

*Annexure I*

*Transformer core loss evaluation under  
harmonics conditions*

## AI.1 INTRODUCTION

Transformer is very important static power system equipment in service for more than a century having highest efficiency. However, of late, the performance of distribution transformer has become critical due to mounting failures. The comforts of new era and super refined control of various power operations is the boon from advancements of power electronics. However, same has added the worst of distortions to the input voltage to transformer and huge harmonics of unprecedented magnitude in the load current supplied by the transformer. Therefore, transformers have become vulnerable; their losses have mounted manifold resulting in very poor efficiency and reduced life.

At the same time, advents of new material and improved core configurations have made it possible to find solutions to the problem. Amorphous magnetic material and wound delta core configuration are typical examples which have offered specific advantages.

It is well known that transformers with amorphous core material are having no load losses lower by 70-80% as compared to transformer with CRGO core at normal power frequency operation. Due to more luxuries and comforts being added through electronics and digital communications, the use of non-linear load world-wide has grown up from less than 5% in 1990 to greater than 50% in 2000. Situation in India is not much different at present. Therefore, utilities in Japan have noticed the increase in losses while supplying nonlinear load. However they have noticed that increase is much more in the case of CRGO core transformers. The increase in load losses due to non-linearity in load current can be evaluated using K-factor, which is well defined in IEEE C57.110-1998. However, the increase in no-load losses due to non-linearity in supply voltage has not been emphasized yet properly.

Based on previous experience through experiments, specific transformers were designed and got manufactured having identical coils losses with CRGO and Amorphous Metal cores with 25 kVA 11000/433 Volts, three phase transformer & 10 kVA, 11000/240 Volts rating. This helped to assign the increase in losses to core and eddy current losses. Authors have measured no-load losses of 10 kVA, 11000/240 Volts rating CRGO and Amorphous Core Transformer (AMT) having identical coil at different frequencies and flux densities. Dedicated and sophisticated equipment is identified having measurement accuracy of 0.03% over entire frequency range of interest. A new mathematical model is developed at author's laboratory for evaluation of transformer core losses in situation where supply voltage is distorted.

## AI.2 MATHEMATICAL MODEL DEVELOPED AT ERDA LABORATORY

ERDA has developed a simple method to find the empirical constants required to evaluate core losses of a distorted Voltage Fed Transformer through experimental results taking only design value of flux density as input. The mathematical model so developed is:

$$W = K_1 \cdot B_s^p \cdot f + K_2 \cdot B_s^q \cdot f^m \quad (1)$$

Where  $W$  is the total iron loss,

$B_s$  is the specified flux density,

$f$  is the applied power frequency

$K_1, K_2, p, q$  and  $m$  are the constants found out experimentally.

For a given design, the empirical co-efficient so derived in above mathematical model through sets of experiments are valid for particular design & rating of

transformer with any given situation having different combinations of voltage and current distortions.

The above mathematical model is independent of design variables like mass, specific resistance, density, thickness of lamination, coefficient of hysteresis loss, etc. used in theoretical formula.

### AI.3 CORE LOSS ANALYSIS OF TRANSFORMER WHILE FED FROM NON-SINUSOIDAL SUPPLY VOLTAGE

The core loss is measured using 0.03% accurate Power Analyzer at 50, 60, 70, 80, 90 & 100 Hz by maintaining rated flux density and half the rated flux density.

The mathematical model used is:

$$W = K_1 \cdot B_s^P \cdot f + K_2 \cdot B_s^Q \cdot f^m \quad (1)$$

Iron loss = Hysteresis loss + Eddy current loss

Where  $W$  is the total iron loss,

$B_s$  is the specified flux density,

$f$  is the applied frequency

$K_1, K_2, P, Q$  and  $m$  are the constants found out experimentally.

Equation 1 can be rewritten as

$$W = A \cdot f + B \cdot f^m \quad (2)$$

Where

$$A = K_1 \cdot B_s^P \quad (3)$$

$$B = K_2 \cdot B_s^Q \quad (4)$$

Dividing equation (2) by  $f$  we get

$$\frac{W}{f} = A + B \cdot f^{m-1} \quad (5)$$

So, if equation (5) is plotted and the constants  $A$  &  $B$  are found from proper curve fitting, the value of  $m$  is calculated by substituting the values of  $A$  and  $B$  in equation (5).

A different sets of experiment with half the rated flux densities were done at same set of frequencies as mentioned above to obtain two different values of constant  $A$ ;  $A_1$  (at rated flux density  $B_{s1}$ ) and  $A_2$  (at half the rated flux density  $B_{s2}$ ) and constant  $B$ ;  $B_1$  (at rated flux density  $B_{s1}$ ) and  $B_2$  (at half the rated flux density  $B_{s2}$ ), which is obtained from equations (3) and (4) as

$$A_1 = K_1 \cdot B_{s1}^P \quad (6)$$

$$A_2 = K_1 \cdot B_{s2}^P \quad (7)$$

$$B_1 = K_2 \cdot B_{s1}^Q \quad (8)$$

$$B_2 = K_2 \cdot B_{s2}^Q \quad (9)$$

Taking ratio of equations (6) and (7) we get

$$\frac{A_1}{A_2} = \left[ \frac{B_{s1}}{B_{s2}} \right]^P$$

$$\therefore P = \frac{\left[ \text{Ln} \left( \frac{A_1}{A_2} \right) \right]}{\left[ \text{Ln} \left( \frac{B_{s1}}{B_{s2}} \right) \right]} \quad (10)$$

Taking ratio of equations (8) and (9) we get

$$\frac{B_1}{B_2} = \left[ \frac{B_{s1}}{B_{s2}} \right]^Q$$

$$\therefore Q = \frac{\left[ \text{Ln} \left( \frac{B_1}{B_2} \right) \right]}{\left[ \text{Ln} \left( \frac{B_{s1}}{B_{s2}} \right) \right]} \quad (11)$$

$K_1$  and  $K_2$  are calculated using equations for  $A_1$  equation no (6) & and  $B_1$  equation no (6).

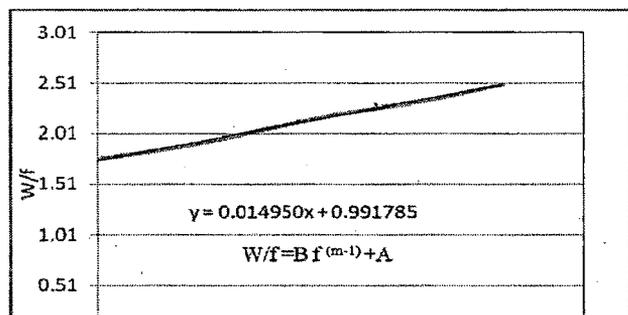
Now, various constants like  $K_1$ ,  $K_2$ ,  $P$ ,  $Q$  &  $m$  have been found. Flux densities corresponding to each individual harmonic voltages and frequency are found out. Eddy current and hysteresis loss is found out by substituting the values of flux densities and above constants in the formula for each individual harmonic frequency. The total core loss is found out by summing up the obtained eddy current loss and hysteresis loss.

### AI.3.1 RESULTS

The core loss is measured at different frequency by maintaining rated flux density at  $V/f=4.8$  &  $2.4$  for Three phase transformer and graph plotted between  $W/f$  and  $f$  is as shown below

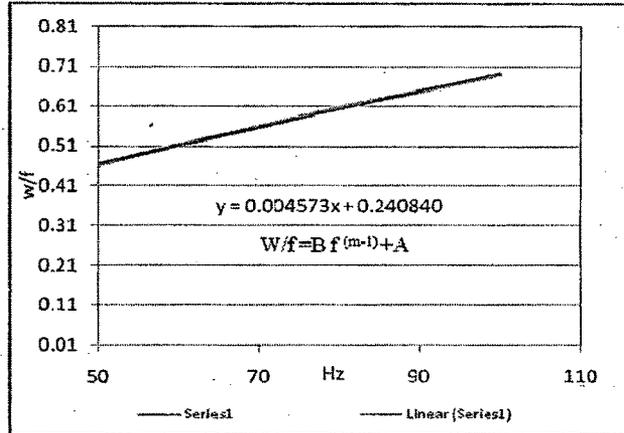
#### CRGO, 3 $\phi$ , 25 kVA, 11000/433Volts

For V/F	8.66	
Freq in Hz	W/f	m1
50	1.75	2.00
60	1.87	2.00
70	2.04	2.00



80	2.20	2.00
90	2.32	2.00
100	2.49	2.00
	Avg	2.00

V/F	4.8	
	.	
Freq in Hz	W/f	m1
50	0.47	2.00
60	0.52	2.00
70	0.56	2.00
80	0.61	2.00
90	0.65	2.00
100	0.70	2.00
	Avg	2.00



Similarly From the above graph the values of A1, A2, B1, B2 and m is obtained for CRGO three phase transformer as explained earlier in the methodology.

	Flux Density		1.54	(as per design)					
<b>A1</b>	<b>A2</b>	<b>B1</b>	<b>B2</b>	<b>p</b>	<b>q</b>	<b>m1</b>	<b>m2</b>	<b>K1</b>	<b>K2</b>
0.9918	0.2408	0.0150	0.0046	2.3985	2.0074	2.0000	1.9999	0.0056	0.0002
								0.0855	0.0019

## AI.4 LOAD LOSS ANALYSIS OF TRANSFORMER WHILE SUPPLYING NON-SINUSOIDAL LOAD CURRENT

The transformer is characterized to find out load loss and ohmic ( $I^2R$ ) loss at linear load condition.

The stray loss is determined by subtracting  $I^2R$  loss from load loss. This stray loss comprises of winding eddy current loss and other stray losses. The winding eddy current loss is taken to be 33% of total stray loss & remaining being other stray loss.

Harmonic loss factor  $F_{HL-EC}$  for winding eddy current loss is determined as given below.

$$F_{HL-EC} = \frac{\sum_{h=1}^{h=h_{max}} I_h^2 \cdot h^2}{\sum_{h=1}^{h=h_{max}} I_h^2}$$

Where  $I_h$  is the individual harmonic current and  $h$  is the harmonic order.

Harmonic loss factor  $F_{HL-STR}$  for other stray loss is determined as given below.

$$F_{HL-STR} = \frac{\sum_{h=1}^{h=h_{max}} I_h^2 \cdot h^{0.8}}{\sum_{h=1}^{h=h_{max}} I_h^2}$$

The loss density is given by

$$P_{LL}(pu) = \sum_{h=h_1}^{h=h_{max}} \left( \frac{I_h}{I_1} \right)^2 \cdot X, \text{ where } X \text{ is the per unit loading.}$$

The  $I^2R$  loss at linear load condition is multiplied with  $P_{LL}(pu)$  to obtain  $I^2R$  loss at non-linear load condition.

The winding eddy current loss at linear load condition is multiplied with  $P_{LL}(pu)$  and  $F_{HL-EC}$  to obtain eddy current stray loss at non-linear load condition.

The other stray loss at linear load condition is multiplied with  $P_{LL}(pu)$  and  $F_{HL-STR}$  to obtain other stray loss at non-linear load condition

The addition of thus obtained  $I^2R$  loss, winding eddy current loss and other stray loss results in total increased load loss at non-linear load condition.

**AI.4.1.1 CRGO TRANSFORMER 3 Ø 25 KVA, 11/0.433 KV, 50**

**HZ, OIL COOLED:**

**1. Linear losses at rated condition and 75 °C**

PARAMETER	HV	LV	HV	LV	
VOLTAGE(Volts)	11000	433			
CURRENT (Amps)	1.31	33.33			
MEASURED RESISTANCE (Ω) AT	30.5°C		75°C		
1U-1V(HV)//2U-2V(LV)	107.36	0.12	125.39	0.14	Ω
1V-1W(HV)//2V-2W(LV)	106.92	0.12	124.87	0.14	Ω
1W-1U(HV)//1W-1U(LV)	107.62	0.12	125.69	0.14	Ω
MEASURED RESISTANCE PER PHASE IN Ω	160.95	0.058728	187.9775	0.06859	Ω
NO LOAD LOSSES			85.1		W
LOAD LOSSES AT 75°C			565		WATTS AT 75°C
TOTAL					
I <sup>2</sup> R LOSSES	277.12	195.77	323.65	228.65	W
TEMPERATURE, °C	30.5		75		
TOTAL I <sup>2</sup> R LOSSES		472.89		552.30	WATTS AT 75°C
STRAY LOSSES & EDDY LOSSES				12.70	WATTS AT 75°C
EDDY CURRENT LOSSES				4.19	WATTS AT 75°C
OTHER STRAY LOSSES				8.51	WATTS AT 75°C
SUMMARY					
TEMPERATURE, °C			75 °C		
NO LOAD LOSSES				85.10	W
I <sup>2</sup> R LOSSES				552.30	W
STRAY & EDDY LOSSES				12.70	W
EDDY CURRENT LOSSES				4.19	W
OTHER STRAY LOSSES				8.51	W
TOTAL LOSSES				650.10	W

## 2. Measured Data at Site

Input		Output	
U-1-Total	10198.00	U-3-Total	389.50
U-2-Total	10195.60	U-4-Total	390.60
I-1-Total	1.07	I-3-Total	25.97
I-2-Total	0.95	I-4-Total	26.40
P-1-Total	3898.20	P-3-Total	9535.30
P-2-Total	9168.80	P-4-Total	3130.20
S-1-Total	10901.60	S-3-Total	10114.20
S-2-Total	9714.30	S-4-Total	10312.30
Q-1-Total	10180.90	Q-3-Total	-3372.90
Q-2-Total	-3209.40	Q-4-Total	9825.70
PF-1-Total	0.36	PF-3-Total	0.94
PF-2-Total	0.94	PF-4-Total	0.30
P-SigmaA-Total	13066.90	P-SigmaB-Total	12665.50
S-SigmaA-Total	17853.90	S-SigmaB-Total	17689.80
Q-SigmaA-Total	6971.40	Q-SigmaB-Total	6452.80
FreqU-1-Total	49.88		

Losses at Site condition	
TEMPERATURE, °C	45
NLL at 390 Volts	62.82 W
I <sup>2</sup> R	498.77 W
STRAY & EDDY LOSSES	14.06 W
EDDY CURRENT LOSSES	4.64 W
OTHER STRAY LOSSES	9.42 W
TOTAL LOSSES	575.65 W

Input Power	Output Power	Difference
13066.90	12665.50	401.40

### 3. Harmonic Analysis of Input & Output Voltage & Current

Harm order	Input				Output				Input				Output			
	volt Mag	% of funda	current mag	% of funda												
1	10170.4	100.0	10166.9	100.0	385.7	100.0	387.5	100.0	1.02	100.0	0.90	100.0	24.73	100.0	25.1	100.0
2	4.0	0.0	4.8	0.0	0.4	0.1	0.3	0.1	0.0	0.6	0.0	1.0	0.1	0.3	0.1	0.5
3	63.0	0.6	24.3	0.2	1.2	0.3	3.7	1.0	0.1	6.4	0.0	2.4	1.3	5.2	1.6	6.5
4	1.4	0.0	0.9	0.0	0.1	0.0	0.1	0.0	0.0	0.2	0.0	0.3	0.0	0.1	0.0	0.2
5	675.6	6.6	654.6	6.4	39.5	10.2	37.1	9.6	0.2	22.3	0.2	24.8	5.9	23.8	5.7	22.8
6	6.5	0.1	2.7	0.0	0.2	0.1	0.3	0.1	0.0	0.2	0.0	0.2	0.0	0.2	0.0	0.2
7	199.9	2.0	234.8	2.3	21.7	5.6	20.7	5.3	0.1	11.4	0.1	14.7	3.1	12.6	3.2	12.7
8	3.1	0.0	5.1	0.1	0.3	0.1	0.3	0.1	0.0	0.1	0.0	0.2	0.0	0.0	0.0	0.1
9	44.8	0.4	10.3	0.1	1.5	0.4	2.5	0.6	0.0	1.6	0.0	0.7	0.3	1.4	0.4	1.5
10	0.8	0.0	2.7	0.0	0.2	0.1	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0
11	223.4	2.2	202.1	2.0	16.7	4.3	15.2	3.9	0.1	5.8	0.1	6.1	1.5	6.1	1.4	5.6
12	3.0	0.0	2.3	0.0	0.1	0.0	0.3	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.1
13	139.3	1.4	181.6	1.8	14.2	3.7	13.3	3.4	0.0	3.3	0.0	4.8	1.0	3.8	0.9	3.6
14	1.4	0.0	2.6	0.0	0.2	0.1	0.2	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
15	42.1	0.4	10.7	0.1	1.3	0.3	2.3	0.6	0.0	0.9	0.0	0.3	0.2	0.7	0.2	0.8
16	0.4	0.0	1.3	0.0	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	121.1	1.2	109.1	1.1	9.9	2.6	8.2	2.1	0.0	2.3	0.0	2.4	0.6	2.5	0.5	2.1
18	2.6	0.0	2.9	0.0	0.1	0.0	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	94.5	0.9	141.9	1.4	10.6	2.7	9.4	2.4	0.0	1.3	0.0	2.4	0.4	1.7	0.4	1.5
20	2.1	0.0	3.4	0.0	0.2	0.1	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	44.8	0.4	14.1	0.1	1.4	0.4	2.4	0.6	0.0	0.6	0.0	0.3	0.1	0.5	0.2	0.6
22	1.3	0.0	1.4	0.0	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	75.7	0.7	72.0	0.7	6.9	1.8	5.0	1.3	0.0	1.2	0.0	1.3	0.3	1.3	0.2	1.0
24	2.5	0.0	2.2	0.0	0.1	0.0	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	51.3	0.5	105.1	1.0	7.9	2.0	6.1	1.6	0.0	0.6	0.0	1.3	0.2	0.9	0.1	0.6
%THD/Amp	10200.22	7.66	10196.70	7.66	389.36	13.82	390.66	12.80	1.06	26.93	0.94	30.22	25.73	28.60	26.07	27.92

#### 4. Calculation of K- Factor

CALCULATION OF HARMONIC LOSS FACTOR & K-FACTOR												
Harm.No	Current (I)	(Ih/I1)	h <sup>2</sup>	(Ih/I1) <sup>2</sup>	(Ih/I1) <sup>2h2</sup>	h0.8	(Ih/I1) <sup>2h0.8</sup>	(Ih/IR)	(Ih/IR) <sup>2</sup>	(Ih/IR) <sup>2h2</sup>	(Ih/IR) <sup>2h0.8</sup>	(Ih) <sup>2</sup>
h	Amps											
1	24.73	1.00	1.00	1.00	1.00	1.00	1.00	0.74	0.55	0.55	0.55	611.75
2	0.08	0.00	4.00	0.00	0.00	1.74	0.00	0.00	0.00	0.00	0.00	0.01
3	1.29	0.05	9.00	0.00	0.02	2.41	0.01	0.04	0.00	0.01	0.00	1.65
4	0.02	0.00	16.00	0.00	0.00	3.03	0.00	0.00	0.00	0.00	0.00	0.00
5	5.88	0.24	25.00	0.06	1.41	3.62	0.21	0.18	0.03	0.78	0.11	34.62
6	0.04	0.00	36.00	0.00	0.00	4.19	0.00	0.00	0.00	0.00	0.00	0.00
7	3.12	0.13	49.00	0.02	0.78	4.74	0.08	0.09	0.01	0.43	0.04	9.72
8	0.01	0.00	64.00	0.00	0.00	5.28	0.00	0.00	0.00	0.00	0.00	0.00
9	0.34	0.01	81.00	0.00	0.02	5.80	0.00	0.01	0.00	0.01	0.00	0.12
10	0.02	0.00	100.00	0.00	0.00	6.31	0.00	0.00	0.00	0.00	0.00	0.00
11	1.51	0.06	121.00	0.00	0.45	6.81	0.03	0.05	0.00	0.25	0.01	2.27
12	0.01	0.00	144.00	0.00	0.00	7.30	0.00	0.00	0.00	0.00	0.00	0.00
13	0.95	0.04	169.00	0.00	0.25	7.78	0.01	0.03	0.00	0.14	0.01	0.90
14	0.00	0.00	196.00	0.00	0.00	8.26	0.00	0.00	0.00	0.00	0.00	0.00
15	0.17	0.01	225.00	0.00	0.01	8.73	0.00	0.01	0.00	0.01	0.00	0.03
16	0.01	0.00	256.00	0.00	0.00	9.19	0.00	0.00	0.00	0.00	0.00	0.00
17	0.61	0.02	289.00	0.00	0.17	9.65	0.01	0.02	0.00	0.10	0.00	0.37
18	0.01	0.00	324.00	0.00	0.00	10.10	0.00	0.00	0.00	0.00	0.00	0.00
19	0.43	0.02	361.00	0.00	0.11	10.54	0.00	0.01	0.00	0.06	0.00	0.19
20	0.00	0.00	400.00	0.00	0.00	10.99	0.00	0.00	0.00	0.00	0.00	0.00
21	0.12	0.00	441.00	0.00	0.01	11.42	0.00	0.00	0.00	0.01	0.00	0.01
22	0.01	0.00	484.00	0.00	0.00	11.86	0.00	0.00	0.00	0.00	0.00	0.00
23	0.31	0.01	529.00	0.00	0.08	12.29	0.00	0.01	0.00	0.05	0.00	0.10
24	0.00	0.00	576.00	0.00	0.00	12.71	0.00	0.00	0.00	0.00	0.00	0.00
25	0.23	0.01	625.00	0.00	0.06	13.13	0.00	0.01	0.00	0.03	0.00	0.05
Σ	25.73	1.61		1.08	4.38		1.34	0.77	0.60	2.41	0.74	661.79

### 5. Calculation of Core Loss Due to Distortion in Supply Voltage

$V_{THD}$	13.817	$B_s=$	1.54000	at $V/f=$	8.80000		
$V_{rms}$	389.36	$k_1=$	0.00559	$k_2=$	0.00020		
Frequency	49.995 Hz	$m=$	2.00003				
		$p=$	2.39854	$q=$	2.00736		
No.	Voltage(V)	Frequency(Hz)	$V/f$	$B_s$	$k_1 B_s^p f$	$k_2 B_s^q f^m$	Calculated Loss
1	379.10	50.00	7.58	1.33	36.06	28.62	64.68
2	0.30	99.99	0.00	0.00	0.00	0.00	0.00
3	1.10	149.99	0.01	0.00	0.00	0.00	0.00
4	0.10	199.98	0.00	0.00	0.00	0.00	0.00
5	42.10	249.98	0.17	0.03	0.02	0.34	0.36
6	0.20	299.97	0.00	0.00	0.00	0.00	0.00
7	23.30	349.97	0.07	0.01	0.00	0.10	0.11
8	0.20	399.96	0.00	0.00	0.00	0.00	0.00
9	1.40	449.96	0.00	0.00	0.00	0.00	0.00
10	0.10	499.95	0.00	0.00	0.00	0.00	0.00
11	16.60	549.95	0.03	0.01	0.00	0.05	0.05
12	0.20	599.94	0.00	0.00	0.00	0.00	0.00
13	14.30	649.94	0.02	0.00	0.00	0.04	0.04
14	0.10	699.93	0.00	0.00	0.00	0.00	0.00
15	1.20	749.93	0.00	0.00	0.00	0.00	0.00
16	0.00	799.92	0.00	0.00	0.00	0.00	0.00
17	9.60	849.92	0.01	0.00	0.00	0.02	0.02
18	0.20	899.91	0.00	0.00	0.00	0.00	0.00
19	10.50	949.91	0.01	0.00	0.00	0.02	0.02
20	0.20	999.90	0.00	0.00	0.00	0.00	0.00
21	1.40	1049.90	0.00	0.00	0.00	0.00	0.00
22	0.10	1099.89	0.00	0.00	0.00	0.00	0.00
23	6.70	1149.89	0.01	0.00	0.00	0.01	0.01
24	0.20	1199.88	0.00	0.00	0.00	0.00	0.00
25	7.40	1249.88	0.01	0.00	0.00	0.01	0.01
Calculated Power (W)					36.08	29.22	65.30

## 6. Loss Calculation Summary

EDDY CURRENT HARMONIC LOSS FACTOR =		4.05				
STRAY LOSSES HARMONIC LOSS FACTOR =		1.24				
K-FACTOR FOR EDDY CURRENT=		2.41				
K-FACTOR FOR STRAY LOSSES=		0.74				
Loading	0.77					
PII(pu)	1.04					
PII(pu) at Particular Loading	0.62					
Temp Correction Factor						
	Rated Loss AT 75°C	Linear Loss at Actual Loading at 45°C	Harmonic Multiplier	Computed Losses	Measured Losses	Deviation in %
No Load Losses	85.10	62.82		65.30		8.28
I <sup>2</sup> R Losses	552.30	308.97		308.97		
Winding Eddy Losses	4.19	2.87	4.05	11.63		
Other Stray Losses	8.51	5.84	1.24	7.22		
Total Losses	650.10	380.50		393.12	401.40	2.11
DIFFERENCE(Non linear-linear)					20.90	5.49