Abstract

The subject of statistical nuclear spectroscopy, that started with Bethe's statistical mechanical level density formula , Wigner's treatment of spectral fluctuations using matrix ensembles and French's embedded ensembles and Gaussian densities, continues to be an important field of research as it has large variety of applications in nuclear structure and nuclear astrophysics. Statistical spectroscopy divides into two parts; one is spectral averages which give smoothed (with respect to energy) forms of observables and the other being spectral fluctuations around these smoothed forms; the later are ignored for most purposes. The smoothed forms of spectroscopic observables follow from the action of Central Limit Theorems (CLT) in nuclear shell model spaces. Recent progress in statistical spectroscopy is in deriving and applying the smoothed forms in indefinitely large spaces with *interactions* by using unitary group decompositions (of hamiltonians and the spectroscopic spaces), CLT's locally, and convolutions - the resulting theory is the Spectral Averaging Theory in Large Shell Model Spaces (SAT-LSS). The aim of the present thesis is to study (and test) some aspects of SAT-LSS for state, expectation value and strength densities in large spaces and apply them to calculate nuclear level densities and β-decay rates. The problems solved to this end are described below along with corresponding results.

In SAT-LSS, (i) using a significant unitary group decomposition under which a one plus two-body nuclear hamiltonian decomposes into orthogonal effective one-body part $\mathbf{h}$ and irreducible two-body part $\mathbf{V}$ ($H \to \mathbf{h} + \mathbf{V}$); (ii) applying the CLT's locally and (iii) decomposing the shell model space

into distant non-interacting S - subspaces (for light nuclei S denotes $\hbar\omega$ excitation), the state densities take a convolution form, $I^H = \sum_S I^{\mathbf{h},S} \otimes \rho_G^{\mathbf{V},S}[E]$ where $I^{\mathbf{h}}$ is renormalized non-interacting particle (NIP) state density and ρ_G is a normalized spreading Gaussian. Similar convolution forms are derived for spin-cutoff and occupancy densities. Firstly, systematic studies of two important aspects of the level density theory are carried out in the thesis and they are: (a) a part of the interaction that produces non-NIP like shifts of the single particle energies, that is neglected in the theory, is conclusively demonstrated (using norms of operators and considering large number of effective interactions) to be indeed small all across the periodic table; (b) moment methods for constructing locally smoothed forms for NIP state, spin-cutoff and occupancy densities which are one of the convolution factors in the IP theory, are comprehensively tested for the three densities and also for their S - decompositions in an example of 12 protons and 12 neutrons in a large shell model space consisting of 16 orbits (s, p, ds, fp, sdg, $1h_{11/2}$) with $0 - 4$ $\hbar\omega$ excitations; moment methods give an excellent representation of exact results.

For a first systematic analysis of experimental data using the convolution forms for state and spin-cutoff densities, in a given region of the periodic table, fp-shell nuclei are chosen as the example. Nuclei in this region of the periodic table have acquired new significance recently as some of them play an important role in nuclear astrophysics problems, double β - decay, problem of quenching of GT sum rule strengths etc. The calculations, using surface delta interaction with strength G, are carried out in large spaces by including configurations upto $2\hbar\omega$ excitations and by employing eight spherical orbits (ds, fp, $1g_{9/2}$) for the five nuclei ^{55}Mn, ^{56}Fe, ^{59}Co, ^{60}Co and ^{60}Ni and by extending for obvious reasons, to ten spherical orbits (ds, fp, $1g_{9/2}$, $2d_{5/2}$,

$1g_{7/2}$) for the three $A > 60$ nuclei ^{62}Ni, ^{63}Cu and ^{65}Cu. In general the convolution form for the densities is seen to provide an excellent representation of the observed total level densities and spin-cutoff factors. The calculations determine a magnitude parameter of the interaction - the strength G which is found to be $\sim 20/A\ MeV$ for all the nuclei, is compatible with the values deduced from low-energy spectroscopy.

In SAT-LSS the bivariate strength density $\mathbf{I}^H_{\mathcal{O}}(E_i, E_f)$ for a transition operator $\mathcal{O}$ takes a convolution form, $\mathbf{I}^H_{\mathcal{O}}(E_i, E_f) = \mathbf{I}^{\mathbf{h}}_{\mathcal{O}} \otimes \rho^{\mathbf{V}}_{BIV-\mathcal{G};\mathcal{O}}[E_i, E_f]$. Leaving aside the spreading bivariate Gaussian $\rho^{\mathbf{V}}_{BIV-\mathcal{G};\mathcal{O}}$, one needs good methods for constructing the NIP density $\mathbf{I}^{\mathbf{h}}_{\mathcal{O}}$ in large spaces. For one-body transition operators, a complete formalism for constructing the exact NIP strength densities using spherical orbits is worked out. For rapid construction of the bivariate NIP densities and also to carry out S-decomposition, trace propagation formulas for bivariate moments M_{rs} with $r + s \leq 2$ are derived for unitary orbit partial densities which determine the bivariate Gaussian representation for the partial densities. This formalism is tested in a large space numerical example with $GT(\beta^-)$ transition operator. Trace propagation formulas for M_{rs} with $r + s \leq 4$ are also derived in m-particle scalar spaces, which are useful for many purposes.

A method to calculate β-decay rates at finite temperature is developed by writing the expression for the rates explicitly in terms of bivariate GT strength densities ($\mathbf{I}^H_{\mathcal{O}(GT)}$) and state densities of parent nucleus besides having the usual phase space factors. For constructing $\mathbf{I}^H_{\mathcal{O}(GT)}$, the method developed in this thesis for constructing NIP strength densities is applied together with a plausible procedure for generating the GT bivariate spreading Gaussians due to interactions. Recently there is considerable interest in evaluating β-decay rates for $A > 60$ neutron excess fp-shell nuclei as these rates play a very important role in determining the structure of the core of massive supernova stars and hence on their subsequent evolution towards gravitational collapse and supernova explosion phases. Therefore as an example, using our method, β-decay rates for $^{61,62}Fe$ and $^{62-64}Co$ isotopes are calculated at presupernova matter densities $\rho = 10^7 - 10^8$ gm/cc, temperatures $T = (3-5) \times 10^9\ ^oK$ and for various values of the electron fraction Y_e. As a biproduct, the calculations produced a value (via observed β-decay half lifes) for the bivariate correlation coefficient $\bar{\zeta}$ between the hamiltonian and the GT operator to be $\bar{\zeta} \sim 0.67$, and thus new information about the nuclear hamiltonians is deduced. In addition an expression for the GT non-energy weighted summed strengths is also deduced.

To summarize, SAT-LSS is explored by carrying out first systematic analysis of fp-shell level density data using the convolution forms for the state and spin-cutoff densities, developed further by deriving an essentially complete theory for constructing NIP strength densities produced by one-body transition operators, and applied to neutron excess fp-shell nuclei relevant for presupernova evolution calculations in nuclear astrophysics by developing a method to calculate β-decay rates at finite temperature in terms of state and bivariate strength densities. The results of the present thesis together with the earlier investigations, show that the convolution forms for state, expectation value and strength densities in large spaces as given by SAT-LSS can be used profitably to study a variety of nuclear physics and nuclear astrophysics problems.