Appendix A

Mixing Between Distant Configurations: $S = 0 \oplus 2$ Shell Model Example

In order to understand the $S = 0 \oplus S = 2$ mixing effects, a simple schematic model is solved in [Fr-83b, Fr-84]. The predictions of this model are tested with a realistic interaction in a shell model $S = 0 \oplus 2$ space and the results are given here.

A.1 Two spike model

In the two spike model considered in [Fr-83a, Fr-84] the hamiltonian (real symmetric) H is a 2×2 block matrix with A and B being the diagonal blocks with dimensions d_A and d_B respectively; $(d_A \leq d_B)$ and X is the off-diagonal block whose transpose is $\widetilde{X}$,

$$H = \left[\begin{array}{c|c} A & X \\ \hline \tilde{X} & B \end{array}\right]\begin{array}{l} d_A \\ d_B \end{array} \longrightarrow \left[\begin{array}{c|c} 0 & X \\ \hline \tilde{X} & \Delta \end{array}\right]$$

$$\quad d_A \quad d_B$$

It is further assumed that $A = [0] \longrightarrow A_{ij} = 0$ and $B = [\Delta] \Longrightarrow B_{ij} = (\Delta)\delta_{ij}$; where Δ is a constant. The matrix X is represented by an EGOE. The partial variance σ_{12}^2 and the mixing parameter τ are,

$$\sigma_{12}^2 = \frac{1}{d_A}\sum X_{ij}^2 \tag{A.1}$$

$$\tau = \sigma_{12}^2/\Delta^2 \tag{A.2}$$

The formal expressions for the state densities $I_\Delta(E)$ and the partial densities $I_\Delta^A(E)$ and $I_\Delta^B(E)$ are given by,

$$I_\Delta(E) = \langle\langle\delta(H-E)\rangle\rangle = I_\Delta^A(E) + I_\Delta^B(E) \tag{A.3}$$

$$\begin{aligned} I_\Delta^A(E) &= \langle\langle\delta(H-E)\rangle\rangle^A \\ &= \sum_{\alpha=1}^{d_A} \left|C_{\alpha(A),E}\right|^2 \; ; \;\; \alpha = 1,2,3,....d_A \\ I_\Delta^B(E) &= \langle\langle\delta(H-E)\rangle\rangle^B \\ &= \sum_{\beta=1}^{d_B} |C_{\beta(B),E}|^2 \; ; \;\; \beta = 1,2,3,....d_B \end{aligned} \tag{A.4}$$

In (A.4), $\alpha(A)$ and $\beta(B)$ are the basis states defining the matrices A and B respectively. Note that $|E\rangle = \sum_{\alpha=1}^{d_A} C_{\alpha(A),E} |\alpha\rangle + \sum_{\beta=1}^{d_B} C_{\beta(B),E} |\beta\rangle$. The moments of $I_\Delta^A(E)$ are

$$\begin{aligned} M_p(I_\Delta^A) &= \langle H^p \rangle^A \\ &= \frac{1}{d_A} \sum_E \sum_{\alpha=1}^{d_A} |C_{\alpha(A),E}|^2 E^p \\ &= \frac{1}{d_A} \sum_E I_\Delta^A(E) E^p \end{aligned} \tag{A.5}$$

Then it is easily seen that $M_0(I_\Delta^A) = 1$, $M_1(I_\Delta^A) = 0$ and $M_2(I_\Delta^A) = \sigma_{12}^2$. The EGOE representation for X gives [Fr-83b, Fr-84],

$$\begin{aligned} I_0^A(E) &= d_A |E| (\sigma_{12}^2)^{-1} exp(-E^2/\sigma_{12}^2) \;\; ; \; -\infty \leq E \leq +\infty \\ I_\Delta^A(E) &= \left| \frac{E-\Delta}{E} \right|^{1/2} I_0^A(\sqrt{E(E-\Delta)}); \;\; E \leq 0, \; E \geq \Delta \end{aligned} \tag{A.6}$$

Thus $I_\Delta^A(E)$ is bimodal and using (A.6), one can calculate the modal centroid and variance for the lower mode. Discrete version of (A.6), gives for $I_0^A(E)$,

$$I_0^A(E_i) dE_i = 1/2 \, , \;\; E_i \neq 0 \tag{A.7}$$

at each non zero eigenvalue E_i of the matrix H. Then with $x_i(x_i + \Delta) = E_i^2$, $I_\Delta^A(E)$ is given by,

$$\begin{aligned} x_i &= -\frac{\Delta}{2} \left\{ 1 - \sqrt{1 + 4E_i^2/\Delta^2} \right\} \\ I_\Delta^A(E = -x_i) &= \sqrt{\frac{x_i + \Delta}{x_i}} \Big/ \left[\sqrt{\frac{x_i}{x_i + \Delta}} + \sqrt{\frac{x_i + \Delta}{x_i}} \right] \\ I_\Delta^A(E = \Delta + x_i) &= \sqrt{\frac{x_i}{x_i + \Delta}} \Big/ \left[\sqrt{\frac{x_i}{x_i + \Delta}} + \sqrt{\frac{x_i + \Delta}{x_i}} \right]. \end{aligned} \tag{A.8}$$

In a shell model example (A.6 - A.8) are tested and the results are given below.

A.2 Numerical test

To test (A.6 - A.8) a shell model calculation is performed in $(ds)^{6\ J=0\ T=0\ ;\ S=0} \oplus [(ds)^4 \times (fp)^2]^{J=0\ T=0\ ;\ S=2}$ space. The dimensions of the $S = 0$ (A block) and $S = 2$ (B block) spaces are $d_A = 71$ and $d_B = 243$. The matrix dimension of H is $d = d_A + d_B = 324$. Employing the Kuo interaction [Ku-71] in the above space, H matrix is constructed using the Rochester - Oak Ridge shell model code [Fr-69b] and then A and B matrices are modified such that $A_{ij} = 0$ and $B_{ij} = (\Delta)\delta_{ij}$. For the above matrix, the partial variance $\sigma_{12}^2 = 5.6044\ MeV^2$. Then the following results are obtained:

[1] Using (A.4) $I^A_{\Delta=0}(E)$ is constructed and verified that $I^A_{\Delta=0}(E) = 1/2$. The histogram for $I^A_{\Delta=0}(E)$ is in good agreement with I^A_0 of (A.6) as shown in Fig. A.1a

[2] For $\tau = 0.1, 0.2, 0.3, 0.5, 1.0, 10.0$ cases $I^A_\Delta(E)$ is constructed and is found to be well represented by (A.6); see for example Figs. A.1b - A.1e.

[3] The ϵ_M/Δ, σ_M^2/σ_{12}^2 for the lower mode are compared with the prediction from (A.6) and the results are given in Table A.1. Thus the shell model calculation confirms that the EGOE results given by (A.6) are good.

[4] The shell model calculations reproduce exactly the results in (A.7, A.8).

[5] Finally it should be mentioned that after completing the calculations, we learnt that the results shown in Figs. A.1 a - e and Table A.1 are also obtained independently by Leclair [Le-94].

Table A.1

τ	ϵ_M/Δ		σ_M^2/σ_{12}^2	
	EGOE[@]	SM[†]	EGOE[@]	SM[†]
0.1	−0.09	−0.8	0.06	0.04
0.2	−0.16	−0.15	0.09	0.07
0.3	−0.22	−0.21	0.11	0.08
0.5	−0.33	−0.31	0.13	0.10
1.0	−0.55	−0.52	0.16	0.12
10.0	−2.37	−2.33	0.20	0.16

[@] The EGOE results follow from Eq. (A.6 - A.8)

[†] Results of the shell model calculations

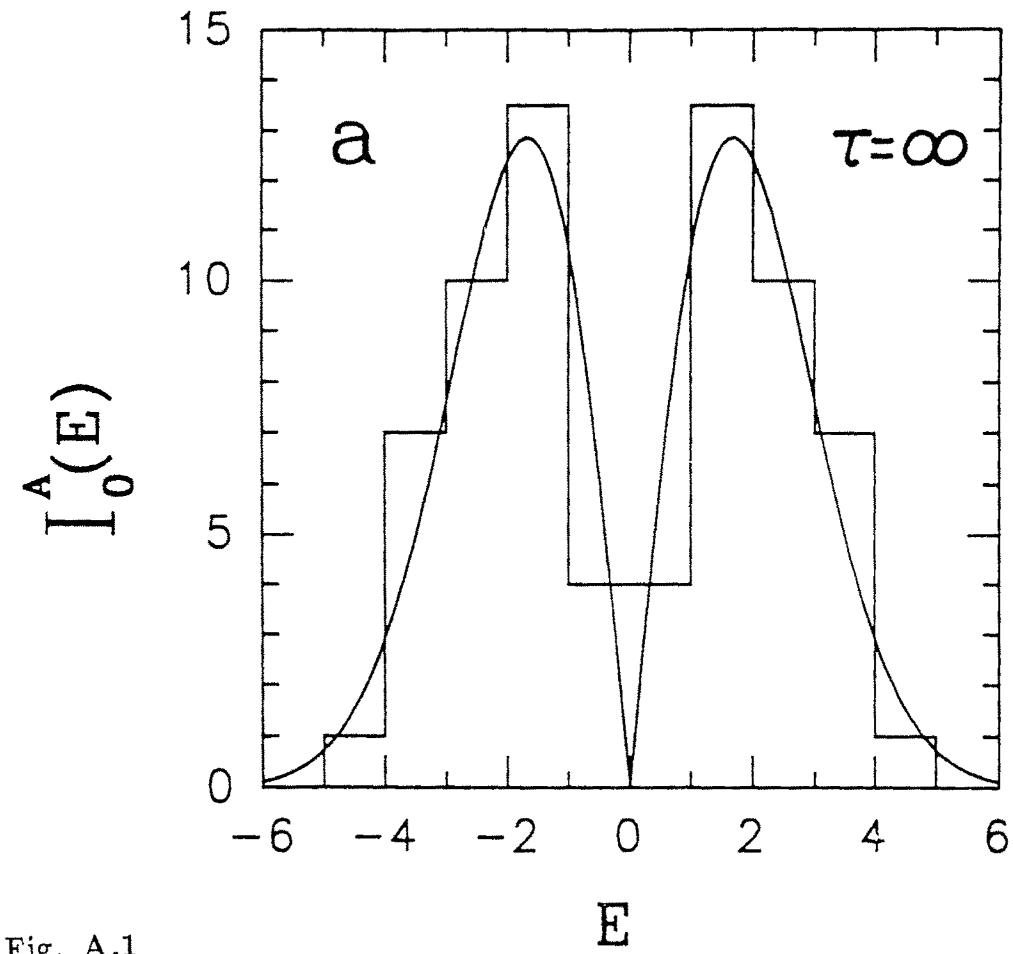

Fig. A.1

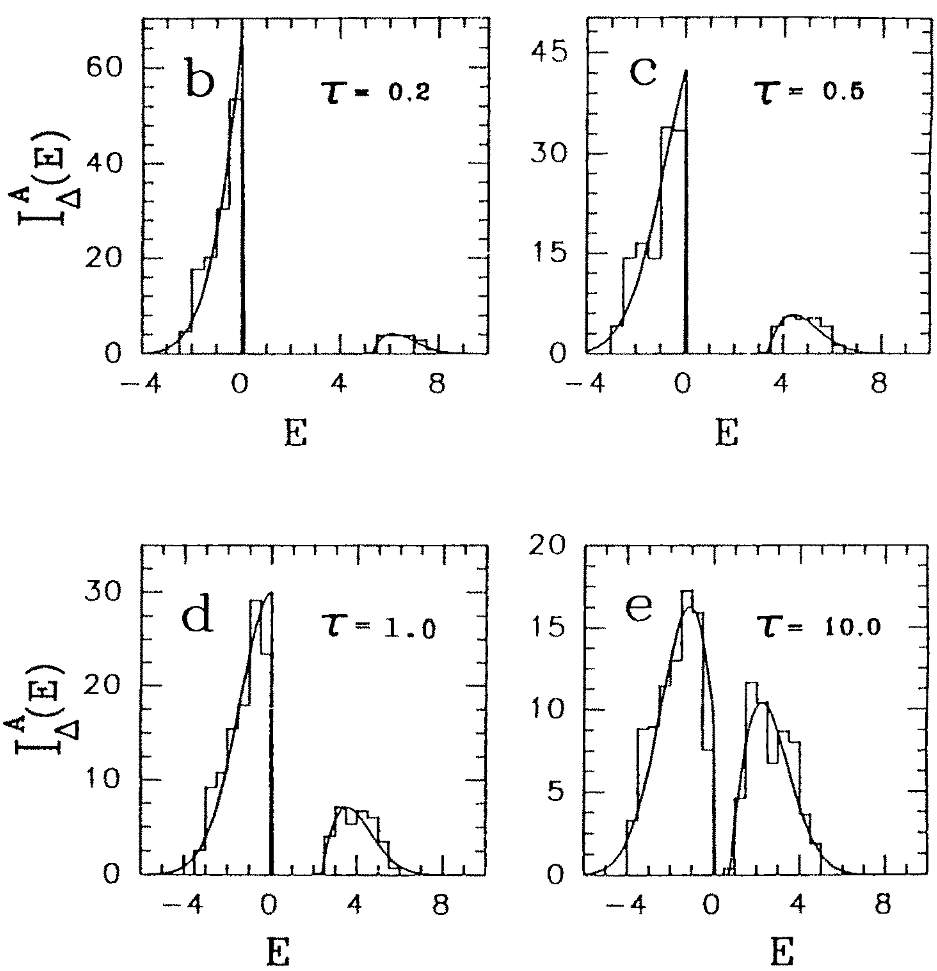

Fig. A.1 (cont'd)