,2,;\ j=1,2)$ and Z be displayed in 8 groups for convenience. The observations of the zth group $(z=1,2,\ldots,8)$ will have the following display:

$$(Y_{zj},\ D_{o1zj},\ D_{11zj},\ D_{21zj},\ D_{o2zj},\ D_{12zj},\ D_{22zj},\ Z)$$

In the above display $j=1,2,\ldots,\ m_{ti}$ where m_{ti} take the values

m_{11}, m_{21}, m_{31}, m_{41}, m_{12}, m_{22}, m_{32}, m_{42} as z takes ~~takes~~ the values 1, 2, 3, 4, 5, 6, 7, and 8 respectively.

The logarithm of the likelihood function for the above sample with little algebra can be written as

$$\text{LogL} = (TG)\log p_0 + (TM)\log p_1 + (TB)\log p_2 \quad \ldots(6.4.17)$$

where
$$(TG) = \sum_{j=1}^{m_{11}} (n_1 - d_{111j} - d_{211j}) + \sum_{j=1}^{m_{21}} (y_{2j} - r_2 - d_{112j})$$
$$+ \sum_{j=1}^{m_{31}} (y_{3j} - r_1) + \sum_{j=1}^{m_{41}} (y_{4j} - r_1)$$
$$+ \sum_{j=1}^{m_{12}} (n_1 + n_2 - d_{215j} - d_{115j} - d_{125j} - d_{225j})$$
$$+ \sum_{j=1}^{m_{22}} (y_{6j} - d_{116j} - r_2 - d_{126j}) + \sum_{j=1}^{m_{32}} (y_{7j} - r_1) + \sum_{j=1}^{m_{42}} (y_{8j} - r_1).$$

$$(TM) = \sum_{j=1}^{m_{11}} d_{111j} + \sum_{j=1}^{m_{21}} d_{112j} + \sum_{j=1}^{m_{31}} (r_1 - d_{213j})$$
$$+ m_{41}(r_1 - r_2) + \sum_{j=1}^{m_{12}} (d_{115j} + d_{125j}) + \sum_{j=1}^{m_{22}} (d_{116j} + d_{126j})$$
$$+ \sum_{j=1}^{m_{32}} (r_1 - d_{217j} - d_{227j}) + m_{42}(r_1 - r_2).$$

$$(TB) = \sum_{j=1}^{m_{11}} d_{211j} + m_{21} r_2 + \sum_{j=1}^{m_{31}} d_{213j} + m_{41} r_2$$

cont...

$$+ \sum_{j=1}^{m_{12}} (d_{215j}+d_{225j}) + m_{22}\, r_2 + \sum_{j=1}^{m_{32}} (d_{217j}+d_{227j}) + m_{42}\, r_2 .$$

Differentiating logL w.r. to p_1 and p_2 and equating to zero we get the MLE's as

$$\hat{p}_1 = \frac{(TM)}{(TU)} \qquad \ldots(6.4.18)$$

and

$$\hat{p}_2 = \frac{(TB)}{(TU)} \qquad \ldots(6.4.19)$$

We note the following results about the expectations of (TB), (TM), and (TG) which are useful in finding the asymptotic variances and covariance of the $\hat{p}_1$ and $\hat{p}_2$.

$$E(TB) = p_2\, E(TU) = p_2(m)(ASN)$$

$$E(TM) = p_1\, E(TU) = p_1(m)(ASN)$$

$$E(TG) = p_o\, E(TU) = p_o(m)(ASN)$$

where in this case ASN = ASN(semi) given by the expression (6.4.15) of Section 6.4.3. Then we find that

$$-E\left(\frac{\partial^2 \log L}{\partial p_1^2}\right) = \frac{p_o^2\, E(TM) + p_1^2\, E(TG)}{p_o^2\, p_1^2}$$

$$= \frac{m(ASN)(1-p_2)}{p_o p_1}$$

$$= \phi_{11}$$

$$- E\left(\frac{\partial^2 \log L}{\partial p_2^2}\right) = \frac{p_o^2\ E(TB) + p_2^2\ E(TG)}{p_o^2\ p_2^2}$$

$$= \frac{m(ASN)(1-p_1)}{p_o p_2}$$

$$= \phi_{22}$$

$$- E\left(\frac{\partial^2 \log L}{\partial p_1 \partial p_2}\right) = \frac{E(TG)}{p_o^2}$$

$$= \frac{m(ASN)}{p_o}$$

$$= \phi_{12} = \phi_{21}$$

$$\therefore \Delta = \begin{vmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{vmatrix} = \frac{m^2(ASN)^2}{p_o p_1 p_2} \qquad \ldots(6.4.20)$$

Then the asymptotic variance - Covariance matrix of $\hat{p}_1$ and $\hat{p}_2$ given by $[\phi_{ij}]^{-1}$ leads to the following asymptotic Variances and Covariance.

$$V(\hat{p}_1) = \frac{\phi_{22}}{\Delta} = \frac{p_1(1-p_1)}{m(ASN)} \qquad \ldots(6.4.21)$$

$$V(\hat{p}_2) = \frac{\phi_{11}}{\Delta} = \frac{p_2(1-p_2)}{m(ASN)} \qquad \ldots(6.4.22)$$

$$\text{and } Cov(\hat{p}_1, \hat{p}_2) = -\frac{\phi_{12}}{\Delta} = -\frac{p_1 \cdot p_2}{m(ASN)} \qquad \ldots(6.4.23)$$

6.5 Extension of the Results to the Other Sampling Plans :

The results of Section 6.4.4 can easily be extended to the other sampling plans. Whether it is uncurtailed DTCAP or fully-curtailed DTCAP, the logL has the similar nature as that of (6.4.17). Hence the expressions for the MLE and for the asymptotic variances and covariance are given by (6.4.18), (6.4.19), (6.4.21), (6.4.22), and (6.4.23). It may be noted that, though, the physical meaning of (TG), (TB), and (TM) in the expression (6.4.17) remains same, the exact expressions for these quantities will differ as one passes from one form of the sampling plan to other. Also ASN in the expressions (6.4.21), (6.4.22), (6.4.23) should be replaced by the expression of the ASN for a sampling which is under consideration.This remark also holds for any form of MTCAP.

TABLE 6.1

Set	Y	D_{o1}	D_{11}
1	2	3	4
$B_{u,11}$ or $B_{s,11}$	n_1	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{u,21}$	n_1	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{u,31}$	n_1	$n_1-d_{11}-d_{21}$	$r_1-d_{21},\ldots,n_1-d_{21}$
$B_{u,41}$	n_1	$n_1-d_{11}-d_{21}$	$r_1-d_{21},\ldots,n_1-d_{21}$
$B_{u,12}$ or $B_{s,12}$	n_1+n_2	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{u,22}$	n_1+n_2	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{u,32}$	n_1+n_2	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{u,42}$	n_1+n_2	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{s,21}$ or $B_{f,41}$	$r_2,\ldots,n_1$	$y-r_2-d_{11}$	$0,1,\ldots,r_1-1-r_2$
$B_{s,31}$ or $B_{f,51}$	$r_1,\ldots,n_1$	$y-r_1$	r_1-d_{21}
$B_{s,41}$ or $B_{f,61}$	$r_1,\ldots,n_1$	$y-r_1$	r_1-r_2
$B_{s,22}$ or $B_{f,42}$	$n_1+r_2-d_{21},\ldots,n_1+n_2$	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{s,32}$ or $B_{f,52}$	$n_1+r_1-(d_{11}+d_{21}),\ldots,n_1+n_2$	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{s,42}$ or $B_{f,62}$	$n_1+r_1-(d_{11}+d_{21}),\ldots,n_1+n_2$	$n_1-d_{11}-d_{21}$	$0,1,\ldots,r_1-1-d_{21}$
$B_{f,11}$	$g_{11},\ldots,n_1$	$g_{o1}+1,\ldots,g_{11}$	$g_{11}-d_{o1}$

TABLE 6.1 (continued)

Set	D_{o2}	D_{12}
	6	7
$B_{u,11}$ or $B_{s,11}$	-	-
$B_{u,21}$	-	-
$B_{u,31}$	-	-
$B_{u,41}$	-	-
$B_{u,12}$ or $B_{s,12}$	$n_2-d_{12}-d_{22}$	$0,1,\ldots,r_1-1-d_{11}-d_{21}-d_{22}$
$B_{u,22}$	$n_2-d_{12}-d_{22}$	$0,1,\ldots,r_1-1-d_{11}-d_{21}-d_{22}$
$B_{u,32}$	$n_2-d_{12}-d_{22}$	$r_1-d_{11}-d_{21}-d_{22},\ldots,n_2-d_{22}$
$B_{u,42}$	$n_2-d_{12}-d_{22}$	$r_1-d_{11}-d_{21}-d_{22},\ldots,n_2-d_{22}$
$B_{s,21}$ or $B_{f,41}$	-	-
$B_{s,31}$ or $B_{f,51}$	-	-
$B_{s,41}$ or $B_{f,61}$	-	-
$B_{s,22}$ or $B_{f,42}$	$y-n_1-(r_2-d_{21})-d_{12}$	$0,1,\ldots,r_1-1-d_{11}-r_2$
$B_{s,32}$ or $B_{f,52}$	$y-n_1-(r_1-d_{11}-d_{21})$	$r_1-(d_{11}+d_{21})-d_{22}$
$B_{s,42}$ Or $B_{f,62}$	$y-n_1-(r_1-d_{11}-d_{21})$	$r_1-d_{11}-r_2$
$B_{f,11}$	-	-
$B_{f,21}$	-	-
$B_{f,31}$	-	-