Chapter - 2

Musical Relationships of Notes & Origin of a Scale

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We have seen that, if the loudness and pitch are within reasonable limits, the quality of a note mainly determines how pleasant or musical it is. Also it was pointed out that human ears have no taste for a *"pure tone"*. A pure tone like that produced by a vibrating tuning fork *(a tuning fork is specially designed to produce a pure tone free from over tones or harmonics)* is too *"shrill"* to be musical. A musical note must have overtones which makes the sound of a musical instrument or a singer's voice pleasant to the ears.

A single note, however, musical, is hardly sufficient to constitute music, though it produces a base for the display of other notes. Such a base *(called tonic)* is the backbone of any recital of Indian classical music which maintains the same pitch *(frequency)* of this main note depends upon the convenience of the performer *(and also suiting the dominant pitch of the surrounding noise, if any)* but once fixed, it remains unaltered during a single recital and is emphasized by means of Tamboora while other notes are played or sung according to the requirement of the composition. This emphasizing all its harmonics before coming to the other notes of the composition. *(This Shadaj is equivalent to the "tonic" of western music).*

Let us now come to other notes which are musical relative to the *"Shadaj"* mentioned above. However, a musical note may be in its own right, its pitch must bear a certain agreeable ratio to that of the *"Shadaj"* otherwise it would not be musical. Once the frequency of Shadaj is fixed, only notes having a certain pitch are allowed in music.

What are these preferred frequencies ? Or, more precisely, what should be the ratio of the frequencies of other notes to that of *"Shadaj"* so that the effect is musical ? A musician knows the answer very well. Broadly speaking, there are seven main notes including Shadaj viz., Shadaj, Rishabh *(Re)*, Gandhar *(Ga)*, Madhyam *(Ma)*, Pancham *(Pa)*, Dhaivat *(Dha)*, Nishat *(Ni)*, in ascending order of frequencies relative to *"Shadaj"*. After Nishad, next note is again *"Shadaj"* but of a higher *"Octave"* the frequency of which is double that of the main base *"Shadaj"*. Apart from these main notes, there are depressed *(Komal)* varieties of Rishabh, Gandhar, Dhaivat, and Nishad having frequencies slightly lower than that of the main notes bearing the same name *(called "Shuddha notes")* and an elevated *(Teevra)* variety of Madhyam having a frequency slightly higher than that of the main one *(Shuddha Madhyam)*.

Thus, there are twelve notes in the frequency range from the base *"Shadaj"* to the *"Shadaj"* of the next higher octave including the base Shadaj, but excluding the higher one. To be more precise, the same interval is divided into twenty-two sub notes or *"Shrutis"* as they are called.

These are the notes and sub notes of the same octave called middle octave or Madhya Saptak. The interval between the Shadaj of next higher octave *(the frequency of which is double that of the base Shadaj)* and the next Shadaj *(this latter Shadaj has four times the frequency of the base Shadaj) (this latter Shadaj has four times the frequency of the base Shadaj)* is divided into twelve notes hearing the same names *(Rishabh, Gandhar etc. with depressed or elevated varieties)* and corresponding sub notes.

Each note or sub notes of this octave *(called Tar Saptak)* is double the frequency of the corresponding note of the Madhya Saptak. Similarly, there are other higher octaves *(Saptakas)* such that each successive Shadaj is double the frequency of the preceding Shadaj. Also there are lower octaves also such that corresponding Shadajs have frequencies 1/2,1/4,1/8..... times the frequency of the base Shadaj *(and of course the interval is divided into eleven more notes)*. The octave starting with the Shadaj as 1/4 the frequency of the base Shadaj is called Ati Mandra Saptak and so on.

The construction of higher and lower octaves with reference to the main *(Madhya Saptak)* octave can go on indefinitely in theory, but in practice, it has practical limits. Usually Mandra Saptak and Tar Saptak only are used *(the octaves one higher and one lower than the main octave containing the base Shadaj apart from the Madhya Saptak)*.

If we introduce Shrutis also, each Octave is divided into twenty two Shrutis *i.e.* the same range which is divided into twelve notes, is divided into twenty -two Shruties.

Regarding the exact frequencies of the notes and sub-notes. *(Once the frequency of the base note is fixed)* there are different scales in vogue and even in Indian Classical Music, different scales have been in use through the ages; but even so, the frequencies of all the twelve notes of an octave are approximately the same in all scales of Western as well as Indian Music. As far as Shrutis are concerned., their exact frequencies can be inferred by the evidence of ancient literature on the subject. There is some controversy regarding the exact frequencies of Shrutis. We shall come to this point in detail later.

We shall return to the detailed discussion of the various scales mentioned in the ancient literature and the exact frequencies of the notes and Shrutis involved. Let us, for the present, consider the origin of a musical scale from a scientific point of view. How is it that certain frequencies only and not all produce a pleasant or musical effect in combination with the base note ? Let us see how the above mentioned notes are derived from the base note.

It has been remarked earlier that the base note *(like any other musical note as a matter of fact)* is accompanied by overtones or harmonics with frequencies twice, thrice, four times etc. of the base note *(Sa)*. A musical ear has pleasant association with such harmonics *(and in fact misses them if they are absent (as in the case of the sound produced by a turning fork)*. After the main frequency of the base note Sa, the second harmonic *(twice the frequency of the base Sa)* is the most prominent and third, fourth, fifth and sixth harmonics *(with three, four, five and six times the frequency of the base Sa respectively)* are less prominent in decreasing order. All these harmonics are familiar to the musical ear because they are present in the base note Sa.

The second harmonic is the most prominent harmonic after the main note which has double the frequency of the main note Sa. In fact it is the same as the Sa of Tar Saptak mentioned earlier. Thus the intimate relationship of the Sa of Tar Saptak with the base Sa *(of Madhya Saptak)* is owing to the fact that Tar Sa is the most prominent overtone present in the fundamental Sa. Whenever these notes are played simultaneously or one after the other, a musical ear immediately recognizes this relationship. The same effect is produced by any two notes with their frequencies in the ratio 1:2. In fact a musician refers to "same notes but in a higher or lower octave". Even a very elementary student of music has little difficulty in recognizing a note 1/2 or double the frequency of the fundamental Sa.

We have seen how Tar Sa *(and so also Mandra Sa)* is derived as the most intimately related note to the fundamental Sa. It is customary to standardize the frequency of Madhya *(fundamental)* Sa as 240 vibrations per second *(though, as a matter of fact, any arbitrarily convenient frequency would do)*. Then the frequency of this newly acquired relative -Tar Sa - is 480 vibrations per second.

Coming to the third harmonic which is next after the second harmonic in prominence, and has the frequency of he fundamental Sa, it is natural to expect the ear to recognize its musical relationship also to the fundamental Sa, though to a lesser degree as compared to that of tar Sa and this note and Madhya Sa, when played simultaneously or one after the other would produce a familiar pleasant effect on a musical ear. The frequency of the third harmonic is 240 x 3 =720 vibrations per second which falls in the tar Saptak. Halving this frequency we get a corresponding note in Madhya Saptak with frequency 360 vibrations per second which *"sounds like"* the third harmonic to the ear owing to the frequency ratio 1:2. This note, with frequency 360 *(vibrations per second)* is called Pancham or Pa which has been mentioned above. Its intimacy to the fundamental note is next only to Tar Sa's.

We remember that when two notes are played together, a note having a frequency equal to the difference of the two frequencies is also produced. Thus, when Madhya Sa *(240)* and P *(360)* are played together, a note with frequency 360 - 240 = 120 is also produced automatically. It can be easily seen that this new note is nothing but Mandra Sa *(being 1/2 the frequency of the Madhya Sa)* and is again closely related to the Madhya Sa.

Now consider the fourth harmonic with frequency 240 x 4 = 960. This does note give any new note, since having this frequency gives Tar Sa only, which we have already discussed. However, it further emphasis the importance of Tar Sa.

The fifth harmonic has frequency 240 x 5 = 1200. Having this frequency, we get 600 and halving it again, we get the corresponding note of the Madhya Saptak halving the frequency 300. This is the note which has been referred to as Shuddha Gandhar above which, produces a pleasing effect on a musical ear.

Also, when Madhya Sa *(240)* and Ga *(300)* are played simultaneously the note equal to the difference of these frequencies *(300-240=60)* is also produced which is Ati Mandra Sa being 1/2 the frequency of Mandra Sa, which itself is half the frequency of the Madhya Sa.

The sixth harmonic does not give us a new note again since its frequency is 240 x 6 = 1440 and halving its successively till we come to a note in Madhya Saptak, we get Pancham only.

Thus, starting with a musical fundamental Sa, we have deduced Shuddha, Gandhar, Pancham and Tar Shadaj. Gandhar is related to fifth harmonic, Pancham is related to third and fifth harmonic, and Tar Sa is the second harmonic itself and is related to fourth harmonic also. *(The higher harmonics are so insignificant that their effect can be neglected).* That is why musicians say that when fundamental Sa is played, Tar Sa, Ga and Pa are also produced along with *(Svayambhu).* Let us see what other notes can be defined with the help of these notes.

Recall that second harmonic is also closely related to the fundamental notes that it is conventional to call it the *"same note in a higher octave"* i.e. Shadaj, of Tar Saptak. Following this convention, we can say that the second harmonic of any note does not give a new note but merely the same note belonging to the next higher octave or Saptak,. So, we must look to the third harmonics to give us new notes.

Let us, for the moment forget Gandhar and see how new notes are produced by considering third harnmoics of Pancham successfully.

The third harmonics of Pancham *(360)* is 360 x 3 = 1080 and the corresponding note in Madhya Saptak is 1080/4 = 270. This new note is called Rishabha. Now the third harmonic of Rishabha is 270 x 3 = 810 and the corresponding note in the Madhya Saptak is 810/2 = 405. This note is called Dhaivat.

Similarly we get new notes following the same technique successively. A complete account of these notes is as given in the *Table No.2.*

Table No.2

Starting note	***Third harmonic Saptak***	***Corresponding note in Madhyam***	***Name of the new note***
(Sa) 240	720	360	Pancham *(Pa)*
(Pa) 360	1080	270	Shuddha Rishabha *(Re)*
(Shuddha Re) 270	810	405	Shuddha Dhaivat *(Dha)*
Shuddha Dha 405	1215	303	3/4 Shuddha Gandhar *(Ga)**
Shuddha Ga 303 3/4	911 1/4	455 5/8	Shuddha Nishad *(Ni)*
(Shuddha Ni) 455 5/8	1366 7/8	341 23/32	Teevra Madhyam *(Ma)*

* *It should be noted that this Shuddha Gandhar is slightly higher than the Shuddha Gandhar defined earlier which is related to the fifth harmonic of the base Sa.*

(Teevra Ma) 341 23/132	1025 5/32	256 37/128	Komal Rishabha *(Re)*
*(Komal Re)*256 37/128	768 111/128	384 111/256	Komal Dhaivat *(Dha)*
(Komal Dha) 384 111/256	1114 61/256	288 333/1024	Komal Gandhar *(Ga)*
(Komal Ga) 288 333/1024	864 999/1024	432 999/2048	Komal Nishad*(Ni)*
(Komal Ni) 432 999/2048	1277 949/2048	324 2997/8192	Shuddha Madhyam *(Ma)*

Thus we get all the twelve notes of the Madhya Saptak starting with the base Shadaj.

What is remarkable is that we have made not arbitrary assumption about the frequency of any note but the frequency of each note is derived by purely mathematical steps using only the following principles.

1. A musical single note is also ways accompanied by harmonics, the effect of which makes the note pleasant to human ears.

2. Second harmonic is the most prominent in the main note and, therefore, appears more closely related to the main note that any other note. In fact this relationship is so close that it is conventional to call a note twice, four times etc. *(or 1/2, 1/4, 1/8...etc.)* the original note *"the same note"* but in a different octave.

3. Next, in the order of importance is the third harmonic which is obtained by multiplying the original note by 3. Then, making use of the principle *(2)* above, we can divide it by 2 or 4 or 8 etc. till we get the *"same note"* on the Madhya Saptak. This gives us a scale of notes which can be called a musical scale.

This looks like a perfect musical scale. Since second harmonic does not give a new note but only defines the same note in a different octave, the method of third harmonic is the best available with us and the series of notes defined by it must be regarded as the *"most musical"*. Let us arrange the notes given by table No. 1 in ascending order of frequency and then compare them with the twelve notes in vogue today in Indian Classical music.

Table No. 3

Name of the note	**Frequency derived by third harmonic method**	**Accepted frequency as in vogue today *(according to BhatKhande)***
Base Shadaj	240	240
Komal Rishabh	256 37/128	256
Shuddha Rishabha	270	270
Komal Gandhar	288 333/1024	288
Shuddha Gandhar	303 3/4	300
Teevra Madhyam	341 23/32	337 1/2
Pancham	360	360
Komal Dhaivat	384 111/256	384
Shuddha Dhaivat	405	405
Komal Nishad	432 999/1024	432
Shuddha Nishad	455 5/8	450

We find that Shadaj, Shuddha Rishabha, Pancham and Dhaivat derived by the third harmonic method are the same as those used today but the other notes are slightly different than the notes in use. The difference is very slight in the case of Komal Rishabha, Komal Dhaivat, Komal Gandhar and Komal Nishad, but more that pronounced in the case of Shuddha Gandhar, Shuddha Madhyam, Teevra Madhyam and Shuddha Nishad. Why should this difference be there ? Why have the musicians not adopted the *"most musically perfect system of notes"* as derived by the third harmonic method ? Is it only the anxiety of instrument makers to avoid cumbersome fractions appearing in some of the notes of the "third harmonic scale" *(as we shall refer to it hereafter)* ? The answer is that our third harmonic scale is not perfect. It contains an inherent contradiction.

It has been remarked that the Shuddha Gandhar derived by the third harmonic method ($303^{3/4}$) is slightly different from Shuddha Gandhar derived by the fifth harmonic of the main Sa *(this Gandhar has frequency 300)* which happens to be the Shuddha Gandhar in vogue also. This calls for a choice; which one of the Shuddha Gandhars should be accept ? The Gandhar *(300)* is directly derived from the fifth harmonic which is present in the base note, while the third harmonic Gandhar ($303\ ^{3/4}$) is related through Pancham-Shuddha Dhaivat-Shuddha Gandhar sequence.

But this is only a minor contradiction. There is a much bigger contradiction also. If we are not too tired of handling the awful fractions involved, we may carry out the calculation of Table No.1 one step further and see what note is yielded by the note named Shuddha Madhyam - the last of the sequence of notes in Table No.1.

The third harmonic of this Shuddha Madhyam comes to

$$324\frac{2997}{8192} \times 3 = 973\frac{799}{8192}.$$

We divide this by two to get a note with frequency $486\frac{8981}{16384}$.

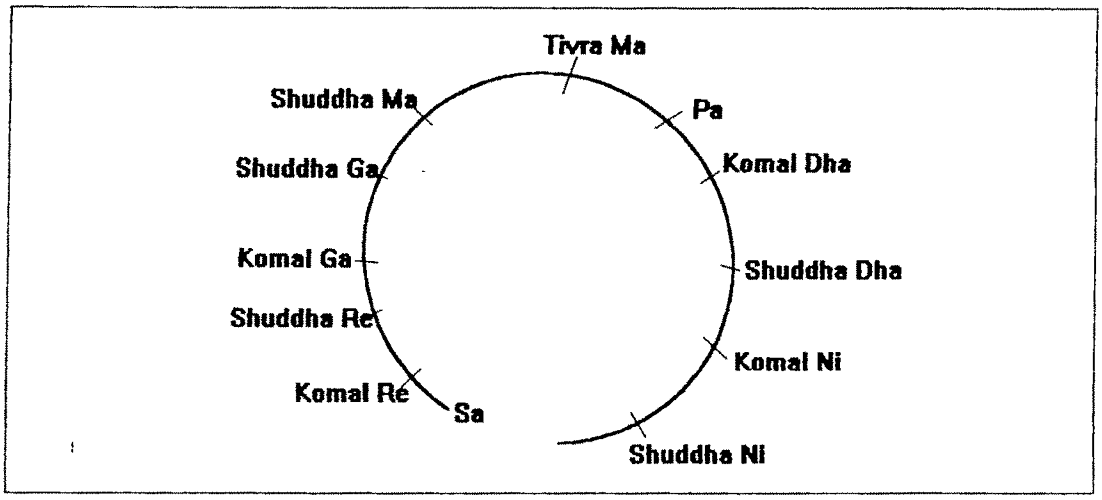
This note is so close to the Shadaj of Tar Saptak *(480)*
that we must either call this note the *"Shadaj of Tar Saptak"*
and forget our old friend *(480 the second harmonic)*
or discard the note $486\frac{8981}{16384}$.

Obviously, whatever may be our faith in the third harmonic method, we cannot discard the Shadaj of Tar Saptak *(480)* because it is the second harmonic of the fundamental Sa (240) and is more closely related to it than any other note. So, we must discard the *"Shadaj of Tar Saptak"* as defined by the process of Table No.1 *i.e.* $486\frac{8981}{16384}$.

The trouble is that our sequence of notes produced by the third harmonic method does not constitute a closed cycle. After completing a cycle, when the sequence returns to the same note, it slightly overshoots it.

This concept will be clear by referring to the diagram below.

Diagram-8



We see that starting from the base Shadaj and following the sequence of the notes as derived by the third harmonic method, we do not come back on the same note but on a note slightly higher and, therefore, if we want to avoid this contradiction, we must either adjust some of the notes in such a way *(of course departing from third harmonic system in the process)* that the sequence closes on itself or else we may continue the process of table No.1. Still further in the hope that at some later stage the sequence closes on itself, i.e. comes back on the Shadaj.

Let us examine the second possibility, it means that we do not discard the note $486 \frac{8981}{16384}$ but halve it to get a note slightly higher than the base Shadaj *(its frequency will be 244 1578/16384 as against 240 of the base Shadaj).*

We name it a new note - say N1 and rank it between base Shadaj and Komal Rishabha and continue the same method of finding the third harmonic of N1, divided it by 2,4,8.. until at some stage the new note say Nn coincides with the original Shadaj. But will thereby any such stage ?

There is no need of performing these tedious calculations till eternity to find out whether the sequence will ever close on itself or not. Let us consult our mathematician friend. These mathematicians are often lazy of performing such calculations but come out with short cuts to provide an answer.

What is the process of dividing third harmonics in mathematical language ?
Simply multiplying the original frequency by 3. Let the starting frequency be N.
Then, its third harmonic is N x 3.

The third harmonic of this is again $N \times 3 \times 3 = N \times 3^2$
and the next third harmonic is $N \times 3 \times 3 \times 3 = = N \times 3^3$

Hence the third harmonic at $p^{th.}$ stage of the original frequency is $N \times 3^p$
(=N x 3 x 3 x 3 .. p times).

Now reducing the newly found note to the original Octave *(Madhya Saptak)* means dividing it by any one of the numbers 2, $2 \times 2 = 2^2$, $2 \times 2 \times 2 = 2^3$ ... until the result lies between N *and* 2 N.

Supposing we have to divide the new note by 2^q *(2 x 2 x 2 ...q times)* in order that the result lies between N and 2 N *(Madhya Saptak)*, our new note at $p^{th.}$ stage of the third harmonic method will be $\frac{N X 3^p}{2^q}$.

(In the actual calculation in Table No. 1 we went on dividing by 2 or 4 each stage because that was convenient and more illustrative, but the result is not altered if we do all the multiplication by 3 x 3 x 3 .. and then divide by the suitable number 2 x 2 x 2 ..)

Now, our process must continue *(for we have now agreed to accommodate more than 12 notes provided that we can come to a close sequence)* till, at some later stage we come back to the same note, which will be the case if, for some value of p $\frac{N X 3^p}{2^q} = N$ or $3^p = 2^q$.

Mathematically this is an impossibility since 3^p is an odd number and 2^q is an even number.
How can an even number be equal to an odd number ?
We did well not to carry on the fruitless calculations any further, didn't we ?

So, the question is settled once and for all. Following the third harmonic method, we cannot derive a sequence of notes which closes on itself no matter how many notes we are willing to admit. We would of course, return to notes very near the original note periodically. We have seen that first such stage comes when we have defined 11 new notes *(12 notes including the original note)* when the 12th. new note comes very near the original Sa. There will be other stages also.

It can be shown Mathematically that for these stages
p/q is approximately equal to Log2/Log3.
But there will be no stage for which p/q = Log2/Log3.

But, in music we cannot do with a sequence of notes which closes on itself approximately but not exactly. We cannot admit two notes which are very close to each other *(in terms of frequency, of course)* but not quite equal. Two almost equal notes may fool us when played one after the other, but never when they are played simultaneously because the beats would make us feel uneasy. In music we cannot accept notes which can be played one after the other but not simultaneously.

Let us now see if, instead of third harmonics, fifth harmonics can lead us to a closed sequence, this means multiplying the original note N by 5 successively and dividing by 2 repeatedly until the note lies between N and 2 N. This will be a closed sequence if for some value, we have
$\frac{N X 5^p}{2^q} = N$ *(q is any positive whole number)* or $5^p = 2^q$ which is again impossible
since the left side is again an odd number and the right side is an even number.

It is a fruitless exercise trying find a closed sequence of notes following the scientific method of harmonics, since we find that the second, fourth, eighth...etc. harmonics do not lead us to new notes *(but to the same notes higher octaves)* and other harmonics lead us to new notes but we can never come back to the same note. Hence we are forced, much against our wishes, to adjust our third harmonic notes in such a way that the sequence closes on itself and thereby strike the best balance between the requirement of third harmonic relationship and the requirement of closed cycle. This has been done in various ways by the musicians of different ages and different schools and accounts for the multiplicity of the systems of scales. However, it must be noted that in any of the systems, the notes do not differ much from those given in Table No.1. All the musical scales have, tried to find notes which are as rich in third harmonic and fifth harmonic relationship only and neglected the fifth harmonic relationship altogether. But in the Indian Classical music, both third harmonic and fifth harmonic relationship *(3/2 and 5/4 respectively, when brought to the Madhya Saptak)* have been taken care of.

Thus the basis of the musical scales and notes are the first six harmonics of the tonic Shadaj. In Sanskrit, Shadaj means **"generator of six"** *(tones).* How true it is mathematically ! Although it seems to generate only three notes - Shadaj, Gandhar and Pancham but they are spread over six harmonics.

We shall follow an abbreviation about the nomenclature of notes from now onwards given below.

A Komal note is denoted by a mark *(')* on the right top of it and a Teevra note by an asterisk *(*).*
A note in Mandra Saptak will be denoted by a lower bar and that in Tar Saptak by an upper bar. Thus Shuddha Re will be denoted as follows in three Saptaks.

<u>Table No. 4</u>

Re	-	*(Madhya Saptak)*
<u>Re</u>	-	*(Mandra Saptak)*
Re	-	*(Tar Saptak)*

and for Komal Re,

Re*	-	(Madhya Saptak)
<u>Re'</u>	-	(Mandra Saptak)
Re'	-	(Tar Saptak)

Also, for Teevra Madhyam

Ma*	-	(Madhya Saptak)
<u>Ma</u>*	-	(Mandra Saptak)
Ma*	-	(Tar Saptak)

The musician of the later era Lochan, Ahobal, Haridaya and Srinivas *(from 15th. to 18th. Century AD.)* have regarded the twelve notes as fundamental. They have defined the frequencies of the twelve notes by means of different lengths of a vibrating string. The frequencies of the notes turn out to be rational as they should be in a musical scale. Shrutis were retained in the same number in between the notes and had been regarded unequal, as they should be.

Srinivas has highlighted the following principle for the construction of his Shuddha notes *(Shadaj gram).*

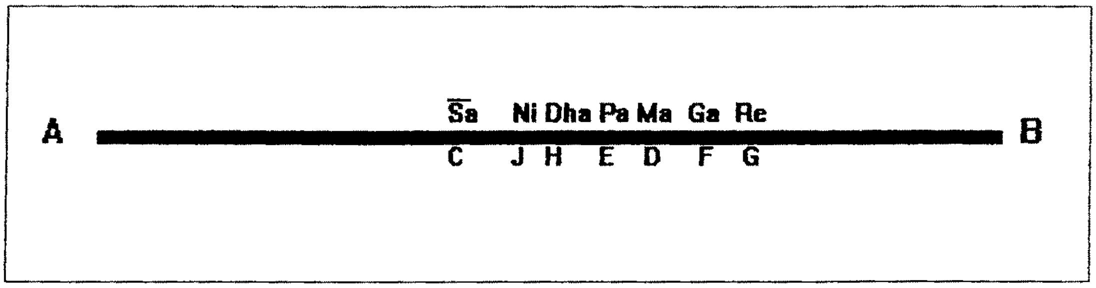
The Shuddha notes of an octave should be such that there is a Shadaj-Pancham relationship between any note and the fifth note after it as in the Shadaj Gram of Bharat.

Sa	-	Pa
Re	-	Dha
Ga	-	Ni
Ma	-	Sa

The following practical method has been given by Srinivas for the construction of his Shuddha notes :

Take a Veena with any arbitrary length as its vibrating chord *(let this length be 36 inches)*. Let the frequency of this chord be taken as the base Shadaj. Now if a bridge is placed on the mid point of this string *(so that only half of the string can vibrate)*, the new note of this halved length *(18 inches)* will be the Shadaj of Tar Saptak. Similarly, halving this length again *(so that the length is 9 inches)* will give Shadaj of Ati Tar Saptak *(fourth harmonic of the original note)*.

Diagram-9



In the above diagram, AB is the original length *(36 inches)* of the Veena which gives the fundamental Shadaj. The note generated by AC or CB *(18 inches)* generates Shadaj or Tar Saptak and the length CD or dB *(9 inches)* gives Shadaj of Ati Tar Saptak. It will be recalled that this is exactly how the second and fourth harmonics are supposed to be produced by a vibrating string.

For the remaining notes the method proceeds further as follows :

The length AD, when vibrated generates Madhyam.
This length is 18 + 9 = 27 inches.
In fact this is why the note is called Madhyam *(middle)* since it is produced when the movable bridge is placed at the mid point of CB - in the midst of the places of Tar and Ati Tar Shadaj.

Now CD *(= 9 inches)* is divided into three equal parts and the point E such that ED-1/3 CD is where the movable bridge must be placed so as to generate Pancham.
Hence the length AE = *(18+6)* = 24 inches, when vibrated, generates Pancham.

Now determine the mid point of EB *(the point F)* which is the mid point of the places of Shadaj and Pancham. The length AF then defines Gandhar. This length is easily seen to be equal to 30 inches. Now the length EB *(the places of Pancham and Shadaj)* is divided into three equal parts and a point G is so determined that GB=1/3 EB = 4 inches. Then the length AG = 36-4 = 32 inches determines Rishabha. Then Dhaivat is defined by the point in between E and C. The definition of Dhaivat is ambiguous. It can either mean that H is the mid point of E and C, which will give AH = 21 inches or it can mean that H is just in between E and C but in such a way that Dhaivat has Shadaj-Pancham relationship with Rishabh defined earlier. The latter interpretation seems to be more sensible, since Shadaj - Pancham relationship between Sa-Pa, Re-Dha, Ga-Ni and Ma-Sa was the basis of the construction of the scale defined by Srinivas. According to this interpretation, the length A H comes to 21 1/3 inches *(while, if H is taken to be the mid point of EC, AH comes to 21 inches)*. Lastly, the length CE is divided into three equal parts and the point J such that CJ = 1/3 CE is determined. The vibrating length AJ then generates Nishad. The length AJ then equals 20 inches.

To recapitulate, let us look at the following table which gives the vibrating length, and the corresponding frequency of the note so define.

Table No. 5

Name of the note	***Generating length***	***Frequency in vibrations per second***
Shadaj	AB = 36"	240 *(Presumed for the sake of convenience)*
Rishabh	AG = 32"	270
Gandhar	AF = 30"	288
Madhyam	AD = 27"	320
Pancham	AE = 24"	360
Dhaivat	AH = 21 1/3"	405
Nishad	AJ = 20"	432
Shadaj of Tar Saptak	AC = 18"	480

The above notes are the Shuddha notes as defined by Srinivas and they correspond to the notes in vogue today according to Bhatkhande School except that the Gandhar and Nishad defined above correspond to Komal and not Shuddha Gandhar and Nishad of the scale in vogue today.

<u>Srinivas goes on to define his Vikrat (elevated or depressed) notes according to the following way :</u>

The length BG may be divided into three equal parts
and a new point G ' is determined in such a way that GG' = 1/3 GB.
Then the length AG' generates Komal Rishabha.

It is easily seen that G'= 33 1/3" and a pint F' is determined so as to be the mid pint of HB.
Then the length FT' *(=28 2/3) generates Teevra Gandhar).*

Then the length CF' is divided into three parts
and a new point D' is so determined that CD' = 2/3 CF'.
Then the length AF' (=25 1/9") generates Teevra Madhyam.

The length AH' generating Komal Dhaivat does not seem to follow by the procedure mentioned in the Shloka but it can be derived by the rule that Komal Dhaivat must bear Shadaj Pancham relationship with Komal Rishabh.

Then finally a point J' is determined such that CJ' = 1/3 CH
and the length AJ' generates Teevra Nishad. *

(Please see the 4th.volume of the series on Indian Classical Music by Pandit Vishnu Narain Bhatkhande).*

Following table summarizes the result about the Vikrat notes of Srinivas.

<u>Table No. 6</u>

Name of the note	**Generating length**	**Frequency in vibration per second**
Komal Rishabha	33 1/3	259 1/5
Teevra Gandhar	28 2/3	301 17/43
Teevra Madhyam	25/1/9	344 8/113
Komal Dhaivat	22 2/9	388 4/5
Teevra Nishad	19/ 1/9	452 4/43

This completes all the twelve notes as defined by Srinivas.
Let us see how far they satisfy the musical requirement of third harmonic relationship.

Shadaj Pancham relationship must exist between all the pairs mentioned below :

Table No.7

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1. Sa - Pa
2. Shuddha Re - Shuddha Dha
3. Shuddha Ga - Shuddha Ni
4. Shuddha Ma - $\overline{\text{Sa}}$
5. Pa - Shuddha $\overline{\text{Re}}$
6. Shuddha Ni - Teevra Ma
7. Teevra $\overline{\text{Ma}}$ - Komal $\overline{\text{Re}}$
8. Komal Re - Komal Dha
9. Komal Dha - Komal $\overline{\text{Ga}}$
10. Komal Ga - Komal $\overline{\text{Ni}}$
11. Komal Ni - Shuddha $\overline{\text{Ma}}$
12. Shuddha Dha - Shuddha $\overline{\text{Ga}}$

In the above table the varieties Shuddha, Teevra and Komal denote the modern convention mentioned in diagram No.1 of the last chapter. Remembering that in the notes defined by Srinivas Shuddha Ga means Komal Ga of today, Shuddha Ni means Komal Ni of today.

Teevra Gandhar means Shuddha Gandhar of today and Teevra Ni means Shuddha Ni of today, we should expect the above relationship to hold with the following substitutions :

1 *Replace Shuddha Ga and Ni by Teevra Ga and Ni*
2 *Replace Komal Ga and Ni by Shuddha Ga and Ni*

Doing this and checking up with the frequencies of the notes *(for Shadaj - Pancham relationship to hold, the frequency ratio must be 3/2 which is the ratio of Pa and Sa).* We find that this relationship holds for all parts except for paid No. 6,7,11 and 12. Of course we have seen above why it is impossible in any scale for this relationship to hold for all the pairs 1 to 12. The fact that for the scale defined by Srinivas, it holds for eight our of twelve possible pairs seems to be reasonably good.

Coming to Shadaj - Gandhar relationships *(5/4 and 6/5)*, the following pairs of notes can be expected to exhibit the Shadaj-Shuddha Gandhar relationship *(5/4).*

Table No.8

1.	Sa - Ga
2.	Re' - Ma
3.	Re - Ma'
4.	Ga' - Pa
5.	Ga - Dha'
6.	Ma - Dha
7.	Ma' - Ni'
8.	Pa - Ni
9.	Dha' - $\overline{\text{Sa}}$
10.	Dha - $\overline{\text{Re'}}$
11.	Ni' - $\overline{\text{Re}}$
12.	Ni - $\overline{\text{Ga'}}$

However, in the scale of Srinivas the pairs 1,2,3,5,6,7,8,9,10 and 12 fail to exhibit this relationship. Only pairs 4 and 11 exhibit the fifth harmonic relationship, the scale is indeed very poor in fifth harmonic relationships.

Coming to the Shadaj-Komal Gandhar relationship (6/5), the following pairs can be expected to exhibit this relationship

Table No.9

1. Sa - Ga'
2. Re' - Ga
3. Re - Ma
4. Ga' - Ma'
5. Ga - Pa
6. Ma - Dha'
7. Ma' - Dha
8. Pa - Ni'
9. Dha' - Ni
10. Dha - $\overline{\text{Sa}}$
11. Ni' - $\overline{\text{Re'}}$
12. Ni - $\overline{\text{Re}}$

Checking up the scale of Srinivas for this relationship, we find that it is exhibited by 1,8 only. The conclusion is that the scale of Srinivas is reasonably rich in Shadaj-Pancham relationships but is very poor in Shadaj Gandhar relationships of both the kind.

As a matter of fact the fifth harmonic Gandhar *(300)* finds no place in the scale of Srinivas at all.

Let us now discuss another scale given by Abhinav Rangmanjarikar. He has adopted the Shuddha note of Srinivas, but has defined a different set of Vikrat notes by a process of construction similar to the one defined by Srinivas. We shall not describe the method of construction but shall only quote the result which is as follows

Table No.10

Name of the note	*Frequency in vibrations per second*
Komal Ra	$254\frac{2}{17}$
Teevra Ga	$301\frac{17}{43}$
Teevra Ma	$338\frac{14}{17}$
Teevra Ma	$338\frac{14}{17}$
Komal Dha	$381\frac{3}{17}$
Teevra Ni	$452\frac{4}{43}$

Checking up these notes for the Shadaj - Pancham relationship of the pairs mentioned in *Table No.5*, we find that in addition to the pairs which satisfied the relationship for the notes of Srinivas, the relationship of pair No. 7 *(Teevra Ma-Komal Re)* also holds. Thus all the pairs except pair No.6, 11 and 12 satisfy the Shadaj - Pancham relationship when Manjarikar's notes are adopted.

As far Shadaj-Gandhar relationships are concerned, we find that there is no improvement on the scale of Srinivas and the same pairs *(4 and 11 for Shadaj - Shuddha Gandhar relationship and 1 and 8 for Shadaj-Komal Gandhar relationship)* exhibit this relationship and no more.

In western music, a scale called diatonic scale has been in vogue. The frequency of its notes *(including the sharps and flats also)* correspond to the following notes.

Table No.11

Frequency corresponding ***to Sa = 240***	**Frequency ratio** ***from Sa***
Sa - 240	1
Re' - 226	16/15
Re - 270	9/8
Ga' - 280	6/5
Ga - 300	5/4
Ma - 320	4/3
Ma' - 337 1/2	45/32
Pa - 360	3/2
Dha' - 384	8/5
Dha' - 400	5/3
Ni' - 432	9/5
Ni - 450	15/8
Sa - 480	2

Checking up this scale for musical relationships, we find that as far as Shadaj-Pancham relationship is concerned, it is exhibited by all pairs except No. 2, 7 and 11. Shadaj Shuddha - Gandhar Bhav *(5/4)* is exhibited by pairs 1,2,3,4,6,8,9,11 that is, all pairs except 5,10,7 and 12. The scale is rich in Shadaj-Gandhar Bhav. Shadaj-Komal Gandhar Bhav *(6/5)* is exhibited by pairs 1,5,6,8,10 and 12, which is an improvement over the scale described earlier.

Thus the scale is rich in Shadaj-Pancham Bhav and Shadaj-Shuddha Gandhar Bhav and moderately, rich in Shadaj-Komal Gandhar Bhav. In Indian Classical music also, these notes are used more often than others. We shall discuss this question in detail later. It should be noted that the fifth harmonic Gandhar *(300)* which was missing in the scales of Srinivas and Manjarikar finds a place in the scale just mentioned.

There is another scale *(the term " scale" has slightly different meaning in western music than the one in which we have been using it, but we shall continue to use " scale" in the same sense in which we have been using it so far)* called ***"Equal Tone Tempered"*** scale which requires rather a detailed discussion since it consists of irrational frequencies and is used widely in Piano, Harmonica and other instruments with fixed keys. In this scale the interval of an octave is divided into twelve notes which have successively equal ratios. The frequency ratios of these notes with the first base note as 1 and the thirteenth note being the counter part of the first note in the next higher octave as 2.

$1,\ 2^{1/12},\ 2^{2/12},\ 2^{3/12},\ 2^{4/12},\ 2^{5/12},\ 2^{6/12},\ 2^{7/12},\ 2^{8/12},\ 2^{9/12},\ 2^{10/12},\ 2^{11/12},\ 2$

These notes correspond to base Sa, Komal Re, Shuddha Re, Komal Ga, Shuddha Ga, Shuddha Ma, Teevra Ma, Pa, Komal Dha, Shuddha Dha, Komal Ni, Shuddha Ni and Sa respectively which have the following frequency ratios in the diatonic scale :

1, 16/15, 9/8, 6/5, 5/4, 4/3, 3/2, 45/32, 8/5, 5/3, 9/5, 15/8, 2

It is seen that except the two Shadajas, all the other notes of the equal tone tempered scale have irrational frequencies while all the frequency ratios of our modern scale are rational fractions. It is a theorem of mathematics that no rational number can ever be equal to an irrational number. It follows that except the two Shadajs, no other note of one scale coincides with that of the other, although, of course, the corresponding notes of both the scale *(bearing the same serial number)* are quite close to each other. In particular, it is found that the coincidence of Shuddha Ma and Pa with the corresponding notes of the Equal Tone Tempered scale is extremely good.

The equal Tone Tempered Scale has a partial advantage that any note can be taken as the starting note without affecting the sequence of notes. For, if the second note is treated as first, the third one will behave as second, fourth as third and so on for the reason that all the consecutive frequency-rations are equal.

Thus, depending upon the convenience of the vocal chord of the singer, any intermediate note can be assumed to be the base note. This cannot be done in diatonic scale since the frequency-ratios are unequal.

For example,
if Komal Re is assumed to be Shadaj,
Shuddha Re will not behave as Komal Re of the new scale, since the ratio :

$\frac{\textit{Shuddha Re}}{\textit{Komal Re}}$ is not equal to but slightly less than the ratio $\frac{\textit{Komal Re}}{\textit{Shadaj}}$

As a matter of fact, there are three elemental frequency-ratios in diatonic scale to which all other ratios can be broken down which are :

X = Ga : Ga' = Ni : Ni' = Dha : Dha' = 25/24

Y = Re : Re' = Ma : Ma* = 135/128

Z = Re' : Sa = Ga' : Re = Ma : Ga = Pa: Ma*
Dha': Pa = Sa : Ni = 16/15

Out of these ratios X, Y, Z, it can be seen that
X is the smallest, Y is bigger than X *and* Z bigger than Y.

In Equal-Tone-Tempered scale, all the above complexities are removed since all consecutive frequency-ratios are equal *(say equal to P),* and any possible frequency-ratio can be only P, P2, p3 ...and so on. This, surely, is a big advantage from the point of view of convenience.

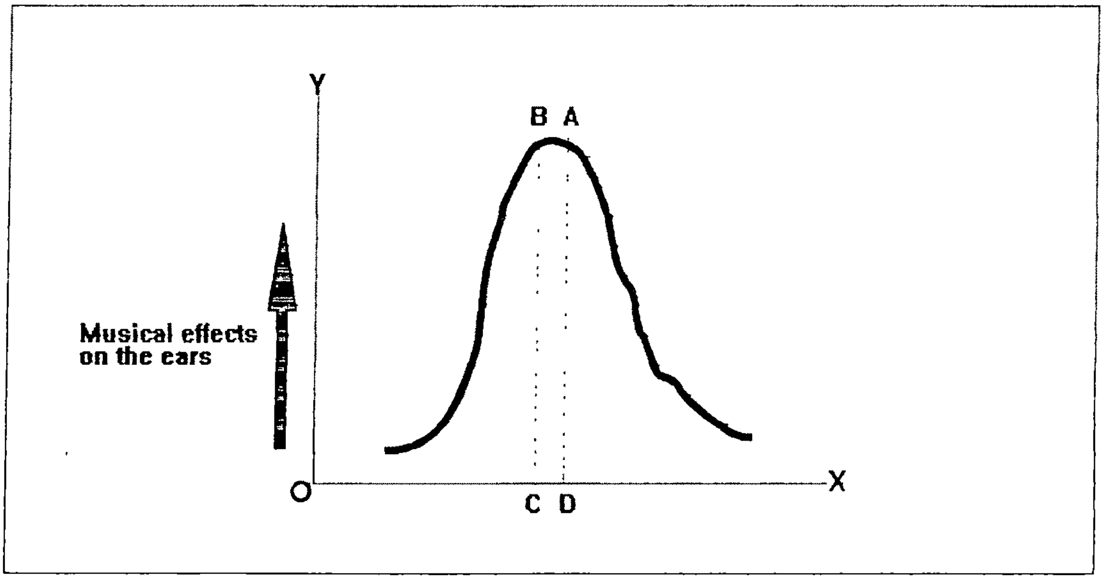
But what about the musical properties of this irrational scale ? Can we call such a scale musical at all ?

'But then' the reader may ask *'if this scale is all that unmusical as claimed by the scientific theory, how is it that it has been so popular and been in use so widely without causing raised eyebrows ?*

The answer is that the scale relies on the imperfection of the human ear and derives rather unfair advantage out of it. Only certain notes are musical *(the base note having been fixed),* but between any two notes, there is a continuous spectrum of infinite notes and the transition from the musical effect to a non-musical effective as one passes to a non-musical note from a musical note is not sudden but gradual.

If we plot a graph showing a variation of musical pleasantness felt by a human ear
as the frequency of a note is varied,
it will show a variation some thing like shown below.

Diagram-10



A denotes a musical note and D a non-musical note in the close neighbourhood of A. It is seen from the graph that, although, the musical effect felt by the ear is maximum for the note A *(A B denotes this musical effect. This is not way to measure this quantity, but the concept has been introduced to bring home the point).* The effect at the neighbouring point C is not much different from that at A. *(In the diagram we can see that AB and CD are roughly equal).* Thus, a non-musical note very close to musical note also produces almost the same musical effect. How close the musical effect would be to the ideal musical effect of a musical note just depends upon how close the note is to the neighbouring musical note.

It follows, therefore, that if instead of a musical note A, another slightly different non-musical note D is used in a musical scale, the human ear will receive almost the same musical effect as if the note A had been used. If D is sufficiently close to A, The human ear may fail to notice that it is not A but a slightly different note D that is being played.

Now the question is, if D is irrational *(as it is in the Equal Tone Tempered scale)* can it lie so close to A *(which must be rational)* to bluff the human ear ? The answer is yes. Mathematics tells us that although an irrational number cannot be equal to a rational number, it can be arbitrarily close to it. There is no dearth of irrational numbers in any region. The notes of the Equal Tone Tempered scale are so close to their counterparts of diatonic scale that the human ear is unable to distinguish them from their counterparts in diatonic scale and therefore the musical effect of the former scale is indistinguishable from that of the latter.

An example will illustrate how close the corresponding notes of the two scales are.

In diatonic scale, the frequency ratios $\frac{Re'}{Sa}$ *and* $\frac{Re'}{Re'}$ **are** $\frac{16}{15}$ *and* $\frac{135}{128}$ *respectively,* which are already so close that it is difficult for most listeners to see the difference when the pair Sa' - Re' and Re' - Re are sounded at different times. In Equal Tone Tempered scale, both these ratios *(pertaining to corresponding notes)* are equal and between 16/15 and 135/128 *(Say P).* It can be guessed how difficult it could be for any human ear to distinguish the ratio P form either 16/15 or 135/128.

It should be carefully noted, however, that the above theory holds only when the notes of Equal Tempered scale alone are used and those of diatonic scale are not touched at all.

We have remarked that however close the notes may be, they cannot fool us when they are played simultaneously *(in fact the closer they are, the more pronounced are the beats).* Even when two very close notes are played at different times during the same recital, keen musicians would easily point out that they are different notes. So, one cannot afford to use sometimes the notes of the Equal Tempered Scale and sometimes those of diatonic scale and hope that the difference would go unnoticed. It is only when only one set of notes is used to the complete exclusion of the other, than the musical effect of one is mistaken for that of the other.

But, any one who is fastidious about the sanctity of musical notes would be unimpressed by this discussion. Why should we bluff the audience by wrong notes just because it saves us the trouble of constructing a musical scale ? A person may have been used to the irrational notes of the Equal Tone Tempered Scale all his life, but when he is confronted with the musical notes, he can be convinced that they have musical relationship with the base note rather than the irrational notes he is used to. A devoted musician would frown at the idea of departing from the correct musical notes for the sake of convenience, however, slight the departure.

This is one of the reason why a beginner is advised not to lean his notes from a harmonium. Most of these instruments are set according to the Equal Tone Tempered Scale which are wrong in Indian classical music. Yet, many musicians, apparently ignorant of this fact, sing on harmonium and think they are singing the notes of the diatonic scale.