CHAPTER II

COLOR HYDRODYNAMIC EQUATIONS

In order to to describe the evolution of QGP we need either kinetic theory or hydrodynamic theory. Classical and quantum, kinetic theories for QGP have already been formulated[1]. Also the equations of classical hydrodynamics have been obtained by taking appropriate moments of the classical kinetic equations[2]. As the classical kinetic theory forms the basis of our discussion of the hydrodynamic equations a brief introduction to the basic kinetic equations will be given in section 2.1, and the color hydrodynamics (CHD) equations will be obtained from them by taking the appropriate moments. These equations, for a cold collisionless plasma, were first written by Kajantie and Montonen[2] from the equation of motion of a classical colored particle[3] by using heuristic arguments. Finally in the last section 2.2, a critical discussion of the derived color hydrodynamic (CHD) equations will be presented.

2.1 KINETIC EQUATIONS AND DERIVATION OF THE HYDRODYNAMICAL EQUATIONS FOR A CLASSICAL QUARK PLASMA :

There are two approaches that have been used to set up the classical kinetic equations:

(i) In the first one, the single particle phase space is augmented by including color degree of freedom for quarks. Further, the single particle distribution function is an invariant under the local gauge transformations. As a

consequence of the augmented phase space, the kinetic equation contains a drift term in the color space, in addition to a drift term in momentum space.

(ii) In the second one, quantum transport equations are used to arrive at the classical kinetic equation for the single particle distribution function. The phase space in this case is not augmented, but the distribution function is a 2x2 matrix in the color (SU(2)) space. The distribution function transforms covariantly under local gauge transformations. The kinetic equation for such a distribution function has a close similarity with the Vlasov equation for a Coulomb plasma[7].

In this chapter, we consider both kinds of kinetic equations to obtain the CHD equations. It ought to be mentioned that the CHD equations are derived in cold collisionless approximation i.e. by dropping collision terms from the kinetic equations and considering the distribution functions as a product of delta functions in the momentum (and also in color space). It will be shown that the resulting CHD equations are the same for both the kinetic equations. However, in the presence of collisions the form of CHD equations may depend upon the nature of the collision terms(see Mrowczynski in Ref.8).

A plasma may be called "cold" if the thermal motion of its particles is negligible This requires a specification of some reference quantity with respect to which the thermal motion may be neglected. The usual way[4] is to compare mean thermal velocity V_T of plasma particles with the phase velocity V_ϕ of the linear plasma waves. Thus, for a cold plasma $V_\phi = \omega/k >> V_T$ where ω is the frequency and k the wave vector of the wave. This comparison is valid only for weakly non-linear waves.[4]

One can understand the cold plasma approximation in physical terms as follows. Usually, in hydrodynamics, one introduces a length L and a time τ which characterize distance and time over which plasma quantities can change significantly. For fluid description we consider length scale associated with fluid element ΔV, satisfying $(\Delta V)^{1/3} << L$. But $(\Delta V)^{1/3} >> \lambda_c$ where λ_c is the mean free path. This implies that particles in ΔV will have to undergo several collisions to leave the volume ΔV. Therefore the fluid element can persist for several time $>> 1/\nu$ where ν is the collision frequency. In general each fluid element will have a random velocity ω and a flow velocity component U. If U is the same for all particles (if the fields acting on them are the same) and $U >> \omega$ then the concept of the fluid element is meaningful. This is quite unlike a neutral gas where there is no long range self-consistent field, which can 'hold' the fluid elements together in the cold collsionless limit and therefore in that case hydrodynamics would be meaningless.[4]

Let us first consider the kinetic equation with color scalar distribution function. Starting from the the equations of motion for a classical colored particle in external color fields, Heinz has obtained the equation for the single particle (antiparticle) distribution function f(x,p,Q) ($\bar{f}$(x,p,Q)). In the absence of collision terms, they can be written as,

$$p^{\mu}\left[\partial_{\mu}-gQ_a\, F^a_{\mu\nu}\, \partial^{\nu}_p - g\varepsilon_{abc}\, A^b_{\mu}\, Q^c\, \partial^a_Q\right] f(x,p,Q) = 0 \qquad (2.1a)$$

$$p^{\mu}\left[\partial_{\mu}+gQ_a\, F^a_{\mu\nu}\, \partial^{\nu}_p - g\varepsilon_{abc}\, A^b_{\mu}\, Q^c\, \partial^a_Q\right] f(x,p,Q) = 0 \qquad (2.1b)$$

where, gQ_a is the triplet color charge (a = 1,2,3 for SU(2) group) and ε_{abc} is completely antisymmetric Levy-Civita tensor. A^a_{μ} is the gauge potentials with the Lorentz indices μ (= 0,1,2,3) and color indices a (=1,2,3 for SU(2)). $F^{\mu\nu}_a$ is the field tensor of the gauge fields defined as

$$F_a^{\ \mu\nu} = \partial A^{\mu}_a \,\underset{-}{\nu}\, \partial A^{\nu}_a + g\varepsilon_{abc}\, A_b^{\ \mu} A_c^{\ \nu}$$

The first two terms on r.h.s. of Eqs. (2.1) are very similar to electrodynamic plasma kinetic (Vlasov-Boltzmann) equation. The third term gives drift in color space due to the color charge exchange between the fields and particles and it is a characteristic of non-abelian nature of the quark plasma. It should be noted that the second and the third terms on the r.h.s. of Eqs. (2.1) take into account the mean field generated by all the plasma particles.

It is interesting to note that characteristics equations of Eq.(2.1) give the equations of motion of a classical color particle moving in an external color fields, given by

$$m\dot{\zeta}^{\cdot\mu}(\tau) = gQ_a\, F^{\mu\nu}_a(\xi(\tau))\dot{\xi}_{\nu}(\tau) \qquad (2.2)$$

and

$$\dot{Q}_a = g\, \varepsilon_{abc}\, \dot{\zeta}^{\mu}(\tau)\, A_{\mu b}(\zeta(\tau))Q_c \qquad (2.3)$$

where τ denotes the proper time and dot denotes the derivative with respect to it.

Eq. (2.2) is the "Lorentz force" equation for the colored particle. Eq. (2.3) describes evolution of the color charge Q_a. The equation shows that unlike electric charge the color charge Q_a can be exchanged with the gauge field potential A_μ. Moreover, Eq. (2.3) conserves $Q_a Q^a$ (summation over a is implied) so that it describes rotation of Q_a in the color space. It ought to be mentioned that Eqs. (2.2)-(2.3) were obtained from the QCD Lagrangian, by writing down Heisenberg's equation of motion for a colored particle in an external color field and then replacing the operators by the C-members[3] (expectation values).

For a self-consistent theory, the dynamics of color fields A^μ_a should be generated by the currents in the Yang-Mills field equations

$$\partial_\mu F^{\mu\nu}_a + g\varepsilon_{abc} A^b_\mu F^{\mu\nu}_c = g J^\nu_a(x) \tag{2.4}$$

where the color current $J^\nu_a(x)$ can be obtained by integrating the distribution function (Eq.(2.1)) over the phase space measure defined as dpdQ with

$$dp = 2\,\theta\left(p_o\right)\,\delta\left(p^2 - m_o{}^2\right)\, d^4p/\left(2\pi\hbar^3\right) \tag{2.5a}$$

$$dQ = \delta\left(Q^a Q_a - q^2\right)\, dQ_1\, dQ_2\, dQ_3 \tag{2.5b}$$

dp and dQ are invariant measures for momentum space and SU(2) color space respectively. The condition $2\ \theta\left(p_o\right)\delta\left(p^2 - m_o{}^2\right)$ selects particles with the positive energy on mass-shell. The condition $\delta\left(Q^a Q_a - q^2\right)$ preserves the bilinear-Casimir invariant in SU(2) color space. Color currents can then be defined as a Lorentz four vector

$$J^{\mu}_a(x) = \iint p^{\mu}\, Q_a \left(f(x,p,Q) - \bar{f}(x,p,Q)\right) dp dQ \tag{2.6}$$

In the the collisionless plasma approximation Eqs.(2.1-2.6), together with gauge fixing conditions, form a closed set of equations which can provide a classical self consistent description of the quark plasma.

One can write, using Eq.(2.5), four current as

$$J^{\mu}_a(x) = \iint p^{\mu}\, Q_a \left(f(x,p,Q) - \bar{f}(x,p,Q)\right) \frac{d^3p dQ}{m_o} \tag{2.7}$$

where $d^3p = \frac{1}{(2\pi\hbar)^3}\, dp_1 dp_2 dp_3$.

In the cold plasma approximation[5] the distribution function is

$$f(x,p,Q) = n(x,t) \prod_{a=1}^{3} \delta\left(Q_a - \bar{Q}_a(x,t)\right) \prod_{i=0}^{3} \delta\left(p_i - \bar{p}_i(x,t)\right) \tag{2.8}$$

where $\bar{Q}_a$ is ath component of the color field and $\bar{p}_i$ is ith component of the "momentum field" . The momentum field is related to the velocity field U(x,t) by the relation $\bar{p}_i = m_o U(x,t)$ where m_o is the rest mass of the particle. Here we have assumed that the mean field velocity does not have any color label. This assumption is necessary to obtain the CHD equations of Kajantie and Montonen. However, without this assumption hydrodynamic equations can be formulated, but the equations are too complicated to study. Next we define moments of f(x,p,Q). A colorless four flux is defined as

$$N^{\mu} = \iint p^{\mu} f(x,p,Q)\ dpdQ \tag{2.9}$$

and a colored four flux is defined by Eq.(2.6)

In the "cold plasma" approximation these will become

$$N^{\mu} = n(x,t)\ U^{\mu}(x,t) \tag{2.10}$$

$$J^{\mu} = \overline{Q}_a(x,t)\ n(x,t)\ U^{\mu}(x,t) \tag{2.11}$$

To obtain moment equations for the colorless flux we integrate Eq. (2.1a) over p and Q:

$$\partial_{\mu}\left[\iint p^{\mu} f(x,p,Q) dpdQ\right] - g\iint Q_a F^{a}_{\mu\nu}\ p^{\mu}\ \partial^{\nu}_{p}\ f(x,p,Q)dpdQ$$

$$-g\iint \varepsilon_{abc} p^{\mu} A_{\mu b}\ Q^{c}\ \partial^{a}_{Q} f(x,p,Q)dpdQ = 0 \tag{2.12}$$

One can show that,

$$Q_a F^a_{\mu\nu} p^\mu \partial^\nu_p f = \partial^\nu_p \left[Q_a F^a_{\mu\nu} p^\mu f \right]$$

and

$$\varepsilon_{abc} p^\mu A_{\mu b} Q^c \partial^a_Q f = \partial^a_Q \left[\varepsilon_{abc} p^\mu A_{\mu b} Q^c f \right]$$

The integrals, in Eq. (2.12), containing these terms will vanish if the volume integrals in momentum and color space are converted to surface ones and if one assumes that as $|p|, |Q| \to \infty$, $f \to 0$. Then using Eq. (2.10), we obtain

$$\partial_\mu \left(nU^\mu \right) = 0 \tag{2.13}$$

This is a continuity equation for colorless flux and can be identified as baryon number (of the quark-fluid) conservation, in the absence of pair production or annihilation. However, a relativistic treatment of the fluid in general does require the inclusion of pair creation and annihilation The neglect of such effects may justified on the physical situations being examined, e.g., when one considers phenomena whose time scale is much shorter than annihilation and creation rate of quarks then we can safely neglect these processes.

In order to get a differential equation for the color flux (or color currents) we multiply equation (2.1a) by Q_d and integrate over p and Q, i.e.

$$\partial_\mu \left[\iint p^\mu Q_d f(x,p,Q)\ dpdQ\right] - g \iint Q_d Q_a\ F^a_{\mu\nu}\ p^\mu \partial^\nu_p\ f(x,p,Q)\ dpdQ$$

$$-g \iint Q_d \varepsilon_{abc}\ p^\mu A_{\mu b} Q^c \partial^a_Q\ f(x,p,Q)\ dqdQ = 0 \qquad (2.14)$$

One also has the equation

$$Q_d \varepsilon_{abc}\ p^\mu A_{\mu b}\ Q_c \partial^a_Q f = \partial^a_Q \left[Q_d \varepsilon_{abc} p^\mu A_{\mu b}\ Q_{cf}\right] - \delta_{ad} \varepsilon_{abc} p^\mu A_\mu Q^c f$$

As we have already argued, the total divergence term in momentum or in color space will vanish. Thus we obtain, (if one uses Eq. (2.13))

$$U^\mu \partial_\mu \bar{Q}_a \ = \ -g\varepsilon_{abc}\ U^\mu A_{\mu b}\ \bar{Q}_c \qquad (2.15)$$

This equation describes precession of the color fluid charge in color space.

To determine the dynamics of $U^\mu(x,t)$, multiply Eq. (2.1a) by p^α and integrate over p and Q. Thus,

$$\partial_\mu \left[\iint p^\alpha p^\mu f(x,p,Q) dpdQ\right] - g \iint p^\alpha Q_a F^a_{\mu\nu} p^\mu \partial^\nu f(x,p,Q) dpdQ \ -$$

$$g\iint p^{\alpha}\varepsilon_{abc}p^{\mu}A_{\mu b}Q^{c}\partial_{Q}^{a}\ f(x,p,Q)\ dpdQ \ = 0$$

This can be written as

$$\partial_{\mu}\left[m_o\ n\ U^{\mu}U^{\alpha}\right] + gQ_a\ F_{\mu}^{a\alpha}\ U^{\mu}\ n = 0$$

Setting $\alpha = \nu$ and using Eq. (2.13) we get

$$U^{\mu}\partial_{\mu}U^{\nu} = \frac{g}{m_o}\ Q_a\ F_{\mu}^{a\nu}\ U^{\mu} \tag{2.16}$$

Eqs. (2.13), (2.15) and (2.16) form a closed set of momentum equations which describe the hydrodynamics of the classical quark plasma.

However, it can be shown, in general, that when the distribution function f(x,p,Q) is separable in p and Q i.e.

$$f(x,p,,Q) = h(x,Q)\ F(x,p) \tag{2.17}$$

then Eqs. (2.13), (2.15) and (2.16) can be obtained by the moments of Eq. (2.1a). In this case the macroscopic "color charge" $\overline{Q}_a$ is defined as,

$$Q_a = \frac{\int Q_a h(x,Q)}{\int h(x,Q) dQ} \tag{2.18}$$

Before, we study Eqs. (2.13), (2.15) and (2.16) further, a derivation of these equation from the matrix kinetic equation will be discussed.

For simplicity we consider spinless quarks of one flavor only. The quark (antiquark) distribution function $f(x,p)(\tilde{f}(x,p))$ is a $2{\times}2$ matrix (for SU(2)) in color space and transforms covariantly under the local SU(2) gauge transformations i.e.

$$f(x,p) = U(x)\ f(x,p)\ U^{-1}(x) \tag{2.19}$$

where U(x) is a local gauge transformation matrix. f and $\tilde{f}$ satisfy the following transport equations in the collisionless case (see S. Mrowczynski in Ref. 2)

$$p^{\mu}\partial_{\mu}f(x,p) + ig\left[A^{\mu}(x),\ f(x,p)\right] - \frac{1}{2}\,p^{\mu}\partial_{p}^{\nu}\left\{F_{\mu\nu}(x),f(x,p)\right\} = 0 \tag{2.20a}$$

$$p^{\mu}\partial_{\mu}\tilde{f}(x,p) + ig\left[A^{\mu}(x),\ f(x,p)\right] - \frac{1}{2}\,p^{\mu}\partial_{p}^{\nu}\left\{F_{\mu\nu}(x),f(x,p)\right\} = 0 \tag{2.20b}$$

where {,} denotes an anticommutator and [,] denotes a commutator.

It should be mentioned that these equations are obtained as the semi classical limit[6] of the original quantum transport equation[1]. f(x,p) is a matrix whose trace is non-zero. A_{μ} are gauge potential and $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + ig\ [A_{\mu},A_{\nu}]$ are the chromo field tensors whose traces are zero. Eqs. (2.20) transform gauge covariantly under the SU(2) local gauge transformation.

However, Eqs. (2.1) and (2.20) are not quite independent. The difference between f(x,p,Q) and f(x,p) is that f(x,p,Q) is a function of the continuous color variable Q_a(a=1,2,3 for SU(2)) whereas f(x,p) has a finite number of color components. We define color components of f(x,p) as,

$$f_o = \mathrm{tr}\ f(x,p) \tag{2.21a}$$

and

$$f_a = \mathrm{tr}\left[\lambda_a f(x,p)\right] \tag{2.21b}$$

where λ_a (a=1,2,3 for SU(2)) are the generators of the SU(2) group. Then we can write f(x,p) as

$$f(x,p) = f_o\, \mathbf{1} + f_a\, \lambda_a \tag{2.22}$$

If we substitute these into Eq. (2.20a) we obtain

$$p^{\mu}\partial_{\mu}f_o - g\ p^{\mu}F^{a}{}_{\nu}\ \partial_p^{\nu}f_a = 0 \tag{2.23a}$$

and

$$p^{\mu}\partial_{\mu}f_a - g\epsilon_{abc}p^{\mu}A_{\mu b}f_c - gp^{\mu}F^{a}_{\mu\nu}\ \partial_p^{\nu}f_o = 0 \tag{2.23b}$$

One can obtain Eqs. (2.23) from Eq. (2.1a) if the color moments of the distribution function in Eq. (2.1a) are taken as shown below.

$$f_o = \int f(x,p,Q)dQ \tag{2.24a}$$

$$f_a = \int Q_a f(x,p,Q)dQ \tag{2.24b}$$

$$f_{ab} = \int Q_a Q_b f(x,p,Q)dQ \tag{2.24c}$$

and other higher color moments can be defined quite similarly. Taking corresponding moments of Eq. (2.1a)[7], we get

$$p^\mu \partial_\mu f_o(x,p) = gp^\mu F_{\mu\nu}(x)\ \partial_p^\nu f_a(x,p) \tag{2.25a}$$

$$p^\mu \partial_\mu f_a - g\varepsilon_{abc} p^\mu A_{\mu b} f_c = gp^\mu F^b_{\mu\nu}(x)\ \partial_p^\nu\ f_{ab} \tag{2.25b}$$

Eqs. (2.25) represent an infinite hierarchy of color moments of the distribution function. From the view point of QCD, one may expect that only color singlet and triplet (Eqs. (2.23a-b)) distribution functions to play a role in the phase space evolution of the system. The hierarchy, however, has all the higher color moments of the distribution function f(x,p,Q). In quantum theory also a similar hierarchy can arise, but one can truncate it by using color algebra of λ_a-matrices. The classical analog of such truncation condition is ($Q_a \to \frac{1}{2}\lambda_a$)[7]. We impose it classically (by hand) so that

$$f_{ab}(x,p) = \delta_{ab}\, f_o(x,p) \qquad (2.26)$$

As a result, Eq. (2.25b) will become

$$p^{\mu}\partial_{\mu}f_a - g\varepsilon_{abc}\, p^{\mu}A_{\mu b}\, f_c = gp^{\mu}F^{a}_{\mu\nu}(x)\, \partial^{\nu}_{p}\, f_o \qquad (2.27)$$

Thus, Eqs. (2.25a) and (2.25b) are the same as Eqs. (2.23a) and (2.23b). In the cold plasma approximation the distribution function (Eq. 2.8) will give

$$\langle Q_a Q_b \rangle = \delta_{ab} \qquad (2.28)$$

After this long digression we return to the question of taking moments of Eq. (2.20a). In the "cold plasma" approximation we can write,

$$f(x,p) = \rho(x,t) \prod_{i=0}^{3} \left(p_i - \bar{p}_i(x,t)\right) \qquad (2.29)$$

where $\rho(x,t)$ is 2x2 matrix in color space and $\bar{p}_i(x,t)$ are momentum fields and p_i are single particle momentum components and $\bar{p}_i$ are assumed not to have any color label. This assumption is the same as the one we have made in the case of color scalar distribution function. We then define the following moments

$$N^{\mu} = \int f(x,p)p^{\mu}dP/m_o$$

using Eq.(2.29)

$$N^{\mu} = \rho(x,t)\ U^{\mu}(x,t) \tag{2.30}$$

and

$$\theta^{\mu\nu} = \int p^{\mu}p^{\nu}f(x,p)\ \frac{dp}{m_o}$$

using Eq.(2.29)

$$\theta^{\mu\nu} = m_o\rho(x,t)U^{\mu}(x,t)\ U^{\mu}(x,t) \tag{2.31}$$

Taking corresponding moments of Eq. (2.20a) we have

$$\partial_{\mu}N^{\mu} + ig\left[A_{\mu},N^{\mu}\right] = 0 \tag{2.32}$$

and

$$\partial_{\mu}\theta^{\mu\nu} + ig\left[A_{\mu},\ \theta^{\mu\nu}\right] - \frac{g}{2}\left\{F^{\nu}_{\mu},N^{\mu}\right\} = 0 \tag{2.33}$$

Next we expand ρ in terms of λ-matrices,

$$\rho(x,t) = n\mathbf{1} + n_a\ \lambda_a$$

$$= n(1 + Q_a \lambda_a) \tag{2.34}$$

where

$$Q_a = n_a/n \tag{2.35}$$

Thus we have, $n = tr(\rho(x,t))$ and $nQ_a = tr[\lambda_a \rho(x,t)]$. If we use decompositions (2.30) and (2.31) then the moment equations (2.32) and (2.33) will give us

$$\partial_\mu (nU^\mu) = 0 \tag{2.36}$$

$$U^\mu \partial_\mu Q_a = g\varepsilon_{abc}\, U^\mu A_{\mu b}\, Q_c \tag{2.37}$$

$$m_o\, U^\mu \partial_\mu\, U^\nu = -gQ_a\, F^{a\nu}_\mu\, U^\mu \tag{2.38}$$

and

$$Q_a\, m_o\, U^\mu \partial_\mu U^\nu - g\, F^{a\nu}_\mu\, U^\mu = 0 \tag{2.39}$$

By using Eq. (2.38) and the condition (2.28) we can demonstrate that Eq. (2.39) is actually an identity. Conditions like (2.39) can also arise when one takes moments like $\theta^{\mu\nu}_a = \int Q_a p^\mu p^\nu$ $f(x,p,Q)dp\ dQ$ of Eq. (2 1a). Eqs. (2.36)-(2.38) form a closed set of equations describing hydrodynamics of classical quark plasma and which are the same as Eqs. (2.13), (2.15) and (2.16). We will call them, hereafter, equations of Chromohydrodynamics (CHD).

2.2 Chromohydrodynamic Equations : In the previous section we have shown that two differnt kinetic equations can give the same set of CHD equations in the cold collisionless approximation so that Eqs. (2.37)-(2.39) are obtained for the quark distribution function. A similar set of equations can be obtained for the antiparticles with Q → -Q. Thus we give an extra label A to all the fluid variables i.e. n_A, U^{μ}_A and $Q_{Aa} \equiv I_{Aa}$. Label A, in general, denotes a 'specie' in the plasma. For example A could denote quarks of different flavors or antiquarks. The four velocity is usually defined as

$$U^{\mu}_A \equiv \left(\frac{1}{\sqrt{1-V_A^2}} \; , \; \frac{V_A}{\sqrt{1-V_A^2}} \right) \tag{2.40}$$

where V_A is three velocity vector of the specie A and $V_A^2 = V_A.V_A$ denotes the scalar product of the 3 velocity vector. Under this substitution and identifying $n_A/\sqrt{1-V_A^2} \rightarrow n_A$ i.e. density in laboratory frame equations (2.36)-(2.38) can be written as:

$$\frac{\partial}{\partial t} n_A + \nabla.\left(n_A V_A\right) = 0 \tag{2.41}$$

$$\left(\frac{\partial}{\partial t} + V_A.\nabla\right)V_A = \frac{g}{m_A} \sqrt{1-V_A^2}\; I_{Aa} \left(E^a + V_A^x\, B^a - V_A\left(V_A.E^a\right)\right) \tag{2.42}$$

$$\left(\frac{\partial}{\partial t} + V_A.\nabla\right) I_{Aa} = g\varepsilon_{abc} \left[A_b^o - V_A.A_b\right] I_{AC} \tag{2.43}$$

where m_A is the rest mass of the specie A. E^a and B^a are a^{th} components of color electric and magnetic field three vectors respectively and their components are defined below:

$$E_a^i = F_a^{io}\ ,\ B_a^j = -\frac{1}{2}\,\varepsilon^{ijk}\,F_a^{jk} \tag{2.44}$$

where i,j (= 1,2,3) are three space components and ε^{ijk} the Levi-Civita tensor. CHD equations in this form were first obtained using heuristic arguments by Kajantie and Montonen[2]. The color current generated by the particles is then

$$J_a^o = g \sum_A I_{Aa}(x,t)\ n_A(x,t) \tag{2.45a}$$

$$J_a = g\sum_A I_{Aa}(x,t)\ n_A(x,t)V_A(x,t) \tag{2.45b}$$

This current will act as a source term on right hand side of SU(2) Yang-Mills equations

$$\partial_\mu F_a^{\mu\nu} + g\varepsilon_{abc}\ A_{\mu b} F_c^{\mu\nu} = J_a^\nu \tag{2.46}$$

Eqs. (2.40)-(2.45) form a self-consistent set of CHD and Yang-Mills equations for the QCD in cold-collisionless limit. In order to determine the effect of gauge-transformations on Eqs. (2.41)-(2.43), we need to know gauge transformation properties of the variables n_A, V_A and I_{Aa}. It is well-known in gauge theory that under the 'local' gauge transformations

$$A_\mu' = U^{-1}(x)A_\mu U(x) - i/g\ U^{-1}\partial_\mu U(x) \qquad (2.47)$$

where $A^\mu = A^\mu_a \lambda^a$, and a summation over a is implied.

We see that the left hand side of Eq. (2.46) transforms covariantly. Since n_A and V_A do not have any color label, they are invariant under the transformations (2.47). The color charge vector I_A, however, transforms covariantly under these transformations i.e. $I_A \to UI_AU^{-1}$. This can be deduced from Eqs. (2.45) and (2.46), since the that left hand side of Eq. (2.46) transforms covariantly. Therefore, the four vector current density must have the same transformation property due to the requirement of covariance of the equation of motion (Eq. (2.46)). In view of these arguments, Eqs. (2.41) and (2.42) are gauge invariant whereas Eq. (2.43) transforms gauge covariantly.

It should, however, be mentioned that Eqs. (2.41)-(2.43) are incomplete in the following sense:

1. Since gluons do interact among themselves they would get

thermalized among themselves and also with the quarks. Hence there ought to be a set of equations to describe the hydrodynamic evolution of the thermal gluons. The contribution of such gluons is not considered in the present work. This is because to date no hydrodynamical theory of thermal gluons has been worked out. One of the difficulties may be in determining the nature of the collision term in the corresponding kinetic equation.

2. The effects due to the spin degree of freedom of quarks are neglected. The spin degree of freedom comes naturally in the quantal formalism of the kinetic theory. To understand the structure of quantum theory it may be useful to have a classical theory which includes spin. The spin degree can be neglected when the kinetic energy of quark is greater than the spin magnetic interaction amongst them in the Dirac Hamiltonian (see H.Th. Elze et. al., in Ref. 1).

3. In the derivation of the moment equation we have made the "cold plasma" approximation. This corresponds to a very idealized situation, where the thermal, or random energies are small enough for the pressure and heat tensors to be neglected. The exhaustive treatment of the hydrodynamics at finite temperature can be found in a beautiful paper by Heinz[2].

We should note, however, that the earlier treatments of hydrodynamics for the QGP discussed so far, assumes a particular model for the collision terms, in order to have local thermodynamic equilibrium The resulting hydrodynamics depends on the form of collision terms i.e. no unique hydrodynamics is available due to a lack of information about these collision terms[8]. In our view all these hydrodynamic equations should have the same cold-collisionless limit.

After considering all these limitations, we should mention that Eqs. (2.41)-(2.43) clearly show how the color of the particles is being exchanged with the field and vice versa. They are simple enough to study the nonlinear non-abelian phenomena.

This completes our discussion of CHD equations and in the subsequent chapters they will be applied to study various nonlinear physical situations.

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