APPENDIX C

Equations for inductance and capacitance for multi-phase transmission system

We know that inductance is given by:

$L = \psi / I$

Where, Ψ = weber turns flux linkage

I = Current flowing through

$Hx = Ix / 2\pi x$ field Intensity

$Ix = \pi x^2 / \pi r^2 I = x^2 / r^2 I$

$Hx = x^2 / 2\pi r^2 x I$

$= x / 2\pi r^2 I$ AT/m

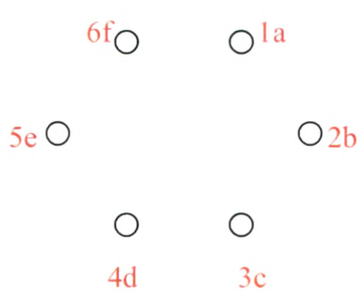
$Bx = \mu_0 \mu r Hx$

$= \mu_0 x I / 2\pi r^2$ As, $\mu r = 1$ for non-magnetic material

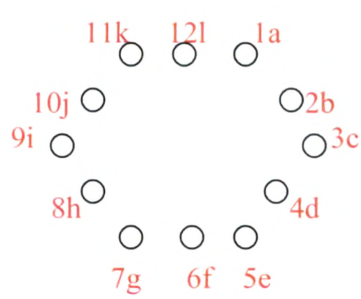
$\psi_{int} = \mu_0 I / 8\pi$

$$\psi_{ext} = \int_r^{\alpha} (\mu_0 I / 2\pi x)\, dx$$

$$\psi = \mu_0 I / 2\pi \{ 1/4 + \int_0^{\alpha} dx/x \}$$



Six-phase configuration
fig. C-1



Twelve-phase configuration
fig. C-2

Inductance for the Six-phase Transmission System can be given by

$$La = \mu_0 I_1/2\pi \{ ¼ + \int_r^{\alpha} dx/x \} + \mu_0 I_2 / 2\pi \int_{d_{12}}^{\alpha} dx/x + \mu_0 I_3/2\pi \int_{d_{13}}^{\alpha} dx/x + \mu_0 I_4/2\pi \int_{d_{14}}^{\alpha} dx/x$$

$$+ \mu_0 I_5/2\pi \int_{d_{15}}^{\alpha} dx/x + \mu_0 I_6/2\pi \int_{d_{16}}^{\alpha} dx/x$$

$$= \mu_0/2\pi \{ (¼ - \log r)I_1 - I_2 \log d_{12} - I_3 \log d_{13} - I_4 \log d_{14} - I_5 \log d_{15} - I_6 \log d_{16} \}$$

$$= \mu_0 / 2\pi \{ (¼ - \log r)I - (0.5 - j0.866)I \log d_{12} - (0.5 - j0.866) I \log d_{13} - ((-1)I \log d_{14})$$
$$-(0.5 - j0.866) I \log d_{15} - (0.5 - j0.866) I \log d_{16} \}$$

$$= \mu o I / 2\pi \{ ¼ - \log_e r - 0.5 \log d_{12} + j0.866 \log d_{12} + 0.5 \log d_{13} + j0.866 \log d_{13} + \log d_{14}$$
$$+ 0.5 \log d_{15} - j0.866 \log d_{15} - 0.5 \log d_{16} - j0.866 \log d_{16} \}$$

$$= \mu o I / 2\pi \{ ¼ - \log_e r - 0.5 \log d_{12} + 0.5 \log d_{13} + 0.5 \log d_{15} - 0.5 \log d_{16}$$
$$+ j0.866 \log d_{12} + j0.866 \log d_{13} - j0.866 \log d_{15} - j0.866 \log d_{16} + \log d_{14} \}$$

$$= \mu o I / 2\pi \{ ¼ - \log_e r + \log[d_{13} d_{15} / d_{16} d_{12}]^{0.5} + \log d_{14} + j0.866 \log[d_{12} d_{13} / d_{15} d_{16}] \}$$

$r' = re^{-1/4} = 0.7788r =$ G.M.R.

$$L_a = 4 \pi 10^{-7} / 2 \pi [¼ + \log (d_{14}/r') (d_{13} d_{15} / d_{16} d_{12})^{0.5} + j0.866 \log (d_{13} d_{12}/d_{15} d_{16})] \quad \_\_\_\_(A)$$

$r' = re^{-1/4} = 0.7788r =$ GMR

$$= 2 \times 10^{-7} [\ln (d_{14}/r') (d_{13} d_{15} / d_{16} d_{12})]^{0.5} + j0.866 \ln (d_{13} d_{12}/d_{15} d_{16})]$$

Similarly,

$L_b = 2 \times 10^{-7} [\ln(d_{25}/r')(d_{24}d_{26}/d_{12}d_{23})^{0.5} + j0.866\ln (d_{23}d_{24}/d_{12}d_{26})]$

$L_c = 2 \times 10^{-7} [\ln (d_{36}/r')(d_{13}d_{35}/d_{23}d_{34})^{0.5} + j0.866\ln (d_{34}d_{35}/d_{13}d_{32})]$

$L_d = 2 \times 10^{-7} [\ln (d_{14}/r')(d_{24}d_{46}/d_{34}d_{45})^{0.5} + j0.866\ln (d_{45}d_{46}/d_{24}d_{34})]$

$L_e = 2 \times 10^{-7} [\ln (d_{25}/r')(d_{15}d_{35}/d_{45}d_{46})^{0.5} + j0.866\ln (d_{15}d_{56}/d_{35}d_{45})]$

$L_f = 2 \times 10^{-7} [\ln (d_{36}/r')(d_{26}d_{46}/d_{16}d_{56})^{0.5} + j0.866\ln (d_{46}d_{56}/d_{16}d_{26})]$

Now for the cyclic transposition for which l is divided in 1/6 and all the six-phases are transposed in such a fashion that each phase occupies every position for equal span during whole length so that inductance of all phases be equal. This would help to achieve balanced voltage at receiving end and to maintain phase sequence too.

$L_{eq} = 1/6 \ln (L_a+L_b+L_c+L_d+L_e+L_f)$

$$= 2\times10^{-7} \ln/6 \, [(d_{14}^2 d_{25}^2 d_{36}^2 / r'^6) [(d_{13}d_{15} / d_{12}d_{16})(d_{24}d_{26} / d_{12}d_{23})(d_{13}d_{35}/d_{23}d_{34}) (d_{24}d_{46} / d_{34}d_{45}) (d_{15}d_{35} / d_{45}d_{56})(d_{26}d_{46} / d_{16}d_{56})]^{1/2} + j0.866 \ln [1]]$$

$$= 2\times10^{-7}\ln/6[1/r'^6(d_{14}^2 d_{25}^2 d_{36}^2)[(d_{13}^2 d_{15}^2 d_{24}^2 d_{26}^2 d_{35}^2 d_{46}^2) /(d_{12}^2 d_{16}^2 d_{23}^2 d_{45}^2 d_{56}^2 d_{34}^2)]^{1/2}] + 0 \quad \text{\_\_\_\_(B)}$$

Where,

$d_{15} = d_{24} = d_{26} = d_{35} \rightarrow (d_{24})^4$

$d_{13} = d_{46} \rightarrow (d_{13})^2$

$d_{12} = d_{16} = d_{23} = d_{45} = d_{56} = d_{34}$

$L \;= 2 \times 10^{-7}\ln/6[1/ r'^6(d_{14}^4 d_{25}^2 d_{13}^2 d_{24}^4 / d_{12}^4 d_{16}^2)$

$\;\;= 2 \times 10^{-7} \ln [1/r' (d_{14}d_{24}/d_{12})^{4/6} (d_{25}d_{13}/d_{16})^{2/6}]$ H/m

$L = 2 \times 10^{-7} \ln [(1/r') (d_{14}d_{24}/d_{12})^{2/3} (d_{25}d_{13}/d_{16})^{1/3}]$ H/m

$=2 \times 10^{-7} \ln [(1/r') (2\sqrt{3}D)^{2/3} (2\sqrt{3}D)^{1/3}]$ H/m

$=2 \times 10^{-7} \ln [2\sqrt{3}D/ r']$ H/m

Which is inductance of Six-phase by cyclic transposition

EQUATION OF INDUCTANCE FOR 12 – PHASE SYSTEM:-

$L_a = \mu_0/2\pi\,(1/4 - \log_e r)I - (0.866 - j0.5)\log d_{12} - (0.5 - j0.866)\log d_{13} - (0 - j1)\log d_{14}$
$-(-0.5 - j0)\log d_{15} - (-0.866 - j0.5)\log d_{16} - (-1)\log d_{17} - (-0.866 + j0.5)\log d_{18}$
$- (0.5 + j0.866)\log d_{19} - (-j1)\log d_{1,10} - (0.5 + j0.866)d_{1,11} - (0.866 + j0.5)d_{1,12}$

$L_a = 2\times10^{-7}\,[-\log_e r' - 0.866\log d_{12} + 0.5\log d_{12} - 0.5\log d_{13} + j0.866\log d_{13} + j\log d_{14}$
$+ 0.5\log d_{15} + j0.866\log d_{15} + 0.866\log d_{16} + j0.5\log d_{16} + \log d_{17} + 0.866\log d_{18}$
$- j0.5\log d_{18} + 0.5\log d_{19} - j0.866\log d_{19} + j\log d_{1,10} - j0.5\log d_{1,11} - j0.866 d_{1,11}$
$-0.866\log d_{1,12} - j0.5\log d_{1,12}]$

$L_a = 10^{-7}\,[\ln (d_{15}d_{17}^2d_{19} / d_{13}d_{1,11}r'^2)\,(d_{16}d_{18} / d_{12}d_{1,12})^{0.866\times 2}$
$+ j\ln(d_{12}d_{14}^2d_{16} / d_{18}d_{1,10}^2d_{1,12})\,(d_{13}d_{15} / d_{19}d_{1,11})^{0.866\times 2}]$
$= 10^{-7}\,[\ln (d_{15}d_{17}^2d_{19} / d_{13}d_{1,11}r'^2)\,(d_{16}d_{18} / d_{12}d_{1,12})^{1.73}$
$+ j\ln(d_{12}d_{14}^2d_{16} / d_{18}d_{1,10}^2d_{1,12})\,(d_{13}d_{15} / d_{19}d_{1,11})^{1.73}]$

$L_a = L_c = L_e = L_g = L_i = L_k$
$= 2\times10^{-7}\,[\ln (d_{15}d_{17}^2d_{19} / d_{13}d_{1,11}r'^2)^{1/2}\,(d_{16}d_{18} / d_{12}d_{1,12})^{1.73/2}$

Where, $(d_{15}d_{17}^2d_{19} / d_{13}d_{1,11}r'^2)^{1/2} = 4\sqrt{3}\,D/r'$ and
$(d_{16}d_{18} / d_{12}d_{1,12})^{1.73/2} = [13]^{0.866}$

So,
$L_a = 2\times10^{-7}\,4\sqrt{3}\,[13]^{0.866} D/r'$

This is an inductance of the conductors, belonging to, say, group1, who are situated at the corners of the hexagon. Inductance of others may be given as:

$L_b = 10^{-7}\,[\ln (d_{26}d_{28}^2d_{2,10} / d_{24}d_{2,12}r'^2)\,(d_{27}d_{29} / d_{23}d_{21})^{1.73}$
$+ j\ln (d_{23}d_{25}^2d_{27} / d_{29}d_{2,11}d_{21})\,(d_{24}d_{26} / d_{2,10}d_{2,12})^{1.73}]$

$L_c = 10^{-7}\,[\ln (d_{37}d_{39}^2d_{3,11} / d_{35}d_{31}r'^2)\,(d_{38}d_{3,10} / d_{34}d_{32})^{1.73}$
$+ j\ln (d_{34}d_{36}^2d_{38} / d_{3,10}d_{3,12}^2d_{32})\,(d_{35}d_{37} / d_{3,11}d_{31})^{1.73}]$

$$L_d = 10^{-7} [\ln (\pi_{48} d_{4,10}^2 d_{4,12} / d_{46} d_{42,} r'^2) (d_{49} d_{4,11} / d_{45} d_{43})^{1.73}$$
$$+ j \ln (d_{45} d_{47}^2 d_{49} / d_{4,11} d_{41}^2 d_{43}) (d_{46} d_{48} / d_{4,12} d_{42})^{1.73}]$$

$$L_e = 10^{-7} [\ln (d_{59} d_{5,11} d_{51} / d_{51} d_{53} r'^2) (d_{5,10} d_{5,12} / d_{56} d_{54})^{1.73}$$
$$+ j \ln (d_{56} d_{58} d_{5,10} / d_{52} d_{52}^2 d_{54}) (d_{57} d_{59} / d_{51} d_{53})^{1.73}]$$

$$L_f = 10^{-7} [\ln (d_{6,10} d_{6,12}^2 d_{62} / d_{68} d_{64} r'^2) (d_{6,11} d_{6,1} / d_{67} d_{65})^{1.73}$$
$$+ j \ln (d_{67} d_{69}^2 d_{6,11} / d_{61} d_{63}^2 d_{65}) (d_{68} d_{61} / d_{62} d_{64})^{1.73}]$$

$$L_g = 10^{-7} [\ln (d_{7,11} d_{71}^2 d_{73} / d_{79} d_{75} r'^2) (d_{7,12} d_{72} / d_{78} d_{76})^{1.73}$$
$$+ j \ln (d_{78} d_{7,10}^2 d_{7,12} / d_{72} d_{74}^2 d_{76}) (d_{79} d_{7,11} / d_{73} d_{75})^{1.73}]$$

$$L_h = 10^{-7} [\ln (d_{8,12} d_{82}^2 d_{84} / d_{8,10} d_{6864} r'^2) (d_{81} d_{83} / d_{89} d_{87})^{1.73}$$
$$+ j \ln (d_{89} d_{8,11}^2 d_{81} / d_{83} d_{85}^2 d_{87}) (d_{8,10} d_{8,12} / d_{81} d_{86})^{1.73}]$$

$$L_i = 10^{-7} [\ln (d_{91} d_{93}^2 d_{95} / d_{9,11} d_{97} r'^2) (d_{92} d_{94} / d_{9,10} d_{98})^{1.73}$$
$$+ j \ln (d_{9,10} d_{9,12}^2 d_{92} / d_{94} d_{96}^2 d_{98}) (d_{9,11} d_{91} / d_{95} d_{97})^{1.73}]$$

$$L_j = 10^{-7} [\ln (d_{10,2} d_{10,4}^2 d_{10,6} / d_{10,12} d_{10,8} r'^2) (d_{10,3} d_{10,5} / d_{10,11} d_{10,9})^{1.73}$$
$$+ j\ln(d_{10,11} d_{10,11}^2 d_{10,3} / d_{10,5} d_{10,7}^2 d_{10,9})(d_{10,12} d_{10,2} / d_{10,6} d_{10,8})^{1.73}]$$

$$L_k = 10^{-7} [\ln (d_{11,3} d_{11,5}^2 d_{11,7} / d_{11,1} d_{11,9} r'^2) (d_{11,4} d_{11,6} / d_{11,12} d_{11,10})^{1.73}$$
$$+ j \ln (d_{11,12} d_{11,2}^2 d_{11,4} / d_{11,1} d_{11,3}) (d_{11,1} d_{11,3} / d_{11,7} d_{11,9})^{1.73}]$$

$$L_l = 10^{-7} [\ln (d_{12,4} d_{12,6}^2 d_{12,8} / d_{12,2} d_{12,10} r'^2) (d_{12,5} d_{12,7} / d_{12,1} d_{12,11})^{1.73}$$
$$+ j\ln(d_{12,1} d_{12,3}^2 d_{12,5} / d_{12,7} d_{12,9}^2 d_{12,11})(d_{12,2} d_{12,4} / d_{12,8} d_{12,10})^{1.73}]$$

First Group

$l_a = l_c = l_e = l_g = l_l = l_k$ (Situated at the corners)

Second Group

$l_6 = l_d = l_f = l_h = l_j = l_i$ (Situated between the corners)

For the first group as derived earlier the inductance will be:

$$= 2 \times 10^{-7}\, 4\, \sqrt{3}\, [13]^{0.866} D/r'$$

For finding inductance of the Second Group, We shall take formulae for any of the conductors in this Group,

$$L_f = 10^{-7} [\ln (d_{6,10} d_{6,12}^2 d_{62} / d_{68} d_{64} r'^2)(d_{6,11} d_{61} / d_{67} d_{65})^{1.73}$$
$$+ j \ln (d_{67} d_{69}^2 d_{6,11} / d_{61} d_{63}^2 d_{65})(d_{68} d_{61} / d_{62} d_{64})^{1.73}]$$

As the imaginary part of the above equation will be zero (as jln(1)=0)we will take the real part only,

$$= 10^{-7} [\ln (d_{6,10} d_{6,12}^2 d_{62} / d_{68} d_{64} r'^2)(d_{6,11} d_{61} / d_{67} d_{65})^{1.73}$$

$$= 2 \times 10^{-7} [\ln [\sqrt{3}\, \sqrt{3} \times D^2 / r'^2]^{1/2}\ [13]^{1.73/2}$$

as, $d_{6,10} / d_{68} = d_{62} / d_{64} \approx \sqrt{3}$

and $(d_{6,12})^2 = 12D^2$

$$= 2 \times 10^{-7} [\ln 6\, [13]^{0.866} D/r'] \text{ H/m}$$

This is the equation of line inductance for Group-2 conductors being situated at the middle of two corners of Hexagon. If cyclic transposition is performed to maintain voltage in balanced condition and to maintain phase sequence the equivalent inductance, which will be equal to sum of all 12 inductances divided by 12.

$L = 2 \times 10^{-7} \ln [(4 \sqrt{3})^6 (6)^6 (13)^{0.866 \times 12}]^{1/12}$

$= 2 \times 10^{-7} \ln [(4 \sqrt{3})^6 (6)^6 013)^{0.866 \times 12}]^{1/12}$

$= 2 \times 10^{-7} \ln [4^6 \times 3^3 \times 2^6 \times 3^6]^{1/12} (13)^{0.866}$

$= 2 \times 10^{-7} \ln [2^{12} \times 2^6 \times 3^9]^{1/12} (13)^{0.866}$

$= 2 \times 10^{-7} \ln [2^{18/12} \times 3^{9/12}] (13)^{0.866}$

$= 2 \times 10^{-7} \ln [2^{3/2} \times 3^{3/4}] (13)^{0.866}$

$= 2 \times 10^{-7} \ln [2 \times \sqrt{3}]^{3/2} (13)^{0.866}$

$= 2 \times 10^{-7} \ln [2\sqrt{3}]^{1.5} (13)^{0.866}$

Capacitance of the Six-phase Line:

Six-phase line has six, phase to ground voltages and 15 phase to phase voltages. Phase to phase voltage characterized by three groups and phase to ground voltage equals adjacent phase voltage.

We know that,

$$V_{ab} = 1/2\pi K[q_a \ln(d_{12}/r) + q_b \ln(r/d_{12}) + q_c \ln(d_{23}/d_{13}) + \ldots + q_n \ln(d_{2n}/d_{1n})] \qquad (A)$$

And

$$Cab = qa/Vab \qquad (B)$$

$$V_{ab} = 1/2\pi K [q_1 \ln(d_{12}/r) + q_2 \ln(r/d_{12}) + q_3 \ln(d_{32}/d_{31}) + q_4 \ln(d_{42}/d_{41}) + q_5 \ln(d_{52}/d_{51}) + q_6 \ln(d_{62}/d_{61})]$$

$$= 1/2\pi K\, q_1[\ln(d_{12}/r) + (0.5-j0.866)\ln(r/d_{12}) + (-0.5-j0.866)\ln(d_{32}/d_{31}) + (-1)\ln(d_{42}/d_{41}) + (-0.5+j0.866)\ln(d_{52}/d_{51}) + (0.5+j0.866)\ln(d_{62}/d_{61})]$$

$$= q_1/2\pi K [\ln(d_{12}/r) + 0.5\ln(r/d_{12}) - j0.866\ln(r/d_{12}) - 0.5\ln(d_{32}/d_{31}) - j0.866)\ln(d_{32}/d_{31}) - 1\ln(d_{42}/d_{41}) - 0.5 \ln(d_{52}/d_{51}) + j0.866\ln(d_{52}/d_{51}) + 0.5 \ln(d_{62}/d_{61}) + j0.866\ln(d_{62}/d_{61})]$$

As $d_{14}=d_{36}$,

$d_{42}=d_{51}=d_{62}$, and

$d_{12}=d_{23}$

$= q_1/2\pi K\ [\ln(d_{31}d_{36}^2)/(d_{16}d_{25}r)]^{0.5} + j\ \ln\ (d_{13}d_{25}/d_{16}r)^{0.866}]$

$C_{an} = 2\pi K/\ [\ln(d_{31}d_{36}^2)/(d_{16}d_{25}r)]^{0.5} + j\ \ln\ (d_{13}d_{25}/d_{16}r)^{0.866}]$

For $C_{bn}=C_{en}$

$$= q/2\pi K[\ln(d_{23}/r)+(0.5-j0.866)\ln(r/d_{23})+(-0.5-j0.866)\ln(d_{42}/d_{42})+(-1)\ln(d_{53}/d_{52}) + (-0.5+j0.866)\ln(d_{63}/d_{62})+ (0.5+j0.866)\ln(d_{13}/d_{12})]$$

Real part of the above equation yields in,

$= \ln[(d_{25}^2d_{13})/(rd_{36}d_{16})]^{0.5}$

Imaginary part,

$= j\ln\ [d_{63}d_{13}/rd_{16}]^{0.866}$ (As $d_{42}=d_{62}$ and $d_{23}=d_{43}$)

So, $C_{bn}=C_{en}$

$$=2\pi K\ /\ \ln\ [(d_{25}^2d_{13})/(rd_{36}d_{16})]^{0.5} + j\ \ln\ [d_{63}d_{13}/rd_{16}]^{0.866}$$