NOTATIONS

1. Lower case letters with a bar below it will denote column vectors while capital letters will denote matrices. $A : p\times q$ means a matrix of p rows and q columns, while $\underline{a} : p\times 1$ means a column vector with p elements. Also

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{matrix} p \\ q \end{matrix}$$
$\quad r \quad s$

will have sub-matrices $A : p\times r$, $B : p\times s$, $C : q\times r$ and $D : q\times s$. All matrices considered here are real.

2. I and O are respectively denoted as identity and null matrices. (Where necessary, I will be denoted by I_n, $n\times n$ being the order of I. The order of null matrix will be understood by context.) $D_\phi : p\times p$ denotes a diagonal matrix with diagonal elements $\phi_1, \phi_2, \ldots, \phi_p$ and $\tilde{A} : p\times p$ means a triangular matrix with zero above the main diagonal.

3. A', A^{-1} and $\mathrm{tr}_i A$ means respectively transpose of A, inverse of A, and sum of the principal minors of order i of A. Usually $\mathrm{tr}_1 A$ will be denoted as $\mathrm{tr}\, A = \sum_i a_{ii}$.

4. $A^{i_1, i_2, \ldots, i_s}_{j_1, j_2, \ldots, j_s}$ is the sub-matrix obtained from $A : p\times p$ by deleting $i_1, i_2, \ldots, i_s$ rows and $j_1, j_2, \ldots, j_s$ columns.

5. $\lambda_{max}\, S$ and $\lambda_{min}\, S$ mean respectively maximum and minimum characteristic roots (or simply roots) of the matrix S.

6. $\Delta : p\times r$ $(r < p)$ is said to be semi-orthogonal if $\Delta'\Delta = I_r$. Also if $\Delta : p\times r$ is semi-orthogonal matrix, then there exists a semi-orthogonal matrix $\Delta_1 : p\times(p-r)$ such that $\underset{r\ \ \ p-r}{(\Delta\ \ \Delta_1)}p$ is a complete orthogonal matrix.

7. If $A_1, A_2, \ldots, A_k, B_1, B_2, \ldots, B_m$ satisfy the following matrix equations $B_i = F_i(A_1, \ldots, A_k)$ for $i = 1, 2, \ldots, m$ and if $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are two respective sets of independent variates in $(A_1, \ldots, A_k)$ and $(B_1, \ldots, B_m)$, then the jacobian of transformation of $(a_1, \ldots, a_n)$ to $(b_1, \ldots, b_n)$ is $\left|\frac{\partial(a_1, a_2, \ldots, a_n)}{\partial(b_1, b_2, \ldots, b_n)}\right|$ and will be denoted by $J(A_1, \ldots, A_k;\ B_1, \ldots, B_m)$.

8. If $f(x_1, \ldots, x_n)$ is a differentiable function in $x_1, \ldots, x_n$, then $df = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\, dx_i$ is called the differential of f and will be denoted as f^* and so dx_i is x_i^*.

9. dX will denote the product of differentials of random variables contained in $X : p\times q$.

10. If the elements of $\underline{x}_i : p\times 1$ are jointly distributed as multivariate normal with means as the elements of $\underline{\mu}_i : p\times 1$ and variances-covariances as the elements of $\Sigma : p\times p$ (symmetric), then we say that $\underline{x}_i$ is multivariate normal with mean $\underline{\mu}_i$ and variance-covariance matrix Σ, and write it as $\underline{x}_i : N(\underline{\mu}_i, \Sigma)$.

Moreover if $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n$ are independently distributed, we say that $X: p\times n = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n)$ is $MN(\mu, \Sigma)$ where $\mu: p\times n = (\underline{\mu}_1, \underline{\mu}_2, \ldots, \underline{\mu}_n)$ and the density function of $MN(\mu, \Sigma)$ is written as

$$MN(\mu,\Sigma) = (2\pi)^{-pn/2} |\Sigma|^{-n/2} \exp\left\{-\tfrac{1}{2}\operatorname{tr} \Sigma^{-1}(X-\mu)(X-\mu)'\right\}.$$

11. The density function of Wishart's distribution

$$2^{-pn/2}\, \pi^{-p(p-1)/4}\, |\Sigma|^{-n/2} \prod_{i=1}^{p} \left\{\Gamma\left(\frac{n-i+1}{2}\right)\right\}^{-1} |S|^{(n-p-1)/2} \cdot$$

$$\exp(-\tfrac{1}{2}\operatorname{tr}\Sigma^{-1} S) \quad \text{if } n \geq p$$

will be denoted as Wishart $(n,p;\Sigma;S)$ or $W(n,p;\Sigma;S)$.

12. $\Pr(\xi = x)$ means the probability of the random variable ξ takes the value x.

13. $\sum_i$ means the summation over i while Σ means a variance-covariance matrix or an orthogonal matrix.

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