Chapter - 5

Notes & Shrutis in actual use today *in* Indian Classical Music

Chapter - V

Notes and Shrutis in actual use today in Indian Classical Music

It is time to apologise to the reader for creating all this confusion about so many systems of Notes, Grams and Scales without setting down on any one chosen system. All the spadework which we came across in the last chapters would not have been necessary if it had been possible to derive a perfect sequence of musical notes by the method of successive fifth harmonics or third harmonics or a combination of both. We have seen that the tonic Shadaj - the generator of six harmonics *(it is not that only six harmonics are generated but other harmonics are so insignificant that they can be neglected)* - gives rise to three more notes-second harmonic Tar Sa, third harmonic Pancham and fifth harmonic Gandhar. We can start from Pancham and go on finding its successive third harmonics as was done in Chapter 2. Thus, we get 12 notes and the 13th. note coincides approximately but not exactly with Shadaj. If we continue the porches further, we can go on adding as many new notes as we like endlessly but we can never return to the Shadaj. The method of successive fifth harmonics is worse, for the third note in the sequence itself comes very close to the Shadaj *(the first note will be the fifth harmonic Gandhar 5/4 x 5/4 = 125/64 which is very close to 2)* and one gets only two notes as against 12 in the third harmonic method.

Thus there can be no scale having 7, 12, 22 or a higher number of notes in which all the notes are related by third harmonic or fifth harmonic relationship and the third harmonic or fifth harmonic of every note is a note within the scale *(even in a different Saptak)*. No matter how we chose a scale, it is bound to contain notes the third harmonic or fifth harmonic of which fall outside the Scale. In other words, we can say that in a Saptak, there are infinite musical notes. The process of finding new musical notes by finding third or fifth harmonics of the notes of any scale never comes to an end. For example, let us take the Shuddha notes derived by the Nishad Murchhana of Madhyam Gram which happens to be the same as the Modern Diatonic scale.

The frequency ratios are :

Sa	Re	Ga	Ma	Pa	Dha	Ni
1	9/8	5/4	4/3	3/2	5/3	15/8

As the first stage, we find the third harmonics and fifth harmonics of these notes as follows :

The original note	Third harmonic	Fifth harmonic
Sa (1)	3/2 (Pancham)	5/4 (Gandhar)
Re (9/8)	27/16 (new note)	45/32 (new note)
Ga (5/4)	15/8 (Nishad)	25/16 (new note)
Ms (4/3)	2 (Tar Sa)	5/3 (Dhaivat)
Pa (3/2)	9/8 (Rishabha)	15/8 (Nishad)
Dha(5/3)	5/4 (Gandhar)	25/24 (new note)
Ni (15/8)	45/32 (new note)	75/64 (new note)

Thus, we got six new notes *(27/16, 45/32, 25/16, 25/24,45/32 and 75/64)* in the first stage. In the second stage, if we try to find the third harmonics and fifth harmonics of these six notes, some of them will certainly add still new notes and so on for all subsequent stages. No matter how many notes we have added in one scale, their third harmonics and fifth harmonics always add yet additional notes. The number of possible musical notes in a Saptak is infinite.

Does it mean that all the notes are musical ? Not at all. In fact, the number of notes not musical are also infinite. We have seen earlier that irrational frequencies are not musical and there are infinite irrational numbers between 1 and 2. Even all rational frequencies are not musical, because the process of finding successive third harmonics or fifth harmonics does not yield all possible rational numbers.

Hence, while there are infinite musical notes in a Saptak, there are infinite non-musical notes also. Moreover, between any two musical notes, there are infinite non-musical notes.

This follows from a well known theorem of Pure Mathematics that between every two rational numbers there are infinite irrational numbers.

Let us ask another question now. Is it possible to find a scale in which all the corresponding pairs are musically related ? *(although the third of fifth harmonics of some of these notes may fall outside the scale)*. The answer is yes, if we are contended with a scale of 1,3 or 5 notes.

The example of 3-note scale and 5-note scales are as follows :

(A)	**Sa**	**Ma**	**Pa**	**(Sa)**		
	1	4/3	3/2	2		
(B)	**Sa**	**Re**	**Ga**	**Ma**	**Dha**	**(Sa)**
	1	10/9	5/4	4/3	5/3	2

In the first example, the relationships are obvious.
In the second one, the following relationship can be verified :

Sa	-	Ga	=	5/4
Sa	-	Ma	=	4/3
Sa	-	Dha	=	5/6 *(Dha-Sa =6/5)*
Re	-	Ma	=	6/5
Re	-	Dha	=	3/2
Ga	-	Dha	=	4/3
Ma	-	Dha	=	5/4

Hence, all the corresponding pairs are related by Shadej-Pancham, Shadaj-Madhyam or one of the Shadaj Gandhar Bhavs.

It is obvious that scales with 2 or 4 notes can always be found with the above properties. However, a musical scale is always chosen to have an odd number of notes since Shadaj is the first note and we must add pairs of notes which are related to Shadaj and Tar Shadaj by a musical relationship. Thus, in the first example, Ma and Pa have been added to the first note Sa. Such that Sa-Ma and Pa-Sa are related by Shadaj-Madhyam Bhav. In the second example, the corresponding pairs are GA-Dha and Re-Ma.

However, the scale with five notes is not regarded as satisfactory as a collection of parent notes from which melodies *(Ragas to be described later)* can be generated, although five notes are considered sufficient for a melody. The reason is that there appears to be gaps between Ma-Dha and Dha-Sa in the scale B cited above. One misses the third harmonics of Sa and Ga *(which would be Pa and Ni having frequencies 3/2 and 15/8 respectively)*. The ancient musicians also included 7 notes in their Grams and Murchhanas. But there can be no scale having 7 notes such that all the corresponding pairs are musically related.

In example B, no matter how we choose Pa, we cannot satisfy the relationships Sa-Pa=3/2 and Re-Pa=4/3 at the same time.

Similarly, no matter how we choose Ni, the relationships Ga-Ni=3/2 and Ma-Ni = 4/3 cannot be taken care of at the same time.

In addition we shall also have to ensure that Pa-Ni is either 5/4 or 6/5. Hence, in a scale of 7 *(or more notes)* we must reconcile ourselves to some of the non-musical pairs. Earlier, we have discussed in detail the Grams or Murchhanas *(7 note scales)* which are rich in musical relationships. Now the question is, why did the ancient musicians choose 7 note scales and not scales with larger number of notes when 12 notes could be easily added to a scale by the simple method of successive third harmonics ? The answer is that they wanted to avoid notes too close to each other for the reason that these notes produce beats when played simultaneously. We have mentioned in the Chapter 3 that ratios 9/8 and 10/9 are most desirable for successive notes because they are sufficiently far apart to avoid beats; but the ratio 16/15 is not so desirable and has been regarded as *"Vivadi"*. The ratio 16/15 *(called half tone in diatonic scale)* is close enough to produce beats but is difficult to avoid. The ancient musicians did not avoid this *"Vivadi"* ratio altogether but they did not allow too many such ratios in their Grams. At least they kept Sa, Ma, Pa - the main notes clear of the *"Vivadi"* ratios in Shadaj and Madhyam Grams. It may be noted that Gandhar Gram avoids a Vivadi ratio altogether, the smallest frequency-ratio between its successive note being 27/25 which is slightly *(by an amount 81/80)* higher than 16/15, Even in the example B of a five-note scale it has not been possible to avoid a Vivadi ratio between Ga and Ma. At any rate, the ancient musicians of Bharat era have completely excluded ratios smaller than 16/15 between successive notes.

Now, let us come back to the method of successive third harmonics for the construction of notes. In the first six stages the following notes are derived Sa-Pa-Re-Dha-Ga-Ni-Ma which can be put in the ascending order as follows :

Sa- Re- Ga- Ma- Pa- Dha- Ni (C)

Which approximately coincides with the Kalyan that of today or the Gandhar Murchhana of Madhyam Gram of Bharat. If the notes are adjusted in such a way that they exactly coincide with the notes of the above Murchhana, the notes become

Sa	Re	Ga	Ma	Pa	Dha	Ni	
1	9/8	5/4	45/32	3/2	27/16	15/8	**(D)**

(The advantage of the adjustment is that Shadaj Gandhar relationships (5/4 and 6/5) are also established between many pairs which were absent in the original notes derived by third harmonic method). If we continue the third harmonic method for the seventh stage. We come to Re' (which is the third harmonic method for the seventh stage, we come to Re' *(which is the third harmonic of Ma)* which falls between Sa and Re. This note coincides approximately with Re' *(16/15)* of diatonic scale and, in any case, comes too close to Re *(Re-Re' is smaller than 16/15)*. Hence, the note Re' does not fit into the requirement that no successive ratio should be smaller than 16/15.

Hence, we find that in the method of successive third harmonics, the first seven notes only are admissible in the scale if we are not going to admit notes too close to each other, the yardstick being that no two notes should be closer than 16/15. However, this sequences of seven notes is *"open"* since the third harmonic of Ma * is a new note Re' *(outside the scale)* and not Sa or Re.

If, we insist on the sequence of notes being closed, we must drop the requirement that two notes should not come closer than 16/15. Then, we come to a closed *(of course, approximately)* sequence of twelve notes but the successive notes *(for example Ga' and Ga in diatonic scale)* may come as close as 25/24. The diatonic scale of 12 notes discussed above is a good adjustment of the 12 notes derived by third harmonic method *(establishing, in the process, many Shadaj-Gandhar relationships)* which forms a closed sequence.

Hence, both seven-note systems and twelve note systems have their virtues and deficiencies. A seven note system is not closed meaning that at least one pair out of seven pairs which could be expected to exhibit Shadaj-Pancham Bahv does not do so even approximately *(For example the pair Ma-Sa in scale C and D above)* but notes closer than 16/15 are totally avoided. On the other hand, in a twelve note scale we must admit closely packed notes *(the smallest ratio between two consecutive notes being 25/24)* in order to make the sequence closed. *(In a closed sequence every fifth note must have a Shadaj-Pancham relationship at least approximately - the approximation being much better than 16/15).*

The exact frequencies of the seven-note scale C are :

Sa	Re	Ga	Ma	Pa	Dha	Ni	(C)
1	9/8	81/64	729/512	3/2	27/16	243/128	

Which differs from D only in the notes Ga, Ma and Ni. It can be verified that each of these notes is higher than the corresponding note of scale D by the same ratio 81/80 which we come across as the "standard Shruti" and which was the note between the Pancham of Shadaj Gram and the same note of Madhyam Gram of Bharat.

The scale C has Shadaj-Pancham relationship between the following of its pairs

Sa	Pa	=	3/2
Re	Dha	=	3/2
Ga	Ni	=	3/2
Pa	Re	=	3/2
Dha	Ga	=	3/2
Ni	Ma	=	3/2

The pair Ma-Sa misses this relationship by a ratio 135/128.
But not a single of the following pairs exhibit Shadaj-Gandhar Bhav *(5/4 or 6/5).*

Sa	-	Ga
Re	-	Ma
Ga	-	Pa
Ma	-	Dha
Pa	-	Ni
Dha	-	Sa
Ni	-	Re

Now, if Ga, Ma* and Ni are depressed by an equal ratio 81/80, all the Shadaj - Pancham relationship remains intact except Dha-Ga *(which misses Shadaj Pancham Bhav by a ratio 81/80)* but all the pair except Dha-Sa exhibit one of the Shadaj-Gandhar Bhav *as follows :*

Sa	-	Ga	=	5/4
Re	-	Ma	=	5/4
Ga	-	Pa	=	6/5
Ma	-	Dha	=	6/5
Pa	-	Ni	=	5/4
Ni	-	Re	=	6/5

We have seen, how in a seven note system derived by third harmonic sequence, the ratio 81/80 plays a crucial role in enriching the scale in Shadaj-Gandhar relationships. The original scale C has Shadaj Pancham relationship in six out of seven pairs but no Shadaj Gandhar relationship in any of the seven pairs. But the converted scale D has 5 out of 7 Shadaj Pancham relationships and six out of 7 Shadaj Gandhar relationships.

Coming to 12 note systems, we have seen above that the diatonic scale is the richest in musical relationships when both Shadaj Pancham and Shadaj Gandhar relationships are taken into account.

Starting from the twelve notes derived in Chapter 2 by the method of successive third harmonics, we can come to the diatonic scale as follows :

Sa 1		**Pa** (x3/2)	**Pa** (3/2) (3/2)
Pa (3/2)	(x3/2)x1/2	**Re** (9/8)	**Re** (9/8)
Re (9/8)	(x3/2)	**Dha** (27/16) *Depress by a Ratio 81/80*	**Dha** (5/3)
Dha (5/3)	(x3/2)	**Ga** (5/4)	**Ga** (5/4)
Ga (5/4)	(x3/2)	**Ni** (15/8)	**Ni** (15/8)
Ni (15/8)	(x3/2)x1/2	**Ma*** (45/32)	**Ma*** (45/32)
Ma* (45/32)	(x3/2)1/2	**Re'** (135/128) *Elevate by a Ratio 2048/2025*	**Re'** (16/15)
Re' (16/15)	(x3/2)	**Dha** (8/5)	**Dha'**(8/5)
Dha' (8/5)	(x3/2)x1/2	**Ga'**(6/5)	**Ga'** (6/5)
Ga' (6/5)	(x3/2)	**Ni'** (9/5)	**Ni'** (9/5)
Ni (9/5)	(x3/2)x1/2	**Ma** (27/20) *Depress by a Ratio 81/80*	**Ma** (4/3)
Ma (4/3)	x3/2	**Sa** (2)	**Sa** (2)

In the process of adjustment, we have depressed Dha and Ma each by a ratio 81/80 with which we are familiar already and elevated Re' by a ratio 2048/2025.

It has been shown earlier that the diatonic scale is rich in Shadaj Pancham relationships *(9 out of 12 pairs).* Shadaj Shuddha Gandhar relationships *(8 out of 12 pairs)* and Shadaj-Komal Gandhar relationships *(6 our of 12 pairs).* If we care only for Shadaj-Pancham Bhav, we can omit the adjustment of Dha and Re' and continue the process of successive third harmonics right upto Ma and then may depress Ma by a ratio *(81/80 x 81/80 x 2045/2048)* to bring it to 4/3. All the notes of this scale will be as mentioned in Chapter 2 except Ma which be 4/3. This scales, however, will not be rich at all in Shadaj-Gandhar relationships and hence can be left out of consideration.

Let us see if there is any 12-note scale which is richer or even as rich as the diatonic scale. Only few scales come into serious consideration. In one of them, all the notes are the same as the diatonic scale but Dha is higher by a ratio 81/80. This Dha, 27/16 times higher then Sa *(diatonic Dha is 5/3 times higher)* has Shadaj-Pancham relationship with Re and Shadaj-Komal Gandhar relationship with Ma* but against that the diatonic Dha has Shadaj Madhyam relationship with Ga, Shadaj-Shuddha Gandhar relationship with Ma and Shadaj-Komal Gandhar relationship with Sa. Testing this scale for musical relationships we find that its performance is

Shadaj-Pancham Bhav	- 9 pairs out of 12
Shadaj-Shuddha Gandhar Bhav	- 7 pairs out of 12

Shadaj-Komal Gandhar Bhav-6 pairs out of 12 which is slightly inferior to that of the diatonic scale *(one more pair exhibits Shadaj Komal Gandhar Bhav in diatonic scale).*

Another scale is obtained by keeping all the notes as the diatonic scale except Ni' which is depressed by a ratio 81/80 bringing it from 9/5 to 16/9. This scale exhibits one less musical relationship as compared to the diatonic scale as can be seen below :

A pair Ni'-Ma exhibits Shadaj-Pancham Bhav in the new scale but the relationship Ga'-Ni' is lost.

The pair Pa-Ni has no longer Shadaj-Shuddha Gandhar Bhav.

The pair Pa-Ni has no longer Shadaj Komal Gandhar Bhav, but to compensate for it, a new pair Ni'-Re has Shadaj Komal Gandhar Bhav.

In this scale (with Ni'-16/9) if we substitute Re-10/9 instead of Re-9/8 the musical proportion remains unaffected since,

The Shadaj-Pancham Bhav Pa-Re is broken but a new pair Re-Dha exhibits this relationship.

Ni-Ma has Shadaj-Komal Gandhar Bhav now but the pair Ni-Re has no longer this relationship.

The conclusion is that the diatonic scale is the most musical 12 note scale, bearing richest in Shadaj-Pancham *(and therefore, Shadaj-Madhyam)* and Shadaj-Gandhar relationships.

<u>The following scales are, however, only slightly inferior to the diatonic scale :</u>

1. The scale Dha =27/16 and other notes the same as in diatonic scale.
2. The scale with Ni'=16/9 and other notes the same as in diatonic scale.
3. The scale with Ni'=16/9 and Re 10/9 and the other note the same as in diatonic scale.

It may be remarked that the scale recommended by Pundit Vishnunarayan Bhatkhande is the (1) above *(with Dha = 27/16).*

Coming to the traditional 22 Shrutis, we have already mentioned the Shrutis of Bharat in the earlier chapter.

Regarding the 22 Shrutis now in use, there is a lot of controversy.
Some of the claims are quoted below :

Pandit Omkarnath Thakur's claims

Table No.16

Sr.No. *of the Shruti*	**Successive** *Frequency ratio*	**Frequency in** *Cycle/second*	**Name** *of the Note*
10	81/80	216	
11	25/24	225	
12	256/243	237/127	
13	81/80	240	Sa
14	81/80	243	
15	25/24	253.1/8	Re'
16	256/243	266.2/3	
17	81/80	270	Re
18	25/24	281.1/4\	
19	256/243	296.8/27	Ga'
20	81/80	300	
21	256/243	316.4/81	
22	81/80	324	
1	81/80	320	
2	25/24	337.1/2	Ma*
3	256/243	355.5/9	
4	81/80	360	Pa
5	25/12	375	
6	256/243	395 15/243	Dha'
7	81/80	400	Dha
8	256/243	421.97/243	
9	81/80	426.2/3	
10	81/80	432	Ni'
11	25/24	450	Ni
12	256/243	474.2/27	
13	81/80	480	Sa

Pandit Omkarnath Thakur has taken 17 Shrutis *(Alapini as the starting point of Madhyam Gram as explained in the last chapter and 13th.Shruti)* where Ni of Madhyam Gram is situated as the starting point of modern Saptak. The Shuddha notes of Pandit Omkarnath Thakur are those generated by Nishad Murchhana of Madyam Gram. All the 12 notes are the same as in the diatonic scale.

If the 4th.Shruti is taken to be the starting point, the Shrutis will be exactly as those is the Shadaj Gram, which have been tabled in the last chapter.

The notes will be situated as given below.

Sa	-	4th.	
Re'	-	6th.	
Re	-	8th.	
Ga'	-	10th.	
Ga	-	11th.	
Ma	-	13th.	
Ma*	-	15th.	
Pa	-	17th.	
Dha'	-	19th.	
Dha	-	20th.	
Ni'	-	1st.	*of a higher octave.*
Ni	-	2nd.	*of a higher octave.*

Pandit Vishnunarayan Bhatkhande has accepted the following frequencies for modern Notes & Shrutis

Table No.17

Sr.No. *of the Shruti*	**Frequency** *(Successive ratio)*	**Frequency**	**Name** *of the Note*
1	1	240	Sa
2	21/20	252	
3	66/63	256	Re'
4	25/24	266.2/3	
5	81/80	270	Re
6	256/243	284.4/9	
7	81/80	288	Ga'
8	25/24	300	Ga
9	81/80	303.3/4	
10	256/243	320	Ma
11	81/80	324	
12	25/24	337.1/2	Ma*
13	2048/2025	341.1/3	
14	135/128	360	Pa
15	75/72	375	
16	128/125	384	Dha'
17	25/24	400	
18	81/80	405	Dha
19	256/243	426.2/3	
20	81/80	432	Ni'
21	25/24	450	Ni
22	81/80	455.5/8	

Table No.18

Sr.No.of Shruti	Pandit K.G.Mule		Prof.Acharckar		Mr.Clement		
	Frequency Ratio *from Sa*	Frequency	Frequency Ratio *from Sa*	Frequency	Frequency Ratio *from A*	Frequency	Name of the Note
22	8/9	214.1/3	8/9	213.1/3	8/9	213.1/3	Ni
1	9/10	216	9/10	216	9/10	216	Ni
2	15/16	225	15/16	225	15/16	225	Ni'
3	80/81	237.1/27	128/135	227.5/9	128/135	227.5/9	
4	1	240	1	240	1	240	Sa
5	256/245	250.38/48	135/128	253.1/8	21/20	252	
6	16/15	256	16/15	256	16/15	256	
7	10/9	266.2/3	10/9	266.2/3	10/9	266.2/3	Re'
8	117/10	280.4/5	9/8	270	9/8	270	Re
9	32/27	284.4/9	160/133	284.4/9	160/133	284.4/9	Sa
10	6/5	288	6/5	288	6/5	288	
11	5/4	300	5/4	300	5/4	300	Ga
12	300/243	316.4/81	81/64	303.3/4	81/64	303.3/4	
					21/10	315	
13	4/3	320	4/3	320	4/3	320	Ma
14	27/20	324	45/32	337.1/2	81/60	324	
15	45/32	337.1/2	-	341.23/32	45/32	337.1/2	
16	40/27	355.5/9	40/27	355.5/9	-	341.1/3	
17	3/2	360	3/2	360	3/2	360	Pa
18	25/16	375	-	379.11/16	-	378	
19	8/5	384	8/5	384	8/5	384	
20	5/3	400	5/3	400	5/3	400	Dha
21	1280/729	421.97/243	27/16	405	27/16	405	
					21/12	420	
22	16/9	426.2/3	16/9	426.2/3	0.16/9	426.3/3	Ni
1	9/5	432	9/5	432	9/5	432	
2	15/8	450	15/8	450	15/8	450	
3	160/81	474.2/3	256/134	455.5/8	256/134	455.5/8	
4	2	480	2	480	2	480	Sa

It should be noted that the Shuddha Dha of Table-17 is higher than the Shuddha Dha of diatonic scale by the ratio of 81/80. The remaining notes are the same as in the diatonic scale. Of course, the intervening Shrutis are much different.

The placement of the 2nd. and 15th.Shrutis could be certainly improved upon. The ratios 21/20, 64/63 and 75/72 appear to be rather arbitrary. If 2nd. and 15th.Shrutis are placed at a ratio of 25/24 from the 1st.and 14th.Shrutis respectively, their frequencies come to 250 and 378 respectively and the ratios become :

Shruti No.2	=	25/24	=	Shruti No.15		
Shruti No.1			=	Shruti No.14		
Shruti No.3	=	128/1256	=	Shruti No.16	=	16/15 / 25/24

All other ratios are musical.

"2048/2025 = 16/15 / 135/128"

Now, the question is which notes and Shrutis are actually used in Indian classical music today ? It has been remarked earlier that musical notes within an octave are infinite and for this reason, it is possible to invent a new note in a musical composition or in the development of a melody exhibiting a new mood. This is what is done in actual practice. Indian classical music cannot be bounded by 12 notes or even 22 Shrutis. For the same of nomenclature the musicians may refer to 12 notes and another 10 rather obscure Shrutis , but even so they recognize that Ga' of Raga Todi is different from that of Raga Bairavi, which in its turn is different from Ga' of Darbari Kanhara and Miyan -Ki-Mallhar. All these notes are called Ga' although they are all different. If we take all these notes the twelve notes used in different Ragas will multiply to much more than the 22 Shrutis.

Broadly speaking, we can say that the 12 notes of diatonic scale are in use today and they are used more often than other notes in the melodies. The majority opinion among the musicians appears to be in favour of regarding these notes as standard notes except that there is scope for controversy over Shuddha Dhaivat. Pandit Bhatkhande has vehemently recommended Dha (27/16); while Pandit Omkarnath Thakur has recommended Dha (5/3) with equal emphasis. Diatonic scale contains Dha (5/3). As a matter of fact both the Dhaivats are valid in their own right and both are actually used according to the musical requirements of a melody of composition. Where Dha is required to be musically related to Ga and Sa, Dha (5/3) is used since it has a Shadaj-Komal Gandhar Bhav with Sa and a Shadaj-Madhyam Bhav with Ga. For example, in Raga Bhopali, Dha (5/3) is used because Dha is required to be related to Ga and Sa. Also when Dha is required to musically related to Ma as in Raga Bageshwari. Dha(5/3) is used since it has Shadaj-Shuddha Gandhar Bhav with Ma. On the other hand, if Dha is required to be musically related to Ma*. as in Raga Yaman or to Re. Dha (27/16) must be used because it has Shadaj-Komal Gandhar Bhav with Ma* and Shadaj-Pancham Bhav with Re.

As far as the Shuddha notes are concerned,
the seven note sales using both the Dhaivats by turn are as follows:

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa-	(1)
1	9/8	5/4	4/3	3/2	5/3	15/8	2	

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa-	(2)
1	9/8	5/4	4/3	3/2	27/16	15/8	2	

Both the scales are musical and have the approval of ancient musicians. The first one is the Nishad Murchhana of Madhyam Gram of Bharat, while the second one is the Nishad Murchhana of the Shadaj Gram. Of course, we have seen that Madhyam Gram is musically richer than Shadaj Gram and therefore, the scale (1) (using Dha 5/3) as more musical than scale (2) using Dha 27/16, but the scale (2) cannot be discarded, and it has not been discarded.

The reader can verify that the Murchhana of the scale (2) starting with Pa generates Khamaj Thata is in use today.

If we consider seven note-scales with Ma,*
both the Dhaivats yield equally musical scale as follows :

Sa	Re	Ga	Ma*	Pa	Dha	Ni	(3)
1	9/8	5/4	45/32	3/2	5/3	15/8	

Sa	Re	Ga	Ma*	Pa	Dha	Ni	(4)
1	9/8	5/4	45/32	3/2	27/16	15/8	

The above two scales are equally rich in musical relationships. In (3) all corresponding pairs except Re-Dhaand Ma*. Sa satisfy Shadaj-Pancham Bhav and all corresponding pairs except Ma*. Dha satisfy Shadaj-Gandhar relationship of one kind of the other. In (4) the only difference is that instead of RE-Dha. Dha-Ga fails to exhibits Shadaj Pancham Bhav and instead of Ma* Dha, Dha-Sa fails to exhibit Shadaj-Gandhar Bhav (6/5). Both the scales are equally rich. If the relationships Re-Dha and Ma* -Dha are to be emphasized, we have to chose Dha (27/16), if the relationships Ga-Dha and Dha-SA are to be emphasized, Dha (5/3) is to be chosen.

In the 12-note scales, we have seen that the diatonic scale is the richest in musical relationships and if we substitute Dha(27/16) for Dha(5/3) , one of the musical relationship breaks down. Hence, we can say that Dha(5/3) has a slight edge over Dha(27/16). We can therefore, regard the 12 notes of the diatonic scale with Dha(5/3) as the *"standard notes"*.

These notes are once more quoted below :

Sa	Re	Re	Ga'	Ga	Ma	Ma*
1	16/15	9/8	6/5	5/4	4/3	45/32

Pa	Dha'	Dha	Ni'	Ni
3/2	9/5	5/3	9/5	15/8

If we must standardize 22-Shrutis also, I would say that the set of Shrutis recommended by Pandit V. Bhatkhande is more in use than any other set (of course, i would suggest frequencies 250 and 378 respectively for the 2nd. and 13th. Shruti) but here the choice is much more difficult. There are many Shrutis which are actually used owing to their musical relationship with certain notes but which have not been included in any of the set of 22 Shrutis suggested by various scholars. For example the Shruti with frequency 135/128 higher than Sa *(which can be loosely called a version of Re')* is used in Ragas Shree, Marva, Puriya Dhanashri etc.

But it does not find a place in Bhatkhande's list. Pandit Omkarnath Thakur's list omits the important note 6/5 itself. As a matter of fact there is no sanctity behind the number 22 except that we get this number of Shrutis when we perform the experiment o Bharat in four stages *(described in Chapter 3)* starting from the Shadaj Gram. However, when we try to include all notes which are equally musical in the same right, we find this number 22 too small.

For example, if we include a Shruti 16/15 higher than Shadaj which is the same as Re' or third Shruti of Pundit Bhatkhande, there is no reason why a Shruti 16/15 below Shuddha Re should not find a place in our set *(there is no such Shruti in the list of Bhatkhande)*, there is no reason why a Shruti 16/15 below Shuddha Re should not find a place in our set *(there is no such Shruti in the list of Bhatkhande, but it would lie between Shruti No.2 and 3 of his list)*. Similarly, if the Shruti 45/32 (Ma* which is 12th. Shruti in Bhatkhande's list) which is 16/15 below PA, finds a place as a note, the claims of a Shruti 16/15 times higher than Ma is also equally strong. This Shruti will be 4/3 x16/15=64/45 times higher than Sa and will be between 11th. and 12th Shruti of Bhatkhande. I think these examples are sufficient to show that it is necessary to approach the problem more systematically and define a larger set of Shrutis. The number 22 is too small.

To define an approximate set of Shrutis, we require (I) a fundamental set of notes which we call the *"note of reference"* and (2) the successive frequency ratios between two consecutive Shrutis. Then we can find out in how many ways the successive frequency-ratios can be arranged between the notes of reference and what is the number of Shrutis thus obtained.

We can, in the first place take the notes of a Gram as the note of reference.

For example let us take the Shadaj Gram of Bharat which has the following notes :

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
1	10/9	32/27	4/3	3/2	5/3	16/9	2

In terms of successive ratios of notes, the scale is :

Sa	Re	Ga	Ma	Pa	Dha	Ni	Sa
	10/9	16/15	9/8	9/8	10/9	16/15	9/8

For the successive frequency-ratios, let us be guided by the Shrutis of Bharat.

We know that are only three kinds of successive Shruti ratios viz.

(1) 81/80 = C *(say)*

(2) 256/243 = L

(3) 25/24 = S

The three successive note ratios can be composed as follows :

Major tone = 9/8 = Ga-Ma = Ma-Pa = Ni-Sa = C x S x L x C

Minor tone = 10/9 = Sa-Re = Pa-Dha = S x L x C

Semi tone = 16/15 = Re-Ga = Dha-Ni = C x L

These are shown diagramatically as follows :

C x S x L x C = Major tone = 9/8

S x L x C = Minor tone = 10/9

L x C = Semi tone = 16/15

Now, if the same successive Shruti intervals are rearranged, we can get more Shrutis. Let us first consider how many more Shrutis we can get by rearranging C, S, L and C in all possible ways in a major tone.

The method will be clear by the following discussion.

Since there are only three unequal successive frequency ratios L S and C, any of them these can be placed first.

*Hence, **the first Shruti*** can be chosen in three ways and its three possible values will be L, S and C.

*For **the second Shruti***, we can have the following four unequal values only.
L x S, C x C, C x S and C x L
(since there are two C's available while there are only one S and L).

*For **the third Shruti***, there can be the following three unequal values.
L x S x C, C x C x L *and* C x C x S

*For **the fourth Shruti***, there is only one choice namely C x C x L x S

(In the above discussion, the second Shruti means the ratio of the second Shruti with respect to the starting point and so on for third and fourth Shrutis).

Hence, the total number of Shrutis obtained by rearranging Bharat's Shrutis in all possible ways is

***In a major tone**, it is 3+4+3+1 = 11.*

In a minor tone, the successive Shruti ratios are C, L and S. If we try to rearrange then we get

1st.Shruti	C, L or S	(3)
2nd.Shruti	CL, LS, SC	(3)
3rd.Shruti	CLS	(1)

Hence, the total number of Shrutis = 3+3+1 = 7

For a semi tone, the successive Shruti ratios are C and L
for which we have the following arrangements :

1st.Shruti	C or L	(2)
2nd Shruti	CL	(1)

Hence, the total number of Shrutis = 3. To recapitulate, by rearranging Bharat's Shrutis in all possible ways within the notes of reference, four Shrutis intervals (major tone) split up into 11 Shrutis, three Shruti intervals to 7 and two Shruti intervals to 3 Shrutis. In Shadaj Gram there are 3 four Shruti intervals, 2 three-Shruti intervals and 2 two-Shruti intervals.

Hence, the total number of Shruti comes to 3 x 11 + 2 x 7 + 2 x 3 = 53 *instead of 22.*

We shall now proceed a step further and show that we should take more successive frequency ratios then the three taken earlier. It can be seen that in diatonic scale, there are the following kinds of successive note ratios.

X = 25/24 *(Ga : Ga')*

Y = 135/128 *(Re : Re)*

Z = 16/15

(The ratio Dha-Ni is Z xY/X = 27/25).

In this notation, the Bharat's successive frequency ratios come to

C = Y/X = 81/80

S = X = 25/24

L = ZX/Y = 256/243

Now, when we have included Y/X as a successive frequency ratio, there is no reason why Z/Y and Z/X also should not be so included. Later on, we shall see that these ratios do appear in practice in some of the Ragas by which some of the notes are shifted relative to their normal position *(frequency).*

Hence, we can include the following ratios as Shrutis.

Z/Y, Z/X, Y/X, X,Y, Z

These have been written in ascending order as Z/Y is the smallest and Z is the largest; though, Z and perhaps Y also may be a bit too large for a Shruti :

A major tone can be broken up into the following Shrutis.

Y,X, X, X, Z/Y and Y/x

These ratios when multiplied together yield a major tone since

$$\frac{Y}{X} \times X \times X \times \frac{Z}{Y} \times \frac{Y}{X} = \frac{135}{128} \times \frac{16}{15} = \frac{9}{8}$$

Similarly, a major tone can be broken up into the following X, X, Z/Y and Y/X

Since $X \times X \times \frac{Z}{Y} \times \frac{Y}{X} = X \times Z = \frac{25}{24} \times \frac{16}{15} = \frac{10}{9}$

And a semi tone can be broken up into the Shrutis denoted by X, Z/Y and Y/X

Since X x Z/Y x Y/X = Z = 16/15

In a Major Tone,
considering the rearrangement of the five Shrutis, we find that

The 1st.Shruti *can be picked up in the following* ***three ways***
X, Z/Y *and* Y/X **(3)**
(since there are only three unequal Shruti ratios).

The 2nd.Shruti *can be picked up in the following* ***five ways***
Y/X x Z/Y, X x X, X x Z/Y, X x Y/X *and* Y/X x Y/X **(5)**

The 3rd.Shruti *can be picked up in the following* ***five ways***
X x X x Y/X, X x X x Z/Y, Y x X x Y/X x X, Y/X x Z/X x Z/Y, X x Z/Y x Y/X **(5)**

The 4th.Shruti *can be picked up in the following* ***three ways***
X x X x Y/X x Y/X, X x X x Y/X x Z/Y, Y/X x Z/Y x X x Y/X **(3)**

The 5th.Shruti *can be picked up in the following way*
X x X x Y/X x Y/X x Z/Y **(1)**

Hence, the total number of Shrutis in a Major Tone comes to 3 + 5 + 5 + 3 + 1 = 17.

Coming to a Minor Tone (10/9),
we have the following ways of arrangements.

The 1st.Shruti X, Z/Y *and* Y/X **(3)**

The 2nd.Shruti X x X x Z/Y, Y x X, X x Z/Y *and* Y/X **(4)**

The 3rd.Shruti X x X x Z/Y, X x X x Y/X, Z/Y x Y/X x X **(3)**

The 4th.Shruti X x X x Z/Y x Y/X **(1)**

Hence, the total number of Shrutis in a Minor Tone comes to 3 + 4 + 3 + 1 = 11.

Lastly, for a Semitone 16/15

The 1st.Shruti X, Z/YU, Y/X **(3)**

The 2nd.Shruti X x Z/Y, Z/Y x Y/X, Y/X x X **(3)**

The 3rd.Shruti X x Z/Y x Y/X **(1)**

Hence, the total number of Shrutis in a Semi Tone comes to 3 + 3 + 1 = 7.

Now, in Shadaj Gram, there are three major tones, two minor tones and two semi tones.
Hence, the total number of Shrutis come to 17 x 3 + 11 x 2 + 7 x 2 = 87.

Then, taking the base Shruti ratios to be Y/X, X, X, Z/Y and Y/X taken by Bharat, we find that a combination of these can produce any desirable ratio, including all the six ratios Z/Y, Y/X, Z/X, X, Y *and* Z.

Rearranging these intervals within major tones, minor tones and semitones, we get 87 Shrutis in Shadaj Gram. The same number will hold for all Grams having 3 major tones, two semitones and two minor tones. The number of Shrutis is the same (87) for Madhyam Gram also and in fact for all Grams described earlier involving only major tones, minor tones and semitones.

But, we come across Grams which involved the ratios 27/25 and 75/64 also.
In the above terminology they are 27/25 = Z x Y x X

75/64 = Z x Y x X

To reduce the number of Shrutis in these intervals, we can break up the interval 75/64 =Z x Y x X into the following elemental successive Shruti ratios.

Y/X, X, X Z/Y, Y/X, X *(Six Shrutis intervals).*

On rearrangement, we have

1st.Shruti Y x X, Z/Y, X (3)

2nd.Shruti X x X, Y/X x X, Z/Y x X, Z/Y x Y/X, Y/X x Y/X (5)

3rd.Shruti X x X x Y/X, X x X x Z/Y, Y/X x Y/X x X, Y/X x Y/X x Z/Y, Y/X x Z/Y x X, X x X x X (6)

4th.Shruti X x X x Y/X x Z/Y, Y/X x Y/X x X x Z/Y, X x X x X x Z/Y, Y/X x Y/X x X x Z, X x X x Y/X x Y/X (5)

5th.Shruti X x X x X x Y/X x Z/Y, Y/X x Y/X x Z/Y x X x X, X x X x X x Y/X x Y/X (3)

6th.Shruti X x X x Y/X x Z/Y x Y/X x X (1)

The total number of Shrutis = 3 + 5 + 6 + 5 + 3 = 23

Coming to the interval 27/25 = Z x Y/X,
it can be broken up into the following elemental intervals : X, Z/Y, Y/X, Y/X

We have, on rearrangement

1st.Shruti X, Z/Y, Y/X (3)

2nd.Shruti X x Z/Y, Z/Y x Y/X, Y/X x X, Y/X x Y/X (4)

3rd.Shruti X x Z/Y x Y/X, Y/X x Y/X x X, Y/X x Y/X x Z/Y (3)

4th.Shruti X x Z/Y x Y/X x Y/X (1)

The total number of Shrutis comes to 3 + 4 + 3 + 1 =11

To summarize, we have the following chart now

Kind of note interval	Frequency ratios	No. of Elemental ratios	Total number of Shrutis when all possible arrangement of elemental Shrutis-ratios are taken into consideration
Major tone	9/8	5	17
Minor tone	10/9	4	11
Semitone	16/15	3	7
(Z/Y/X)	27/25	4	11
(Y x Y x X)	75/64	6	23

With the help of the above chart, we an calculate the number of Shrutis in any Gram.

Two examples are given below :

Gandhar Gram contains one major tone, two 27/25 intervals and four minor tones.
Hence the number of Shruti comes to 17 x 1 **plus** 11 x 2 **plus** 11 x 4 = 83.

Bhairav That has four semitones, two 75/64 intervals and one major tone.

Sa	Re'	Ga	Ma	Pa	Dha'	Ni
1	16/15	5/4	4/3	3/2	8/5	15/8

Thus the number of Shrutis comes to 4 x 7 **plus** 2 x 23 **plus** 17 = 91

It may be wondered why the number of Shrutis become different for different Grams when the octave is the same. The answer is that the number of Shrutis, obtained by rearranging the elemental Shruti ratios, depends upon the notes of reference which are differently located in different Grams. For instance, in the original Shadaj Gram, each of the 22 Shrutis is regarded as a note of reference. No rearrangement is possible and in the same octave we get only 22 Shrutis (and not 53). The note of reference determine the range available for the rearrangement of the intervening elemental Shruti ratios between two successive notes.

Let us see now how many Shrutis we get when we consider the 12 notes of a diatonic scale as the notes of reference.

There are following kinds of successive frequency ratios between consecutive notes

(1) X

(2) Y

(3) Z

(4) Z x Y/X *(=27/25, between Dha : Ni')*

The interval X can be broken up into smaller ratios in the following ways.
(Of course, we are confirming our scales to Z/Y, Y/X, Z/X, X, Y and Z only).

(a) $Z/Y \times X\ Y/Z$ *(Two elemental Shrutis)*

(b) $Z/X \times X^2$ *(Two elemental Shrutis)*

(c) $Y \times X^2/Y$ *(Two elemental Shrutis)*

By rearranging each of the above, we get 3 more Shrutis *(by simply reversing the order).*
Hence, including the starting point, there are 7 Shrutis in all.
(The two elemental Shrutis in a, b, and c including the starting point. By rearrangement, we get three more).

The interval Y can be broken up as follows :

(d) $Z/Y \times Y^2/Z$

(e) $Z/X \times X^2/Z$

(f) Y/X x X

And, similarly we get 7 Shrutis in all, on rearrangements.

The interval Z has been already seen to contain 7 Shrutis *(see the chart given earlier for semitone)* and the interval Z x Y/X has been seen to contain 11 Shrutis *(Z x Y/X = 27/25).*

Now a diatonic scale contains

Six Z intervals

Two Y intervals

Three X intervals

One Z/Y/X intervals

Hence, the number of Shrutis comes to 6x7 + 2x7 + 3x7 + 11 = 88.
This is the total number of Shrutis in a diatonic scale.

The above mentioned systems of Shrutis are a complete set in the sense that starting from a note of reference, all Shrutis corresponding to each of the basic ratios Y/X,Z/Y,Z/X,X,Y and Z are included in these sets. For example, in Shadaj Gram between Sa and Re *(notes of reference)* there are no longer 3 Shrutis but 11 and any Shruti corresponding to the above six ratios is included in the above 11 Shrutis.

Also of a particular Shruti ratios *(say 16/15)* from Sa is included, so also the ratio starting backward from RE is also included. These sets of Shrutis, therefore, include all the Shrutis which may be required actually in any musical recital when the notes of reference are determined by the Gram chosen. Needless to say, these sets include almost all of the 22 Shrutis quoted by different scholars.

To recapitulate, if we abandon our obsession with the number 22 for Shrutis, we can derive in a much more systematic way, a set of Shrutis with a larger number which takes care of all the fundamental Shruti-ratios and all possible rearrangements within the notes of reference depending upon the seven-note scale or 12 more scale chosen. The number of Shruti is different from different scales *(seven note scale or 12 note scale)* although the octave is the same. This is because the range available for rearrangement of the elemental Shruti ratios is different with different notes of reference.

It must be asserted that the above discussion is not a contradiction of the earlier statement that Shrutis are infinite.

It has not been suggested anywhere that more Shrutis than given by the above sets are not possible. If each of the Shrutis derived above is regarded as a new reference point, new Shrutis can always be derived by successive application of the elemental frequency-ratios. One example will be sufficient to clarify the point.

Starting from any note of reference, say Re we come to a Shruti Y/X (81/80). If this Shruti is taken as a new note of reference, we can derive another Shruti Y/XxY/X. Similarly, taking this second Shruti as the note of reference, we can come to a Shruti Y/X x Y/X x Y/X, Y/X and the process can go on indefinitely *(when the ratio Y/X x Y/X x Y/X x Y/X x ... becomes larger than two, we can always divide by 2 to keep it within the octave)* and we get an infinite sequence of Shrutis.

Therefore, the sets of Shrutis derived above not are all the Shrutis possible in an octave, but they are the *"first-stage"* Shrutis derived with respect to certain notes of reference.

Usually, only the first stage Shrutis are useful in a musical recital as we shall see more clearly later. Therefore, the sets of Shrutis derived above include all the useful Shrutis.

One more remark about the derivation of the above sets of Shrutis. The number of possible rearrangements could be derived more elegantly in each of the above cases using stronger tools of mathematics, but the above procedure has been preferred because this way all the individual Shrutis are also evidenced.

To sum up the discussion of this Chapter, two types of scale have been in vogue : seven-note scales or Grams and Murchhana of the ancients and twelve note systems of middle and modern era. A seven note scale can avoid too closely packed notes *(consecutive note ratios less than 16/15)* and therefore, produce a pleasant effect when produced one after the other in succession, but the sequence of the seven notes of this scale is open *(in the sense that one pair which should exhibit Shadaj Pancham Bhav does not do so even approximately)* but does not avoid closely packed notes. For this reason, if all the twelve notes are produced one after the other in ascending order, the effect is not pleasant.

These days, a twelve note scale is taken to be more fundamental and even the seven note scales in use today *(called Thatas, to be discussed in detail in the following Chapter)* are derived from a twelve note scale in the ancient times, the individual seven note scales were regarded as more fundamental and there was no concept of a twelve note scale.

The notes of the diatonic scale, which happens to be the richest in musical relationships among the twelve note scales, can be regarded as the "standard notes" in use today which are the following :

Sa	Re'	Re	Ga'	Ga	Ma	Ma*
1	16/15	9/8	6/5	5/4	4/3	45/32

Pa	Dha'	Dha	Ni'	Ni
3/2	8/5	5/3	9/5	15/8

However, the following notes are also used quite frequently depending upon the particular requirements of the melody (Raga) and the composition.

Re (10/9)	*which has* Shadaj Komal Gandhar Bhav *with* Ma (4/3).
Re' (135/128)	*which has* Shadaj Madhyam Bhav *with* Ma* (45/32).
Ga' (32/27)	*which is* 9/8 below Ma (4/3).
Ga (81/64)	*which has* Shadaj Madhyam Bhav *with* Dha (25/16).
Ma (27/20)	*which has* Shadaj Komal Gandhar Bhav *with* Re (9/8).
Dha' (45/32)	*which has* Shadaj Madhyam Bhav *with* Re' (135/128).
Dha (27/16)	*which has* Shadaj Pancham Bhav *with* Re (9/8).
Ni' (16/9)	*which has* Shadaj Madhyam Bhav *with* Ma (4/3).
Ni (243/128)	*which has* Shadaj Pancham Bhav *with* Ga (81/64).

Out of these, Re, Ga' and Ni are 81/80 times lower than the corresponding standard notes, Ga, Ma, Dha and Ni are higher than the corresponding standard notes by the same amount.

Re' and Dha' are lower than the corresponding standard notes by a ratio 2048/2025 (Z/Y).

Re' and Dha' Shrutis are concerned, the traditional 22 Shrutis are not sufficient to include all elemental Shruti ratios with respect to the notes of a seven scale or twelve scale or twelve note scale.

In fact, when we choose the 22 Shrutis some important ones actually in use also are left out. When care is taken to include all types of elemental Shrutis ratios (Z/Y, Y/X, Z/X, X, Y and z) a scale breaks up into a much larger number of Shrutis. The standard twelve note scales breaks up into 88 Shrutis and, generally, different scales break up into different number of Shrutis.

But this is not all. *Shrutis are infinite.* Mathematically speaking, the musical notes within one octave a discreet but infinite sequence. The above sets of Shrutis represent only some more important ones out of them which are used in recitals, but since the possible Shrutis are infinite, there is always a scope for inventing a new Shruti in the same range to produce an entirely new shade of musical effect.

Indian classical music cannot be bound in 12 notes. 22 Shrutis or even in 88 Shrutis. In different melodies, and in different moods, new notes ad Shrutis are always employed according to the exact musical relationship required between the relevant notes. No musician calculates these relationships and very few even realize which note or Shruti is being played or sung, but owing to the musical sense with which every musicians equipped, the right note or Shruti is always employed to fir in with the requirements of the mood and the rules of the melody.