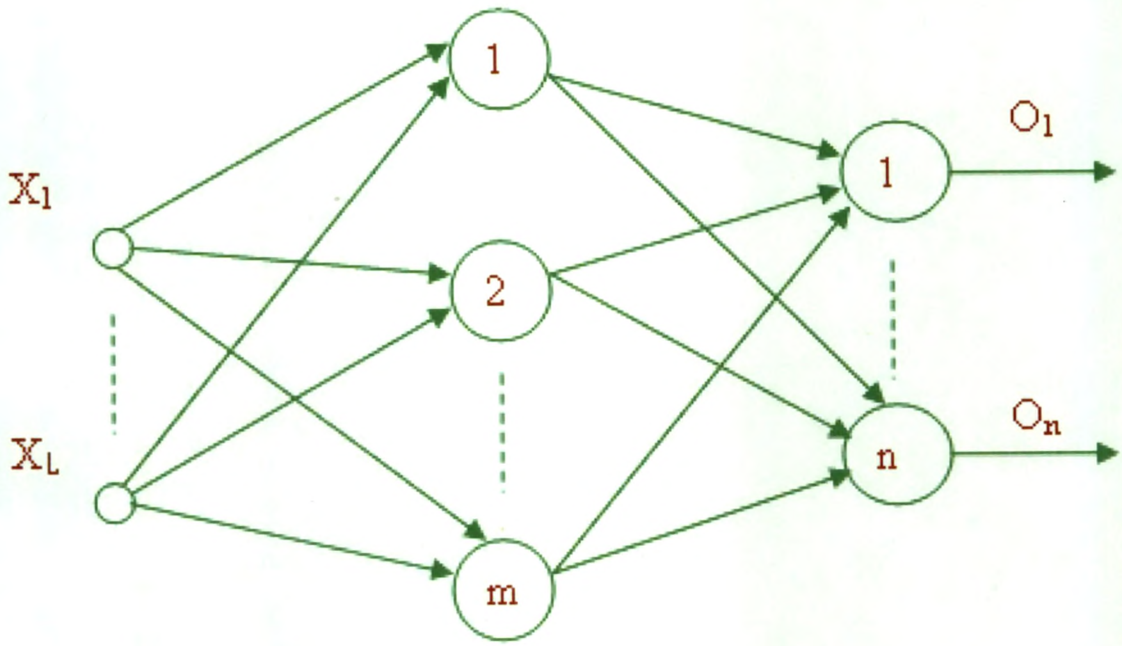
Chapter 5

ANN Based Steering Control of Semilinear Discrete Time System



Chapter 5

ANN BASED STEERING CONTROL OF SEMILINEAR DISCRETE TIME SYSTEM

5.1 Introduction

In Chapter 4, we derived the steering control for the continuous time-invariant semilinear system. In this chapter, we investigate the the controllability results for the discrete semilinear dynamical system in the finite dimensional space, since the main aim of this work is to emphasize the realization of the steering control using Artificial Neural Networks for the automated semilinear dynamical system.

The continuous time variant semilinear system in the differential form is represented as

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(x(t), u(t))$$
$$x(0) = x_0 \tag{5.1.1}$$

Suppose the system (5.1.1) becomes the part of the automated process in which the change in the system are at the discrete times and remains constant in between.

Then representation of such a system would be

$$x(k+1) = F(k)x(k) + G(k)u(k) + f_d(x(k), u(k))$$

$$x(0) = x_0 \tag{5.1.2}$$

where, $F(k)_{n\times n}, G(k)_{n\times m}$ are time dependent matrices and $F(k)$ is non-singular for all k. The state $x(k) \in X \subseteq R^n$, $u(k) \in U \subseteq R^m$ is the control input to the system and $f_d : R^n \times R^m \rightarrow R^n$ is the nonlinear function. The system (5.1.2) is in the discrete semilinear form.

For the semilinear system given by (5.1.2), we want to obtain sufficient conditions on the nonlinear term such that the system is controllable. To proceed for it, we first investigate the local controllability results about the equilibrium and then give the conditions on the nonlinearity that allows us to extend the domain of controllability.

In our analysis, we will employ the Inverse Function Theorem and Implicit Function Theorem (refer Chapter 2 Theorems 2.4.1 and 2.4.2) to establish the existence of controller. The implementation of steering controller is done using feed-forward Artificial Neural Network, with the help of MATLAB.

5.2 Controllability: Discrete Semilinear System

For the linear systems, local controllability and global controllability are equivalent whereas, for nonlinear system they are not the same and global controllability is hard to establish. However, in our approach we will first prove local controllability of the semilinear system and then expand the set of controllable states, almost to the complete state space. Without loss of generality, we will assume the equilibrium state as $(0,0)$ to simplify the derivation.

Controllability of Associated Linear System :

The linearized system corresponding to the nonlinear system (5.1.2) about the equilibrium solution $(0,0)$ is given by

$$x(k+1) = (F(k) + F_0)x(k) + (G(k) + G_0)u(k) \tag{5.2.3}$$

where, $F_0 = \frac{\partial f_d}{\partial x} |_{(0,0)}$ and $G_0 = \frac{\partial f_d}{\partial u} |_{(0,0)}$.

As stated in Chapter 2, the system (5.2.3) is controllable if and only if n rows of matrix function $\Phi(k_0, k+1)(G(k) + G_0)$ are linearly independent, where,

$$\Phi(k_0, k+1) = \prod_{i=k_0}^{k-1} (F(i) + F_0)$$

For the sake of simplicity we take $u(k)$ as scalar for each k in our system.

The state of the system (5.1.2) after n steps can be given as:

$$x(k+n) = \Phi(k+n,k)x(k) + \sum_{m=1}^{n} \Phi(k+n,k+m)G(k+m-1)u(k+m-1)$$

$$+ \sum_{m=1}^{n} \Phi(k+n,k+m)f_d(x(k+m-1),u(k+m-1)) \quad (5.2.4)$$

$$\equiv F_F[x(k),\cdots,x(k+n-1),u(k),\cdots,u(k+n-1)] \quad (5.2.5)$$

Using (5.2.4) and taking derivative of F_F w.r.t. $u(i), i = k,\cdots,k+n-1$ at (0,0) we get

$$\frac{\partial x(k+n)}{\partial u(k+n-1)} = G(k+n-1) + G_0$$

$$\frac{\partial x(k+n)}{\partial u(k+n-2)} = \frac{\partial x(k+n)}{\partial u(k+n-1)} \frac{\partial x(k+n-1)}{\partial u(k+n-2)}$$

Hence,

$$\frac{\partial x(k+n)}{\partial u(k+n-2)} = F(k+n-1)G(k+n-2)$$
$$+F_0G(k+n-2) + F(k+n-1)G_0 + F_0G_0$$
$$= \Phi(k+n-1,k+n-1)(G(k+n-2)+G_0)$$

Similarly,

$$\frac{\partial x(k+n)}{\partial u(k)} = F(k+n-1)\cdots F(k)G(k-1) + F_0F(k+n-2)\cdots F(k)F(k-1)$$
$$+\cdots + F_0^{n-2}F(k)G_0 + F_0^{n-1}G_0$$
$$= \Phi(k+n-1,k)(G(k-1)+G_0)$$

Defining $U_n(k) = [u(k),u(k+1),\cdots,u(k+n-1)]$, we get the Jacobian matrix as

$$D_{U_n(k)}^{x(k+n)} \equiv \frac{\partial x(k+n)}{\partial U_n(k)} \quad (5.2.6)$$

It can be shown easily that the rank of the controllability Grammian of the linearized system (5.2.3) and the rank of the Jacobian matrix (5.2.6) are the same. That is, if the linearized system is controllable then the Jacobian matrix $D_{U_n(k)}^{x(k+n)}$ will have n linearly independent rows. Therefore, the rank of $D_{U_n(k)}^{x(k+n)}$ is n if linearized system is controllable.

Local Controllability of SemiLinear System :

We now state a theorem which establishes the local controllability of the nonlinear system (5.1.2).

THEOREM 5.2.1 *If the linearized system (5.2.3) is controllable then the semilinear system (5.1.2) is locally controllable around the equilibrium state (0,0).*

Proof : Define a map $H : R^{2n} \to R^{2n}$ by $H(x(k), U_n(k)) = (x(k), x(k+n))$. We will show that there exist an mapping $\psi = H^{-1}$ such that $(x(k), U_n(k)) = \psi(x(k), x(k+n))$, whenever $(x(k); x(k+n))$ lie in the neighborhood of the equilibrium (0,0). Thus $U_n(k) = \psi\mid_u (x(k), x(k+n))$ will drive the system (5.1.2) from the state $x(k)$ to $x(k+n)$ for any k, where $\psi\mid_u (x_i, x_f)$ is the projection of $\psi(x_i, x_f)$ on the control space. The Jacobian matrix of the matrix H at (0,0) is given by

$$D_{(0,0)}H = \begin{bmatrix} I & 0 \\ D^{x(k+n)}_{x(k)} & D^{x(k+n)}_{U_n(k)} \end{bmatrix}$$

where, I is an identity matrix of order n. Also $D^{x(k+n)}_{U_n(k)}$ is of rank n because the linearized system is controllable . Therefore, the Jacobian matrix $D_{(0,0)}H$ is full rank $2n$. Hence given any two states x_i and x_f, an input sequence $U_n(k) = \psi\mid_u (x_i, x_f)$ will drive the system from x_i to x_f in n steps. For x_i and x_f lying in the neighborhood V_x of origin and by continuity of f_d and ψ, the system is transferred from x_i and x_f without leaving W_x.

The following corollary is immediate consequence of the above theorem.

COROLLARY 5.2.2 *If the linear system $x(k+1) = F(k)x(k) + G(k)u(k)$ is controllable then the system is (5.1.2) is locally controllable around the equilibrium state (0,0).*

The above result holds for the non-zero equilibrium point with slight modifications.

Mixtank Revisited :

Recall that the mathematical representation of the mixtank phenomenon (refer Section 4.2) is given as:

$$\dot{x}(t) = Bu(t) + f(x(t), u(t)) \tag{5.2.7}$$

where,

$$x(t) = \begin{bmatrix} V(t) \\ C(t) \end{bmatrix}, \; u(t) = \begin{bmatrix} Q_1(t) \\ Q_2(t) \end{bmatrix}, \; B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

And the nonlinearity $f(x(t), u(t))$ is given as:

$$f(x(t), u(t)) = \begin{bmatrix} -k\sqrt{\frac{V(t)}{A_c}} \\ \frac{(C_1 - C(t))Q_1(t)}{V(t)} + \frac{(C_2 - C(t))Q_2(t)}{V(t)} \end{bmatrix}$$

The linearization of (5.2.7) at the equilibrium point $(3600, 15)$ in continuous form is given by

$$\dot{x}(t) = \begin{bmatrix} -0.0042 & 0 \\ 0 & -0.0083 \end{bmatrix} x(t) + \begin{bmatrix} 1.0000 & 1.0000 \\ -0.0017 & 0.0008 \end{bmatrix} u(t) \tag{5.2.8}$$

$$\text{with} x(t_0) = x_0.$$

That is,

$$\dot{x}(t) = A_L x(t) + B_L u(t), \; x(t_0) = x_0 \tag{5.2.9}$$

where, $A_L = \begin{bmatrix} -0.0042 & 0 \\ 0 & -0.0083 \end{bmatrix}$, $B_L = \begin{bmatrix} 1.0000 & 1.0000 \\ -0.0017 & 0.0008 \end{bmatrix}$

Assuming, that the system (5.2.7) forms part of a automatized process due to which the valve settings of the inlets change at discrete instants only and remain constant in between. If these instants are separates by time period $T = 1$ seconds, i.e. taking the sampling period to be 1 seconds then the equivalent linearized discrete-time system is given by

$$x(k+1) = Fx(k) + Gu(k), x(0) = x_0 \tag{5.2.10}$$

where,

$$F = e^{A_L \times ST}$$

and

$$G = \int_0^{ST} e_L^A \tau B_L d\tau.$$

Thus, $F = \begin{bmatrix} 0.9958 & 0 \\ 0 & 0.9917 \end{bmatrix}$, $G = \begin{bmatrix} 0.9979 & 0.9979 \\ -0.0017 & 0.0008 \end{bmatrix}$.

The state of the system (5.2.10) at time k is given

$$x(k) = F^2x_0 + U_n u(k)$$

where, $U_n = [G|GF]$ given as

$$U_n = \begin{bmatrix} 0.9979 & 0.9979 & 0.9938 & 0.9896 \\ -0.0017 & 0.0008 & -0.0017 & 0.0008 \end{bmatrix}$$

is the controllability matrix for the discrete linearized state.

We see that the first two columns of U_n are linearly independent hence the system can be steered in two steps from the deviated state x_0 in the neighborhood of equilibrium to the equilibrium $x_1 = (3600, 15)$ as desired by the phenomenon and have the consistent concentration of the mixture at the desired consistent flow rate.

The control signal at time k is given by

$$u(k) = U_n^{-1}(x_1 - F^2x_0).$$

5.3 Simulations - Discrete Linearized :

1. 5-Step ANN Steering Controller :

The steering control for the system (5.2.10) is implemented using Neural Network NN_u with the architecture $NN^3_{2,30,20,10}$ as shown in the Figure 5.1. All the nodes in NN_u have tan-sigmoidal as activation function. We use Back-propagation algorithm to train NN_u. Using the definition for steering control (4.3.10), 25 input-output patterns are generated:

Input: The arbitrary initial state, in the neighborhood of the equilibrium (3600, 15).

Output: The control signal which steers the given initial state to the the final state, the equilibrium at $t = 1$.

The patterns are so generated that the steering is done in 5-steps, each of duration 0.2 seconds.

The training for NN_u converged in 173 epochs, as shown in Figure 5.2.

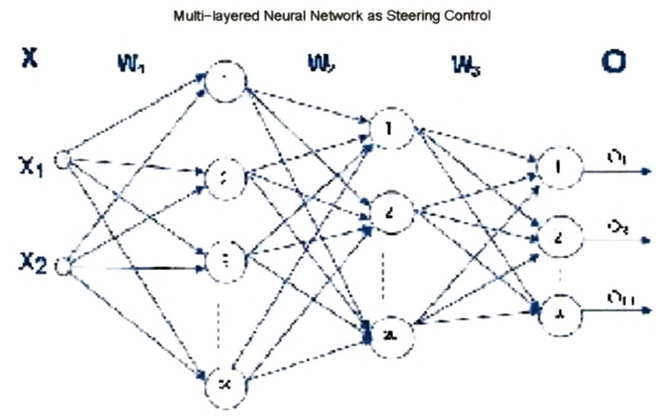


Figure 5.1: The ANN steering control with two hidden layers h_1 and h_2 having 30 and 20 nodes respectively and an output layer O with 10 nodes.

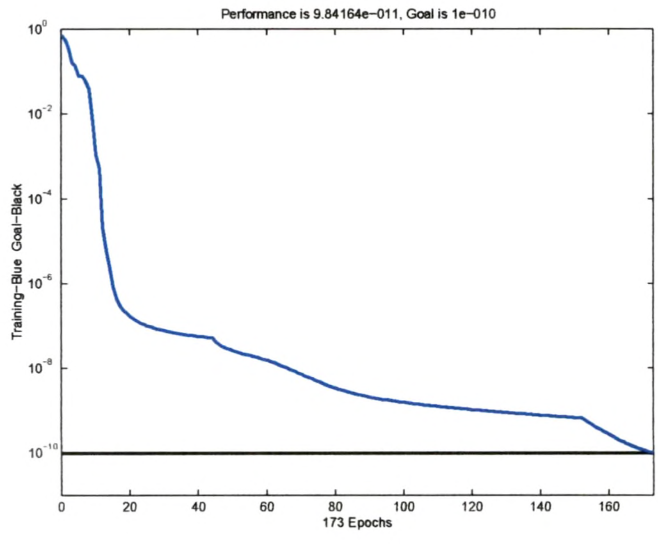


Figure 5.2: The convergence of training of $NN^3_{u(2,30,20,10)}$

The verification shows the approximation of steering control by NN_u to the acceptable error. For example, for the initial state $(3580, 14)$ the control signal by NN_u at each time step is:

$$\begin{bmatrix} -391.0984 & -391.6453 & -391.9524 & -393.5645 & -394.4029 \\ 427.0424 & 427.0149 & 427.3864 & 427.2683 & 427.5064 \end{bmatrix}$$

The evolution of the state components $(V(t), C(t))$ due to the control signals at these time steps is:

$$1.0e+003 * \begin{bmatrix} 3.5842 & 3.5880 & 3.5923 & 3.5947 & 3.5981 \\ 0.0142 & 0.0135 & 0.0145 & 0.0131 & 0.0149 \end{bmatrix}$$

The Figures 5.3 and 5.4 shows, the trajectory for the volume $V(t)$ and the concen-

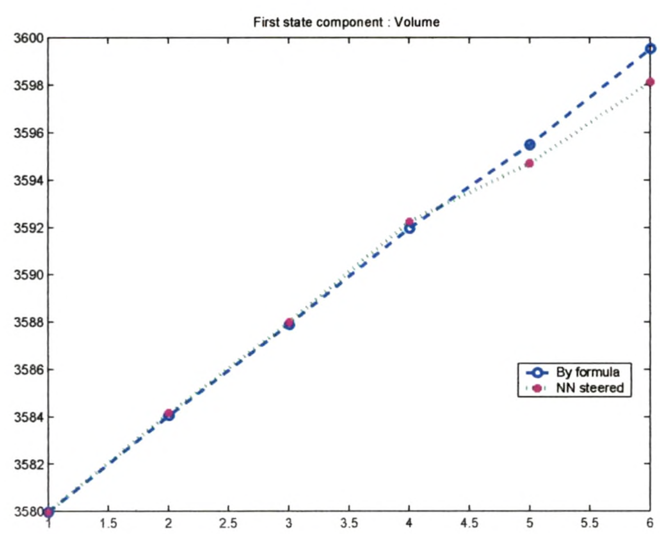


Figure 5.3: The trajectory for volume steered by formula and NN_u

tration $C(t)$ of the state variable for the initial state $(3580, 14)$ to the steered final state $(3598.1, 14.9)$.

For computation refer Appendix-A (Program : nnMTD_5.m).

2. Closed loop 2-step controller: The above simulation result shows the simulation of 5-step controller using ANN, the results are not compromisable. Our aim is to implement the Steering Controller in the form of Neural Network so that it can be

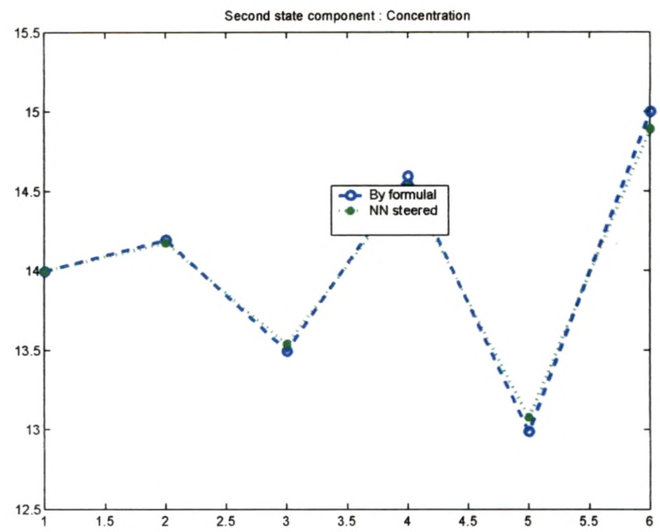


Figure 5.4: The trajectory for concentration steered by formula and NN_u

installed on VLSI chip and hence can be embedded in an automated plant.

To have an efficient Artificial Neural Network Steering Control we train the ANN with the architecture $N^3_{2,10,20,4}$ using Back-propagation algorithm. The training patterns are generated using definition for the 2-step controller where, the state is brought to the equilibrium in two steps each of 0.5 second.

The training converged in 7340 epochs. The trained NN works in the following manner for different initial states (see Appendix-A : program NNgDMT.m):

- For the initial state $(V(0), C(0))$ in $[3575, 3625], [10, 20]$ the state is steered to the equilibrium $(3600, 15)$ in 2-steps.

  Example-1 : As shown in the Figure 5.5, for the initial state $(3595, 15)$, the 2-step control signal by NN_u is

$$\begin{bmatrix} -44.6530 & -45.1512 \\ 62.1867 & 62.5868 \end{bmatrix}$$

  The steered state is given by

$$\begin{bmatrix} 3575.5 & 3600 \\ 15 & 15 \end{bmatrix}$$

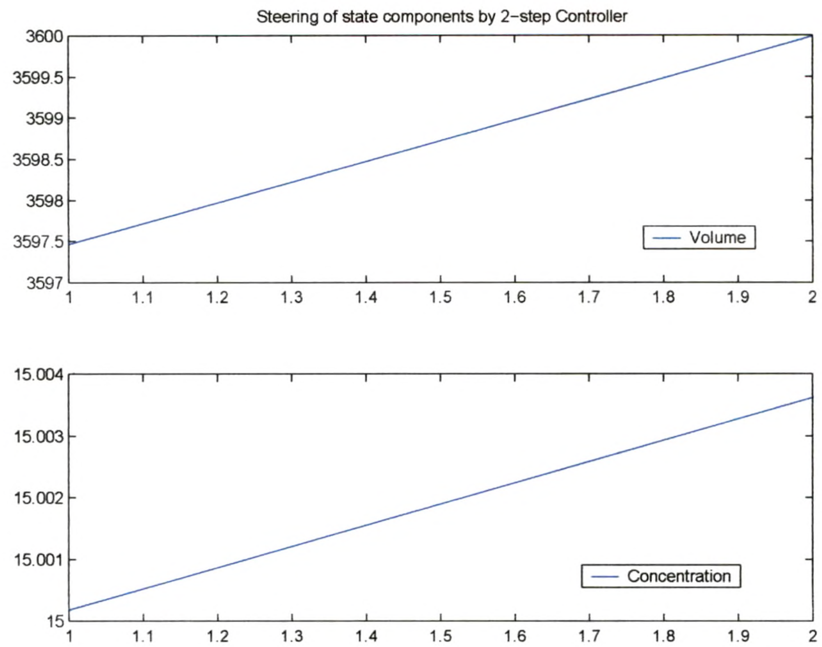


Figure 5.5: The steering produced due to single 2-step control signal.

- For the initial state $(V(0), C(0))$, slightly far from the equilibrium, the state is steered to the equilibrium $(3600, 15)$ in more than 2-steps, in multiple of 2. In this case, the steered state is repeatedly feedback to the network to produce the control signal until the state reached is equilibrium.

  Example-2 : The Figure 5.6 shows steering of the initial state $(3580, 14)$, due to the two consecutive 2-step control signals by NN_u are

$$\begin{bmatrix} -239.7477 & -242.0961 & -43,7331 & -44.1990 \\ 264.8041 & 266.5858 & 58.9593 & 59.3268 \end{bmatrix}$$

  The steered state is given by

$$\begin{bmatrix} 3590 & 3599.6 & 3598 & 3600 \\ 14.5 & 15 & 15 & 15 \end{bmatrix}$$

- For the initial state that is far from the equilibrium the signal for the divergence of the state is given.

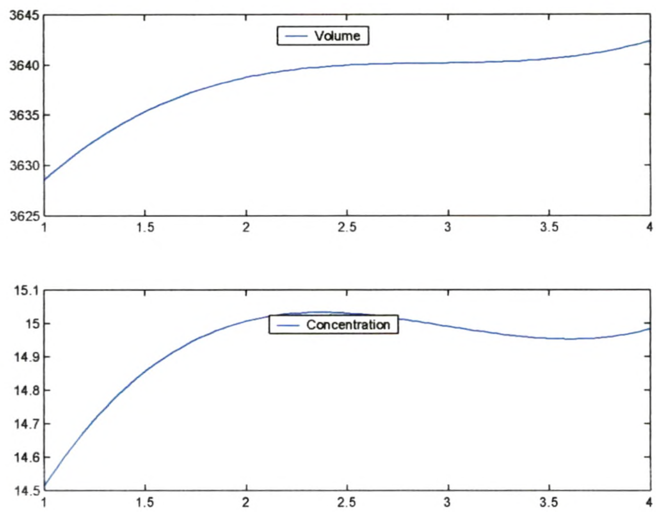


Figure 5.6: The steering produced due to multiple 2-step control signal.

Example-3 : The Figure 5.7 shows the divergence of the state for the initial state $(3500, 10)$.

3. ANN Controller trained without using definition:

We demonstrated above the realization of ANN controller for the discrete linearized mixtank problem in which the I/O patterns used for the Neural Network training were generated using the definition for controller.

Practically, it may happen that for a system we do not have the suitable mathematical model. We only have control inputs and their respective responses from the system. Hence, definition for the computations of control signal and state cannot be applied. Our next simulation demonstrates the use of ANN as controller to the system when it does not have the mathematical representation for controller.

For the simulation in this case, we generated I/O pairs for the arbitrary control input to the system and the corresponding state and trained the network NN_{ud} with the architecture $NN^4_{4,20,15,10,2}$. The ANN gives the control signals that brings the state to

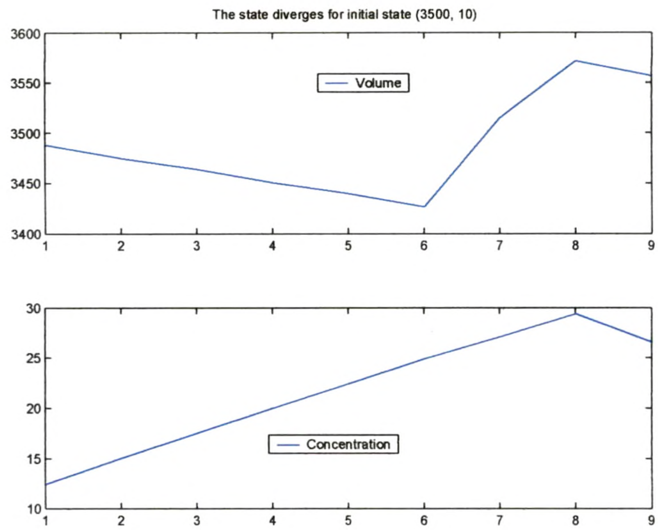


Figure 5.7: The state far from equilibrium (3600, 15) diverges.

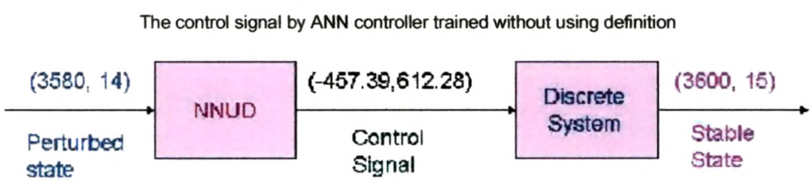


Figure 5.8: ANN Controller for Linearized Discrete, I/O generated without definition

the equilibrium for the perturbed state in the small neighborhood of the equilibrium.

For the larger deviations the controller fails to produce the proper signal (refer program: arb_u_NN.m in Appendix-A). For example, for the initial state $(3580, 14)$, NN_{ud} produces output signal $(-457.39, 612.28)$ which when given to the discrete system steers the system to the equilibrium $(3600, 15)$.

The above simulation results demonstrates the application of Neural Network to act as steering control in the Automated systems.

5.4 Feedback Controller

In the feedback controller setup, we look for a control which can be represented in terms of the state. We have the following result for the feedback controller establishing local controllability of system (5.1.2).

THEOREM 5.4.1 *If the linearized system (5.2.3) is controllable, then the nonlinear system (5.1.2) is controllable in a neighborhood V_x of the origin, with a feedback control of the form $u(k) = h(x(k))$ which steers any state $x_0 \in V_x$ to the origin in at most n - steps.*

Proof : The local controllability of the nonlinear system is guaranteed by Theorem 5.2.1 and the Inverse Mapping Theorem implies the existence of the controller $u(k) \in W_u$ for every initial and final states $x_0, x_f \in W_x$ where, $W_x \times W_u$ is neighborhood of (x = 0, u = 0). Moreover, we get a unique sequence of controller

$$u(k) = h_k(x_0, x_f) \; k = 0, 1, \cdots, n-1.$$

which steers the system from x_0 to x_f in n - steps. If we choose x_f to be zero, without loss of generalization, then the controller is $u(k) = h_k(x_0)$.

Applying the inputs for $k = 0, 1, 2, \cdots, n-1$ to the system started with $x(0) = x_0 \in V_x \subset W_x$, we get the next state as

$$x_1 = F(0)x_0 + G(0)h_0(x_0) + f_d(x_0, h_0(x_0)) \tag{5.4.11}$$

and successively,

$$x_{k+1} = F(k)x_k + G(k)h_k(x_0) + f_d(x_k, h_k(x_0)), \text{ for } k = 1, 2, \cdots, n-1 \tag{5.4.12}$$

Because $x(0) = x_0 \in V_x \subset W_x$ and $x_f = 0$ the Implicit Function Theorem guarantees that $h_k(.)$ are continuous function of $x(k)$ for all $k < n$.

Now assume that, the system is initiated at x_1 i.e. $x(0) = x_1$ where, x_1 is the state that is reached by the system from the state $x(0) = x_0$ on 1^{st} step by applying the input $h_0(x_0)$. Since $x_1 \in W_x$ the sequence of inputs $u(k) = h_k(x_1)$ will drive it to origin in n - steps. On other hand, the original input sequence $u(k) = h_{k+1}(x_0)$ will drive it to the origin in $n-1$ - steps. But as the origin is an equilibrium state the system will remain at the origin with zero input. Thus the input sequence $(h_1(x_0), h_2(x_0), \cdots, h_{n-1}(x_0), 0)$ will also drive x_1 to the origin in n - steps.

Also, from Theorem 5.2.1 for any $x \in W_x$, the input sequence that steers the system to the origin in n - steps is unique and thus we get $h_0(x_1)$ must be equal to $h_1(x_0)$. The same reasoning, applied to each of the $x_i, i = 2, 3, \cdots, n-1$ when the system is started from x_0, we get $h_0(x_i) = h_i(x_0)$.

Thus for any initial state $x \in V_x$, the system:

$$x(k+1) = F(k)x_k + G(k)h_0(x_k) + f_d(x_k, h_0(x_k))$$

will be steered to the origin in at most n - steps.

Hence, Theorem 5.4.1 gives the sufficient condition for the existence of feedback controller for the system (5.1.2).

Extending the set of controllable states :

Let $\overline{f}_h(x(k)) \equiv F(k)x(k) + G(k)h(x(k)) + f_d[x(k), h(x(k))]$ and $\overline{F}_h(.) \equiv \overline{f}^n_h(.)$. Theorem 5.4.1 states that there exists feedback controller that makes the neighborhood of the origin V_x, n - step stable. That is, for all $x \in V_x, \overline{F}_h(.) = 0$.

Since $\overline{F}_h(.)$ is a continuous function we can obtain a larger $W_x \supset V_x$ such that for all $x \in W_x$,

$$\| \overline{F}_h(x) - \overline{F}_h(0) \| = \| \overline{F}_h(x) - 0 \| < \| x \| . \tag{5.4.13}$$

Thus, for any $x \in W_x$ by contraction mapping theorem

$$\lim_{k \longrightarrow \infty} \overline{f}^k_h(x) = \lim_{k \longrightarrow \infty} f^{kn}_h(x) = 0.$$

The following example, illustrates how gradually the domain of controllability can be increased using the contraction.

EXAMPLE 5.4.2 *Consider the first order system*

$$x(k+1) = x(k) + u(k) + au^2(k) + bx^2(k) \tag{5.4.14}$$

where, a and b are scalars.

For the system represented by (5.4.14) we gradually proceed for the robust controller having the largest possible set of controllable states.

Stabilizing the system using linear controller :

The linearized system around the equilibrium $(x = 0, u = 0)$ is given by

$$\delta x(k+1) = \delta x(k) + \delta u(k) \tag{5.4.15}$$

For the system (5.4.15), $u(k) = -x(k)$ will stabilize it at origin. Applying this linear feedback law to the nonlinear system (5.4.14) we get an autonomous system given by

$$x(k+1) = x(k) - x(k) + ax^2(k) + bx^2(k)$$

That is,

$$x(k+1) = ax^2(k) + bx^2(k)$$

which is stable in the interval $(-\frac{1}{a+b}, \frac{1}{a+b})$. Thus $u(k)$ acts as the feedback controller for the linearized system, although for small neighborhood of origin.

Stabilizing the system (5.4.14) using nonlinear controller :

Theorem 5.4.1 guarantees the existence of 1 - step feedback local controller $u_1(k) = h[x(k)]$ for the original system (5.4.14).

$$\text{That is, } x + u_1(x) + (x+b)u_1^2(x) = 0 \tag{5.4.16}$$

Solving for u_1 we get $u_1(x) = \frac{-1+\sqrt{1-4(a+b)x}}{2}$, this control law is defined on the interval $(-\infty, \frac{1}{4(a+b)}]$.

Extending the set of controllable states for the nonlinear controller :

To extend the range of nonlinear controller instead of controlling the system to zero in one step we choose a contraction coefficient ρ and design the controller to move the system from x to ρx in one step when $x > \frac{1}{4(a+b)}$. To achieve this, we need to determine u_2 so that,

$$x + u_2(x) + (a+b)u_2^2(x) = \rho x \tag{5.4.17}$$

And again solving for u_2 we obtain , $u_2 = \frac{-1+\sqrt{1-4(a+b)(1-\rho)x}}{2}$. For a given ρ this control law will stabilize the system (5.4.17) on the interval $(-\infty, \frac{1}{4(a+b)(1-\rho)})$ approaching $(-\infty, \infty)$ as $\rho \to 1$. Thus, combining the two control laws we get, a global law

$$u(x(k)) = \begin{cases} u_1(x) & \text{if } x \in (-\infty, \frac{1}{4(a+b)}] \\ u_2(x) & \text{otherwise} \end{cases}$$

EXAMPLE 5.4.3 *Consider the system given by*

$$\begin{aligned} x_1(k+1) &= u(k) - x_2(k)u(k) \\ x_2(k+1) &= -x_1(k) + 0.5x_1(k)x_2(k) \\ x_3(k+1) &= x_1(k) - x_2(k) - u^2(k) \end{aligned} \tag{5.4.18}$$

Solution : The system (5.4.18) can be written in the form $x(k+1) = Ax(k)+Bu(k)+f(x(k), u(k))$ where,

$$A = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{and} f(x(k), u(k) = \begin{bmatrix} -x_2(k)u(k) \\ 0.5x_1(k)x_2(k) \\ u^2(k) \end{bmatrix}$$

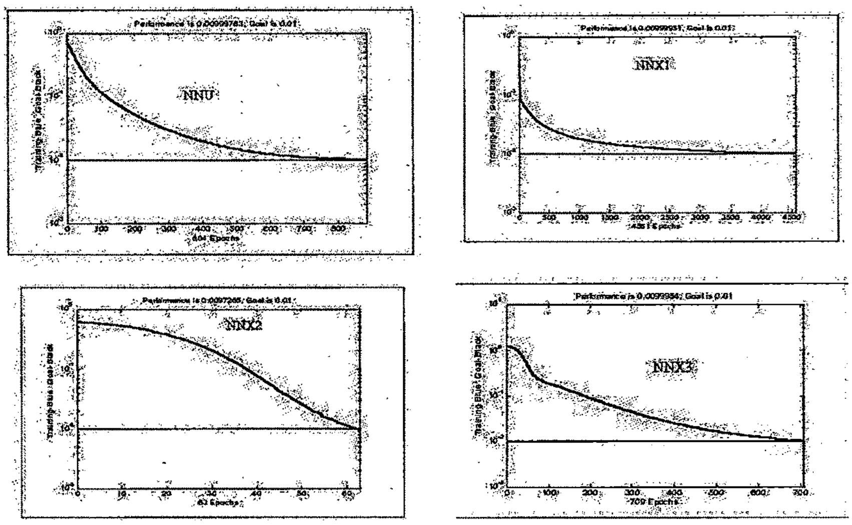
We can see that the linear part of (5.4.18) satisfies the controllability condition $rank = [B|AB|A^2B] = 3$ as,

$$rank \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = 3$$

hence, as stated by the Theorem 5.2.1 the system (5.4.18) is approximately controllable and by the Theorem 5.4.1 its feedback controller exists. The nonlinear controller $u(k) = -sin(x_1(k) + x_3(k))$ for the system stabilizes the system to the equilibrium.

We provide here the simulation of above system. The system has controllable linear part hence it is locally controllable. For it a feedback nonlinear controller is simulated in using feedforward Neural Network .

The feedforward Neural Networks with the configuration $N^2_{4,10,1}$ were used to train the state components and $N^2_{3,10,1}$ was used for training it to act as an artificial feedback controller. The training performances of each NNs are as shown in the following figure.



The convergence of error in training the Neural Networks

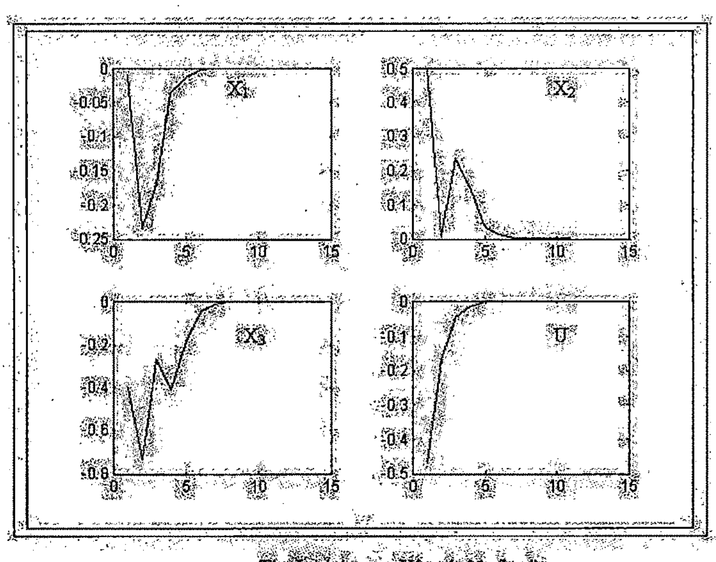


Figure 5.9: The convergence for the state near equilibrium

Once the training of the system and the controller was completed, the performance of the simulator was verified for the initial state (-0.01,0.5, -0.4). The successive inputs from the simulated neural networks controller [-0.2059, -0.0752, -0.0478, -0.0428, -0.0421, -0.0419] brought the system to the state (-0.0087, 0.0094, -0.083), the state near to the equilibrium, which can be seen in Figure 5.9.

However, it is observed that when the system is initiated form state (4,15,2), the

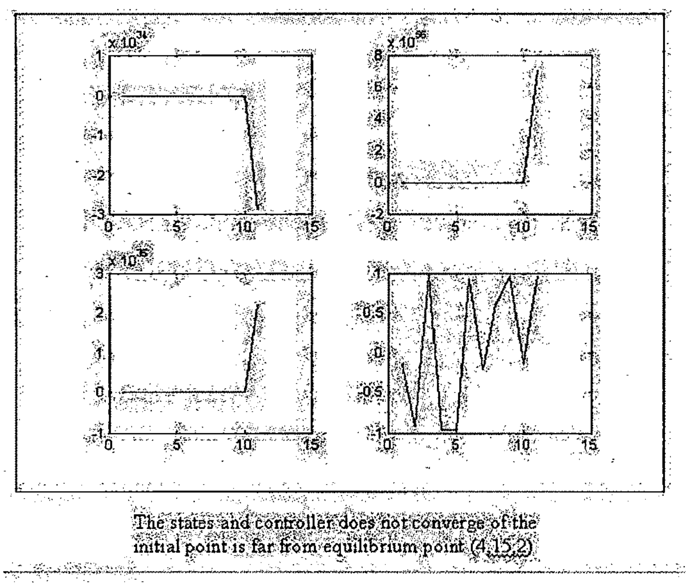


Figure 5.10: The divergence for the state far equilibrium

state far from equilibrium, it diverges as shown in the Figure 5.10.

Mixtank Revisited - Discrete Semilinear form :

The discretized semilinear form of dynamical system representing Mixtank formulation is obtained by taking sampling time as 1 second. Using this sampling factor we get the equivalent discrete form as

$$x(k+1) = Fx(k) + Gu(k) + f_d(x(k), u(k))$$
$$x(0) = x_0 \tag{5.4.19}$$

That is,

$$\begin{bmatrix} V(k+1) \\ C(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V(k) \\ C(k) \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Q_1(k) \\ Q_2(k) \end{bmatrix} + \begin{bmatrix} -1\sqrt{\frac{V(t)}{4}} \\ \frac{(9-C(t))10}{V(t)} + \frac{(18-C(t))20}{V(t)} \end{bmatrix}$$

The rank of the controllability matrix U_n for the above system is 1. Hence, the linear part of discretized system (5.4.19) is not controllable. Therefore, the discrete semilinear system is not controllable.

Illustrative Example: ANN controller for the semilinear system :

To demonstrate the ANN controller for semilinear discrete system consider the system given by

$$x(k+1) = Fx(k) + Gu(k) + f_d(x(k), u(k))$$
$$x(0) = x_0$$

That is,

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9958 & 0 \\ 0 & 0.9917 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.9979 & 0.9979 \\ -0.0002 & 0.0001 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}$$
$$+ \begin{bmatrix} -\sqrt{\frac{x_1(t)}{4}} \\ \frac{(9-x_2(t))10}{x_1(t)} + \frac{(18-x_2(t))20}{x_1(t)} \end{bmatrix} \tag{5.4.20}$$

The controllability matrix for linear part of system (5.4.20) has rank 2 and the nonlinear function is lipschitz except at zero. Therefore, the semilinear discrete system is controllable except at zero. The state and the steering control for the system (5.4.20) can be computed using the coupled equations

$$u^{i+1}(k+1) = U_n^{-1} \left[x_1 - F^n x_0 - \sum_{j=1}^{n} F^{n-j} f(x^i, u^i) \right] \tag{5.4.21}$$

and

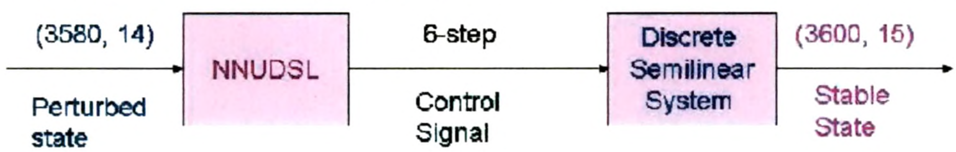
$$x^{i+1}(k+1) = F^n x_0 + \sum_{j=1}^{n} F^{n-j} G u(k) + \sum_{j=1}^{n} F^{n-j} f(x^i, u^i) \tag{5.4.22}$$

Our aim is to steer the perturbed state of the system (5.4.20)to the state $(3600, 15)$.

For the system (5.4.20), it is observed that the coupled equations converges with the accuracy 0.01 in 6 iterations, hence the 6-step steering controller is implemented using the feedforward neural network having architecture $NN^3_{4,15,12,12}$.

The training patterns are generated using the coupled equations (5.4.22) and (5.4.21), for the state and controller in the discrete form.

The steering of the state due to signal by 6−step NN steering controller for discrete semilinear system



6-step control signal:

0.9968	-654.473	-685.182	-685.325	-685.297	-685.294
0.9972	682.3191	706.6499	706.5944	706.5912	706.5915

State at each step: 1.0e+003 *

3.58	3.5777	3.6096	3.6034	3.6032	3.6032
0.014	0.0138	0.0154	0.0154	0.0154	0.0154

Figure 5.11: The 6-step ANN controller

Once ANN is trained to act as 6-step controller. It is verified for different perturbed states in the neighborhood of the point (3600, 15), and is found to steer the initial state to the desired state (3600, 15).

For example, for the initial state (3580, 14) the 6-step control signal is produced which when given to the system steers the state (3603, 15.4), near to the equilibrium (3600, 15) as in Figure 5.11. The steering of the state components is as shown in the Figure 5.12.

5.5 Summary

In this chapter we developed the controllability results for the semilinear discrete dynamical system, first by linearizing it and then extending the domain of controllability to almost complete state space. The mixtank process problem revisited, this time in the discrete form. For this form of the mixtank problem the local steering control is realized using Artificial Neural Networks.

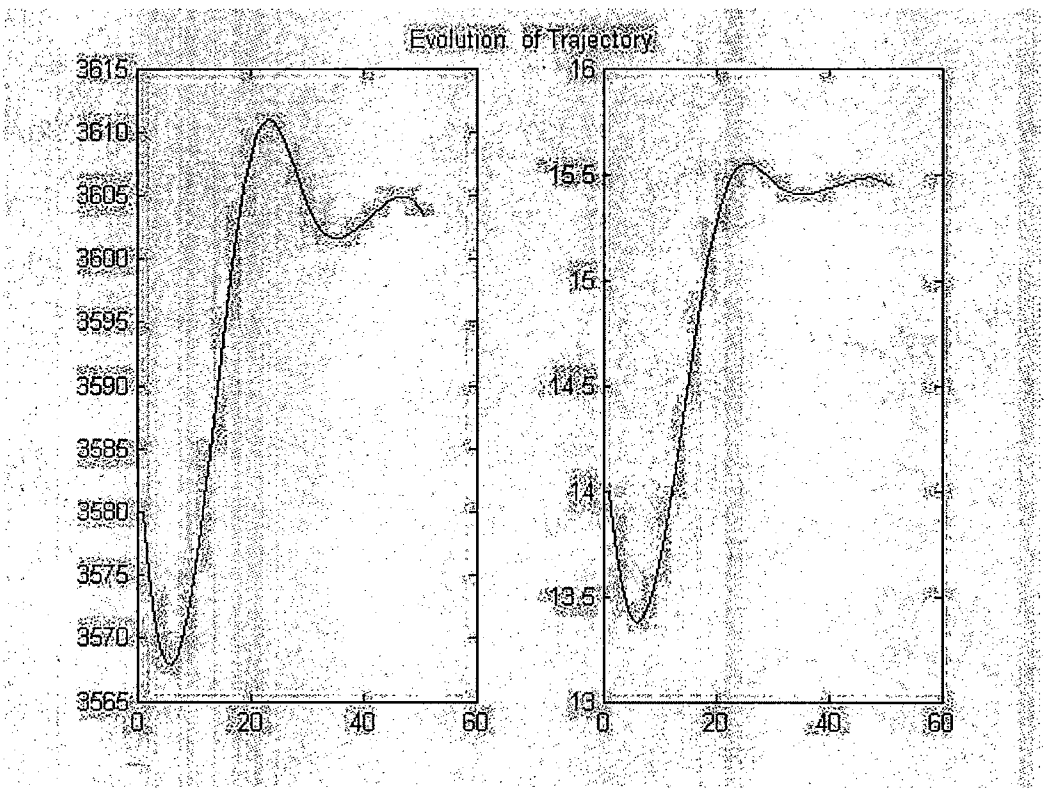


Figure 5.12: Steering of the state from initial to final by 6-step ANN controller