APPENDIX

Some of the integrals used frequently throught the thesis are given below. They are evaluated using standard integral techniques (Grandshteyn and Ryzhik 1965).

(A) $$I_1(q^2, y_1^2, y_2^2) = \int \frac{d\underline{p}}{(p^2 + y_1^2)(|\underline{q} - \underline{p}|^2 + y_2^2)}$$

$$= \frac{\pi}{E} \log\left[\frac{(y_2^2+q^2)(y_2^2+q^2+E)-y_1^2(y_2^2-q^2)}{y_1^2(E + y_2^2 - y_1^2 - q^2)} \right]$$

where $E^2 = y_1^4 - 2y_1^2(y_2^2 - q^2) + (y_2^2 + q^2)^2$

The special case of $I_1(q^2, y_1^2, y_2^2)$ with $y_1^2 = \beta_i^2 + \lambda^2$ and $y_2^2 = \beta_i^2$ has been denoted as $I_1(\beta_i^2, \lambda^2)$ Yates (1979).

(B) $$I_2(q^2, \beta^2, y^2) = \mathcal{P} \int d\underline{p} \int_{-\infty}^{\infty} \frac{dp_z}{(p_z-\beta)(|\underline{q}-\underline{p}|^2+p_z^2)}$$

$$\frac{1}{(p^2 + p_z^2 + y^2)}$$

$$= \frac{-\pi^3}{[(q^2 + y^2)^2 + 4q^2\beta^2]^{1/2}} \left[1 - \mathrm{Sgn} \right.$$

$$\left. (y^2 - q^2) \left\{ \frac{1}{2} - \frac{\mathrm{Sin}^{-1} A_1}{\pi} \right\} \right]$$

where $A_1 = 1 - \dfrac{2\beta^2(\lambda^2 - q^2)^2}{(y^2 + q^2)^2 (y^2 + \beta^2)}$

and Sgn $X = \dfrac{x}{|x|}$

(C) $I_3(\beta_i^2, y) = \mathcal{P} \int \int_{-\infty}^{\infty} \dfrac{d\underline{p} \quad dp_z}{(p_z - \beta_i)(p^2 + p_z^2 + y^2)}$

$$= -2\pi^2 \left[\frac{\pi}{2} - \tan^{-1} \frac{y}{\beta_i} \right]$$

(D) $I_4 (q^2, \beta_i^2, y_i^2, y_j^2) = \mathcal{P} \int dp \int_{-\infty}^{\infty} \dfrac{dp_z(|q-p|^2 + y_2^2 + p_z^2)^{-1}}{(p_z - \beta_i)(p^2 + p_z^2 + y_1^2)}$

is obtained by putting $y_1^2 = \beta_i^2 + y_i^2$

and $y_2^2 = \beta_i^2 + y_j^2$ in $I_1(q^2, y_1^2, y_2^2)$

given above(A).

(E) $X_1 (b, r_1, r_2) = I_1(b, \beta_i, z_1) - \dfrac{\underline{b} \cdot \beta_1}{b^2 \beta_1^2}$

$$I_2(b\ ,\beta_1,z_1) \ + \ I_1(b,\ \beta_2,\ z_2) - \frac{\underline{b}.\beta_2}{b^2\ \beta_2^2}$$

$$I_2(b,\ \beta_2,\ z_2) + I_1\ (b,\beta_2,z_2) - \frac{b.\beta_2}{b^2\beta_2^2}\ I_2(b,\beta_2,z_2)$$

$$+ I_1(\ \beta_1,\beta_2,(z_2-z_1) \ - \frac{\beta_1.\beta_2}{\beta_1^2\beta_2^2}\ I_2\ [\beta_1,\beta_2(z_2-z_1)]$$

Here $\beta_1 = \underline{b} - \underline{b}_1 \quad ; \quad \beta_2 = \underline{b} - \underline{b}_2$

The detailed calculations for $I_1(b\ ,\ \beta_1,\ z_1)$ and $I_2(b\ ,\ \beta_1,\ z_1)$ are given by Byron et al.

The integral $I_1[\ \beta_1,\beta_2\ (z_2-z_1)\ ]$ will be obtained by putting $b = \beta_i$; $\beta_1 = \beta_2$ and $z_1 = (z_2-z_1)$ in $I_1(b,\ \beta,\ z_1)$.

The integral $I_2\left[\beta_1,\ \beta_2(z_2-z_1)\right]$ will be obtained by putting $b = \beta_1$; $\beta_1 = \beta_2$ and $z_1=(z_2-z_1)$ in $I_2(b\ ,\beta_1,\ z_1)$.

(F) In the fifth chapter for the calculation of X_{abs} , the integrals are of the following type.

$$A = \frac{1}{\pi} \int dv_1\ \ K_o(\beta_i|\underline{b} - \underline{b}_1|)\ \ K_o(\beta_i|\underline{b}-\underline{b}_1|)e^{-2r_1}$$

The solution of the this integral can be obtained as

$$A = \frac{1}{\pi^2} \int dv_1 \int dv_1 \, e^{-2r_1} \, K_o(\beta_i|\underline{b} - \underline{b}_1|)$$

$$K_o \, (\beta_i|\underline{b} - \underline{b}_1| e^{-2r_1}$$

$$= \left| \int_0^\infty \int_{-\infty}^\infty \int_0^\infty db_1 \, b_1 \, dz_1 \, e^{-2\sqrt{b_1^2 + z_1^2}} \right.$$

$$\left. \frac{J_o(xb) \, J_o(xb_1) \, dx}{(x^2 + \beta_i^2)} \right|^2$$

$$= 16 \left| \int_0^\infty (- \frac{\partial}{\partial \lambda}) \, x \, \frac{J_o(xb) \, dx}{(x^2 + \beta_i^2)(x^2 + \lambda^2)} \right|^2$$

$$= 16 \left[(- \frac{\partial}{\partial \lambda}) \frac{1}{(\lambda^2 - \beta_i^2)} \quad K_o(\beta_i b) - K_o(b) \right]^2$$

Here $\lambda = 2$.