Appendix C

Compilation of Level Density and Spin-Cutoff Factor Data for fp-Shell Nuclei

Experimental data for level densities (also for spin-cutoff factors) is normally available in two forms: (i) values for the level densities at some select energies; (ii) values for the level density parameters assuming a form for the level densities. In the later case, usually the NIP Fermi gas forms (reviews of NIP level density theories are given for example in [Er-60, Er-66, Hu-72, Da-80]) are used and this is always the case with the data available in literature for fp - shell nuclei. Therefore, in Sect. C.1 an overview of the Fermi gas forms for level densities and spin-cutoff factors is given. Sect. C.2 describes briefly the various experimental methods for obtaining level density and spin-cutoff factor data. Finally in Sect. C.3 the data compilation for the eight fp - shell nuclei ^{55}Mn, ^{56}Fe, ^{59}Co, ^{60}Co, ^{60}Ni, ^{62}Ni, ^{63}Mn and ^{65}Cu (their selection follows from Sect. 4.2.1) is given.

C.1 Fermi gas forms for level densities and spin-cutoff factors

Fermi gas forms for state densities

For identical particle systems, the celebrated formula $I(E) = \frac{1}{\sqrt{48E}} exp\ 2\sqrt{aE}$ for state densities is due to Bethe [Be-36] and it is derived by considering the nucleus to be degenerate Fermi gas and assuming that the density depends only on the single particle state density $g(\epsilon_f)$ at the Fermi surface. Replacing the true s.p. spectrum by a uniform spectrum with spacing g^{-1} and then using Hardy Ramanujan partition formula [Ab-64; p. 825, Va-37] immediately gives the above Bethe formula with $a = \frac{\pi^2}{6} g(\epsilon_f)$. For the realistic pn systems, the form for the state densities follows easily by using the fact that the pn NIP density is a simple convolution of the proton and neutron state densities, $I_{pn}(E) = I_p \otimes I_n[E]$. Using the Bethe form for I_p and I_n and evaluating the convoluting integral using saddle point approximation, the formula for the state density (with $g_p(\epsilon_f) = g_n(\epsilon_f) = g/2$ and dropping the pn indices from now onwards) is $I(E) = (\sqrt{\pi}/12a^{1/4}E^{5/4}) exp 2\sqrt{aE}$. However in practice the energies should be back-shifted with $E \rightarrow E - \Delta$ in the above Fermi gas formula for $I(E)$. Hurwitz and Bethe [Hu-51] for the first time pointed out the importance of shifting the energies. The origin of the energy shift lies in (i) shell effects: for example Rosenzweig [Ro-57] demonstrated this by dealing with periodic s.p. spectrum with or without degeneracies; (ii) pairing effects: by solving the so called temperature dependent gap equation, Sano and Yamasaki [Sa-63] showed that pairing interaction produces a shift of energies in the Fermi gas formula. However as Vonach and Hill [Vo-69] pointed out in general it is good to treat Δ as a free parameter (Δ is usually smaller than the pairing shift). The explicit back shifted Fermi gas form for state density $I(E)$, in terms of the parameters a and Δ, is

$$I(E) = \frac{\sqrt{\pi}}{12a^{1/4}(E-\Delta)^{5/4}} exp\ 2\sqrt{a(E-\Delta)} \tag{C.1}$$

Given the density $I(E)$, nuclear temperature T is defined as $T^{-1} = \frac{\partial}{\partial E} \ell n\, I(E)$ and on the other hand the thermodynamic (statistical) temperature t is defined in terms of the entropy S, $t^{-1} = \frac{\partial S}{\partial E}$. In general one can write E in powers of T as $E = T^n/\bar{a}$ where $\bar{a}$ is a constant. For a degenerate Fermi gas, $n = 2$ and using this gives $E = aT^2 \implies I(E) = Cexp2\sqrt{aE}$ where a and C are constants. This Weisskopf formula [We-37] is same as Bethe's formula (C.1) but with no energy denominator. Gilbert and Cameron [Gi-65] argued that in general it is good to use constant temperature formula in analyzing low energy level density data and on the other hand Maruyama [Ma-69, Ig-70] showed that for the nuclei near closed shells, constant temperature formula is relevant. The constant temperature formula (with back shifting denoted by E_0), used recently by Von Egidy et al [Vo-86] in analyzing large amount of low energy data, is

$$I(E) = \frac{1}{T} exp\{(E-E_0)/T\}. \tag{C.2}$$

The Lang LeCouteur (LLC) level density formula [La-54], which is commonly used in data analysis, was derived by first considering free energy and it differs from (C.1) in the energy denominator (it involves the temperature 't'),

$$I(E) = \frac{\sqrt{\pi}}{12a^{1/4}(E-\Delta+t)^{5/4}} exp\ 2\sqrt{a(E-\Delta)}\,,$$

$$U = E - \Delta = at^2 - t\,. \tag{C.3}$$

$$t = \frac{1 \pm \sqrt{4a(E-\Delta)+1}}{2a} \simeq \sqrt{\frac{E-\Delta}{a}} \tag{C.4}$$

The result in (C.4) that $t \simeq \sqrt{(E-\Delta)/a}$ also follows from (C.1) and the definition of T ($T \sim t$).

Angular momentum decomposition: fixed-J densities

In order to analyze the data on observable level densities, it is necessary to decompose state densities into fixed-J level densities. Starting with fixed (E, M) bivariate densities $\rho(E,M)$ where M is a eigenvalue of the J_Z operator and recognizing that $\rho(E,M)$ can be written as $\rho(E,M) = \rho(E)\rho(M|E)$, then applying CLT for the conditional $\rho(M|E)$ density gives the J - decomposition of $I(E)$. Noting that the centroid $\langle J_Z\rangle^E$ of $\rho(M|E)$ is always zero and denoting the variance $\langle J_Z^2\rangle^E$ by $\sigma_J^2(E)$ (the spin-cutoff factor), $I(E,M) = \dfrac{I(E)}{\sqrt{2\pi}\sigma_J(E)} exp - \dfrac{M^2}{2\sigma_J^2(E)}$. Now the elementary subtraction $I(E,M=J) - I(E,M=J+1)$ gives the fixed-J level densities which are denoted hereafter by $I_\ell(E,J)$. The formulas for $I_\ell(E,J)$, for the level densities $I_\ell(E,J_1:J_2)$ in the spin window $J_1 - J_2$ ($I_\ell(E,J_1:J_2)$ gives number of levels per MeV at energy E with spins between J_1 and J_2) and total level density $I_\ell(E)$ (which gives number of levels per MeV at energy E irrespective of the J value) are,

$$
\begin{aligned}
I_\ell(E,J) &= \frac{1}{\sqrt{2\pi}\sigma_J(E)}\left[exp\left\{-\frac{J^2}{2\sigma_J^2(E)}\right\} - exp\left\{-\frac{(J+1)^2}{2\sigma_J^2(E)}\right\}\right] I(E) \\
&\simeq -\frac{\partial}{\partial M} I(E,M)\bigg|_{M=J+1/2} \\
&= \frac{2J+1}{\sqrt{8\pi}\sigma_J^3(E)} exp\left\{-\frac{(J+1/2)^2}{2\sigma_J^2(E)}\right\} I(E) \\
&= C_J(E)I(E) \qquad \text{(C.5)}
\end{aligned}
$$

$$
I_\ell(E,J) = (2J+1)exp\left\{-\frac{(J+1/2)^2}{2\sigma_J^2}\right\} I_\ell(E,0)\ ;
$$

$$I_\ell(E,0) = \frac{1}{\sqrt{8\pi}\sigma_J^3} I(E) \tag{C.6}$$

$$\begin{aligned} I_\ell(E,J_1:J_2) &= \frac{1}{\sqrt{2\pi}\sigma_J(E)}\left[exp\left\{-\frac{(J_1)^2}{2\sigma_J^2(E)}\right\} - exp\left\{-\frac{(J_2+1)^2}{2\sigma_J^2(E)}\right\}\right] I(E) \\ &\simeq \left[\sum_{J=J_1}^{J_2} C_J(E)\right] I(E) \\ &= C_{J_1:J_2}(E) I(E)\ ; \end{aligned} \tag{C.7}$$

$$I_\ell(E) = \sum_J I_\ell(E,J) = I_\ell(E, 0:\infty) = C_{0:\infty} I(E)$$

$$= \frac{1}{\sqrt{2\pi}\sigma_J(E)} I(E) = [\sqrt{2\pi}\sigma_J(E) C_{J_1:J_2}(E)]^{-1} I_\ell(E,J_1:J_2) = 2\sigma_J^2(E) I_\ell(E,0) \tag{C.8}$$

Formulas for spin-cutoff factors

The spin-cutoff factors can be written in terms of a and Δ and rigid body moment of inertia $\mathcal{I}_{rig}$ for a 'A' nucleon system. Adding the rotational hamiltonian $V_R = J^2\hbar^2/(2\mathcal{I}_{rig})$ to the NIP hamiltonian h gives $H = h + V_R$ and then assuming that h and V_R are independent leads to $E = E_h + E_{V_R}$. Using the Fermi gas form (C.1) for $I_h(E)$ immediately gives $I_H(E) \sim exp 2\sqrt{a(E - E_{V_R})}$, where $E_{V_R} = \hbar^2 J(J+1)/(2\mathcal{I}_{rig})$. Writing $I_\ell(E,J)$ given in (C.5) in the form of $exp\ 2\sqrt{a(E-E')}$ by using (C.1) for $I(E)$, it is easily seen that $E' = \frac{J(J+1)}{2\sigma_J^2}\sqrt{\frac{E}{a}}$. Then by equating the expressions for E_{V_R} and E' yields a formula for $\sigma_J^2(E)$, $\sigma_J^2(E) = \frac{\mathcal{I}_{rig}}{\hbar^2}\sqrt{\frac{E-\Delta}{a}}$. In practice $\mathcal{I}_{rig}$ is replaced by $\chi\mathcal{I}_{rig}$ with χ a free parameter. With this and also noting from (C.4) that $t \simeq \sqrt{(E-\Delta)/a}$, a formula for $\sigma_J^2(E)$ is (with m the nucleon mass) [Wo-80a],

$$\sigma_J^2(E) = \frac{\chi\mathcal{I}_{rig}}{\hbar^2}\sqrt{\frac{E-\Delta}{a}} \simeq \frac{\chi I_{rig}}{\hbar^2} t \tag{C.9}$$

With $\mathcal{I}_{rig} = 2/5mAR^2$ and $R = r_0 A^{1/3} fm$ ($r_0 \simeq 1.2$), a simpler formula is

$$\sigma_J^2(E) = 0.0096(\chi)(r_0^2)A^{5/3}t \tag{C.10}$$

Unless otherwise specified, $\chi = 1$ in (C.9, C.10) from now onwards. A simpler expression in terms of mass number A follows by recognizing that (i) $\mathcal{I}_{rig}$ can be written in terms of $\langle J_Z^2 \rangle^1$ and 'a' as given for example in [La-66]; (ii) using Fermi gas model to derive expressions for $\langle J_Z^2 \rangle^1$ in terms of A, $\langle J_Z^2 \rangle^1 \propto A^{2/3}$ as follows for example from [Je-52]. The resulting formula for $\sigma_J^2(E)$, as given by Gilbert and Cameron [Gi-65], is

$$\sigma_J^2(E) = 0.0888\sqrt{a(E-\Delta)}A^{2/3} \tag{C.11}$$

Parity decomposition

The state density is sum of fixed parity (π) densities $I^\pi(E)$ and assuming the equilibration (as commonly used in data analysis [1]) of $I^\pi(E)$ densities gives,

$$\begin{aligned} I(E) &= I^+(E) + I^-(E) \\ I^+(E) &\simeq I^-(E) \simeq \frac{1}{2}I(E) \end{aligned} \tag{C.12}$$

Similar expressions for $I_\ell^\pi(E,J)$, $I_\ell^\pi(E,J_1:J_2)$ and $I_\ell^\pi(E)$ can be written down in terms of $I_\ell(E,J)$, $I_\ell(E,J_1:J_2)$ and $I_\ell(E)$ respectively. Given fixed parity spin-cutoff factors $\sigma_J^2(E;\pi)$, following (C.12) it is assumed that,

$$\sigma_J^2(E) = \frac{1}{2}\{\sigma_J^2(E;+) + \sigma_J^2(E;-)\}\,;$$

[1]Using parity equilibration in data analysis demands that this feature should come out in a IP theory but this need not be the case with NIP densities. In fact in the NIP calculations presented in Sect. 3.3 (both in the exact and moment methods) it is found that the parity ratio $w = (I^+ - I^-)/(I^+ + I^-)$ fluctuates around zero and tends to zero only at higher energies. In a recent NIP study, Cerf [Ce-93; also Pi-94] produced a theory for understanding this result. It should be mentioned that in our IP calculations (reported in Chapter 4) we do find parity equilibration. But however, in order to firmly establish this result it is necessary to use two different reference energies one for each parity. Good experimental data for this is not available for the fp - shell nuclei considered in Chapter 4.

$$\sigma_J^2(E;+) \simeq \sigma_J^2(E;-) \tag{C.13}$$

a and Δ parameters

Using the oscillator s.p. spectrum and filling the shells with A nucleons upto shell number N, it is easy to derive that $g(\epsilon_f) \sim 2(N+3/2)^2/(\hbar\omega)$, $A = (2/3)(N+2)^3$ and $\hbar\omega = 41A^{-1/3}\ MeV$. This then gives $a \simeq 0.105A[1 - \left(\frac{2}{3}\right)^{1/3} A^{-1/3}]$. Extending this to pn systems, leads to $I_Z = (N-Z)/2$ dependence for the 'a' parameter, $a = 0.015A[1 - (2/3)^{1/3}A^{-1/3} - (2/9)I_Z^2]$. Thus $a \sim A/10$. By considering more general potentials it is shown that [Ig-76, Ka-80, Il-92] 'a' in general takes the form that $a = \alpha A[1 + \gamma B_s A^{-1/3}]$ where α and γ are constants and B_s is the surface energy. There are several other variants to this formula where 'a' is related to for example neutron and proton separation energies [Ka-80], shell correction energies [Gi-65], excitation energy E dependence [Ba-70] etc.

There are various parametrization for 'Δ' available in literature and the simplest one is where the formula [Bo-69] for odd-even mass difference $\bar{\Delta} \sim 12/A^{1/2}\ MeV$ is used and this gives $\Delta = 2\bar{\Delta}$ for even-even nuclei, $\bar{\Delta}$ for odd-even and even-odd nuclei and zero for odd-odd nuclei. Several other parametrization are given for example in [Gi-65, Di-73, Ig-83, Il-92].

C.2 Brief description of the experimental methods

Direct counting of levels

At low excitation energies, the nuclear energy levels can be measured - in some cases with the spin and the parity and in some cases without spin and/or parity. In an energy domain $E - \frac{\epsilon}{2}$ to $E + \frac{\epsilon}{2}$ with $\epsilon \sim 1 MeV$, assuming that all the levels are identified experimentally, simple counting of the number of levels gives $I_\ell(E)$. This method is severly constrained by the requirement that all the levels in a given energy domain must be identified experimentally. Because of this, direct counting yields level density data only upto 3-4 MeV excitation (in many cases at energies lower than this). It is useful to mention that Von Egidy et al [Vo-86] compiled complete set of low energy data for a number of nuclei but they constrain the spin and parity window. This data is not used in the present compilation as its analysis requires a knowledge of spin-cutoff factors at low energies (spin-cutoff factors are poorly defined and determined at low energies).

Neutron or proton resonance experiments

In the neutron resonance experiments, a nucleus in its ground state is bombarded with a slow neutron (with energy ranging from few eV to a keV) as a result of which an excited compound nucleus is formed with the excitation energy equal to the neutron binding energy (B_n). Near the neutron binding energy (resonance energy), 100-200 resonances can be observed in the compound nucleus and the width of these resonances is very small compared to the mean resonance spacing. Thus in the slow neutron experiments, at the resonance energy, the number of resonances can be counted in a small energy window and this then gives the mean level spacing (D) at B_n. If the angular momentum and parity of the target nucleus is J^π, then the spins and parities of the resonances are $(J + \frac{1}{2})^\pi$ and $(J - \frac{1}{2})^\pi$. For a target with zero spin, the resonance spin and parity is $\frac{1}{2}^+$. Thus from the mean level spacing D, $I_\ell^\pi(E; J_1 : J_2)$ can be calculated from which $I_\ell(E)$ is deduced using (C.3 - C.8, C.10, C.12, C.13) at neutron binding energy (at one energy only). The neutron resonance data is available [Mu-81, Mu-84, Ro-92] for ^{60}Co, ^{60}Ni and ^{62}Ni.

A proton resonance experiment is basically a (p, p') reaction [Mi-80]. For the resonances to be observed, the level (resonance) width Γ has to be much less than the mean spacing (between the resonances) D. In the actual experiment the target (even-even nucleus) is bombarded with protons of energy $\sim$ 2 - 3 MeV and then the energies, spins and parities of about 100-200 compound states are determined. The reaction cross-section is plotted against the proton energy and the resonances are counted. Here the resonance data are restricted to the medium and light nuclei due to the Coulomb barrier. With B_p the proton separation energy and that the resonances are observed with incident protons having energy in the range E_{p_1} to E_{p_2}, the resonance energy is then taken to be $E = B_p + (E_{p_1} + E_{p_2})/2$. Thus the average spacing between the resonances is calculated by direct observation of the resonances and it is converted into level density and interpreted as density at the energy E given above. In practice this will be in error (as $I(E)$ changes like $2\sqrt{aE}$) as $E_{p_1} - E_{p_2} \sim 1 MeV$. Some authors take this error into account and in fact produce $I_\ell(E)$ at two energies around E given above. Wherever this analysis is available, those values are taken or otherwise the above procedure is followed to get proton resonance density. The proton resonance data is available for the odd nuclei ^{55}Mn, ^{59}Co and $^{63,65}Cu$.

Charge particle spectra

In these experiments the level densities are not measured directly but the level density parameters are extracted out by analyzing the data from charge particle nuclear reactions of the type $A(a,b)B$ where a and b represent charge particles (i.e. α particles, proton etc.). The differential cross sections ($\frac{d^2\sigma_{ab}}{d\epsilon_b d\Omega_b}$) data are analyzed using an extended form of Weisskopf formula [We-37] where the angular momentum is incorporated fully [Lu-72a, Hu-72],

$$\frac{d^2\sigma_{ab}(\epsilon_b,\theta)}{d\epsilon_b d\Omega_b} = \sum_{L=0\ (L\, even)}^{\infty} B_L(\epsilon_b) P_L(cos\theta)$$

$$B_L(\epsilon_b) = \tfrac{1}{4}(2K_a+1)^{-1}(2i_a+1)^{-1}k_a^{-2} \sum_{S_a,S_b,K_b,\ell_a,\ell_b,J} (-1)^{S_a-S_b} T_a^{\ell_a}(\epsilon_a) T_b^{\ell_b}(\epsilon_b)$$
$$\times Z(\ell_a J \ell_a J; S_a L) Z(\ell_b J \ell_b J; S_b L) I_{\ell;b}(U_b, K_b)[G(J)]^{-1}$$
$$G(J) = \sum_{b'} \int_0^{(U_{b'})_{max}} dU_{b'} \sum_{\ell_{b'}=0}^{(\ell_{b'})_{max}} T_{b'}^{\ell_{b'}}(\epsilon_{b'}) \sum_{S_{b'}=|J-\ell_{b'}|}^{J+\ell_{b'}} \sum_{K_{b'}=|S_{b'}-i_{b'}|}^{S_{b'}+i_{b'}} I_{\ell;b'}(U_{b'}, K_{b'}). \qquad (C.14)$$

In (C.14) K_a, i_a, J, K_b and i_b are the spins of the target, projectile, compound nucleus, residual nucleus and emitted particle respectively. S_a and S_b are the channel spins in the incident and outgoing channels respectively; ℓ_a and ℓ_b are the orbital angular momenta of the incident and outgoing particles respectively; k_a is the wave number of the incident particle; $P_L(cos\theta)$ is the Legendre polynomial of order L. $T_a^{\ell_a}(\epsilon_a)$ and $T_b^{\ell_b}(\epsilon_b)$ are transmission coefficients for the projectile and emitted particles respectively with the channel energies ϵ_a and ϵ_b (the channel energy ϵ is defined as the sum of the kinetic energies with respect to the center of mass of the emitted particle and recoil nucleus). The $Z(\ell_a\, J\, \ell_a J;\, S_a\, L)$ and $Z(\ell_b\, J\, \ell_b J;\, S_b\, L)$ are Z coefficients and are defined as sums of products of Racah W coefficients and Clebsch-Gordon coefficients; $Z(\ell\, J\, \ell\, J;\, S\, L) = (2J+1)(2\ell+1)W(\ell\, J\, \ell\, J;\, S\, L)\langle \ell\, 0\, \ell\, 0 | L\, 0\rangle$. $I_\ell(E, J)$ is level density of the residual nucleus formed by the emission of particle b with channel energy ϵ_b. The sum over b' refers to the sum over all the different types of emitted particles. In the 'exact' analysis using (C.14), spectra from different reactions leading to the same final nucleus are analyzed with one set of

$(a,\ \Delta)$ values and then a complicated search programme produced the values of $(a,\ \Delta)$. In the analysis, the LLC formula (C.3, C.4), angular momentum decomposition formulas (C.5 - C.8) and the spin-cutoff factor formula (C.10) with $r_0 = 1.2$ are used. The charged particle experiments are due to Lu et al [Lu-72a] and they are carried out for the nuclei ^{56}Fe, ^{59}Co, $^{60,62}Ni$ and $^{63,65}Cu$:

Residual Nucleus	Reaction	Residual Nucleus	Reaction
^{56}Fe	$^{56}Fe(\alpha,\alpha')^{56}Fe$	^{63}Cu	$^{63}Cu(p,p')^{63}Cu$
	$^{59}Co(p,\alpha)^{56}Fe$		$^{60}Ni(\alpha,p)^{63}Cu$
^{59}Co	$^{59}Co(p,p')^{59}Co$		$^{63}Cu(\alpha,\alpha')^{63}Cu$
	$^{56}Fe(\alpha,p)^{59}Co$	^{65}Cu	$^{65}Cu(p,p')^{65}Cu$
	$^{59}Co(\alpha,\alpha')^{59}Co$		$^{62}Ni(\alpha,p)^{65}Cu$
	$^{62}Ni(p,\alpha)^{59}Co$		$^{65}Cu(\alpha,\alpha')^{65}Cu$
^{60}Ni	$^{60}Ni(\alpha,\alpha')^{60}Ni$	^{62}Ni	$^{62}Ni(p,p')^{62}Ni$
	$^{63}Cu(p,\alpha)^{60}Ni$		$^{59}Co(\alpha,p)^{62}Ni$
			$^{62}Ni(\alpha,\alpha')^{62}Ni$
			$^{65}Cu(p,\alpha')^{62}Ni$

Lu et al [Lu-72a] deduced one set of $(a,\ \Delta)$ values for the nuclei ^{56}Fe, ^{59}Co, ^{60}Ni, $^{63,65}Cu$ and two sets of $(a,\ \Delta)$ values for ^{62}Ni. The level densities and spin-cutoff factors calculated at different energies using the deduced $(a,\ \Delta)$ parameters and the smoothed NIP forms (C.3 - C.8, C.10) with $r_0 = 1.2$ are treated as experimental data. This data is called Lu et al data in the compilation given in Sect. C.3 (for ^{62}Ni there are Lu at al (a) and Lu at al (b) data corresponding to the two sets of $(a,\ \Delta)$ parameters deduced for this nucleus).

Ericson fluctuations

In the excitation energy domain around 20 MeV of the compound nuclei (in compound nucleus reaction) the cross-sections fluctuate strongly and here the level width Γ is larger than the average spacing D between the resonances. This region is called the Ericson fluctuation region. Using a statistical theory

[Er-66], the average cross-section in Ericson fluctuation domain is related to $\Gamma/D_{J=0}$; $D_{J=0} = [I_\ell(E,0)]^{-1}$. Measurement of average cross-sections and independent knowledge of Γ, then yield a value of $D_{J=0}$ or equivalently (using C.6, C.8) for the total level density $I_\ell(E)$ at various energies in the Ericson fluctuation domain. Using the (p,p') and (p,α) reactions, Ericson fluctuation experiments are carried out and at some energies around 20 MeV excitation in ^{55}Mn, ^{56}Fe, ^{60}Co and ^{60}Ni the total level density $I_\ell(E)$ is deduced. These experiments are due to Katsanos et al [Ka-66b, Ka-70] and Huizenga et al [Hu-69].

Dilg et al data, Katsanos et al and Iljinov et al data

First systematic analysis of the level density data was carried out by Gilbert and Cameron [Gi-65] in 1965. They have used a constant temperature formula (C.2) for level densities at low energy domain and the back-shifted fermi gas form (C.1) for high energy domain and gave a prescription to join them to obtain a smoothed (continuous) curve for level densities. A more recent analysis of this type is due to Dilg et al [Di-73]. By fitting the low energy level density data and the data from neutron/proton resonance experiments, using the LLC formulas (C.3, C.4) and (C.5 - C.8, C.10) with $r_0 = 1.25$, Dilg et al deduced the values for the level density parameters (a, Δ) for about 200 nuclei. The Dilg et al (a, Δ) values are available for the six nuclei ^{55}Mn, $^{59,60}Co$, ^{62}Ni, $^{63,65}Cu$ in the set considered in Sect. C.3. Using (a, Δ) values that are deduced for $\sim$ 200 nuclei, Dilg et al constructed smoothed formulas for (a, Δ) as a function of the mass number A. The smoothed formulas are

$$a(MeV^{-1}) = 2.40 + 0.067A$$
$$\Delta(MeV) = P - 130/A$$

$$
\begin{aligned}
P &= 25.6A^{-1/2} \text{ for even-even nuclei} \\
&= 12.8A^{-1/2} \text{ for odd-mass nuclei} \\
&= 29.4A^{-1} \text{ for odd-odd nuclei}
\end{aligned} \tag{C.15}
$$

Using (C.15), the (a, Δ) values for ^{56}Fe and ^{60}Ni in the set of nuclei considered in Sect. C.3 are calculated and they are called Dilg et al (a, Δ) values for these two nuclei. The level densities and spin-cutoff factors calculated at different energies using the Dilg et al (a, Δ) parameters and the smoothed NIP forms (C.3 - C.8, C.10) with $r_0 = 1.2$ are treated as experimental data. This data is called Dilg et al data in the compilation given in Sect. C.3.

Katsanos et al [Ka-70] fitted the Ericson fluctuation data (they measured the same) at certain energies (around 20 MeV excitation) and the data at low energies obtained by direct counting, using (C.3 - C.8, C.10) with $r_0 = 1.2$ and deduced (a, Δ) values. The level densities and spin-cutoff factors calculated at different energies using the deduced (a, Δ) parameters and the smoothed NIP forms (C.3 - C.8, C.10) with $r_0 = 1.2$ are treated as experimental data. This data is called Katsanos et al data in the compilation given in Sect. C.3.

Most recent exercise of selection and compilation of level density data from direct counting, proton/neutron resonances and Ericson fluctuations is due to Iljinov et al [Il-92] and wherever possible, this data is used in the compilation given in Sect. C.3.

Data for spin-cutoff factors

The measurement of spin-cutoff factor $\sigma_J^2(E)$, the quantity which characterizes the spin distribution of nuclear levels, cannot be carried out by a commonly available method. However information about $\sigma_J(E)$ basically come from three kinds of methods. They are: 1) the experiments to measure the isomer ratio; 2) the measurement of angular distribution of particles from charged particle reactions and 3) using the analytical formulas (C.4, C.9 - C.11, C.13) and the deduced values of a and Δ. These three methods are briefly described below.

Long lived excited states are known as isomers. By measuring cross-sections for two isomeric states with quite different angular momenta and comparing their ratios with a statistical theory of nuclear reaction that involves σ_J, a value for $\sigma_J(E)$ is deduced by best fit to data (in the analysis energy dependence of σ_J is ignored). For example in $^{197}Au(p,n)^{197}Hg$, $^{197}Au(d,2n)^{197}Hg$, $Pt(\alpha,xn)^{197}Hg$, $^{196}Hg(n,\gamma)^{197}Hg$, $^{196}Hg(d,p)^{197}Hg$ etc. reactions, for spin $1/2^-$ ^{197}Hg ground state (life time 65 hours) and the isomeric states with spin $13/2^+$ at 0.297 keV excitation (life time 24 hours), the cross sections are measured and by fitting the cross-section ratio to a statistical theory, $\sigma_J(E)$ value is deduced by Vandenbosch and Huizenga [Va-60]. However for the eight fp-shell nuclei of interest to us, there are no $\sigma_J(E)$ data from isomeric ratio measurements.

In the second method, the spin-cutoff factors are obtained from angular distribution of particles emitted in charged particle experiments. In these experiments, the differential cross-section at various angles (denoted by $W(\theta)$) are measured and then using (C.14) and fitting the data for $W(\theta)/W(0)$ to theory with an assumed form for $\sigma_J(E)$, the values for $\sigma_J(E)$ are deduced. Lu et al [Lu-72b] carried out these experiments for fp-shell nuclei and deduced $\sigma_J(E)$ values. In fact they give the following results: (I) using $E^{1/2}$ energy dependence; (II) with no energy dependence,

Reactions	Bombarding energy (MeV)	Final nucleus	excitation energy of the final nucleus $E(MeV)$	Deduced $\sigma_J(E)$	
				I	II
$^{56}Fe(\alpha,\alpha')^{56}Fe$	17	^{56}Fe	5.5 − 9.5	$(3.5-3.8)\pm 0.1$	3.8 ± 0.2
$^{59}Co(\alpha,\alpha')^{59}Co$	17	^{59}Co	5.5 − 9.5	$(3.9-4.4)\pm 0.1$	$4.2\pm 0\ 2$
$^{60}Ni(\alpha,\alpha')^{60}Ni$	17	^{60}Ni	5.5 − 9.5	$(3.4-4.0)\pm 0.1$	3.4 ± 0.2
$^{62}Ni(\alpha,\alpha')^{62}Ni$	17	^{62}Ni	5.5 − 9.5	$(3.6-3.7)\pm 0.1$	$3.8\pm 0\ 2$
$^{63}Cu(\alpha,\alpha')^{63}Cu$	17	^{63}Cu	5.5 − 9.5	$(3.9-4.2)\pm 0.1$	4.1 ± 0.2
$^{60}Ni(\alpha,p)^{63}Cu$	16.6	^{63}Cu	4.1 − 8.3	$(3.4-3.9)\pm 0.2$	3.7 ± 0.3
$^{62}Ni(\alpha,p)^{65}Cu$	16.2	^{65}Cu	3.3 − 7.3	$(3.8-4.0)\pm 0.2$	4.0 ± 0.3

In the compilation given ahead in Sect. C.3 only the results of set I given in the above table are used. As the energy changes from E_1 to E_2, the $\sigma_J(E)$ changes from σ_1 to σ_2 and in the compilation given in Sect. C.3 it is assumed that at $E = E_1$, $E_1+\delta$, $E_1+2\delta$ and E_2, $\sigma_J(E) = \sigma_1$, $\sigma_1+\delta'$, $\sigma_1+2\delta'$ and σ_2 where $\delta = (E_2-E_1)/3$ and $\delta' = (\sigma_2-\sigma_1)/3$; this is similar to what was done in [Be-73] for ^{56}Fe nucleus. The spin-cutoff values $\sigma_J(E)$ thus obtained at these four energies are referred to as experimental spin-cutoff data (though experiments give only spin-cutoff datum value as shown in the table above).

There are analytical formulas available for $\sigma_J(E)$ as given by (C.9 - C.11, C.4). Knowing (a, Δ) parameters and $\mathcal{I}_{rig}$ or r_0 value, σ_J at different energies can be calculated. Using (C.10) and the a and Δ values given by Dilg et al [Di-73], Lu et al [Lu-72a] and Katsanos et al [Ka-70], $\sigma_J(E)$ values are calculated and compiled in Sect. C.3.

C.3 Compilation of data

The compilation of data for the eight fp-shell nuclei $^{55}_{25}Mn_{30}$, $^{56}_{26}Fe_{30}$, $^{59}_{27}Co_{32}$, $^{60}_{27}Co_{33}$, $^{60}_{28}Ni_{32}$, $^{62}_{28}Ni_{34}$, $^{63}_{29}Cu_{34}$ and $^{65}_{29}Cu_{36}$ are given in this section. The data are obtained from various sources as described earlier. The exact values of errors in level densities and spin-cutoff factors are compiled if they are available in literature. If errors are not available, as in the case of proton resonance data for ^{65}Cu and the spin-cutoff data from Lu et al [Lu-72b], we put 30% error for $I_\ell(E)$ and 10% error for $\sigma_J(E)$ (they are consistent with the errors that are generally believed to be in the deduced values of $I_\ell(E)$ and $\sigma_J(E)$ respectively).

Nucleus ^{55}Mn

For ^{55}Mn the most recent compilation of level densities deduced from direct counting, Ericson fluctuations and proton resonance data is due to Iljinov et al [Il-92]. Their compilation of data in Ericson fluctuation domain is same as given in [Ka-70]. In the proton resonance experiments [Di-73], 48 resonances are observed with spin $1/2^+$ using protons in the energy range $2.1 - 2.7$ MeV (proton separation energy is 8.07 MeV). For the level densities from the above three sources of data, Iljinov et al compilation is adopted and they are given Table C.1. The Dilg et al and Katsanos et al data for energies upto 24 MeV are given in Table C.2; the corresponding (a, Δ) values are shown below the table.

Table C.1 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
3.5^a	40	12
10.0^b	3000	800
15.1^c	110000	30000
18.0^c	490000	150000
23.1^c	7300000	2000000

a) direct counting b) proton resonances c) Ericson fluctuations

Table C.2 Dilg et al and Katsanos et al data

E	Dilg et al			Katsanos et al		
(MeV)	$I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	$I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	9	71	3.2	9	61	2.8
4.0	46	410	3.6	59	474	3.2
6.0	193	1854	3.8	308	2673	3.5
8.0	703	7153	4.1	1333	12300	3.7
10.0	2306	24578	4.3	5061	49082	3.9
12.0	6973	77241	4.4	17413	175926	4.0
14.0	19733	225971	4.6	55359	579285	4.2
16.0	52843	623095	4.7	164937	1779678	4.3
18.0	135026	1634272	4.8	465266	5158878	4.4
20.0	331318	4105805	4.9	1252308	14230104	4.5
22.0	784618	9934711	5.1	3235796	37596835	4.6
24.0	1800606	23254296	5.2	8065205	95642640	4.7

$a = 5.45\ MeV^{-1}$, $\Delta = -1.20\ MeV$	$a = 6.3\ MeV^{-1}$, $\Delta = -0.5\ MeV$

Nucleus ^{56}Fe

No resonance data are available for ^{56}Fe nucleus. The Katsanos et al and

Table C.3 Katsanos et al and Lu et al data

E	Katsanos et al			Lu et al		
(MeV)	$I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	$I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	1	4	2.0	2	10	2.5
4.0	9	64	2.8	12	94	3.1
6.0	64	515	3.2	65	563	3.4
8.0	337	2951	3.5	283	2631	3.7
10.0	1476	13772	3.7	1064	10457	3.9
12.0	5678	55664	3.9	3592	36940	4.1
14.0	19763	201851	4.1	11150	119150	4.3
16.0	63508	671823	4.2	32343	357252	4.4
18.0	191119	2084760	4.4	88672	1008373	4.5
20.0	544228	6100538	4.5	231711	2704451	4.7
22.0	1477992	16978739	4.6	580896	6941267	4.8
24.0	3851586	45242809	4.7	1404340	17144504	4 9

$a = 6.4\ MeV^{-1}$, $\Delta = 1.5\ MeV$	$a = 5.7\ MeV^{-1}$, $\Delta = 0.7\ MeV$

Lu et al data for energies upto 24 MeV are given in Table C.3 and the Dilg et al data for energies upto 24 MeV are given in Table C.4. The most recent compilation of level density data deduced from direct counting and Ericson fluctuations is due to Iljinov et al [Il-92]; their compilation of data in Ericson fluctuation domain is same as given in [Ka-70]. The Iljinov et al data are given in Table C.5. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.6; see Sect. C.2.

Table C.4 Dilg et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	1	7	2.4
4.0	11	86	3.1
6.0	69	601	3 5
8.0	337	3152	3.7
10.0	1385	13750	4.0
12.0	5050	52575	4.2
14.0	16786	181823	4.3
16.0	51791	580449	4.5
18.0	150252	1735048	4 6
20.0	413765	4906938	4.7
22.0	1089444	13233948	4.9
24.0	2758314	34247030	5.0

$a =$ 6.15 MeV^{-1}, $\Delta =$ 1.1 MeV

Table C.5 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})	E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
5.0[a]	24	7	19.0[b]	260000	70000
5.5[a]	32	9	20.0[b]	400000	120000
6.0[a]	41	12	21.0[b]	600000	220000
17.0[b]	130000	30000	22.0[b]	800000	300000
18.0[b]	160000	50000	23.0[b]	1200000	400000

a) direct counting *b*) Ericson fluctuations

Table C.6 Experimental spin-cutoff data

E (MeV)	$\sigma_J(E)$
5.50	3.5 ± 0.35
6.83	3.6 ± 0.36
8.16	3.7 ± 0.37
9.50	3.8 ± 0.38

Nucleus ^{59}Co

For ^{59}Co proton resonance data was measured by Lindstrom et al [Li-71, Li-72]. In [Li-71] 32 resonances with spin $1/2^+$ are observed with protons in the energy range $2.14 - 2.65\,MeV$ and in [Li-72] 71 resonances are observed

Table C.7 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
9.60	1700	400
10.00	3000	900

Table C.8 Lu et al and Dilg et al data

E	Lu et al			Dilg et al		
(MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	10	80	3.1	8	59	3.1
4.0	67	574	3.4	53	465	3.5
6.0	329	3069	3.7	276	2635	3.8
8.0	1371	13588	4.0	1196	12172	4.1
10.0	5060	52588	4.2	4555	48725	4.3
12.0	16986	183748	4.3	15708	175101	4.5
14.0	52869	591900	4.5	50046	577853	4.6
16.0	154589	1783575	4.6	149391	1778754	4.8
18.0	428765	5081176	4.7	422131	5165236	4.9
20.0	1136405	13796208	4.8	1137973	14270322	5.0
22 0	2894900	35925212	5.0	2944576	37758200	5 1
24.0	7120894	90168296	5.1	7349086	96183216	5.2

$a = 6.2\ MeV^{-1}$, $\Delta = -0.8\ MeV$	$a = 6.31\ MeV^{-1}$, $\Delta = -0.47\ MeV$

with spin $1/2^+$ with protons in the energy range $2.66 - 3.10\,MeV$ (the proton separation energy being $7.37\,MeV$). By analyzing this resonance data, Iljinov et al obtained $I_\ell(E)$ at $9.6\,MeV$ and $10\,MeV$ as given in Table C.7. The Lu et al and Dilg et al data for energies upto 24 MeV are given in Table C.8. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.9; see Sect. C.2.

Table C.9 Experimental spin-cutoff factor data

E (MeV)	$\sigma_J(E)$
5.50	3.90 ± 0.39
6.83	4.07 ± 0.41
8.16	4.23 ± 0.42
9.50	4.40 ± 0.44

<u>Nucleus ^{60}Co</u>

For ^{60}Co, neutron resonance density was measured by Baba [Ba-70] and this datum is recently compiled by Iljinov et al [Il-92]. The resonance energy E_{res} is 7.491 MeV, the resonance spins are $(3^-, 4^-)$ and the mean spacing at E_{res} is (1400 ± 200) eV (Rohr [Ro-92] gives $E_{res} = 7.492\,MeV$ and $D_{res} = (1600 \pm 160)\,eV$). Using this data, (C.3 - C.8, C.10, C.12, C.13) with $r_0 = 1.2$ and $(a,\,\Delta) = (8.01\,MeV^{-1}, 0\,MeV)$ as given in [Il-92], the following neutron resonance data is deduced,

Table C.10 Neutron resonance data

E_{res}	=	7.491 MeV
J_1^π, J_2^π	=	$3^-, 4^-$
$I_\ell(E_{res})$	=	$(4389 \pm 627)\,MeV^{-1}$

The Dilg et al data for energies upto 24 MeV are given in Table C.11. Iljinov et al [Il-92] compiled data from Ericson fluctuation measurements (the experiments are due to Kopsch and Cierjacks [Ko-72]) and their values are adopted in Table C.12.

Table C.11 Dilg et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	E (MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	62	531	3.4	14.0	297296	3496139	4.7
4.0	367	3438	3.7	16.0	891450	10779080	4.8
6.0	1764	17657	4 0	18.0	2540453	31495226	4.9
8.0	7337	77307	4.2	20.0	6927024	87846392	5.1
10.0	27342	300558	4.4	22.0	18168622	235238780	5.2
12.0	93418	1064605	4.6	24.0	46038104	607590530	5.3

$a = 6.86\ MeV^{-1}, \quad \Delta = -2.16\ MeV$

Table C.12 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
9.50	29000	12000
11 50	100000	40000
13.50	240000	110000

Nucleus ^{60}Ni

For ^{60}Ni neutron resonance data is compiled by Rohr [Ro-92] and Iljinov et al [Il-92]. The resonance spins and the resonance spacing are $J^{\pi}_{res} = (1^-, 2^-)$ and $D_{res} = (1400 \pm 300)\, eV$. Note that Iljinov et al give $E_{res} = 11.399\, MeV$ while Rohr [Ro-92] and Mughabghab et al [Mu-81] give $E_{res} = 11.389\, MeV$ and we adopt the later. Using this data, (C.3 - C.8, C.10, C.12, C.13) with $r_0 = 1.2$ and $(a, \Delta) = (7.47\, MeV^{-1}, 3.098\, MeV)$ as given in [Il-92], produce the neutron resonance data given in Table C.13. The Lu et al and Dilg et al data for energies upto 24 MeV are given in Table C.14.

Table C.13 Neutron resonance data

E_{res}	=	11.389 MeV
J^{π}	=	$1^-, 2^-$
$I_\ell(E_{res})$	=	$(6006 \pm 1286)\ MeV^{-1}$

Table C.14 Lu et al and Dilg et al data

E (MeV)	Lu et al $I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	Dilg et al $I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	1	6	2.3	1	7	2.5
4.0	11	81	3.0	12	98	3.2
6.0	72	621	3.4	81	731	3 6
8.0	371	3470	3.7	411	4025	3.9
10.0	1603	15928	4.0	1761	18291	4.1
12.0	6098	63599	4.2	6658	72524	4.3
14.0	21039	228454	4.3	22865	259184	4.5
16.0	67128	754589	4.5	72701	852774	4.7
18.0	200817	2326832	4.6	216899	2621691	4.8
20.0	568956	6772494	4.8	613209	7612653	5.0
22.0	1538392	18761962	4.9	1655235	21049702	5.1
24.0	3993622	49792876	5.0	4291116	55779876	5.2

$a = 6.4\ MeV^{-1}$, $\Delta = 1.3\ MeV$	$a = 6.42\ MeV^{-1}$, $\Delta = 1.14\ MeV$

The most recent compilation of level density data deduced from direct counting and Ericson fluctuations measurements is due to Iljinov et al [Il-92]; their compilation of data in Ericson fluctuation domain is same as given in Huizenga

Table C.15 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})	E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
4.5^a	27	5	18.5^b	500000	200000
5.0^a	32	6	19.5^b	750000	220000
15.5^b	120000	36000	20.5^b	1100000	330000
16.5^b	190000	57000	21.5^b	1800000	540000
17.5^b	310000	. 90000	22.5^b	2500000	700000

a) direct counting *b*) Ericson fluctuations et al [Hu-69]. The Iljinov compilation is given in Table C.15. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.16; see Sect. C.2.

Table C.16 Experimental spin-cutoff data

E (MeV)	$\sigma_J(E)$
5.50	3.4 ± 0.34
6.83	3.6 ± 0.36
8.16	3.8 ± 0.38
9.50	4.0 ± 0.40

Nucleus ^{62}Ni

For ^{62}Ni the neutron resonance data compiled by Mughabghab et al [Mu-84] is used. The $E_{res} = 10.597\,MeV$, $J^{\pi}_{res} = (1^-, 2^-)$ and $D_{res} = (1.8 \pm 0.3)\,keV$. (Rohr [Ro-92] gives $D_{res} = (1.6 \pm 0.2)\,keV$ while Iljinov et al [Il-92] gives $D_{res} = (1.4 \pm 0.2)\,keV$). Using Mughabghab data, $(a,\,\Delta) = (8.05\,MeV^{-1},\,3.1\,MeV)$, as given by Iljinov et al, and (C.4 - C.8, C.10, C.12, C.13) with $r_0 = 1.2$ produce the neutron resonance data given in Table C.17.

Table C.17 Neutron resonance data

E_{res}	=	$10\,597\,MeV$
$J^{\pi}_1,\, J^{\pi}_2$	=	$1^-, 2^-$
$I_\ell(E_{res})$	=	$(4552 \pm 761)\,MeV^{-1}$

The Lu et al (a), Lu et al (b) data for energies upto 24 MeV are given in Table C.18 and the Dilg et al data for energies upto 24 MeV are given in Table C.19. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.20; see Sect. C.2.

Table C.18 Lu et al (a) and Lu et al (b) data

E (MeV)	Lu et al (a) $I_\ell(E)$ (MeV^{-1})	Lu et al (a) $I(E)$ (MeV^{-1})	Lu et al (a) $\sigma_J(E)$	Lu et al (b) $I_\ell(E)$ (MeV^{-1})	Lu et al (b) $I(E)$ (MeV^{-1})	Lu et al (b) $\sigma_J(E)$
2.0	3	18	3	1	6	2.4
4.0	23	192	3	10	81	3.1
6.0	138	1272	3.7	70	621	3.5
8.0	660	6510	3.9	361	3470	3.8
10.0	2698	28117	4.2	1560	15928	4.1
12.0	9840	107272	4.4	5934	63599	4.3
14.0	32840	371807	4.5	20472	228454	4.5
16.0	101972	1192931	4.7	65319	754589	4.6
18.0	298176	3590401	4.8	195404	2326832	4.8
20 0	828481	10236756	4.9	553620	6772494	4.9
22.0	2202501	27856996	5.0	1496925	18761962	5.0
24.0	5633102	72781472	5.2	3885974	49792876	5.1

$a = 6.4\ MeV^{-1},\quad \Delta = 0.5\ MeV$	$a = 6.4\ MeV^{-1},\quad \Delta = 1.3\ MeV$

Table C.19 Dilg et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	E (MeV)	$I_\ell(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2.0	2	11	2.5	14.0	75392	851067	4.5
4.0	23	183	3.2	16.0	260725	3045518	4.7
6.0	175	1576	3.6	18.0	841705	10131017	4.8
8.0	1007	9817	3.9	20.0	2563920	31694960	4.9
10.0	4809	49744	4.1	22.0	7429486	94079760	5.1
12.0	20057	217590	4.3	24.0	20610884	266778270	5.2

$a = 7.27\ MeV^{-1},\quad \Delta = 1.07\ MeV$

Table C.20 Experimetal spin-cutoff data

E (MeV)	$\sigma_J(E)$	E (MeV)	$\sigma_J(E)$
5.50	3.60 ± 0.36	8.16	3.67 ± 0.37
6.83	3.63 ± 0.36	9.50	3.70 ± 0.37

Nucleus ^{63}Cu

The Lu et al and Dilg et al data for energies upto 24 MeV are given in Table C.21.

Table C.21 Lu et al and Dilg et al data

E (MeV)	Lu et al $I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$	Dilg et al $I_t(E)$ (MeV^{-1})	$I(E)$ (MeV^{-1})	$\sigma_J(E)$
2 0	11 .	82	3.1	11	91	3 3
4.0	81	711	3 5	79	730	3 7
6.0	457	4348	3.8	423	4251	4 0
8.0	2120	21468	4.0	1893	20202	4.3
10 0	8572	91224	4.2	7424	83166	4.5
12.0	31224	346224	4.4	26327	307100	4.7
14.0	104649	1201905	4.6	86149	1040404	4.8
16.0	327545	3879295	4.7	263844	3284704	5.0
18.0	967868	11780130	4.9	764200	9774401	5.1
20.0	2722307	33957264	5.0	2109864	27650506	5.2
22.0	7335278	93562424	5.1	5586861	74855888	5 3
24.0	19031592	247765220	5.2	14258997	19496662	5.5

$a = 6.8\ MeV^{-1}$, $\Delta = -0.5\ MeV$	$a = 6.63\ MeV^{-1}$, $\Delta = -0.67\ MeV$

For ^{63}Cu proton resonance data is available [Br-70, Di-73]. There are 63 resonances observed with spin $1/2^+$ using protons in the energy range 2.1 – 2.7 MeV (the proton separation energy being 8.07 MeV). The resonance experiment is due to Browne et al [Br-70]. Recently Iljinov et al [Il-92] compiled the $I_t(E)$ values (at two energies) obtained by analyzing the proton resonance data and also data at low energies from direct counting. The Iljinov et al data is given in Table C.22. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.23; see Sect. C.2.

Table C.22 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
3.5^a	40	7
4.0^a	50	8
8.6^b	2100	600
9.0^b	4100	1200

a) direct counting b) proton resonances

Table C.23 Experimental spin-cutoff data

E (MeV)	$\sigma_J(E)$	E (MeV)	$\sigma_J(E)$
5.50	3.9 ± 0.39	4.1	3.40 ± 0.34
6.83	4.0 ± 0.4	5.5	3.57 ± 0.36
8.16	4.1 ± 0.41	6.9	3.73 ± 0.37
9.50	4.2 ± 0.42	8.3	3.90 ± 0.39

Nucleus ^{65}Cu

For ^{65}Cu proton resonance data is available [Br-70, Di-73]). There are 15 resonances observed with spin $1/2^+$ using protons in the energy range $3.15 - 3.25\ MeV$. With the proton separation energy being $7.45\ MeV$ the resonance density at $E = E_{res} = 7.45 + (3.25 + 3.15)/2 = 10.65\, MeV$ is calculated which gives $I_\ell^+(E_{res}; 1/2) = 15/0.1 = 150\, MeV^{-1}$. The $I_\ell^+(E; \frac{1}{2})$ is converted into $I_\ell(E_{res})$ using (C.3-C.8, C.10, C.12, C.13) with $r_0 = 1.25$ and (a, Δ) values given by Dilg et al [Di-73]. In the level density obtained from proton resonances we put an error of 30% (normally this much error one expects) and the data is shown in Table C.24.

Table C.24 Proton resonanace data

E_{res}	=	$10.65\ MeV$
J^π	=	$\frac{1}{2}^+$
$I_\ell(E_{res})$	=	$(6854 \pm 2056)\, MeV^{-1}$

The Lu et al and Dilg et al data for energies upto 24 MeV are given in Table C.25. Low energy level density data compiled by Iljinov et al is given in Table C.26. The experimental spin-cutoff data from Lu et al [Lu-72b] are collected in Table C.27; see Sect. C.2.

Table C.25 Lu et al and Dilg et al data

E (MeV)	Lu et al $I_\ell(E)$ (MeV^{-1})	Lu et al $I(E)$ (MeV^{-1})	Lu et al $\sigma_J(E)$	Dilg et al $I_\ell(E)$ (MeV^{-1})	Dilg et al $I(E)$ (MeV^{-1})	Dilg et al $\sigma_J(E)$
2.0	9	73	3.2	9	79	3.4
4.0	67	606	3.6	60	578	3.9
6.0	364	3587	3.9	297	3122	4.2
8.0	1644	17224	4.2	1249	13935	4.5
10.0	6485	71391	4.4	4637	54300	4.7
12.0	23093	264893	4.6	15660	190871	4.9
14.0	75798	900549	4.7	49003	618171	5.0
16.0	232662	2850448	4.9	143984	1871943	5.2
18.0	674989	8498223	5.0	401160	5357308	5.3
20.0	1865776	24073836	5.2	1067739	14608063	5.5
22.0	4944672	65238848	5.3	2730816	38192120	5.6
24.0	12627143	170039040	5.4	6742644	96222600	5.7

Lu et al	Dilg et al
$a = 6.6\ MeV^{-1}$, $\Delta = -0.5\ MeV$	$a = 6.24\ MeV^{-1}$, $\Delta = -0.77\ MeV$

Table C.26 Iljinov et al data

E (MeV)	$I_\ell(E)$ (MeV^{-1})	$\Delta I_\ell(E)$ (MeV^{-1})
3.5	45	9
4.0	56	16

Table C.27 Experimental spin-cutoff data

E (MeV)	$\sigma_J(E)$	E (MeV)	$\sigma_J(E)$
3.30	3.80 ± 0.38	5.96	3.93 ± 0.39
4.63	3.87 ± 0.39	7.30	4.00 ± 0.40