Chapter 1

Introduction

1.1 Introduction

Fixed point theory is a powerful and productive tool for non-linear analysis. It is a rapidly expanding area in non-linear analysis and non-linear operators, because of its importance in the existence theory of differential equations and integral equations, partial differential equations, random differential equations, fluid flows, chemical reactions, nonlinear oscillations, approximation theory, economic theories, steady state temperature distribution and other related fields.

Nonlinear analysis is concerned with the solution of nonlinear problems. It is used to examine the conditions under which solutions to mappings exist. The equation $x = Tx$, where T is a nonlinear operator defined on a metric space, can be used to model a variety of problems in various disciplines of mathematics and x is a solution of this equation, which is called the fixed point of T.

The simple way to find whether an equation has a solution or not is to present it as a fixed point problem.

Example 1.1.1. *Assume we have a set of n equations with n unknowns of the form:*

$$A_i(x_1, x_2, \ldots, x_n) = 0 \quad ; i = 1, 2, \ldots n,$$

where the A_i *are real-valued continuous functions of the real variables* x_i*. Let* $b_i(x_1, x_2, \ldots, x_n) = A_i(x_1, x_2, \ldots, x_n) + x_i$ *and for any* $x = (x_1, x_2, \ldots x_n)$*, define* $b(x) = (b_1(x), b_2(x), \ldots, b_n(x))$*. Suppose* b *has a fixed point* $\bar{x} \in \mathbb{R}^n$*. Then it is clear that* $\bar{x}$ *is a solution to the system of equations.*

The results which are connected to the existence of fixed points are called fixed point results. We know from fixed point results that a mapping T of X allows one or more fixed points under certain conditions on the mapping T and the space X. Poincaré (1886) is the first to use the concept of a fixed point. Poincare's last geometric theorem claims that "there exist at least two fixed points for an area preserving twist homeomorphism of an annulus". Dutch mathematician Brouwer (1912) gave the first fixed point result for a topological space, stating that "continuous self mapping defined on the closed unit ball in Euclidean space has at least one fixed point". His result is applicable to finite-dimensional spaces and serves as the foundation for numerous fixed-point results. Because of its use in different areas of mathematics and economics, this finding proved to be a crucial theorem. This theorem was further generalized for set-valued functions by Kakutani (1941). There are numerous generalizations to Brouwer's fixed point theorem that exist at present.

Metric fixed point theory has been essential in the advancement of nonlinear functional analysis. It has several utilization in fields like economics and computer science (see Border (1985),Matthews (1992)), etc. The credit, however, goes to Polish Mathematician Banach (1922) for putting the above theory into an abstract framework suited for broad applications. Kannan (1968) established a fixed point result for a different contractive condition for the mappings which are discontinuous in their domain but still have a fixed point, although such mappings are continuous at their fixed point. This result of Kannan led to the turning point in fixed point theory. After that, many authors improved and extended the Banach Contraction Principle.

1.2 Preliminaries

Definition 1.2.1. Fixed point:

Let X be a non-empty set and $T : X \to X$. If $x = Tx$ for some $x \in X$, then x is called a fixed point of T.

Definition 1.2.2. Common fixed point:

Let X be a non-empty set and $T_1, T_2 : X \to X$. If $x = T_1x = T_2x$ for some $x \in X$, then x is called a common fixed point of T_1 and T_2.

Fréchet (1906) introduced the term metric space in his doctoral thesis. On the other hand, the explanation proposed by German mathematician Hausdorff (1914), is widely used and stated as below:

Definition 1.2.3. Metric space:

Let X be a non-empty set, and the mapping $\hat{d} : X \times X \to [0, \infty)$ satisfies the following:

(1) $\hat{d}(x, y) = 0$ if and only if $x = y$,

(2) $\hat{d}(x, y) = \hat{d}(y, x)$ for all $x, y \in X$,

(3) $\hat{d}(x, y) \leq [\hat{d}(x, z) + \hat{d}(z, y)]$, $\forall x, y, z \in X$ (triangle inequality).

Then $\hat{d}$ is called a metric on X and $(X, \hat{d})$ is called a metric space.

In metric space, the concepts of Cauchy sequence, convergence, and completeness are defined as follows:

Definition 1.2.4. *Let $(X, \hat{d})$ be a metric space and $\{x_n\}$ be a sequence in X. Then*

(i) A sequence $\{x_n\}$ is called convergent to $u \in X$ if $\lim_{n\to\infty} \hat{d}(x_n, u) = 0$.
That is, for each $\epsilon \geq 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, we have $\hat{d}(x_n, u) \leq \epsilon$.

(ii) A sequence $\{x_n\}$ is called a Cauchy sequence if $\lim_{n\to\infty} \hat{d}(x_n, x_m) = 0$.
That is, for each $\epsilon \geq 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$, we have $\hat{d}(x_n, x_m) \leq \epsilon$.

(iii) $(X, \hat{d})$ is said to be a complete metric space if every Cauchy sequence in $(X, \hat{d})$ is convergent.

Definition 1.2.5. Orbitally continuous:*(Ćirić (1971))*
If f is a self-mapping of a metric space $(X, \hat{d})$, then the set $O(x, f) = \{f^n x : n = 1, 2, ...\}$ is called the orbit of f at x and f is called orbitally continuous if $\lim_{k\to\infty} f^{n_k} x = x$, for some $x \in X$ implies $\lim_{k\to\infty} f(f^{n_k} x) = fx$.

Remark: It is obvious that a continuous function is always orbitally continuous but the converse may not be true.

The following example justifies this fact.

Example 1.2.6. *Define the map $T : X \to X$ on $X = [0, 2]$ by*

$$T(x) = \frac{1}{2} \;\; if \;\; x \in [0, 1], T(x) = 0 \;\; if \;\; x \in (1, 2].$$

It is clear that f is orbitally continuous but not continuous at $x = 1$.

Definition 1.2.7. Asymptotic regularity: *(Browder and Petryshyn (1966))*
Let $(X, \hat{d})$ *be a metric space. A mapping* $T : X \to X$ *satisfying the condition* $\lim_{n\to\infty} \hat{d}(T^{n+1}x, T^n x) = 0$ *for all* $x \in X$ *is called asymptotically regular.*

Definition 1.2.8. Coincidence point: *(Jungck (1986))*
Let X *be a non-empty set and* $T_1, T_2 : X \to X$. *If* $w = T_1 x = T_2 x$ *for some* $x \in X$, *then* x *is called a coincidence point of* T_1 *and* T_2, *and* w *is called a point of coincidence of* T_1 *and* T_2.

Definition 1.2.9. Weakly compatible: *(Jungck and Rhodes (1998)) Let* X *be a non-empty set and* $T_1, T_2 : X \to X$. *The pair* $\{T_1, T_2\}$ *is said to be weakly compatible if* $T_1T_2t = T_2T_1t$, *whenever* $T_1t = T_2t$ *for some* t *in* X.

Definition 1.2.10. Commutative map: *(Jungck (1976)) Suppose that* $T_1, T_2 : X \to X$ *are two mappings in metric space* $(X, \hat{d})$. *Then* T_1, T_2 *are called commutative mappings if for all* $x \in X$, $T_1T_2x = T_2T_1x$.

1.3 A Review on Fixed Point Results

Theorem 1.3.1. *(Brouwer (1912)) Let* C *be a closed unit ball in* R^n *and let* f *be a continuous self-mapping on* C. *Then* f *has fixed point in* C.

Banach (1922) gave the essential result known as the Banach Contraction Principle.

Definition 1.3.2. Banach contraction: *A mapping* T *from a metric space* X *into itself is said to be a contraction if*

$$\hat{d}(Tx, Ty) \leq a\hat{d}(x, y) \qquad ; \forall x, y \in X \quad \text{and} \quad 0 < a < 1.$$

It is clear that a contraction mapping is continuous but the converse need not be true. The Banach Contraction Principle is as follows:

Theorem 1.3.3. *(Banach (1922)) A contraction mapping of a complete metric space X into itself has a unique fixed point in X.*

This finding has now become one of the well-known and successful tools for solving existing problems in a wide range of mathematical fields due to its simplicity and utility. The requirement of continuous mapping is a disadvantage of this famous finding. Kannan (1968) solved this issue by employing adjusted contraction condition.

Theorem 1.3.4. *(Kannan (1968)) A mapping T from a complete metric space X into itself such that there exists $\lambda \in [0, \frac{1}{2})$ and*

$$\hat{d}(Tx, Ty) \leq \lambda[\hat{d}(x, Tx) + \hat{d}(y, Ty)] \qquad ; \forall x, y \in X.$$

Then T has a unique fixed point in X.

Following his work, extensive research was initiated in this direction and the researchers presented many contractive conditions in the next two decades. Later on, Chatterjea (1972) and Reich (1971) presented some results obtained through the use of modified contractive conditions.

Theorem 1.3.5. *(Chatterjea (1972)) A mapping T from a complete metric space X into itself such that there exists $\lambda \in [0, \frac{1}{2})$ and*

$$\hat{d}(Tx, Ty) \leq \lambda[\hat{d}(x, Ty) + \hat{d}(y, Tx)] \qquad ; \forall x, y \in X.$$

Then T admits a unique fixed point in X.

Theorem 1.3.6. *(Reich (1971)) A mapping T from a complete metric space X into* *itself satisfies the following:*

$$\hat{d}(Tx, Ty) \leq \alpha \hat{d}(x, Tx) + \beta \hat{d}(y, Ty) + \gamma \hat{d}(x, y),$$

$\forall x, y \in X$ and α, β, γ non negative with $\alpha + \beta + \gamma < 1$. Then T has a unique fixed point in X.

Motivated by these results, Hardy and Rogers (1973) established the following contractive inequality to prove their result.

Theorem 1.3.7. *(Hardy and Rogers (1973)) A mapping T from a complete metric space X into itself satisfies the following:*

$$\hat{d}(Tx, Ty) \leq \alpha_1[\hat{d}(x, Tx) + \hat{d}(y, Ty)] + \alpha_2[\hat{d}(x, Ty) + \hat{d}(y, Tx)] + \alpha_3 \hat{d}(x, y),$$

$\forall x, y \in X$ and $\alpha_1, \alpha_2, \alpha_3$ non negative with $2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$. Then T has a unique fixed point in X.

Later on, a more generalized contractive condition was obtained by Ćirić (1974) to prove the uniqueness.

Theorem 1.3.8. *(Ćirić (1974)) A mapping T from a complete metric space X into itself satisfies the following:*

$$\hat{d}(Tx, Ty) \leq a\ m(x, y), \tag{1.1}$$

$\forall x, y \in X$ and $a \in [0, 1)$, where

$$m(x, y) = max\{\hat{d}(x, Tx), \hat{d}(y, Ty), \hat{d}(x, y), \hat{d}(x, Ty), \hat{d}(y, Tx)\}.$$

Then T has a unique fixed point in X.

As we know, Kannan (1968) proved the result for discontinuous mappings but these maps are continuous at their fixed points. Later on, Rhoades (1988) posed an open problem of whether there exists a contractive definition that yields a fixed point but do not require the mapping to be continuous at the fixed point. Pant (1999) solved the Rhoades problem by making the mapping discontinuous at the fixed point.

Theorem 1.3.9. *(Pant (1999)) Let $(X, \hat{d})$ be a complete metric space. Let T be a self map on X which satisfies the following:*

(i) $\hat{d}(Tx, Ty) \leq \phi(m(x, y)) \qquad ; \forall x, y \in X,$
where, $m(x, y) = max\{\hat{d}(x, Tx), \hat{d}(y, Ty)\}$.

(ii) given $\epsilon > 0$ *there exists* $\delta > 0$ *such that* $\epsilon < m(x, y) < \epsilon + \delta \implies \hat{d}(Tx, Ty) \leq \epsilon$.

Then T has a unique fixed point, say z. Moreover, T is continuous at z if and only if $\lim_{x \to z} m(x, z) = 0$.

For a long time, it was unknown whether two commuting mappings of a convex and compact set into itself had a common fixed point.

Theorem 1.3.10. *(Boyce (1967)) There exist two commuting continuous mappings S and T from [0,1] into itself without fixed points.*

Jungck (1976) derived the following generalization of the Banach Contraction Principle which confirm common fixed point under certain condition.

Theorem 1.3.11. *(Jungck (1976)) Let T and S be two commuting self maps of a complete metric space $(X, \hat{d})$ such that S is continuous and satisfies the following:*

(i) $T(X) \subset S(X)$, *and*

(ii) $\hat{d}(Tx, Ty) \leq k \ \hat{d}(Sx, Sy) \quad ; \forall x, y \in X, x \neq y,$

where, $k \in (0, 1)$ a real number. Then T and S have a unique common fixed point.

1.4 Some Abstract Spaces

Now some abstract spaces are discussed as follows:

1.4.1 b - metric space

Bakhtin (1989) introduced b-metric space by multiplying the right hand side of the triangle inequality with some real number.

Definition 1.4.1.1. *Let X be a non-empty set with the coefficient $s \geq 1$, and the mapping $\hat{d} : X \times X \to [0, \infty)$ satisfies the following:*

(1) $\hat{d}(x, y) = 0$ *if and only if* $x = y$,

(2) $\hat{d}(x, y) = \hat{d}(y, x)$ *for all* $x, y \in X$,

(3) $\hat{d}(x, y) \leq s[\hat{d}(x, z) + \hat{d}(z, y)]$ $\forall x, y, z \in X$.

Then $\hat{d}$ is called a b-metric on X and $(X, \hat{d})$ is called a b-metric space with coefficient s.

1.4.2 Rectangular metric space (generalized metric space)

The generalized metric space introduced by Branciari (2000) by replacing triangular inequality with rectangular one in the context of fixed point theorem.

Definition 1.4.2.1. *Let X be a nonempty set. Suppose that the mapping $\hat{d} : X \times X \to [0, \infty)$, satisfies:*

(1) $\hat{d}(x, y) \geq 0$, *for all* $x, y \in X$ *and* $\hat{d}(x, y) = 0$ *if and only if* $x = y$,

(2) $\hat{d}(x, y) = \hat{d}(y, x)$ *for all* $x, y \in X$,

(3) $\hat{d}(x, y) \leq \hat{d}(x, w) + \hat{d}(w, z) + \hat{d}(z, y)$ *for all* $x, y \in X$ *and for all distinct points*

$w,z \in X - \{x, y\}$ [rectangular property].

Then $(X, \hat{d})$ is called a generalized metric space (rectangular metric space) (RMS).

Example 1.4.2.1. *(Branciari (2000)) Let $X = R$ and $0 \neq \alpha \in R$. Define $\hat{d}$: $X \times X \to [0, \infty)$ as follows:*

$$\hat{d}(x,y) = \begin{cases} 3\alpha & ; x, y \in \{1,2\} \\ 0 & ; x \text{ and } y \text{ can not both at a time in} \{1,2\} \\ 0 & ; x = y \end{cases}$$

Then it is clear that $(X, \hat{d})$ is a rectangular metric space. Also $(X, \hat{d})$ does not hold the triangular property: $3\alpha = \hat{d}(1,2) \geq \hat{d}(1,3) + \hat{d}(3,2) = \alpha + \alpha = 2\alpha$.
So, it is not a standard metric space.

1.4.3 2 - metric Space

Definition 1.4.3.1. *(Gahler (1963)) Let X be a nonempty set, a function $\hat{d} : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) For distinct points $x, y \in X$, there is $z \in X$ such that $\hat{d}(x, y, z) \neq 0$,

(2) $\hat{d}(x, y, z) = 0$ if two of the triple $x, y, z \in X$ are equal,

(3) $\hat{d}(x, y, z) = \hat{d}(x, z, y) = \ldots$ (symmetry in all three variables),

(5) $\hat{d}(x, y, z) \leq \hat{d}(x, y, a) + \hat{d}(x, a, z) + \hat{d}(a, y, z)$.

Then $(X, \hat{d})$ is called a 2-metric space.

1.4.4 D - metric space

Definition 1.4.4.1. *(Dhage (1992)) Let X be a nonempty set, a D-metric on X is a function $D : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) $D(x, y, z) \geq 0$,

(2) $D(x, y, z) = 0$ *if and only if* $x = y = z$,

(3) $D(x, y, z) = D(x, z, y) = \dots$ *(symmetry in all three variables),*

(5) $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$.

Then (X, D) is called a D-metric space.

1.4.5 G - metric space

Definition 1.4.5.1. *(Mustafa and Sims (2006)) Let X be a nonempty set. A G-metric on X is a function $G : X^3 \to [0, \infty)$ that satisfies the following conditions,*

(1) $G(x, x, y) > 0$ *for all* $x, y \in X$ *with* $x \neq y$,

(2) $G(x, y, z) = 0$ *if* $x = y = z$,

(3) $G(x, x, y) \leq G(x, y, z)$ *for all* $x, y, z \in X$ *with* $x \neq y$,

(4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$ *(symmetry in all three variables),*

(5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ *for all* $x, y, z, a \in X$.

Then (X, G) is called a G-metric space.

1.4.6 D*- metric space

Definition 1.4.6.1. *(Sedghi et al. (2007)) Let X be a nonempty set. A D^*-metric on X is a function $D^* : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) $D^*(x, y, z) \geq 0$,

(2) $D^*(x, y, z) = 0$ *if and only if* $x = y = z$,

(3) $D^*(x, y, z) = D^*(p\{x, y, z\})$, *(symmetry), where p is a permutation function,*

(5) $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

Then (X, D^*) is called a D^*-metric space.

1.4.7 S - metric space

Sedghi et al. (2012) introduced a three dimensional metric space, and it is called S-metric space, which is defined by modifying D-metric and G-metric spaces.

Definition 1.4.7.1. *Let X be a nonempty set, an S-metric on X is a function $S : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) $S(x, y, z) \geq 0$,

(2) $S(x, y, z) = 0$ *if and only if* $x = y = z$,

(3) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$.

Then (X, S) is called an S-metric space.

Every metric is known to produce an S-metric. It is stated in Gupta (2013) that every S-metric defines a metric. On the other hand, Hieu et al. (2015) provided a counter example, which shows that every S-metric is not generated by metric space. The relation between an S-metric and a metric is defined as follows:

Lemma 1.4.7.1. *(Hieu et al. (2015)) Let $(X, \hat{d})$ be a metric space and $S_{\hat{d}}$ be a S-metric generated by $\hat{d}$. Then $S_{\hat{d}}(x, y, z) = \hat{d}(x, z) + \hat{d}(y, z)$, $\forall\ x, y, z \in X$ is an S-metric on X.*

After that, Özgür and Tas (2017) gave the following another example of an S-metric that is not generated by a metric (i.e. $S \neq S_{\hat{d}}$).

Example 1.4.7.1. *Let $X = \mathbb{R}$ and $S(x, y, z) = |x + z - 2y| + |x - z|\ \ ;\forall x, y, z \in \mathbb{R}$. Here, (X, S) is an S-metric but there is no such metric $\hat{d}$ exists such that $S = S_{\hat{d}}$.*

1.4.8 S_b - metric space

Souayah and Mlaiki (2016) introduced an S_b- metric space which is a combination of b-metric space and S-metric space.

Definition 1.4.8.1. *Let X be a nonempty set and let $s \geq 1$, an S_b-metric on X is a function $S_b : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) $S_b(x, y, z) = 0$ *if and only if* $x = y = z$,

(2) $S_b(x, x, y) = S_b(y, y, x)$,

(3) $S_b(x, y, z) \leq s[S_b(x, x, a) + S_b(y, y, a) + S_b(z, z, a)]$.

Then (X, S) is called an S_b-metric space.

Remark: The S_b -metric space is larger than the S-metric space. Actually, S_b-metric space with $s = 1$ turns into S-metric space. However, S-metric space may not be S_b-metric space.

Condition(2) is not true in the definition of S_b-metric space in general. To make it more generic, Rohen et al. (2017) changed the definition as follows:

Definition 1.4.8.2. *Let X be a nonempty set and let $s \geq 1$, an S_b-metric on X is a function $S_b : X^3 \to [0, \infty)$ that satisfies the following conditions, for each $x, y, z, a \in X$,*

(1) $S_b(x, y, z) = 0$ *if and only if* $x = y = z$,

(2) $S_b(x, y, z) \leq s[S_b(x, x, a) + S_b(y, y, a) + S_b(z, z, a)]$.

Then (X, S) is called S_b-metric space.

1.4.9 A - metric space

Abbas et al. (2015) presented the concept of A-metric space, which is a generalization of S-metric space and defined it as follows:

Definition 1.4.9.1. *Let X be a nonempty set and the function $A : X^n \to [0, \infty)$ that satisfies the following conditions, for all $x_1, x_2, ..., x_n, a \in X$,*

(1) $A(x_1, x_2, ..., x_n) \geq 0$,

(2) $A(x_1, x_2, ..., x_n) = 0$ *if and only if* $x_1 = x_2 = ... = x_n$,

(3) $A(x_1, x_2, ..., x_n) \leq$
$A(x_1, x_1, .., x_{1_{(n-1)}}, a) + A(x_2, x_2, .., x_{2_{(n-1)}}, a) + \ldots + A(x_n, x_n, ...x_{n_{(n-1)}}, a)$.

Then (X, A) is called an A-metric space.

Example 1.4.9.1. *Let $X = R$ and $A(x_1, x_2, x_3, ..., x_n) = |x_1 - x_n| + |x_2 - x_n| + ... + |x_{n-1} - x_n|$. Then, (X, A) is an A-metric space.*

1.5 Methodology

For mappings satisfying particular contraction type conditions, there exist various metrical fixed point theorems. In all of these results, a sequence of iterates is considered, which becomes a Cauchy sequence due to the contraction condition and whose limit is a fixed point of the mapping. A joint sequence of iterates is appropriate for common fixed point theorems.

The thesis is organized as follows:

1.6 Layout of The Thesis

The thesis consists of six chapters, chapter 1 contains an introduction and preliminaries. In subsequent chapters, chapters 2-6, we begin with some preliminaries which are required to prove the author's own results.

Chapter 1 contains an introduction to fixed point theory and the theoretical background needed for the problems studied in the subsequent chapters. It also contains the summary of each chapter of the thesis. We have attempted to provide a concise explanation of the subject's historical evolution, some abstract spaces, essential definitions, and significant results. This chapter's major goal is to make the current text as self-contained as possible.

Chapter 2 The purpose of this chapter is to find a common fixed point for two maps in which one map is orbitally continuous and then we have extended some fixed point theorems with some additional conditions such as compactness, asymptotic regularity. We have provided an example that gives strength to our result.

Chapter 3 is devoted to studying the Wardowski F-contraction and rectangular b-metric space. We provide some new fixed point theorems for F-contraction on rectangular b-metric spaces in which maps need not be continuous. We assume two pairs of weakly compatible mappings satisfying the new contractive condition in rectangular b-metric spaces and derive a unique common fixed point. Some corollaries are also obtained from the main result. Our results not only generalize many known results in the literature but also improve some of the results therein. In addition, the results are justified by appropriate examples and deployed to examine the existence and uniqueness of solutions for a system of Volterra integral equation which is used to model many real life problems.

Chapter 4 deals with the common fixed point theorem for F-contraction by assuming that one map is orbitally continuous in $\Omega-$extended rectangular b-metric space (not necessarily continuous). Furthermore, we identify a fixed point for Banach's and Kannan's type F- contraction inequality without consideration of orbital continuity. The appropriate examples are also provided for all the results.

Chapter 5 In this chapter, we present the essential results of A_b-metric space and then we obtain the Banach type contraction principle and Kannan type fixed point theorem as corollaries. An example is also provided for the utility of our result. Our theorems generalize different results from the existing literature, especially the results of Ughade et al. (2016). In addition, we provide examples for the justification of our results.

Chapter 6 The sixth and last chapter is divided into two sections. In the first section, we prove the primary result (Theorem 2.11) of Mustafa et al. (2019b) without assuming the continuity of the class of functions defined by Jleli et al. (2014).

In the second section, we prove a common fixed point theorem by taking one map is orbitally continuous in A_p-metric space (not necessarily continuous) using altering distance function ϕ. Consequently, we derive a fixed point for Banach and Kannan type contraction inequality without taking the orbitally continuous map. A suitable example is also included.