Chapter 6

Interacting Particle Bivarite GT Strength Densities in SAT-LSS and β-Decay Rates for Some $A > 60$ fp-Shell Nuclei Relevant for Presupernova Stars

6.1 Introduction

Given a one plus two-body hamiltonian (H) and a transition operator ($\mathcal{O}$), in SAT-LSS the smoothed form of the IP transition strength densities ($\mathbf{I}_{\mathcal{O}}^{H}(E_i, E_f)$) in general takes a bivariate convolution form [Fr-87a, Fr-88b] with the NIP strength density being convoluted with a spreading bivariate Gaussian due to interactions. The construction of IP bivariate strength densities, allows one to address a wide variety of nuclear physics and nucler astrophysics problems; earlier applications, of the convolution form, in nuclear physics are given in [Fr-87a, Fr-88b, Po-91]. Applying the construction, explicitly in terms of the (partial) bivariate moments, of the IP bivariate strength densities, for the first time to the problems of interest in nuclear astrophysics (it is well known [Wo-80a, Wo-80b, Ra-84, Pi-94], that knowledge of level densities, partition functions, orbit occupancy numbers, β-decay strength densities and other related statistical quantities is essential for a quantitative study of various problems in nuclear astrophysics – therefore SAT-LSS is ideally suited for applications in this important area of physics), this chapter deals with the construction of β-decay Gamow - Teller (GT) strength distributions and in particular for $A > 60$ neutron excess fp-shell nuclei that are of interest in presupernova evolution calculations. The β-decay rates for these nuclei play a very important role [Be-79, Au-90] in determining the structure of the core of massive presupernova stars and hence on their subsequent evolution towards gravitational collapse and supernova explosion phases.

The earlier calculations of β-decay rates at zero temperature mainly used the "gross theory" (which is indeed a statistical theory of β-decay) due to Takahashi and Yamada [Ta-69] and the microscopic quasiparticle random phase approximation (QRPA) [Kl-84]; shell model calculations are feasible and available for lighter ($A < 40$) nuclei [Br-85]. Fuller, Fowler and Newman [Fu-80] (hereafter referred as FFN) for the first time dealt with in detail the problem of calculating β-decay rates (and also other steller weak interaction rates which include electron capture rates, neutrino energy loss rates etc.) at non-zero temperatures appropriate for presupernova evolution studies and tabulated the rates for 226 ds and fp-shell nuclei with $21 \leq A \leq 60$. They (FFN) have used experimental data for energy levels and transition matrix elements (comparative half lifes $\ell og(ft)_{ij}$) where available and for the unmeasured matrix elements, zeroth order shell model calculations and also detailed shell model results (for GT strengths) wherever available are employed; recently Oda et al

[Od-94] extended for ds-shell nuclei the FFN tabulations by using the results of complete shell model calculations with Wildenthal's interaction [Wi-84]. Steller evolution calculations including FFN rates showed that the FFN rates affect significantly the evolution of presupernova stars - they led to greater reduction of electron fraction throughout the core [We-85]. However more recently Aufderheide et al [Au-90] pointed out that the calculation of presupernova evolution generates neutron rich cores and therefore emphasized the need for reliable β-decay rates for neutron rich $A > 60$ fp-shell nuclei; they tentatively conclude that β-decay of neutron rich nuclei (particularly ^{63}Co; they in fact studied $^{60,62,63,64}Co$ isotopes) prevents the decrease of overall electron fraction and affects substantially the post-core-silicon-burning behavior. With this observation, the calculation of β-decay rates for $A > 60$ neutron excess nuclei has acquired new impetus. Calculations of the rates involve the construction of the bivariate GT strength densities. So far only [Au-90, Au-94, Ka-91] dealt with the neutron excess $A > 60$ fp-shell nuclei; in Ref. [Au-90, Au-94] Aufderheide et al used methods similar to that of FFN and in Ref. [Ka-91] Kar, Sarkar and Ray (hereafter referred as KSR) introduced a method, combining the ideas of the "gross theory" with a zeroth order SAT result (described ahead in Sect. 6.4.3). In this chapter a method is developed for β-decay rates calculations where GT bivariate strength densities are explicitly constructed, starting with an effective nucleon - nucleon interaction, using the bivariate convolution form (Eq. (6.2) ahead) for $\mathbf{I}^{H}_{\mathcal{O}(GT)}$. The theory for constructing NIP GT strength densities, which enter into the IP theory as one of the convolution factors, is given in the preceding chapter. We will now give a preview.

Sect. 6.2 gives the IP theory for constructing GT strength densities and

Sect. 6.3 gives the formalism for calculating β-decay rates where $\mathbf{I}^{H}_{\mathcal{O}(GT)}$ and the state density $I^{H}(E)$ enter explicitly. In Sect. 6.4 the calculational procedure is described. Sect. 6.5 deals with GT NEWSR strength. Sect. 6.6 gives the results of β-decay rates at densities and temperatures typical of presupernova stars. Finally a summary is given in Sect. 6.7. The results given in this chapter are first reported in [Ko-94b].

6.2 IP strength densities for Gamow - Teller transition operator

Given $H = \mathbf{h} + \mathbf{V}$ the IP strength densities $\mathbf{I}_{\mathcal{O}}^{H=\mathbf{h}+\mathbf{V}}(E_i, E_f)$ produced by a transition operator $\mathcal{O}$, as derived in [Fr-87a, Fr-88b], will take a bivariate convolution form with the two convoluting functions being the NIP strength density $\mathbf{I}_{\mathcal{O}}^{\mathbf{h}}$ and a normalized spreading bivariate Gaussian ($\rho_{BIV-\mathcal{G}}$) due to $\mathbf{V}$ (interactions),

$$\mathbf{I}_{\mathcal{O}}^{H=\mathbf{h}+\mathbf{V}}(E_i, E_f) = \mathbf{I}_{\mathcal{O}}^{\mathbf{h}} \otimes \rho_{\mathcal{O};BIV-\mathcal{G}}^{\mathbf{V}}[E_i, E_f] \tag{6.1}$$

In large spectroscopic spaces, just as in the case of state densities (Chapter 3, Eqs. (3.3, 3.4)), one can use an extended version of (6.1) with some plausible approximations [Fr-87a, Fr-88b, Ko-89] by decomposing the space into S-subspaces and unitary configurations. For the $GT(\beta^{\pm})$ operator, using pn unitary configurations and noting that GT operator does not connect (5.3) two different S-subspaces, the IP strength density with partitioning, following [Fr-87a, Fr-88b, Ko-89] together with an additional approximation described ahead, is,

$$\mathbf{I}_{\mathcal{O}(GT)}^{H=\mathbf{h}+\mathbf{V}}(E_i,E_f)=\sum_{S}\sum_{[\mathbf{m}_p^i,\mathbf{m}_n^i],[\mathbf{m}_p^f,\mathbf{m}_n^f]\in S}$$

$$\mathbf{I}_{\mathcal{O}(GT)}^{\mathbf{h};[\mathbf{m}_p^i,\mathbf{m}_n^i],[\mathbf{m}_p^f,\mathbf{m}_n^f]}\otimes\rho_{\mathcal{O}(GT)}^{\mathbf{V};[\mathbf{m}_p^i,\mathbf{m}_n^i],[\mathbf{m}_p^f,\mathbf{m}_n^f]}[E_i,E_f]\,; \qquad (6.2a)$$

$$\rho_{\mathcal{O}(GT)}^{\mathbf{V};[\mathbf{m}_p^i,\mathbf{m}_n^i],[\mathbf{m}_p^f,\mathbf{m}_n^f]}(x,y)=$$

$$\rho_{\mathcal{O}(GT);BIV-\mathcal{G}}^{\mathbf{V}}(x,y;0,0,\sigma_{\mathbf{V}}([\mathbf{m}_p^i,\mathbf{m}_n^i]),\sigma_{\mathbf{V}}([\mathbf{m}_p^f,\mathbf{m}_n^f]),\bar{\zeta}) \qquad (6.2b)$$

In (6.2b) one is assuming that for $\rho_{\mathcal{O}(GT)}^{\mathbf{V};[\mathbf{m}_p^i,\mathbf{m}_n^i],[\mathbf{m}_p^f,\mathbf{m}_n^f]}$, the marginal centroids are $M_{10}=\langle\mathcal{O}^\dagger\mathcal{O}\mathbf{V}\rangle^{[\mathbf{m}_p^i,\mathbf{m}_n^i]}/\ \langle\mathcal{O}^\dagger\mathcal{O}\rangle^{[\mathbf{m}_p^i,\mathbf{m}_n^i]}$, $M_{01}=\langle\mathcal{O}\mathcal{O}^\dagger\mathbf{V}\rangle^{[\mathbf{m}_p^f,\mathbf{m}_n^f]}/\langle\mathcal{O}\mathcal{O}^\dagger\rangle^{[\mathbf{m}_p^f,\mathbf{m}_n^f]}$, and the marginal variances are $\mathcal{M}_{20}=\langle\mathcal{O}^\dagger\mathcal{O}\mathbf{V}^2\rangle^{[\mathbf{m}_p^i,\mathbf{m}_n^i]}/\langle\mathcal{O}^\dagger\mathcal{O}\rangle^{[\mathbf{m}_p^i,\mathbf{m}_n^i]}-\{M_{10}\}^2$, $\mathcal{M}_{02}=\langle\mathcal{O}\mathcal{O}^\dagger\mathbf{V}^2\rangle^{[\mathbf{m}_p^f,\mathbf{m}_n^f]}/\ \langle\mathcal{O}\mathcal{O}^\dagger\rangle^{[\mathbf{m}_p^f,\mathbf{m}_n^f]}-\{M_{01}\}^2$ and they in turn are assumed to be close to the corresponding state density centroids and variances. This approximation is extremely well satisfied by fp-shell interactions. In $S=0$ spaces, with one unitary orbit each for protons and neutrons and $(m_p,m_n)\longrightarrow(m_p\pm1,m_n\mp1)$ for $\beta^\mp$ transitions, the above approximations are equivalent to assuming that $M_{10}=\langle\mathcal{O}^\dagger\mathcal{O}\mathbf{V}\rangle^{m_p,m_n}/\langle\mathcal{O}^\dagger\mathcal{O}\rangle^{m_p,m_n}\simeq\langle\mathbf{V}\rangle^{m_p,m_n}=0$, $\mathcal{M}_{01}=\langle\mathcal{O}^\dagger\mathbf{V}\mathcal{O}\rangle^{m_p,m_n}/\ \langle\mathcal{O}^\dagger\mathcal{O}\rangle^{m_p,m_n}=\langle\mathcal{O}\mathcal{O}^\dagger\mathbf{V}\rangle^{m_p\pm1,m_n\mp1}/\ \langle\mathcal{O}\mathcal{O}^\dagger\rangle^{m_p\pm1,m_n\mp1}\simeq\langle\mathbf{V}\rangle^{m_p\pm1,m_n\mp1}=0$, $\mathcal{M}_{20}=\langle\mathcal{O}^\dagger\mathcal{O}\mathbf{V}^2\rangle^{m_p,m_n}/\ \langle\mathcal{O}^\dagger\mathcal{O}\rangle^{m_p,m_n}-M_{10}^2\simeq\langle\mathbf{V}^2\rangle^{m_p,m_n}$ and $\mathcal{M}_{02}=\langle\mathcal{O}^\dagger\mathbf{V}^2\mathcal{O}\rangle^{m_p,m_n}/\ \langle\mathcal{O}^\dagger\mathcal{O}\rangle^{m_p,m_n}-M_{01}^2=\langle\mathcal{O}\mathcal{O}^\dagger\mathbf{V}^2\rangle^{m_p\pm1,m_n\mp1}/\ \langle\mathcal{O}\mathcal{O}^\dagger\rangle^{m_p\pm1,m_n\mp1}-M_{01}^2\simeq\langle\mathbf{V}^2\rangle^{m_p\pm1,m_n\mp1}$ for $GT(\beta^\mp)$ operators; see also (2.38). Using (2.58, 2.71, 2.73), one can calculate $\langle\mathcal{O}^\dagger\mathcal{O}\mathbf{V}^2\rangle^{[\mathbf{m}]}$ and $\langle\mathcal{O}^\dagger\mathcal{O}\rangle^{[\mathbf{m}]}$ traces and test the above approximations. We have verified explicitly that for the fp-shell interactions MWH2 [Mc-70] and FPM13 [Ri-91], the centroid shifts are $\lesssim 0.1\ MeV$ and variances differ from $\langle\mathbf{V}^2\rangle$ by $\lesssim 3\%$.

Besides the assumptions for the marginal centroids and variances, which are well justified for fp-shell nuclei, (6.2) also assumes that the bivariate correlation coefficient $\zeta = \bar{\zeta}$ is independent of the configurations involved. At present, because of the complexities in the proper definition of ζ as a function of the initial and final configurations, one has to make this assumption [Fr-88b]. From NIP calculations in ds and fp-shells (carried out using the results of Chapter 5), one sees that $0.6 \leq \zeta \leq 0.9$. For example for $\mathcal{O}(GT(\beta^-))$ in ds-shell with ^{17}O energies, $\zeta = 0.78$ for $(m_p = 4, m_n = 4)$ and $\zeta = 0.8$ for $(m_p = 6, m_n = 6)$. Similarly in fp-shell with ^{41}Ca energies, $\zeta = 0.84$ for $(m_p = 6, m_n = 16)$, and $\zeta = 0.87$ for $(m_p = 6, m_n = 10)$. In addition it is known from [Dr-77b, Fr-88b, Sa-93] that for various other operators, with realistic ds-shell interactions $0.6 \leq \zeta \leq 0.9$. Moreover in the two-body EGOE model [Fr-88b], which extends the $\zeta \simeq 1 - 1/m$ result given in Chapter 5 for random one-body hamiltonians, one has the result $\zeta \simeq 1 - 2/m$ where m is the number of active nucleons. Thus in EGOE, $\zeta \simeq 0.8$ for $m = 10$ with a one-body transition operator and a two-body hamiltonian. Therefore in the present calculations, (6.2) is used with $\bar{\zeta}$ a free parameter whose value will be determined by demanding that certain experimental data is reproduced by the theory but with the constraint $0.6 \leq \bar{\zeta} \leq 0.9$.

In this chapter, as we are dealing with fp-shell nuclei, (6.2) is used for constructing $\rho^{\mathbf{V}}_{\mathcal{O}(GT);BIV-\mathcal{G}}$ *and* $\mathbf{I}^{H}_{\mathcal{O}(GT);BIV}$.

6.3 Formalism for β-decay rates at finite temperature

The β-decay rate $T_\beta(E_i \rightarrow E_f)$ is the number of β-decays per second from a given initial state $|E_i\rangle$ of the parent nucleus to the final nuclear state $|E_f\rangle$ and

$T_\beta(E_i \to E_f) \propto [g_V^2 B_F(E_i \to E_f) + g_A^2 B_{GT}(E_i \to E_f)]$, where g_V and g_A are respectively the vector and axial vector coupling constants and B_F and B_{GT} are the Fermi and Gamow - Teller transition strengths respectively. Including the phase space factor f that incorporates the dependence of the rate on nuclear charge Ze and the available β-decay energy (phase space available for the leptons), T_β takes the form $T_\beta(E_i \to E_f) = Cf(Z, Q_i - E_f)[g_V^2 B_F(E_i \to E_f) + g_A^2 B_{GT}(E_i \to E_f)]$ where C is a constant and with Q the Q-value for β-decay from GS, $Q_i = Q + E_i$. Fixing the value of C [Br-77, Wu-66] by using the $\ell og ft$ values ($ft = (\ell n 2) f / T_\beta$) for pure Fermi transitions, one can write down the expressions for ground state half lifes and β-decay rates at finite temperature. Before going further it is important to note that in practice in β-decay rates calculations for fp-shell nuclei the B_F term is neglected as the Fermi strength is concentrated in a narrow energy domain (width $\sim 0.157 Z A^{-1/3} MeV$) and high up in energy (centroid $\sim 1.44 Z A^{-1/3} MeV$) [Ta-69, Ta-73]. With this, by writing B_{GT} in terms of GT strength density $\mathbf{I}^H_{\mathcal{O}(GT)}$ and the state density $I^H(E)$, the expression for GS half lifes is,

$$t_{1/2}(GS) = \{6250 \quad (s)\} \times$$

$$\left\{ \int_0^Q \left[\left(\frac{g_A}{g_V}\right)^2 3\mathcal{L} \right] \left[\frac{\mathbf{I}^H_{\mathcal{O}(GT)}(E_{GS}, E_f)}{I^H(E_{GS})} \right] f(Z, Q - E_f) dE_f \right\}^{-1} \tag{6.3}$$

In (6.3), the factor 3 comes due to our definition of $\mathbf{I}^h$ in Chapter 5 [1] and $\mathcal{L}$ is the so called quenching factor which is required because the calculated (shell model) GT sum rule strength is always found to be larger than the observed strength [Ho-83]. The usual value of 0.6 for $\mathcal{L}$ [Ho-83] and $\left(\frac{g_A}{g_V}\right)^2 = 1.4$ as given

[1] In discrete version $B_{GT}(J_i E_i \to J_f E_f) = (2J_i + 1)^{-1} \sum_{M_i, M_f(\mu)} |\langle E_f J_f M_f| (\mathcal{O}_{GT})^k_\mu |E_i J_i M_i\rangle|^2$ which in the continuous version becomes $\{I(E_i)\}^{-1} \sum_{\alpha \in E_i, \beta \in E_f, \mu} |\langle E_f \alpha | (\mathcal{O}_{GT})^k_\mu | E_i \beta\rangle|^2$. Noting that $\sum_\alpha |E_i \alpha\rangle \Longrightarrow |E_i\rangle I(E_i)$ and replacing the sum with $\mathbf{I}^h_{\mathcal{O}(GT)}$ using (2.37), gives rise to factor 3 in (6.3).

in [Br-77] are adopted. Our primary concern in this chapter is to calculate β-decay rates $\lambda(T)$ at finite temperature and they are defined by the thermal average (with weight factor $e^{-E/k_BT}I(E)$) of the rates from all parent nucleus states [Fu-80]. Then the expression for $\lambda(T)$ is,

$$\lambda(T) = \frac{\ell n2(s^{-1})}{6250}\left[\int e^{-E_i/k_BT} I^H(E_i)dE_i\right]^{-1} \times$$
$$\left[\int dE_i e^{-E_i/k_BT} I^H(E_i)\left[\int_0^{Q_i} dE_f \left\{\left(\frac{g_A}{g_V}\right)^2 3\pounds\right\}\right.\right.$$
$$\left.\left.\left[\frac{\mathbf{I}^H_{O(GT)}(E_i,E_f)}{I^H(E_i)}\right] f(Z,T,Q_i - E_f)\right]\right]$$

$$= \frac{\ell n2(s^{-1})}{6250}\left[\int e^{-E_i/k_BT} I^H(E_i)dE_i\right]^{-1} \times$$
$$\left[\int dE_i e^{-E_i/k_BT}\left[\int_0^{Q_i} dE_f \left\{\left(\frac{g_A}{g_V}\right)^2 3\pounds\right\} \mathbf{I}^H_{O(GT)}(E_i,E_f) f(Z,T,Q_i - E_f)\right]\right] \tag{6.4}$$

The second equality in (6.4) shows that in order to calculate β-decay rates one needs to construct explicitly the bivariate GT strength densities $\mathbf{I}^H_{O(GT)}$ and the stae densities $I^H(E)$. The phase space factor $f(Z,T,Q_i - E_f)$ appearing in (6.4) is given by the integral [Au-90, Fu-80, Wu-66],

$$f(Z,T,E_0 = Q_i - E_f) = \int_1^{\epsilon_0} \frac{F(Z,\epsilon_e)\epsilon_e(\epsilon_e^2 - 1)^{1/2}(\epsilon_0 - \epsilon_e)^2}{\{1 + exp[(\mu_e - \epsilon_e)/(k_BT)_e]\}} d\epsilon_e. \tag{6.5}$$

In (6.5) $\epsilon_0 = E_0/m_e$, $\mu_e = \mu/m_e$ and $(k_BT)_e = k_BT/m_e$ where m_e is the rest mass of the electron in MeV. For the Coulomb factor $F(Z,\epsilon)$, the Schenter and Vogel [Sc-83] expression is used,

$$F(Z,\epsilon) = \frac{\epsilon}{\sqrt{\epsilon^2 - 1}} exp[\alpha(Z) + \beta(Z)\sqrt{\epsilon - 1}];$$

$$
\begin{aligned}
\alpha(Z) &= -0.811 + 4.46(-2)Z + 1.08(-4)Z^2 & (\epsilon - 1) < 1.2 \\
&= -8.46(-2) + 2.48(-2)Z + 2.37(-4)Z^2 & (\epsilon - 1) \geq 1.2 \\
\beta(Z) &= 0.673 - 1.82(-2)Z + 6.38(-5)Z^2 & (\epsilon - 1) < 1.2 \\
&= 1.15(-2) + 3.58(-4)Z - 6.17(-5)Z^2 & (\epsilon - 1) \geq 1.2
\end{aligned}
\tag{6.6}
$$

It should be mentioned that f in (6.3) follows from (6.5) by dropping the denominator in the integral in the R.H.S. of (6.5). The only unknown quantity left to be determined is the electron chemical potential μ. In the case of presupernova core where the density $\sim 10^8$ gms/cc, the electrons are close to being relativistic and hence μ is found by inverting the expression for the lepton number density. Given the density ρ_7 (ρ in units of 10^7 gms/cc), temperature $\bar{T}$ (T in MeV is denoted by $\bar{T}$) and the electron fraction Y_e in the presupernova star, there are hierarchy of approximate expressions that can be used for μ (see [Au-90, Ra-92]) and in the present calculations Eq. (14) of [Au-90] is used,

$$
\mu = 1.11(\rho_7 Y_e)^{1/3} \left[1 + \left(\frac{\pi}{1.11}\right)^2 \frac{\bar{T}^2}{(\rho_7 Y_e)^{2/3}}\right]^{-1/3} . \tag{6.7}
$$

Using (6.4 - 6.7) β-decay rates can be calculated by constructing state densities $I^H(E)$ and the strength densities $\mathbf{I}^H_{\mathcal{O}(GT)}(E_i, E_f)$. The procedure for calculation of these densities is given in the following section together with comments on some of its variants. As an example, the results for five neutron excess fp-shell nuclei $^{61,62}Fe$, $^{62-64}Co$ ($^{61}Fe \rightarrow {}^{61}Co$, $^{62}Fe \rightarrow {}^{62}Co$, $^{62}Co \rightarrow {}^{62}Ni$, $^{63}Co \rightarrow {}^{63}Ni$, $^{64}Co \rightarrow {}^{64}Ni$) are given in Sect. 6.6.

6.4 Calculational procedure

Our primary concern in this chapter is to demonstrate that the theory for β-decay rates calculation given by (6.3 - 6.7) combined with (6.2) and the methods of Chapters 3 - 5 (i.e. SAT-LSS) is applicable in practice. Because of this limited aim, we followed simpler methods for constructing state and strength densities unlike the more elaborate procedure used for example in Chapter 4 in the case of level densities, and they are described in Sect. 6.4.1 and 6.4.2 respectively. Some useful comments are given in Sect. 6.4.3.

6.4.1 State Densities

For the calculation of state densities $I^H(E)$ the following procedure is adopted. The s.p. orbits are chosen to be the five orbits $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ and $1g_{9/2}$ with $s = 0, 0, 0, 0$ and 1 respectively. Following the calculations in Chapter 4 for fp-shell level density data analysis, the traceless SPE for the four fp-orbits are chosen to be -2.664 MeV, -0.644 MeV, 3.526 MeV and 1.366 MeV respectively and the $(fp) - g_{9/2}$ separation $\Delta_{(fp)-g_{9/2}}$ is chosen to be $6 MeV$; for ^{64}Co however it is taken to be 7 MeV. The calculations are performed in $S = 0 \oplus 1 \oplus 2$ spaces using pn unitary configurations with (fp) and $g_{9/2}$ orbits as unitary orbits; $I^{\mathbf{h}}$ is constructed using the above SPE (i.e. no renormalizations due to the interactions are included - their main effect is assumed to be in fixing the value of $\Delta_{(fp)-g_{9/2}}$) and spherical orbits. The density $\rho_G^{\mathbf{V}}$ is constructed by calculating the spreading variances using surface delta interaction (SDI) with strength $G = 20/A$ MeV (this value follows from Chapter 4). The GS is fixed by demanding that the total level density $I_\ell(E)$ at 8 MeV excitation is same as the value given by the (a, Δ) values that follow from Dilg et al smoothed expression (C.15). This choice is made because for the nuclei under consideration ($^{61-62}Fe$, $^{61-64}Co$, $^{62-64}Ni$) , the low - energy spectrum is not known with certainty to be complete. For example, with Dilg et al values for (a, Δ) together with the back shifted LLC Fermi gas formula

(C.3, C.4), for ^{62}Fe, ^{61}Co and ^{64}Co, the $I_\ell(E)$ (in MeV^{-1}) values at (4, 8) MeV are (13, 455), (61, 1452) and (180, 3765) respectively. In general it is found that with the calculated GS, the density at 4 MeV is quite close (within 30%) to the value predicted by Dilg et al (a, Δ) values. It is also verified that the calculated spin-cutoff factors $(\sigma_J(E))$ are quite close to the values given by (C.10). For example for ^{62}Fe, ^{61}Co ,^{64}Co and ^{62}Ni nuclei the calculated and Dilg et al values at 8 MeV are (3.9, 3.99), (3.72, 4.15), (4.24, 4.40) and (3.40, 3.99) respectively. Also for even-even and odd-odd nuclei for positive parity states and for odd-A nuclei negative parity states, the $S = 2$ intensities in the GS domain are $\leq$ 30% of the $S = 0$ intensities (thus the $g_{9/2}$ orbit is seen to be important). In addition to these tests, the GT NEWSR as predicted by the present calculations are compared with shell model results as described in Sect. 6.5 ahead. The level density calculations here are carried out in the same spirit as in [Fr-88b] and it is seen that as long as the densities are constructed well upto 8 MeV (the β^--decay Q-values and Eqs. (6.3, 6.4) show that densities only upto $\sim$ 8 MeV are needed in the present exercise), the details of the SPE and interaction **V** do not alter the final results significantly. It should be stressed that, as in the studies of [Fr-89a, Fr-89b, Ko-91, Ko-93a, Fr-94] and Chapter 4, there are many other variants of the above procedure which can be used in practice.

6.4.2 Strength densities

The GT strength densities $\mathbf{I}^H_{\mathcal{O}(GT)}(E_i, E_f)$ are constructed for the $GT(\beta^-)$ operator using (6.2) with $\bar{\zeta}$ as a free parameter. In the present study the coulomb correction to GT centroids and the variances [Ta-69] are ignored and it is assumed that the parametrization of $\bar{\zeta}$ takes care of the same [illegible] ; they can be easily put back. $\mathbf{I}^{h}_{\mathcal{O}(GT)}(E_i, E_f)$ is constructed using the results of Chapter 5. In the first calculation $\bar{\zeta}$ is determined for each nucleus by demanding the observed half lifes are reproduced within the constraint $0.6 \leq \bar{\zeta} \leq 0.9$. The deduced values of $\bar{\zeta}$ are 0.6, 0.7, 0.61, 0.68 and 0.72 and the corresponding half lifes (in seconds) are 244, 69.4, 88.8, 31 and 0.3 for ^{61}Fe, ^{62}Fe, ^{62}Co, ^{63}Co and ^{64}Co respectively. The half lifes are calculated using (6.3). These $\bar{\zeta}$ values change quite substantially (also fluctuates) as we go from ^{61}Fe ($\bar{\zeta} = 0.6$) to ^{64}Co ($\bar{\zeta} = 0.72$) while the (valence) particle number m changes only by 3 units. The EGOE form $\bar{\zeta} \simeq a + b/m$ immediately shows that it is not possible to reproduce the above variation within the framework of SAT. Therefore the above values of $\bar{\zeta}$ are not acceptable in a SAT calculation. The plausible alternative is to seek a best fit solution to the observed half lifes of the nuclei under consideration (in the present exercise they are $^{61-62}Fe$, $^{62-64}Co$ isotopes) by assuming the form of $\bar{\zeta}$ to be $\zeta_0 + \zeta_1/m$ where m is the number of valence nucleons/holes, i.e. by minimizing $\sum_{i=nuclei} (\ell og(\tau^i_{1/2})_{cal} - \ell og(\tau^i_{1/2})_{expt})^2$. The resulting $\bar{\zeta}$ values and the corresponding calculated half lifes are given in Table 6.1. The $\bar{\zeta}$ values are around 0.67 (changing from 0.68 for ^{61}Fe to 0.668 for ^{64}Co). It should be obvious that the calculated half lifes, with these $\bar{\zeta}$ values, are the predictions of SAT-LSS. The differences between the calculated and experimental values are of the same order of magnitude as in other models [Ta-69, Kl-84, Ka-91]. Before going further it should be pointed out that in general for a given nucleus, as $\bar{\zeta}$ decreases the calculated half life increases and sometimes the increase is fast.

Using the $\bar{\zeta}$ values given in Table 6.1, the IP strength densities $\mathbf{I}^{H}_{\mathcal{O}(GT)}$ are constructed using (6.2). Using these $\mathbf{I}^{H}_{\mathcal{O}(GT)}$ densities, state densities $I^{H}(E)$ and Eqs. (6.4 - 6.7), the β-decay rates are calculated.

Table 6.1. Calculated and experimental β^--decay half lifes. The method of determining the correlation coefficient $\bar{\zeta}$ is given in the text. The calculated half lifes are given in the fourth column. The Q-values and experimental data for half lifes are taken from [Le-78]. The KSR results are from [Ka-91, Ra-92] and the QRPA result, which is available only for ^{64}Co in the set of nuclei considered in the table, is from the first reference of [Kl-84].

Nucleus	Q		half life (s)			
	(MeV)	$\bar{\zeta}$	Calc.	Expt.	KSR	QRPA
^{64}Co	7.307	0.668	1.9	0.3	3.5	10.0
^{63}Co	3.662	0.671	52.7	27.5	52.1	–
^{62}Co	5.315	0.675	16.5	90	15.1	–
^{62}Fe	2.327	0.675	267.2	68	183.4	–
^{61}Fe	3.890	0.680	23.0	360	34.5	–

6.4.3 Comments

Firstly, it is useful to recognize that the formalism, for β-decay rates calculations, given by (6.2 - 6.7) together with the procedure outlined above for calculating state and bivariate GT strength densities, allows one to investigate the effects of nucleon-nucleon interaction on the β-decay rates as $\mathbf{I}^H_{\mathcal{O}(GT)}$ and I^H are constructed explicitly in terms of hamiltonian parameters.

Secondly, it should be pointed out that in going from (2.37) to (6.1) and then to (6.2a, 6.2b), in order to construct IP strength densities, involve some general principles/approximations and some of them are yet to be understood well; for example the correlation coefficient is assumed to be independent of the configurations involved.

Thirdly, with the final interest is in calculating β-decay rates for nuclear astrophysics studies, it is plausible that one may adopt a phenomenological/empirical viewpoint in applying (6.2 - 6.7), i.e. in the construction of $\mathbf{I}^{H}_{\mathcal{O}(GT)}$ and I^{H}. This is what was done earlier by KSR: The KSR method of constructing $\mathbf{I}^{H}_{\mathcal{O}(GT)}$ and I^{H} is based on a simpler version of (6.2), in parts it is an extended version of the "gross theory" given in [Ta-69] and in addition it uses experimental data also. The basic difference beetween the present method and KSR method is that the KSR method assumes single unitary orbit in applying (6.2) so that the summation in (6.2) disappears. Without several unitary orbits the agreements found for level densities, as given in [Ko-91, Fr-94, Ko-93a, Chapter 4], and Sect. 6.4, could not have been obtained and it is well known that the assumption of single unitary orbit is unsatisfactory [Ha-76, Fr-88b, Fr-89a, Fr-89b, Ko-89, Ko-91, Ko-93a, Fr-94] - however this is ignored. With a single unitary orbit $\mathbf{I}^{H}_{\mathcal{O}(GT)} = \mathbf{I}^{\mathbf{h}}_{\mathcal{O}(GT)} \otimes \rho^{\mathbf{V}}_{\mathcal{O}(GT);\mathcal{G}}$ will be a bivariate Gaussian with CLT applied to $\mathbf{I}^{\mathbf{h}}$. Then the ratio $\mathbf{I}^{H}/I^{H}$ in (6.4) is the conditional density $\rho^{H}_{21;\mathcal{O}(GT)}(E_f|E_i)$ (see (2.23)) which will be a Gaussian (as $\mathbf{I}^{H}$ is a bivarite Gaussian) with normalization given by GT NEWSR strength $M_{\mathcal{O}(GT)}(E)$. The NEWSR strength is calculated using an expression similar to (6.8) given ahead. The conditional density $\rho^{H}_{21;\mathcal{O}(GT)-\mathcal{G}}(E_f|E_i)$ is constructed by parametrizing its centroid and variance and in order to correct for the departures from this single Gaussian approximation in some cases KSR *add* [Ra-92, Ka-94] Edgeworth corrections with (γ_1, γ_2) values chosen such that $|\gamma_1| \leq 0.3$ and $|\gamma_2| \leq 0.3$. As can be seen from (2.42) the conditional centroid ϵ_c changes linearly with energy, $\epsilon_c(E_i) = \epsilon_0 + \bar{\zeta} E_i$ and the conditional width is a constant, $\sigma_c = \sigma_0(1 - \bar{\zeta}^2)^{1/2}$ where $\sigma_0 \simeq \sigma_H = (\sigma^2_{\mathbf{h}} + \sigma^2_{\mathbf{V}})^{1/2}$. The centroid ϵ_c is parametrized (assuming $\bar{\zeta} = 1$) and the value of σ_c is fixed, after adding

Coulomb correction as in [Ta-69], by minimizing square of the difference in the logarithm of the calculated and experimental half lifes exactly as in the "gross theory". In addition, instead of calculating $I^H(E)$ and carrying out integration over all E_i's in applying (6.4), KSR use the experimental energies and spins (wherever observed) so that the calculation of state densities is completely avoided. In the formalism presented in Sects. 6.2 - 6.3 and 6.4.1 - 6.4.2, the densities $\mathbf{I}^H$ and I^H are explicitly constructed and used alongwith (6.4) within a single framework unlike in the KSR method where the explicit construction, with interactions, is avoided as described above. It should be added that the s.p. spectrum and the $\mathbf{V}$ used in Sect. 6.4 give $\sigma_H^2 = \sigma_{\mathbf{h}}^2 + \sigma_{\mathbf{V}}^2 \simeq 8\ MeV$ and the expression $\sigma_c = \sigma_H(1-\bar{\zeta}^2)^{1/2}$ gives, with $\bar{\zeta} \simeq 0.67$, $\sigma_c \simeq 6\ MeV$ which is compatible with the value of σ_c deduced by KSR. In summary KSR approach is a semiempirical approach taking lightly into account some aspects of SAT, and their approach to the construction of strength densities is in the same spirit as the approach to state densities proposed in [Ha-82b]. In addition, the KSR method is a limiting case of the present method.

Finally a biproduct of (6.2) and the results of Chapter 5 is that they give a formalism for calculating GT NEWSR strengths. This is described in the next section before giving the results for β-decay rates.

6.5 *GT* NEWSR strength

Using (6.2) and the explicit form for $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}(GT)}$ as given in Chapter 5, a simple expression for NEWSR strength $M_{\mathcal{O}(GT)}(E)$ for $\mathcal{O}(GT)$ can be written down,

$$
\begin{aligned}
M_{\mathcal{O}(GT)}(E) &= \sum_{(\mathbf{m}_p,\mathbf{m}_n)} \frac{I^{H;(\mathbf{m}_p,\mathbf{m}_n)}(E)}{I^{H,(m_p,m_n)}(E)} \left\{ 3\sum_{\alpha,\beta}[\epsilon^2_{\alpha\beta}(\mathcal{O}(GT))] \frac{m_\beta(\mathbf{m}_p,\mathbf{m}_n)}{N_\beta} \times \right. \\
& \left. \frac{N_\alpha - m_\alpha(\mathbf{m}_p,\mathbf{m}_n)}{N_\alpha} \right\} ; \\
I^{H,(\mathbf{m}_p,\mathbf{m}_n)}(E) &= d(\mathbf{m}_p,\mathbf{m}_n)\rho_{\mathcal{G}}(E;\epsilon_{\mathrm{h}}(\mathbf{m}_p,\mathbf{m}_n),\sigma_{\mathrm{V}}(\mathbf{m}_p,\mathbf{m}_n)) \\
I^{H,(m_p,m_n)}(E) &= \sum_{(\mathbf{m}_p,\mathbf{m}_n)\in(m_p,m_n)} I^{H,(\mathbf{m}_p,\mathbf{m}_n)}(E) \qquad (6.8)
\end{aligned}
$$

In (6.8) $\epsilon_{\alpha\beta}$ define $\mathcal{O}(GT)$ and α and β are proton and neutron spherical orbits respectively for $GT(\beta^-)$ and vice versa for $GT(\beta^+)$. The formula (6.8) whose origin lies in the NIP expression (5.10) for $\mathbf{I}^{\mathrm{h}}_{\mathcal{O}(GT)}$, is quite similar to the sum rule formulas used in earlier studies (though derived using different methods) [Ma-86, Ka-91; Appendix A of [Au-90]] of fp-shell nuclei; thus (6.8) provides a proper justification and corrections to the formulas given in [Ma-86, Ka-91; Appendix A of [Au-90]]. The corrections to (6.8) come from the centroid shifts and the variance corrections to $\rho_{\mathcal{G}}$ as discussed below (6.2) and they are small for fp-shell nuclei. In order to test the applicability of (6.8) for fp-shell nuclei, the $GT(\beta^-)$ NEWSR strengths are calculated for $^{54,56,60}Fe$ and $^{58,60}Ni$ isotopes by first carrying out level density calculations and determining the ground state, exactly as described in Sect. 6.4. These nuclei are chosen as there are corresponding shell model results available in literature [Bl-85, Ra-83], though calculated using different interactions and including only fp-orbits (i.e. without the $g_{9/2}$ orbit). The results shown in Table 6.2 clearly demonstrate that (6.8) gives results that are in close agreement with shell model values. Thus (6.8), which is easy to apply, can be used in future studies of GT NEWSR and quenching of GT strengths. The calculated sum rule strengths (from GS) with quenching factor $\mathcal{L} = 0.6$ for ^{61}Fe, ^{62}Fe, ^{62}Co, ^{63}Co

and ^{64}Co nuclei are 20, 18.9, 18.8, 19.7 and 20.8 respectively. The $^{61-62}Fe$ sum rule strengths are compatible with the ^{60}Fe results given in Table 6.2. We conclude from these results that the various choices made in Sect. 6.4 are quite reasonable.

Table 6.2. Calculated and shell model (SM) results for $GT(\beta^-)$ NEWSR.

Nucleus	NEWSR	
	Calc.	SM
^{54}Fe	17.8	15.1^a
^{56}Fe	22.6	22.1^a
^{60}Fe	31.6	33.5^b
^{58}Ni	20.2	16.6^a
^{60}Ni	24.2	24.6^a

a) [Ra-83], *b*) combining $GT(\beta^+)$ NEWSR value 9.47 given in [Bl-85] with the sum rule result $S_{\beta^-} - S_{\beta^+} = 3(N - Z)$.

6.6 Results for β-decay rates at finite temperature

Using the formalism given in Sects. 6.2, 6.3 and the procedure outlined in Sect. 6.4, which is described in the flow chart in Fig. 6.1, β-decay rates for the five nuclei ^{61}Fe, ^{62}Fe, ^{62}Co, ^{63}Co and ^{65}Co are calculated at densities $\rho = 10^9$ gm/cc, 10^8 gm/cc, 10^7 gm/cc, temperatures $T = 3 \times 10^9$, 4×10^9, 5×10^9 ^oK ($T = 4 \times 10^9$ ^oK compared to 0.345 MeV) and electron fractions $Y_e = 0.5$, 0.47 and 0.43. The results are given in Table 6.3. It is observed that the excitation energy (of the parènt nucleus) contribution to the decay rate is quite significant upto about $(6-8) \times k_BT$ and beyond that their importance ($\sim 10-15\%$) goes down rapidly; note that with $E_i = 5k_BT$, $e^{-E_i/k_BT} \simeq 1/150$.

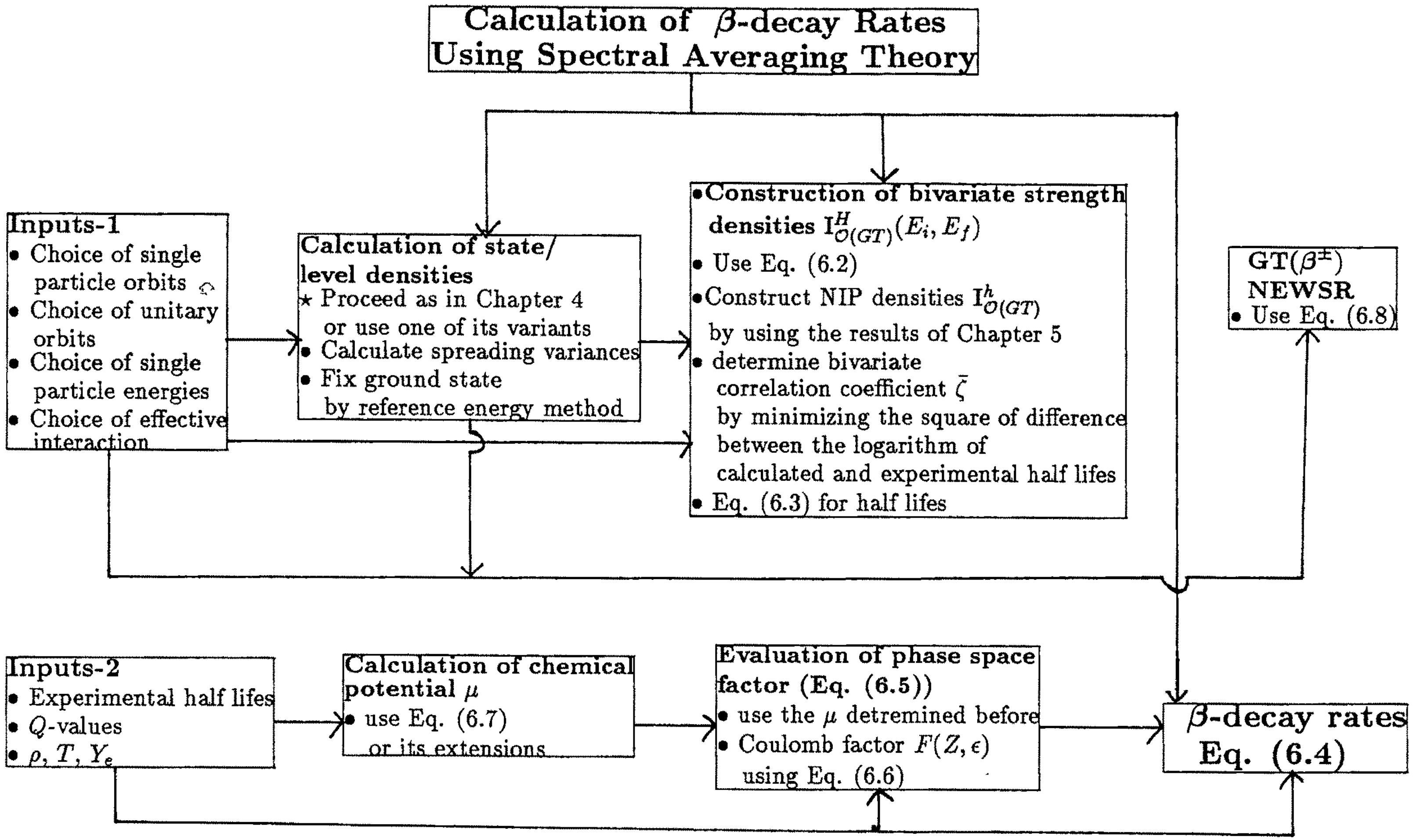

Fig. 6.1 Flow chart describing various steps involved in β - decay rates calculations

It is also observed that for a given nucleus the rates go down as the correlation coefficient $\bar{\zeta}$ decreases. For example for ^{64}Co, with $\bar{\zeta} = 0.668$, $\lambda(T) = 0.345\, s^{-1}$ while with $\bar{\zeta} = 0.72$, $\lambda(T) = 1.33\, s^{-1}$ for $\rho = 10^8$ gm/cc, $T = 4 \times 10^9\ ^oK$ and

Table 6.3. β^--decay rates for ^{64}Co, ^{63}Co, ^{62}Co, ^{62}Fe and ^{61}Fe nuclei

Nucleus	ρ(gms/cc)	Y_e	Temperature (^oK)		
			3×10^9	4×10^9	5×10^9
			Rate (s^{-1})		
^{64}Co	10^9	0.50	0.85×10^{-1}	1.04×10^{-1}	1.30×10^{-1}
		0.47	0.91×10^{-1}	1.11×10^{-1}	1.38×10^{-1}
		0.43	1.00×10^{-1}	1.21×10^{-1}	1.49×10^{-1}
	10^8	0.50	3.10×10^{-1}	3.43×10^{-1}	3.84×10^{-1}
		0.47	3.14×10^{-1}	3.47×10^{-1}	3.87×10^{-1}
		0.43	3.19×10^{-1}	3.52×10^{-1}	3.93×10^{-1}
	10^7	0.50	3.83×10^{-1}	4.14×10^{-1}	4.54×10^{-1}
		0.47	3.83×10^{-1}	4.14×10^{-1}	4.54×10^{-1}
		0.43	3.84×10^{-1}	4.15×10^{-1}	4.55×10^{-1}
^{63}Co	10^9	0.50	0.12×10^{-3}	0.65×10^{-3}	2.26×10^{-3}
		0.47	0.15×10^{-3}	0.76×10^{-3}	2.54×10^{-3}
		0.43	0.21×10^{-3}	0.94×10^{-3}	2.98×10^{-3}
	10^8	0.50	0.85×10^{-2}	1.31×10^{-2}	2.09×10^{-2}
		0.47	0.88×10^{-2}	1.35×10^{-2}	2.14×10^{-2}
		0.43	0.93×10^{-2}	1.41×10^{-2}	2.21×10^{-2}
	10^7	0.50	1.63×10^{-2}	2.17×10^{-2}	3.06×10^{-2}
		0.47	1.64×10^{-2}	2.18×10^{-2}	3.07×10^{-2}
		0.43	1.65×10^{-2}	2.19×10^{-2}	3.08×10^{-2}
^{62}Co	10^9	0.50	0.29×10^{-2}	0.50×10^{-2}	0.84×10^{-2}
		0.47	0.34×10^{-2}	0.56×10^{-2}	0.92×10^{-2}
		0.43	0.41×10^{-2}	0.65×10^{-2}	1.04×10^{-2}
	10^8	0.50	3.06×10^{-2}	3.64×10^{-2}	4.41×10^{-2}
		0.47	3.12×10^{-2}	3.71×10^{-2}	4.48×10^{-2}
		0.43	3.22×10^{-2}	3.80×10^{-2}	4.58×10^{-2}
	10^7	0.50	4.41×10^{-2}	4.97×10^{-2}	5.74×10^{-2}
		0.47	4.42×10^{-2}	4.99×10^{-2}	5.76×10^{-2}
		0.43	4.44×10^{-2}	5.00×10^{-2}	5.77×10^{-2}

Table 6.3 (cont'd)

Nucleus	ρ(gms/cc)	Y_e	Temperature (^oK) 3×10^9	4×10^9	5×10^9
			Rate (s^{-1})		
^{62}Fe	10^9	0.50	0.04×10^{-4}	1.16×10^{-4}	9.99×10^{-4}
		0.47	0.06×10^{-4}	1.42×10^{-4}	11.50×10^{-4}
		0.43	0.09×10^{-4}	1.88×10^{-4}	14.00×10^{-4}
	10^8	0.50	2.40×10^{-3}	6.56×10^{-3}	1.62×10^{-2}
		0.47	2.57×10^{-3}	6.86×10^{-3}	1.67×10^{-2}
		0.43	2.83×10^{-3}	7.29×10^{-3}	1.73×10^{-2}
	10^7	0.50	8.11×10^{-3}	1.43×10^{-2}	2.71×10^{-2}
		0.47	8.19×10^{-3}	1.44×10^{-2}	2.72×10^{-2}
		0.43	8.30×10^{-3}	1.46×10^{-2}	2.74×10^{-3}
^{61}Fe	10^9	0.50	0.36×10^{-3}	1.51×10^{-3}	4.52×10^{-3}
		0.47	0.45×10^{-3}	1.76×10^{-3}	5.07×10^{-3}
		0.43	0.62×10^{-3}	2.18×10^{-3}	5.93×10^{-3}
	10^8	0.50	1.89×10^{-2}	2.71×10^{-2}	3.98×10^{-2}
		0.47	1.96×10^{-2}	2.78×10^{-2}	4.07×10^{-2}
		0.43	2.06×10^{-2}	2.89×10^{-2}	4.19×10^{-2}
	10^7	0.50	3.44×10^{-2}	4.33×10^{-2}	5.72×10^{-2}
		0.47	3.46×10^{-2}	4.35×10^{-2}	5.74×10^{-2}
		0.43	3.48×10^{-2}	4.37×10^{-2}	5.76×10^{-2}

$Y_e = 0.5$ and for the same set of (ρ, T, Y_e) values, for ^{62}Co, $\lambda(T) = 3.64 \times 10^{-2}$ s^{-1} for $\bar{\zeta} = 0.675$ and $\lambda(T) = 0.46 \times 10^{-2}$ s^{-1} for $\bar{\zeta} = 0.610$. By comparing the rates in Table 6.3 with the results given by Aufderheide et al [Au-90], one observes that the numbers for ^{63}Co are quite similar (in [Au-90] the rates are tabulated for $\rho = 10^8$ gm/cc, 10^7 gm/cc, $T = 3 \times 10^9$, 4×10^9, 5×10^9 ^oK and $Y_e = 0.5$) while for ^{62}Co the numbers in Table 6.3 are smaller by a factor 5 and that for ^{64}Co by a factor 8. The numbers given in Table 6.3 are much closer to KSR results [Ka-91, Ka-94] and this should not be surprising as the method employed by KSR derives from (6.2). More importantly, the variation in $\lambda(T)$ with T as can be seen from our results in Table 6.3 is not to be found in the first KSR tabulation [Ka-91] but similar variation is found in their latest compilation of the rates [Ka-94]. For example for ^{63}Co nucleus with $\rho = 10^9$ gm/cc, $Y_e = 0.5$, for $T = 3 \times 10^9\ ^oK$, $4 \times 10^9\ ^oK$, $5 \times 10^9\ ^oK$ the rates $\lambda(T)$ from [Ka-94] and Table 6.3 are (in s^{-1}) (0.37×10^{-3}, 0.94×10^{-3}, 2.07×10^{-3}) and (0.12×10^{-3}, 0.65×10^{-3}, 2.26×10^{-3}). Similarly for ^{63}Co with $\rho = 10^8$ gm/cc, $Y_e = 0.5$ the numbers are (3.6×10^{-2}, 4.11×10^{-2}, 4.66×10^{-2}) and (3.06×10^{-2}, 3.64×10^{-2}, 4.41×10^{-2}) respectively. In the case of ^{62}Fe with $\rho = 10^8$ gm/cc, $Y_e = 0.47$ the numbers are (5.79×10^{-3}, 7.31×10^{-3}, 9.46×10^{-3}) and (2.57×10^{-3}, 6.86×10^{-3}, 16.7×10^{-3}) respectively. Thus it is plausible to conclude that the method proposed in this chapter, where IP state and strength densities are constructed and used for the first time in β-decay rates calculations, is reliable. The goodness of the present method, which is based on the smoothed form for GT densities, derives partly from the fact that the β-decay rates involve an averaging (thermal average to be precise). In the present method, as the densities are constructed by superposing unitary orbit densities, it is easy to go beyond $0\hbar\omega$ spaces.

6.7 Summary

In this chapter a method based on smoothed forms, derived using the principles of SAT-LSS, for GT strength densities is developed for calculating β-decay rates. This method is simple as the earlier methods [Au-90, Ka-91] but at the same time it incorporates much more microscopic informations as the desities $\mathbf{I}^H_{\mathcal{O}(GT)}$ and $I^H(E)$ are explicitly constructed with *interactions* and used. The present exercise represents a first application of the bivariate convolution form for strength densities in nuclear astrophysics problems. A different and new result that is derived from the β-decay rates calculations is that they determine a value for the bivariate correlation coefficient (between the nuclear hamiltonian and GT operator) $\bar{\zeta}$; $\bar{\zeta} \simeq 0.67$. In addition, an expression for GT NEWSR strength is deduced. Applications are carried out for some $A > 60$ neutron excess fp-shell nuclei.