C H A P T E R VI

HARDNESS ANISOTROPY OF RHOMBOHEDRAL CRYSTALS : SODIUM NITRATE AND CALCITE

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6.1 INTRODUCTION:

In the earlier chapter the hardness of $NaNO_3$ and $CaCO_3$ were experimentally studied by considering their variation with applied load and quenching temperature for different but constant orientations of the indenter with the crystal lattice. The present work aims at studying the microhardness anisotropy of $NaNO_3$ and $CaCO_3$ by employing Knoop indenter of low symmetry. An important feature of the Knoop hardness test is that the hardness value is dependent on the orientation of the major axis of the indenter in a given plane as well as on the orientation of the plane itself with reference to the principal axis of anisotropy /1/. Further the depth of penetration of the indenter is shallow. Hence brittle materials like glass or mineral could be indented without causing premature fracture. Besides the indenter shape is relatively non-symmetric, the variations in hardness along different directions on a given surface can be determined. For such a study, single crystals can serve as ideal materials to establish the orientation-dependence of hardness values. It is from this point of view that hardness anisotropy study of synthetic single crystals of $NaNO_3$ and natural $CaCO_3$ crystal is carried out and reported here.

It is apparent from the hardness studies presented here that macroscopically there are four parameters affecting hardness: (i) Applied load, (ii) Quenching temperature, (iii) Orientation of the indenter diagonal (major) with reference to the crystal lattice, (iv) Crystal plane under indentation. The empirical formulae derived in the present work should be valid for majority of crystals of different materials.

6.2 OBSERVATIONS:

For studying the anisotropic behaviour of rhombohedral crystals $NaNO_3$ and $CaCO_3$ the observations recorded in chapter

V are used for considering in a quantitative manner the effect of four major factors, namely, (i) Applied load (P), (ii) Quenching temperature (Tq), (iii) Orientation of the major diagonal of indenter with reference to [100] direction in the cleavage plane (A), and (iv) Crystal plane/face for indentation (F). For the purpose of quantitative study of the relations amongst P, T_q, A, H and F; variations between any two factors are considered by keeping remaining parameters constant. Since the observations of chapter V are to be used, the crystal plane/face (F), which in the present case is a cleanage plane of $NaNO_3$ and $CaCO_3$ is fixed. Further the applied load P should be considered as constant. However it was shown (vide chapter V) that it represents a range of applied loads in HLR where hardness (H) is constant and independent of load. The range of applied loads for $NaNO_3$ is from 20 to 160 gm and for $CaCO_3$ 40 to 80 gm. In case of $CaCO_3$ the actual range was from 40 to 160 gm. However the microstructures produced by indentation loads beyond 80 gm made the measurements of length of diagonal of the indentation mark difficult and to a certain extent not dependable. Hence the load range for $CaCO_3$ was reduced. In this range of applied loads, there is a slight change in values of hardness. In the discussion, mean value of hardness is considered. The hardness anisotropy for different orientations is studied at different constant temperatures, viz., room temperature T and different quenching temperatures T_q'S. Thus the approach to hardness study is basically phenomenalogical. This approach is likely to be useful for development of model theory of hardness of crystalline materials.

6.3 RESULTS AND DISCUSSION:

6.3.1. Variation of hardness with orientation of major diagonal with direction [100] on cleavage faces {100} of $NaNO_3$ and $CaCO_3$ for constant temperature:

It is observed from the plots of H Vs. P (Fig.5.1) for $NaNO_3$ that for higher loads greater than 20 gm in HLR, Knoop hardness is almost constant whereas for $CaCO_3$ (Fig.5.2) Knoop hardness is constant for all loads greater than 40 gm in HLR. Considering the mean hardness value, the plots of $\bar{H}$ Vs. A for room temperature and different quenching temperatures are shown in Fig.6.1a and 6.1b, where 'A' is the orientation of the longer diagonal of the Knoop indenter from [100] direction on cleavage face (100) of $NaNO_3$ and $CaCO_3$. The plots of $\bar{H}$ Vs. A depict the hardness anisotropy as the orientation changes. Maximum values of hardness occur at [100] and [0$\bar{1}$0] directions whereas minima at direction [1$\bar{1}$0] for hardness determination at room temperature and quenching temperatures. The plots indicate that the variation of hardness on either side of direction [1$\bar{1}$0] is symmetrical. This direction represents projection of the optic axis [111] on cleavage plane (100) of $NaNO_3$ and $CaCO_3$. For $NaNO_3$, observations were taken for all values of A, i.e., from 0° to 78° and for calcite from 0° to 35° and by invoking the symmetry considerations the remaining observations from 39° (direction [1$\bar{1}$0]) are assumed to be the same; that is observations for 35°, 28°, 21°, 14°, 7° and 0° correspond with those of 43°, 50°, 57°, 64°, 71° and 78°, respectively. This was also confirmed by taking observations for some values of A, chosen at random. The plots show very clearly that hardness changes with A and quenching temperature. In this section only the variation of H with A at constant temperature will be considered.

There are certain basic characteristics of these plots (Fig. 6.1a and 6.1b). They are as follows:

(i) For $NaNO_3$, the hardness range is from 16 to 23 $kg.mm^{-2}$ whereas for $CaCO_3$ it is from 100 to 150 $kg.mm^{-2}$

(ii) The minimum value of hardness occurs at 39°, i.e., along the direction [1$\bar{1}$0] whereas the maximum values on either side of the above direction are at 0° and 78°, i.e., along directions [100] and [0$\bar{1}$0]. Since both these crystals have perfect cleavages {100}, it can safely be conjectured that along [001] direction hardness should have a maximum value. Thus it can be said that hardness is maximum along directions <100> and has a minimum value along [$1\bar{1}0$]. It can be guessed that it should be minimum along direction [111] for which the projection on (100) is [$1\bar{1}0$].

(iii) The plots do not exhibit sharp minima.

(iv) For $NaNO_3$, hardness decreases with increasing T_q whereas for $CaCO_3$ reverse is the case. However for both these crystals the form of the curve and the nature of variation are not different from each other.

(v) The curve is an imperfect parabola.

Since hardness is inversely proportional to the square of the major diagonal of the Knoop indentation mark, it is also possible to draw a plot of d Vs. A. Because of the increase proportion, these plots should exhibit maximum values corresponding to minimum values of H in the various plots of H Vs. A for various, but constant, room temperature and quenching temperatures. This is indeed found to be the case and the above three conclusions are valid. Since due to broadening of the curve, it is desirable to have a change in the mathematical aproach so as to obtain straight line plots.

A careful study of the curve of H Vs. A indicates that to a greater extent it corresponds to a parabola. The curvature near 0° and 78° shows that the latter part of the parabolic curve turns into a bell-shape. It is thus a mixture of these two curves. Hence by following mathematical combinations in a judicious manner, it is possible to obtain a straight line plot. The graph of $\sqrt{HA}$ Vs. A is a straight line with intercepts on the axis. For $NaNO_3$ and $CaCO_3$ the plots are straight lines (vide table 6.1 a,b) (Fig.6.2a,b) (Fig.6.2 a,b). The general equation for such a straight line plot is,

$$\sqrt{HA} = mA + C \qquad \ldots \qquad \ldots \qquad (1)$$

where $\sqrt{HA}$ and A are along Y and X-axis respectively and m and c are the slope and intercept respectively. Squaring both sides of (1) yields,

$$HA = m^2A^2 + 2mAc + c^2 \quad \ldots \quad \ldots \quad (2)$$

Division of both sides by A gives,

$$H = m^2A + 2mc + c^2/A \quad \ldots \quad \ldots \quad (3)$$

Differentiation of (2) and (3) with reference to A yields-

$$\frac{dH}{dA}.A + H.1 = 2m^2A + 2mc \quad \ldots \quad \ldots \quad (4)$$

and

$$\frac{dH}{dA} = m^2 - c^2/A^2 \quad \ldots \quad \ldots \quad (5)$$

For an extremum (maximum or minimum) of the curve at a point, say, $H = h_o$, $A = a_o$

$$\frac{dH}{dA} = 0 \quad \text{at} \quad H = h_o, \quad A = a_o$$

Substitution of this value in (4) and (5) yields,

$$o.a_o + h_o.1 = 2m^2a_o + 2mc$$

$$h_o = 2m^2 a_o + 2mc \quad \ldots \quad \ldots \quad (7)$$

and

$$m^2 - c^2/a_o = 0$$

$$a_o = \pm c/m \quad \ldots \quad \ldots \quad (8)$$

Elimination of a_o from (7) by using (8) gives,

$$h_o = 2m^2 \left(\pm \frac{c}{m}\right) + 2mc$$

$$= \pm 2mc + 2mc$$

$$= 4mc \quad \text{or} \quad 0.$$

Obviously zero value of h_o is inadmissible,

Hence,

$$h_o = 4mc \qquad \ldots \qquad \ldots \qquad \ldots \qquad (9)$$

Multiplying (8) and (9), one gets-

$$a_o h_o = \pm \frac{c}{m} \cdot 4mc$$

$$= 4c^2$$

$$\therefore c = \pm \sqrt{\frac{a_o h_o}{4}} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (10)$$

Division of (8) by (9) gives,

$$\frac{a_o}{h_o} = \frac{1}{4m^2}$$

$$\therefore m = \pm \sqrt{h_o/4a_o} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (11)$$

Thus the following values are of importance for testing straight line and parabolic plots (Fig.6.1 a,b & 6.2 a,b):

$$a_o = c/m \qquad \ldots \qquad \ldots \qquad \ldots \qquad (12)$$

$$h_o = 4mc \qquad \ldots \qquad \ldots \qquad \ldots \qquad (13)$$

$$c = + \tfrac{1}{2} \sqrt{a_o h_o} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (14)$$

$$m = + \tfrac{1}{2} \sqrt{h_o / a_o} \qquad \ldots \qquad \ldots \qquad (15)$$

The values of a_o and h_o can be obtained from the parabolic plot where $dH/dA = 0$ at the point P having co-ordinates (a_o, h_o). These values can be utilized to obtain the slope m and intercept c of the straight line plot.

For a straight line plot following a well-established relation between two variables in the plot, the conventional method is to select any two points on the straight line, say, (A_1, H_1) and (A_2, H_2) and to follow the normal procedure of calculating the slope and intercept from the general equation-

$$\sqrt{HA} = mA + c \quad \text{of the straight line.}$$

Thus,

$$m = \frac{\sqrt{H_1A_1} - \sqrt{H_2A_2}}{A_1 - A_2} \qquad \ldots \qquad \ldots \qquad \ldots \qquad (16)$$

$$c = \frac{\sqrt{H_1A_1} - \sqrt{H_2A_2}}{2} - m(A_1 + A_2) \qquad \ldots \qquad (17)$$

The values of m and c obtained from (16) & (17) should agree with the values obtained from (12), (13), (14) and (15) and also from the statistical method of the best fit of straight line. In the present case more emphasis is given on the statistical method rather than the conventional method in view of the fact that the relations between H and A are in the developmental stage. For $NaNO_3$ and $CaCO_3$, the values of m and c determined by using statistical method are compared with the values obtained from a few distinguishing characteristics of the parabolic plots. These values obtained at room temperature and different quenching temperatures are given in table (6.2). A glance at these values indicates a fairly good agreement between values calculated by statistical method and determined from parabolic plots. It should be noted that due to combination of variables in the general equation (1), it is clear that the entire analysis becomes invalid for H = 0 and A = 0.

The above formulae which are based on experimental observations for cleavage faces of $NaNO_3$ and $CaCO_3$ are derived without any direct reference to crystal structure or microstructures developed on a crystal surface due to indentation. The basic requirement is the availability of a highly clean crystal face of low indices free from different growth features. Hence these formulae should be applicable to similar type of hardness studies of different crystals reported in the literature. The crystals for which such studies were carried out are as follows:

(i)	Tantalum carbide,	TaC	/1/
(ii)	Aluminium,	Al	/1/
(iii)	Calcium Fluoride,	CaF_2	/1/
(iv)	Tungsten,	W	/1/
(v)	Iron,	Fe	/1/

The hardness observations reported by different workers are reproduced here for the purpose of analysis. It is necessary to provide explanation for the symbols used in eleven tables numbered from 6.3(i) to 6.3(xi); Fig.6.3 a(i) to (iv); Fig.6.3 b(i) to 6.3 (xi).

A - Angle between major diagonal of Knoop indenter and reference direction on a crystal plane under observation and indentation, expressed in degrees.

H - Knoop hardness number in the HLR where H is constant and independent of applied load P, expressed in $kg.mm^{-2}$.

a_o - Minimum value of A in the parabolic curve of H Vs. A.

h_o - Minimum value of hardness in the parabola of H Vs. A.

m_c - Calculated value of slope by using formula (16), namely, $m = \frac{1}{2}\sqrt{h_o / a_o}$ for straight line plot of $\sqrt{HA}$ Vs. A.

m_s - Statistically determined value of the slope of the straight line plot of $\sqrt{HA}$ Vs. A.

c_c - Calculated value of intercept by using formula, (17), namely $C = \frac{1}{2}\sqrt{h_o a_o}$ for straight line plot of $\sqrt{HA}$ Vs. A.

c_s - Statistically determined value of the slope of the straight line plot of $\sqrt{HA}$ Vs. A.

For m, m_s, c, c_s the same straight line plot of $\sqrt{HA}$ Vs. A is used. m and c are calculated from the data of a_o and h_o obtained from the parabola of H Vs. A; m_s and c_s are determined by using statistical considerations for obtaining the best fit of a straight line.

It is clear from Figs.(6.3(i)), (6.3(ii)) .... to ...(6.3(xi)) that in all cases straight line plots are obtained. A careful study of the tables for different planes of crystals indicate that the agreement between m and m_s and also between c and c_s at a constant temperature is fairly good considering the accuracy of measurements and preparing data of A and H from the curved plots of H Vs. A and the experimental set up for study reported in the literature mentioned above. It should be noted that in case of $NaNO_3$ and $CaCO_3$ the symmetry direction in the parabolic curve of H Vs. A was $[1\bar{1}0]$, i.e., the angle between indenter and [100]. This direction is the projection of optic axis [111] on cleavage plane of $NaNO_3$ and $CaCO_3$. Hence $[1\bar{1}0]$ is also the direction corresponding to that of homogeneous isotropic material. Such a direction exists for curved plots of H Vs. A for crystals studied by different workers. However, this direction may not correspond to that of a homogeneous isotropic material because for determining such equivalent directions additional information is required, e.g., for $NaNO_3$ and $CaCO_3$, optical study indicates [111] to be such a direction. Hence its projection should also possess this property. Such additional information appears to be wanting for other crystals mentioned above. Further the initial direction from where the angular rotation is considered is also important. Thus in the present study two directions forming a plane angle are important:

(i) the direction indicating the symmetrical disposition on either side of it, i.e,, the line of symmetry of the curve of H Vs. A. This direction is different from the crystallographic line of symmetry of etch pits, viz., [110].

(ii) the initial direction for considering angular rotation of the indenter.

The plots of $\sqrt{HA}$ Vs. A for iron (Fe) crystals deserve special reference (Fig.6.3 A(iv)). The line of symmetry and reference direction are taken to be the same, viz., [110] in these plots. Hence the angle between these directions, namely, a_0, which is different from zero in other crystals has zero value here. Therefore there will be two straight line plots representing the curved parts of the curve H Vs. A, on either side of symmetry direction [110].

6.3.2 Variation of hardness with orientation (A) and quenching temperature (T_q):

The earlier studies were carried out by considering the variation between two parameters, $\bar{H}$ and A, out of $\bar{H}$, P, T_q, A and F, by keeping remaining parameters P, T_q, & F constant. In the present case the variation of hardness with other parameters is reported. In this case the values of hardness corresponding to those values in the HLR where hardness is constant and independent of load, but depends on T_q and A are taken and that their mean hardness value ($\bar{H}$) is used here.

Several combinations of $\bar{H}$, T_q and A were tried to obtain the straight line plot. A plot of log $T_q \sqrt{HA}$ Vs. log T_q Fig.6.4 A & B (vide table 6.4 a & b) is a straight line. The regression coefficient based on the statistical consideration for obtaining the best fit of a straight line has a value much nearer unity. Further the graph (Fig.6.4 A & B) consists of a series of parallel straight lines corresponding to different orientations A with respect to direction [100].

The general equation for such a straight line plot is,

$$\log T_q \sqrt{HA} = m_1 \log T_q + \log c_1$$

where, as usual, m_1 and c_1 represent slope and intercept. Simplification of the above yields,

$$\bar{H} T_q^{2(1-m_1)} = \frac{C_1^2}{A}$$

$$\bar{H} T_q^{p} = \frac{C_1^2}{A} ; \qquad p = 2(1-m_1)$$

or

$$\bar{H} = \frac{C_1^2}{A} \left(\frac{1}{T_q^p} \right) \qquad \ldots \quad \ldots \qquad (1)$$

From the earlier studies on the variation of H with T_q by keeping parameters A and P constant, the general equation was derived. It is,

$$\bar{H} T_q^{K} = C$$

or $$\bar{H} = \frac{C}{T_q^K} \qquad \cdots \qquad \cdots \qquad (2)$$

Comparison of the above two equations for $\bar{H}$ suggests that-

$$\frac{C_1^2}{A}\left(\frac{1}{T_q^p}\right) = \frac{C}{T_q^K}$$

$$\frac{\left(\frac{C_1^2}{A}\right)}{C} = \frac{T_q^p}{T_q^K} = T_q^{p-K}$$

Since p and k which depend on A, are obtainable from the graphs (Figs.6.4,A,B, 5.4, 5.5), T_q^{p-k} for different values of T_q can be calculated. Similarly using (C_1^2/A) and C values obtained from the graphs (Fig.6.4,A,B, 5.4, 5.5), $(C_1^2/A)/C$ can be calculated. These two sides should be equal. This is actually found to be the case (cf. table 6.4a).

It is thus obvious that when the orientation A and quenching temperature T_q are simultaneously changed, the hardness number follows the equation-

$$\bar{H} A T_q^p = \text{constant, say, } B$$

where the exponent p is given by,

$$p = 2(1 - m_1)$$

where m_1 is the slope of the straight line plot between Log $T_q \sqrt{\bar{H}A}$ Vs. Log T_q.

6.4 CONCLUSIONS:

(1) $NaNO_3$, the hardness range expressed by Knoop hardness numbers is from 16 to 23 $kg.mm^{-2}$, for a range of applied loads from 20 to 160 gm in HLR. For calcite the range of hardness numbers is from 100 to 150 $kg.mm^{-2}$ for loads ranging from 40 to 80 gm in HLR.

(2) At a constant temperature the Knoop hardness number $\bar{H}$ varies with orientation, A, of the major diagonal of Knoop indenter. $\bar{H}$ attains maximum values for $A = 0°$ and 78°, i.e., along directions [100] and $[0\bar{1}0]$ and minimum value along direction $[1\bar{1}0]$, i.e., $A = 39°$. Further, the variation of H on either side of $[1\bar{1}0]$ is symmetrical. This direction represents projection of optic axis [111] on a cleavage plane {100} of $NaNO_3$ and $CaCO_3$.

(3) Hardness $\bar{H}$ changes with A and quenching temperature T_q. For $NaNO_3$, $\bar{H}$ decreases with increasing T_q, whereas reverse is the case for $CaCO_3$. Plots between $\sqrt{HA}$ and A are straight lines. The slope and intercept are related to minimum values of H and A. Excellent correlation between the calculated values of slope and intercept from the actual plot and the statistically determined values. Further this relation is applicable to other crystals TaC, Al, CaF_2, W and Fe for hardness data reported in the literature.

(4) The simultaneous variations of $\bar{H}$ with orientation A and quenching temperature T_q follow the experimentally observed relation:

$$\bar{H}\, A\, T_q^p = \text{constant, say, } B$$

where,

$$p = 2(1 - m_1) \quad \text{and}$$

m_1 = slope of straight line plot of $\log T_q \sqrt{\bar{H}A}$ Vs. $\log T_q$.

TABLE 6.1 (A) : $NaNO_3$

Orientation A in degrees	Temperature in °K				
	298°K	343°K	393°K	443°K	493°K
	$\sqrt{\bar{H}_A}$				
13	16.49	16.28	15.34	15.97	16.37
26	21.59	21.81	21.13	20.73	21.01
39	26.04	27.43	25.46	25.19	25.25
52	30.13	30.75	29.87	29.42	30.44
65	35.02	37.01	35.91	35.41	34.13
78	41.67	41.53	39.79	39.65	39.46

TABLE 6.1 (B) : $CaCO_3$

Orientation A in degrees	$\sqrt{\bar{H}_A}$				
	303°K	498°K	573°K	698°K	773°K
7	29.24	30.25	30.79	30.99	31.42
14	42.64	42.35	43.04	43.24	43.72
21	50.26	50.94	52.16	52.00	52.63
28	57.74	57.84	59.12	59.39	59.93
35	59.59	62.58	64.10	65.53	65.81
42	65.19	68.56	70.23	71.78	72.09
49	69.28	76.53	78.20	78.57	79.27
56	83.95	83.18	85.18	84.92	85.94
63	92.01	89.83	91.29	91.73	92.75
70	96.46	95.64	97.36	98.01	99.36
78	101.79	102.61	103.84	104.98	106.39

TABLE 6.2(a)

$NaNO_3$

Values of slope m (calculated by formula, statistical, observed) intercept C (calculated by formula, statistical, observed) of the plot $\sqrt{\bar{H}A}$ Vs. A; hardness h_o in $kg\text{-}mm^{-2}$ at a_o = 39° from the plot of $\bar{H}$ Vs. A.

Temp	Slope : m			Intercept : C			h_o
	$\frac{1}{2}\sqrt{\frac{h_o}{a_o}}$	Statistically obtained	Observed	$\frac{1}{2}\sqrt{a_o h_o}$	Statistically obtained	Observed	
°K	$= m_c$	m_s	m_o	C_c	C_s	C_o	
298	0.3339	0.3742	0.3692	13.025	11.46	11.7	17.4
343	0.3397	0.3849	0.3942	13.248	11.63	11.37	18.0
393	0.3265	0.3758	0.3717	12.73	10.82	11.0	16.63
443	0.3230	0.3663	0.3654	12.59	11.06	11.25	16.28
493	0.3238	0.3516	0.3576	12.63	11.78	11.6	16.36

TABLE 6.2(b)

$CaCO_3$

Values of slope m (calculated by formula, statistical, observed) intercept C (calculated by formula, statistical, observed) of the plot $\sqrt{\overline{H}A}$ Vs. A; hardness h_o in kg-mm^{-2} at $a_o = 39°$ from the plot of $\overline{H}$ Vs. A.

Temp	Slope : m			Intercept : C			h_o
	$\frac{1}{2}\sqrt{\frac{h_o}{a_o}}$	Statistically obtained	Observed	$\frac{1}{2}\sqrt{a_o h_o}$	Statistically obtained	Observed	
°K	$= m_c$	m_s	m_o	C_c	C_s	C_o	
303	0.8026	0.9905	0.9894	31.303	26.31	28.21	100.5
498	0.8435	0.9755	0.9636	32.89	28.059	28.13	111.0
573	0.8680	0.9882	0.9647	33.77	28.885	29.88	117.0
698	0.8825	0.9962	0.9636	34.41	29.07	30.73	121.5
773	0.8897	0.9901	0.9797	34.70	28.57	31.33	123.5

TABLE 6.3 (i)

Crystal : TaC
Temp : R.T.
Plane : (001)

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	1645.81	-
10	1593.73	126.24
20	1572.90	177.36
30	1510.42	212.86
40	1489.58	244.09
45	1468.76	257.08
50	1541.66	277.63
60	1531.25	303.10
70	1583.32	332.91
80	1624.98	360.55
90	1635.39	383.64

a_o = 45°

h_o = 1468.76

m_c = 2.8565

m_s = 3.1274

m_o = 3.0344

c_c = 128.54

c_s = 112.73

c_o = 119.48

Range of Hardness = 1468.76 - 1645.81 $kg\text{-}mm^{-2}$

Reference direction for A = [100]

Ref. /1/, Pg.35

TABLE 6.3 (ii)

Crystal : Al
Temp : R.T.
Plane : $\{100\}$

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	23.3	-
10	22.6	15.03
18.33	21.3	19.75
27.47	19.3	23.02
37.47	18.0	25.97
45.0	17.0	27.65
54.98	18.0	31.45
63.32	19.3	34.95
71.66	21.3	39.06
80.0	22.6	42.52
90.0	23.3	45.79

$a_o = 45°$

$h_o = 17.0\ kg\text{-}mm^{-2}$

$m_c = 0.3073$

$m_s = 0.3744$

$m_o = 0.3928$

$c_c = 13.83$

$c_s = 11.86$

$c_o = 11.07$

Range of Hardness = 17.0 - 23.3 $kg\text{-}mm^{-2}$

Reference direction for A = $\langle 100 \rangle$

Ref. /1/, Pg.171

TABLE 6.3 (iii)

Crystal : Al
Temp : R.T.
Plane : {210}

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	23.2	-
10	22.3	14.93
20	21.0	20.49
30	19.0	23.87
40	16.6	25.77
49	17.15	28.99
63.32	18.0	33.76
66.66	19.0	35.58
70.0	18.6	36.10
80.0	19.0	38.98
90.0	19.3	41.63

a_o = 40°

h_o = 16.6 $kg\text{-}mm^{-2}$

m_c = 0.2958

m_s = 0.3036

m_o = 0.3

c_c = 14.49

c_s = 14.25

c_o = 15.125

Range of Hardness = 17.15 - 23.2 $kg\text{-}mm^{-2}$

Reference direction for A = ⟨100⟩

Ref. /1/, Pg.171

TABLE 6.3 (iv)

Crystal : Al
Temp : R.T.
Plane : $\{110\}$

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	23.3	-
10	23.0	15.16
15	22.0	18.16
20	21.0	20.49
25	20.0	22.36
30	19.0	23.87
35	18.3	25.31
40	17.6	26.53
45	17.3	27.9
50	17.0	29.15
55	16.75	30.35
60	16.8	31.74
65	16.8	33.04
70	16.75	34.24
75	16.8	35.49
80	17.0	36.87
85	17.0	38.01
90	17.0	39.11

a_o = 55°
h_o = 16.75 $kg\text{-}mm^{-2}$
m_c = 0.2759
m_s = 0.2804
m_o = 0.2727
c_c = 15.17
c_s = 14.67
c_o = 15.18
Range of Hardness = 16.75 - 23.3 $kg\text{-}mm^{-2}$
Reference direction for A = 100
Ref. /1/, Pg.171

TABLE 6.3 (v)

Crystal : Calcium Fluoride

Temp : R.T.

Plane : (001)

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	176.25	-
9.72	173.75	41.09
19.44	167.50	57.06
29.16	161.25	68.57
38.88	157.5	78.25
48.86	157.5	87.72
58.32	160.0	96.59
68.04	162.5	105.15
77.76	170.0	115.97
87.48	180.0	125.45
97.2	182.5	133.18

a_o = 45°

h_o = 157.5 $kg\text{-}mm^{-2}$

m_c = 0.9351

m_s = 1.016

m_o = 0.9733

c_c = 42.09

c_s = 36.45

c_o = 38.4

Range of Hardness = 157.5 - 176.25 $kg\text{-}mm^{-2}$

Reference direction for A = [100]

Ref. /1/, Pg.205

TABLE 6.3 (vi)

Crystal : Tungsten
Temp : 77°K
Plane : $\{110\}$

Orientation A in degrees	Knoop Hardness H in kg-mm^{-2}	$\sqrt{HA}$
0	558	-
15	545	90.42
37	530	140.03
50	540	164.31
60	550	181.65
64	557	188.8
71	570	201.17
80	600	219.08
90	665	244.64

a_o = 37°
h_o = 530 kg-mm^{-2}
m_c = 1.8923
m_s = 1.9853
m_o = 1.8222
c_c = 70.02
c_s = 62.85
c_o = 72.22
Range of Hardness = 530 - 665 kg-mm^{-2}
Reference direction for A = $\langle 110 \rangle$

TABLE 6.3 (vii)

Crystal : Tungsten
Temp : 203°K
Plane : {110}

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	401	-
8	406	56.99
18	405	85.38
27	410	105.21
38	403	123.74
48	400	138.56
56	407	150.97
62	423	161.94
70	444	176.29
81	470	195.12
90	480	207.85

a_o = 48°
h_o = 400 $kg\text{-}mm^{-2}$
m_c = 1.4423
m_s = 1.7728
m_o = 1.74
c_c = 69.28
c_s = 51.92
c_o = 54.5
Range of Hardness = 400 - 480 $kg\text{-}mm^{-2}$
Reference direction for A = ⟨110⟩

TABLE 6.3 (viii)

Crystal : Tungsten
Temp : 296°K
Plane : {110}

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
0	300	-
10	315	56.12
20	311	78.86
30	318	97.67
40	319	112.96
50	320	126.49
60	331	140.93
70	355	157.64
80	385	175.49
90	390	187.35

m_s = 1.6044

m_o = 1.5714

c_s = 45.72

c_o = 48.57

Range of Hardness = 300 - 390 $kg\text{-}mm^{-2}$

Reference direction for A = ⟨110⟩

TABLE 6.3 (ix)

Crystal : Iron
Temp : R.T.
Plane : (110)

Orientation A in degrees	Knoop Hardness H in kg-mm^{-2}	$\sqrt{HA}$
90	101.6	95.62
75	98.4	85.91
60	88.4	72.82
45	76.4	58.63
30	74.8	47.37
15	73.2	33.13
0	71.0	110
15	71.6	32.77
30	73.2	46.86
45	74.8	58.01
60	79.6	69.10
75	86.8	80.68
90	103.2	96.37

m_s = 0.8008 (LHS); 0.8201 (RHS)

m_o = 0.8222 (LHS); 0.8888 (RHS)

c_s = 23.54 (LHS); 20.91 (RHS)

c_o = 20.83 (LHS); 19.66 (RHS)

Range of Hardness = 71.0 - 103.2 kg-mm^{-2}

Reference direction for A = $\langle 110 \rangle$

Ref. /1/, Pg.215

TABLE 6.3 (x)

Crystal : Iron
Temp : R.T.
Plane : (100)

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
44	106	68.29
40	104	64.49
36	100	60.00
32	94	54.84
28	90	50.12
24	84	44.89
20	80	40.00
16	78	35.33
12	76	30.19
8	74	24.33
4	73	17.08
0	72	-
4	73	17.08
8	75	24.49
12	79	30.79
16	84	36.66
20	90	42.43
24	95	47.75
28	100	52.92
32	103	57.41
36	105	61.48

m_s = 1.2619 (LHS); 1.3821 (RHS)
m_o = 1.25 (LHS); 1.425 (RHS)
c_s = 14.21 (LHS); 13.58 (RHS)
c_o = 15.0 (LHS); 12.875 (RHS)
Range of Hardness = 72 - 106 $kg\text{-}mm^{-2}$
Reference direction for A = $\langle 110 \rangle$
Ref. /1/, Pg.214

TABLE 6.3 (xi)

Crystal : Iron
Temp : R.T.
Plane : (110)

Orientation A in degrees	Knoop Hardness H in $kg\text{-}mm^{-2}$	$\sqrt{HA}$
90	108	98.59
75	103	87.89
60	89	73.08
45	83	61.11
30	79	48.68
15	76	33.76
0	75	-
15	76	33.76
30	80	48.98
45	83	61.11
60	91	73.89
75	103	87.89
90	108	98.59

m_s = 0.8643 (LHS); 0.8546 (RHS)

m_o = 0.8888 (LHS); 0.8947 (RHS)

c_s = 21.81 (LHS); 22.50 (RHS)

c_o = 20.838 (LHS); 20.39 (RHS)

Range of Hardness = 75 - 108 $kg\text{-}mm^{-2}$

Reference direction for A = $\langle 110 \rangle$

Ref. /1/, Pg.214

TABLE 6.4(a)

$NaNO_3$

$\log T_q$	$\log T_q \sqrt{\bar{H} A}$ 13°	26°	39°	52°	65°	78°
2.4742	3.6914	3.8084	3.8898	3.9532	4.0185	4.0940
2.5353	3.7469	3.8759	3.9735	4.0231	4.1036	4.153
2.5943	3.7802	3.9192	4.0002	4.0696	4.1496	4.1942
2.6464	3.8497	3.9630	4.0476	4.1150	4.1955	4.2446
2.6928	3.8697	3.9688	4.0486	4.1298	4.1795	4.2425
Slope: m_1	0.8	0.8	0.8	0.82	0.79	0.8
C_1	52.48	70.79	84.33	88.00	125.45	131.82
$P=2(1-m_1)$	0.4	0.4	0.4	0.36	0.42	0.4
*K=(1-m)	0.175	0.162	0.125	0.150	0.182	0.196
* C	57.21	45.43	35.53	43.15	59.07	68.11

* K, C values are taken from the plots of $\log \bar{H}T_q$ Vs. $\log T_q$

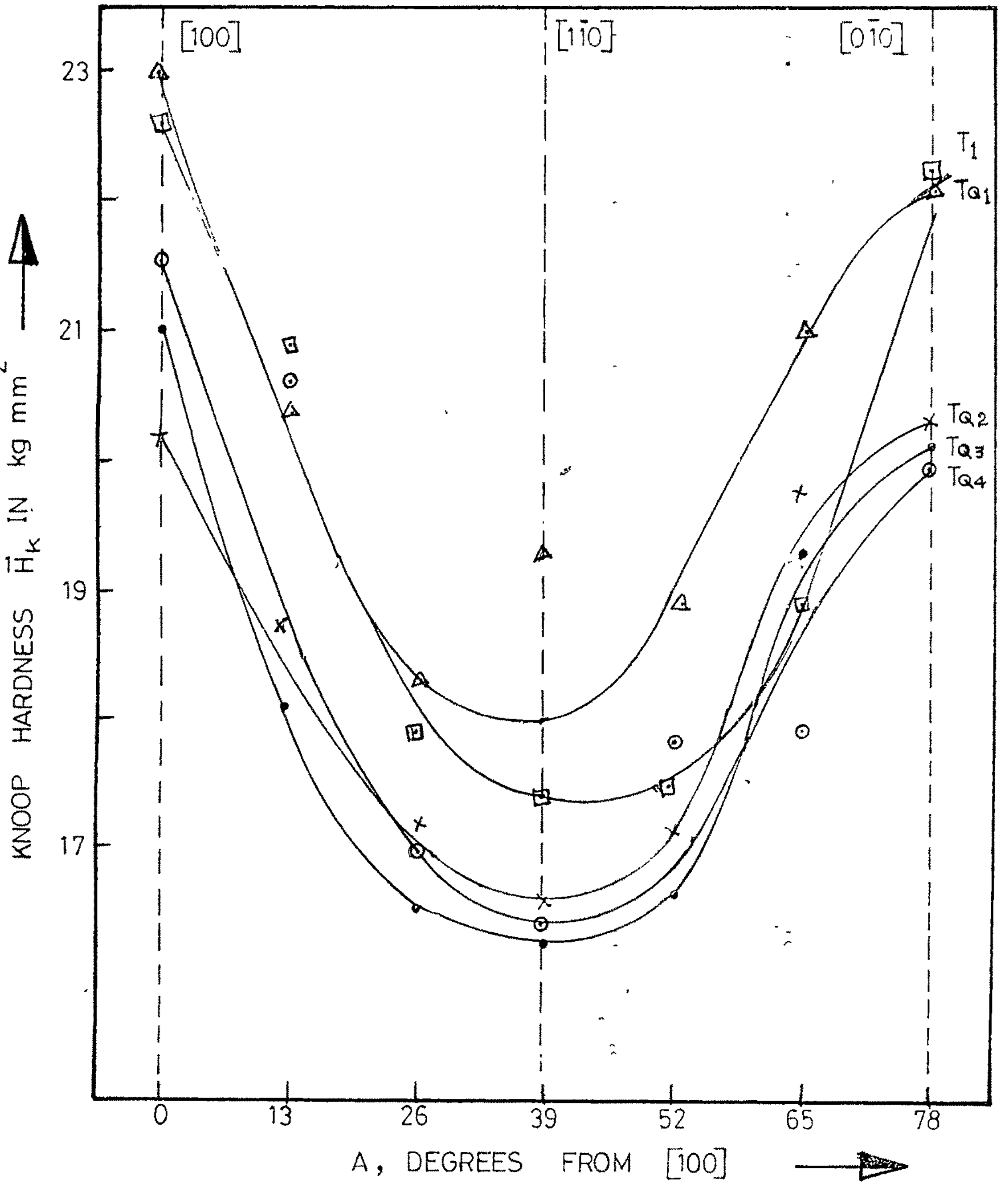
Temp °K T_q	$T_q^{(P-K)}$	$\dfrac{\left(\frac{C_1^2}{A}\right)}{C}$
298	3.603	3.702
343	4.012	4.241
393	5.1696	5.1646
443	4.5877	3.453
493	3.8640	4.1019

TABLE 6.4(b)

$CaCO_3$

Orientation A in degrees	$\log T_q \rightarrow$ 2.4814	2.6972	2.7581	2.8438	2.8881	Slope m_1	C_1	$p=2(1-m_1)$
	← $\log T_q \sqrt{\bar{H} A}$ →							
7	3.9474	4.1779	4.2465	4.3351	4.3853	1.0769	19.09	-0.1538
14	4.1112	4.3241	4.3920	4.4797	4.5289	1.08	26.06	-0.16
21	4.1827	4.4043	4.4754	4.5598	4.6094	1.0769	31.91	-0.1538
28	4.2429	4.4595	4.5298	4.6175	4.6658	1.08	36.31	-0.16
35	4.2566	4.4936	4.5650	4.6603	4.7064	1.0666	42.35	-0.1332
42	4.2956	4.5333	4.6046	4.6998	4.7461	1.0666	46.43	-0.1332
49	4.3220	4.5811	4.6513	4.7391	4.7873	1.05	56.89	-0.1
56	4.4055	4.6172	4.6884	4.7728	4.8223	1.028	70.34	-0.056
63	4.4453	4.6506	4.7185	4.8064	4.8554	1.033	75.43	-0.066
70	4.4658	4.6779	4.7465	4.8351	4.8854	1.055	69.57	-0.11
78	4.4891	4.7084	4.7745	4.8649	4.9151	1.044	79.45	-0.08

Plot of knoop hardness number (H) vs orientation (A) of the longer diagonal of knoop indenter with respect to direction (100) on (100) cleavage planeo of $NaNO_3$ single crystal at room temp. T & quenching temp. T_{Q_1}, T_{Q_2}, T_{Q_3}, T_{Q_4}.



T_1 = 298° k
T_{Q_1} = 343° k
T_{Q_2} = 393° k
T_{Q_3} = 443° k
T_{Q_4} = 493 k

Fig: 6·1 (a)

Plot of knoop hardness number (H) vs orientation (A) of the longer diagonal of knoop indenter with respect to direction (100) on (100) cleavage plane of $CaCO_3$ single crystal at room temp. T_1 & quenching temp. T_{Q_1}, T_{Q_2}, T_{Q_3}, T_{Q_4}.

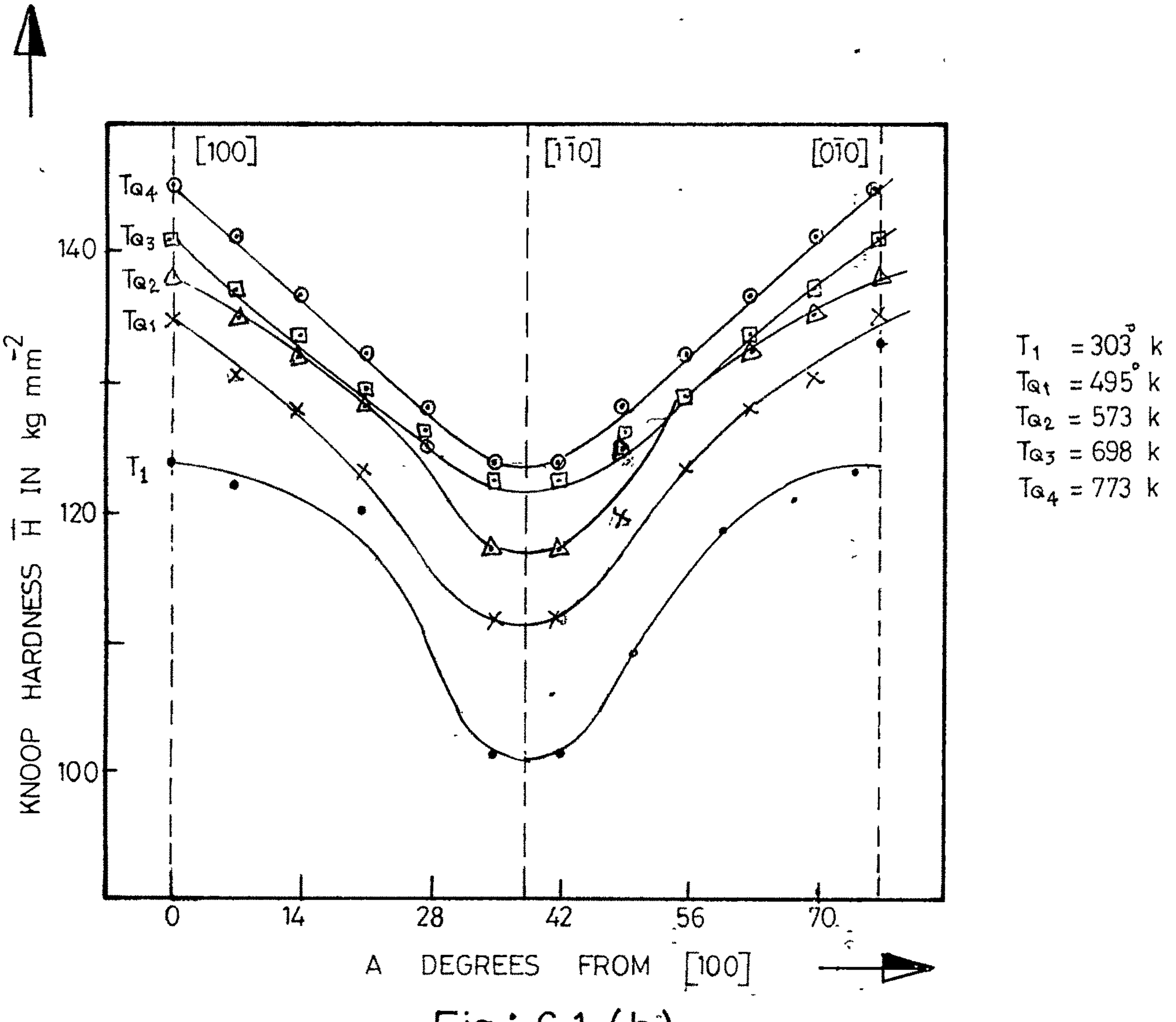
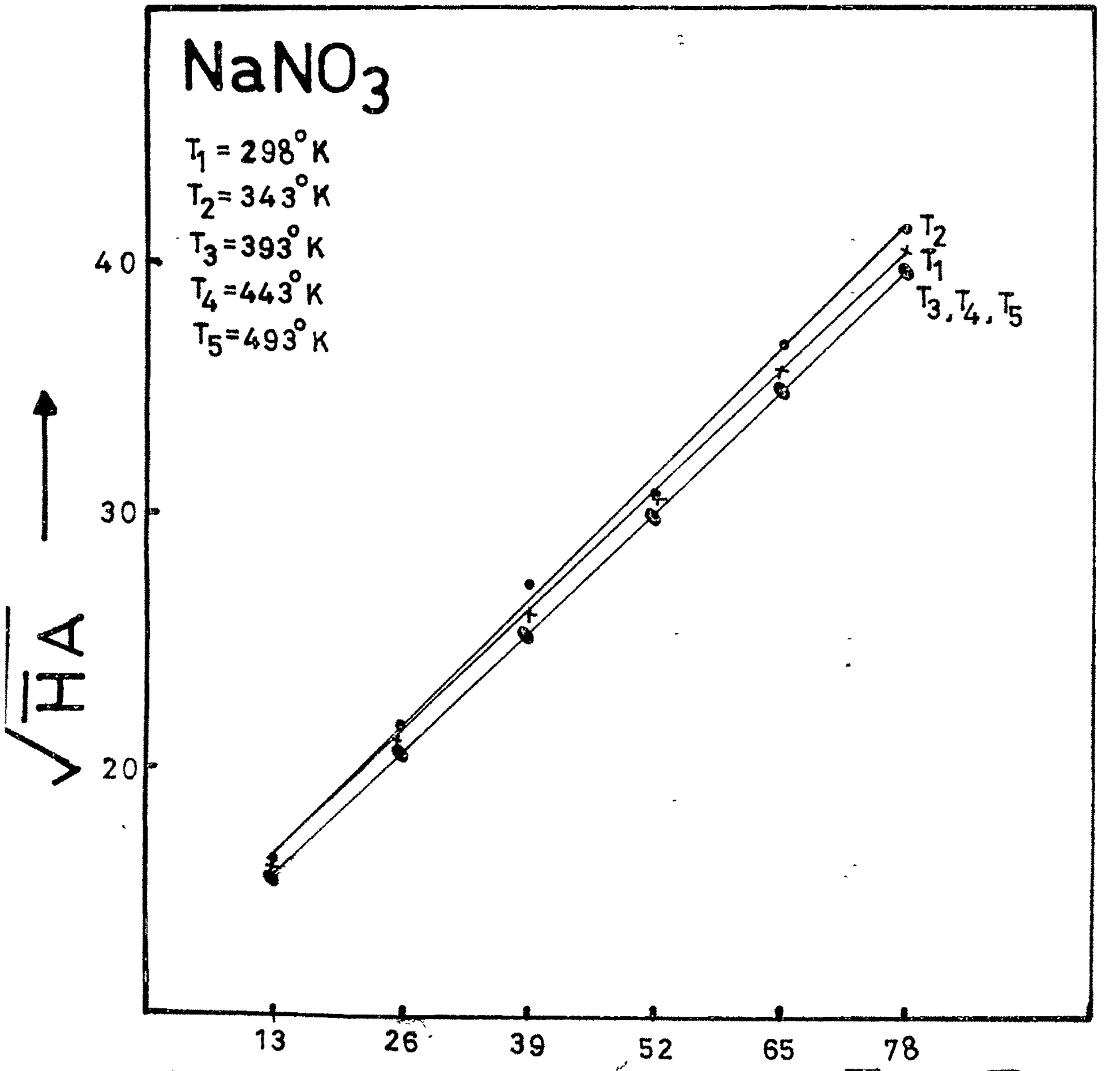
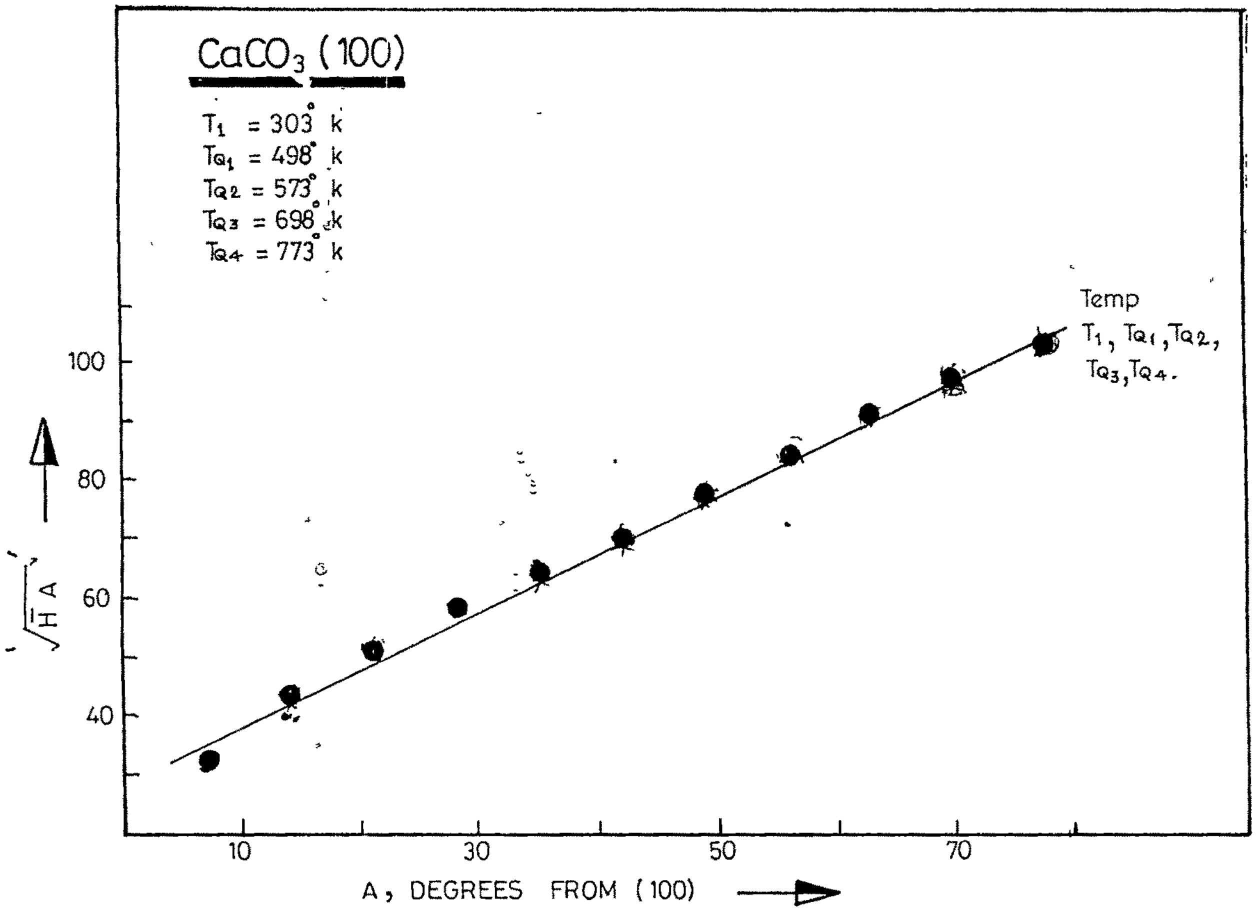


Fig : 6·1 (b)



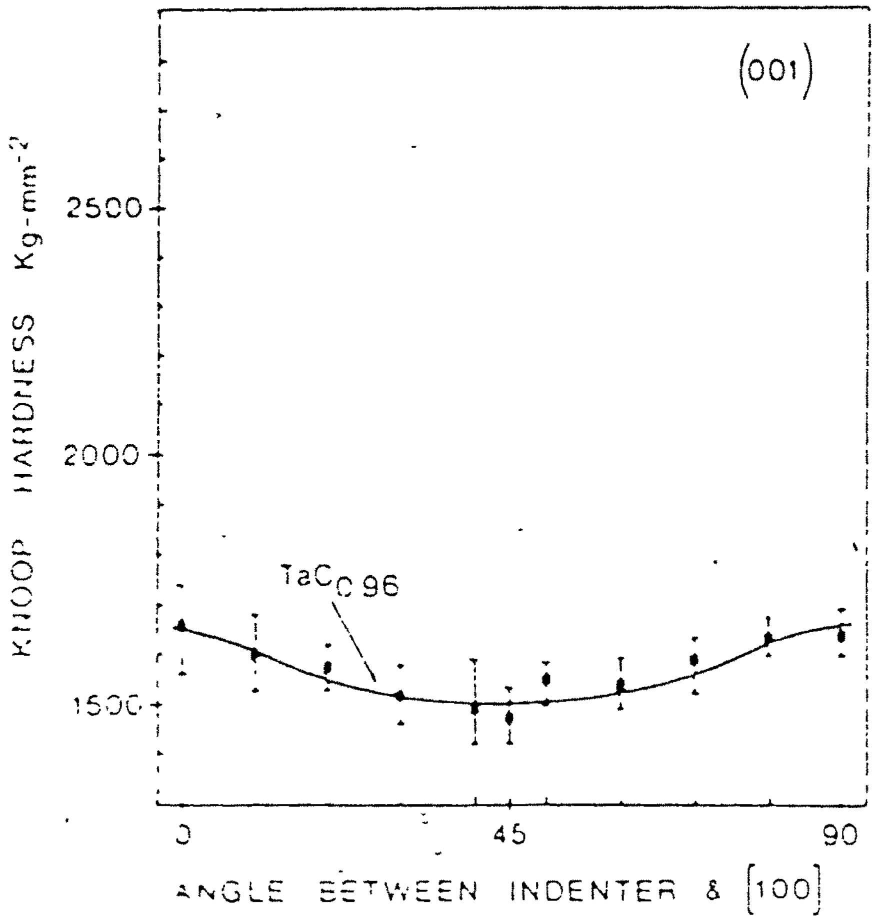
PLOT OF $\sqrt{HA}$ Vs A FOR (100) AT ROOM TEMPERATURE T_1 AND QUENCHING TEMPERATURES T_2, T_3, T_4, T_5.

FIG: 6.2 (a)



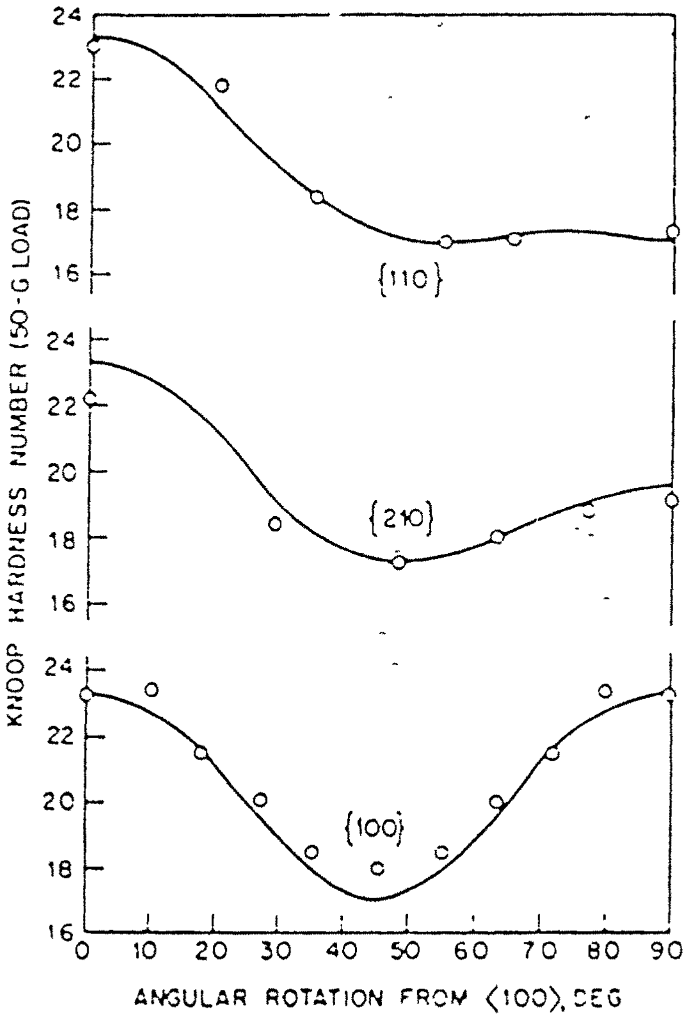
Plot of $\sqrt{\bar{H}A}$ vs A, for (100) at room temp. T_1 & quenching temps. T_{Q1}, T_{Q2}, T_{Q3}, T_{Q4}.

Fig.: 6·2 (b)



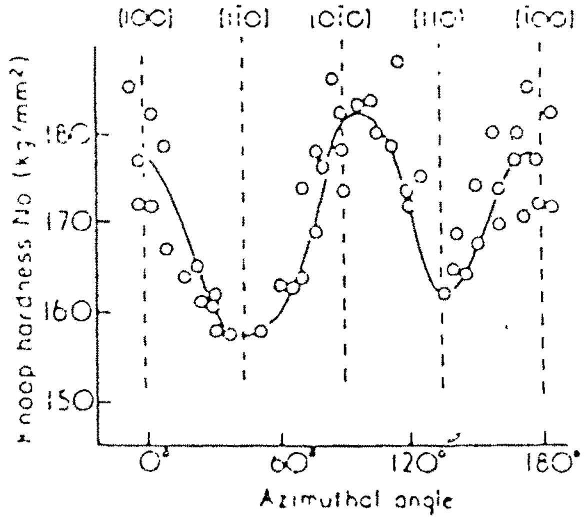
Variation of Knoop hardness (100-gram load) with indenter orientation on (100) surface of TaC.

FIG: 6.3a(i)



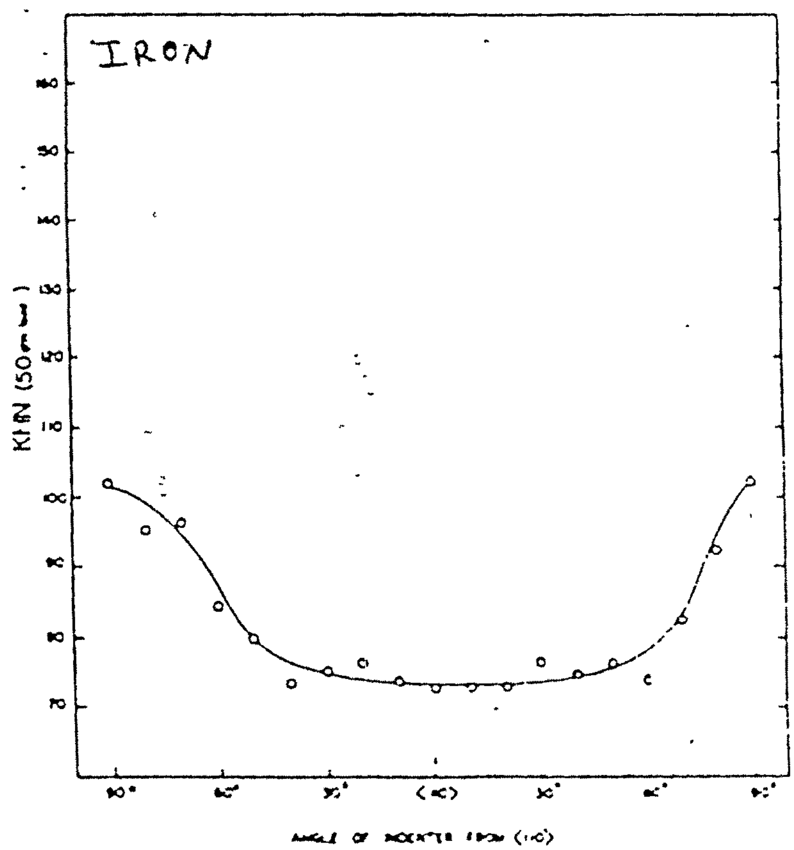
Theoretical and experimental Knoop hardness anisotropy for three crystallographic planes in aluminum as a function of long-axis orientation.

FIG: 6.3a (ii)



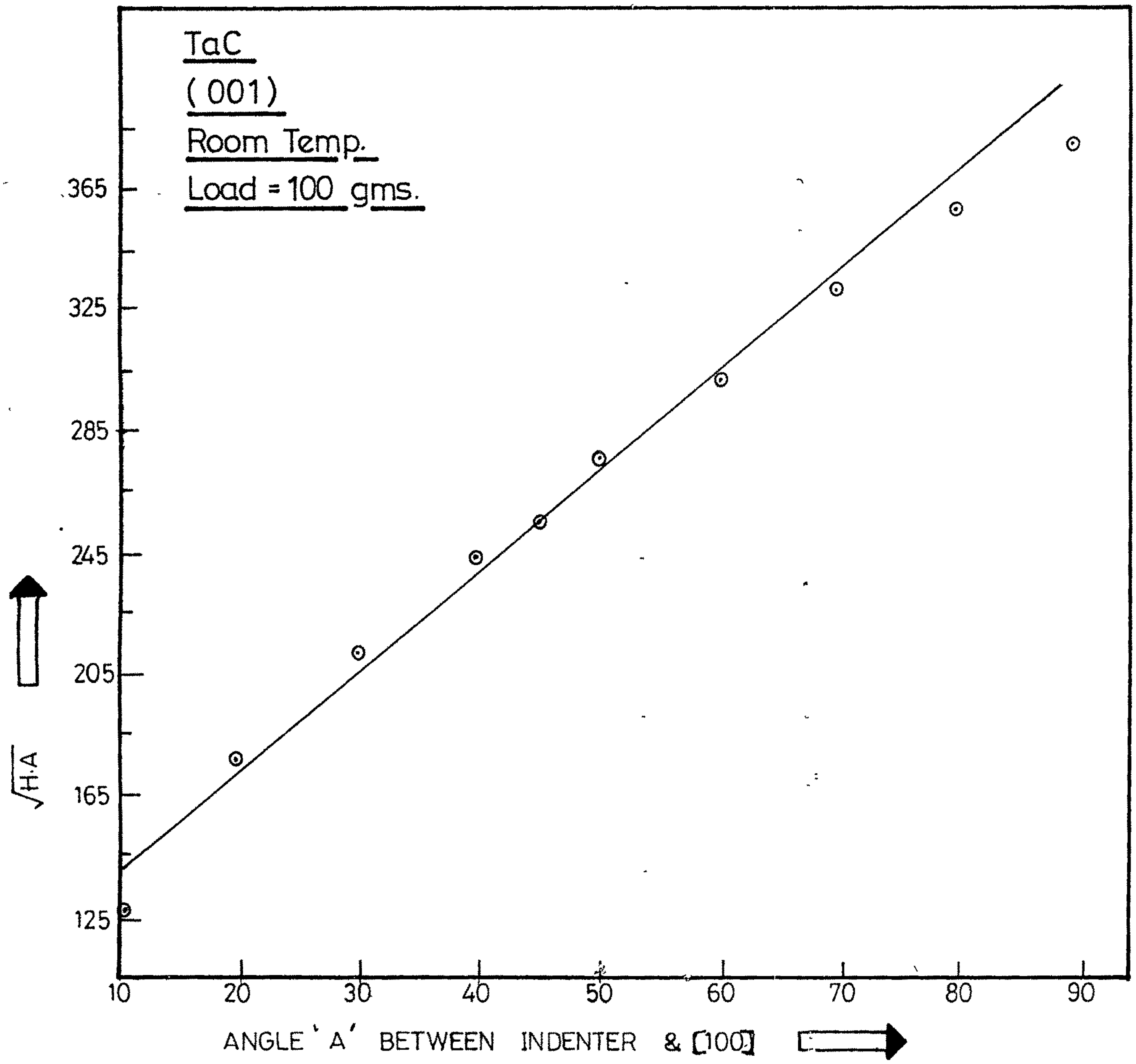
Anisotropy in the hardness of calcium fluoride crystals on the (001) plane

FIG: 6·3 a (iii)



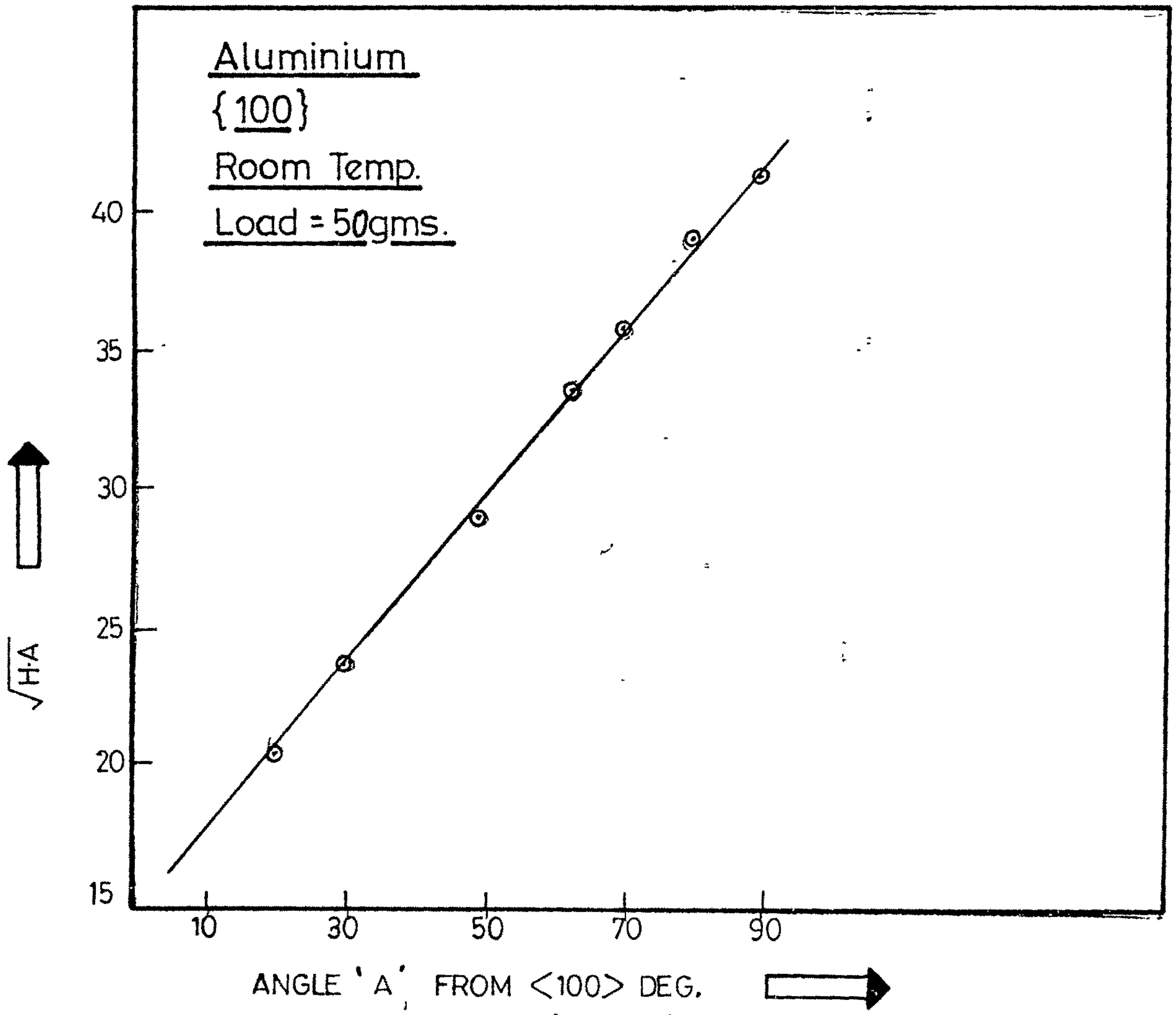
Microhardness anisotropy of as-grown and axisymmetrically drawn crystal of near-(110) orientation

FIG: 6.3 a (iv)



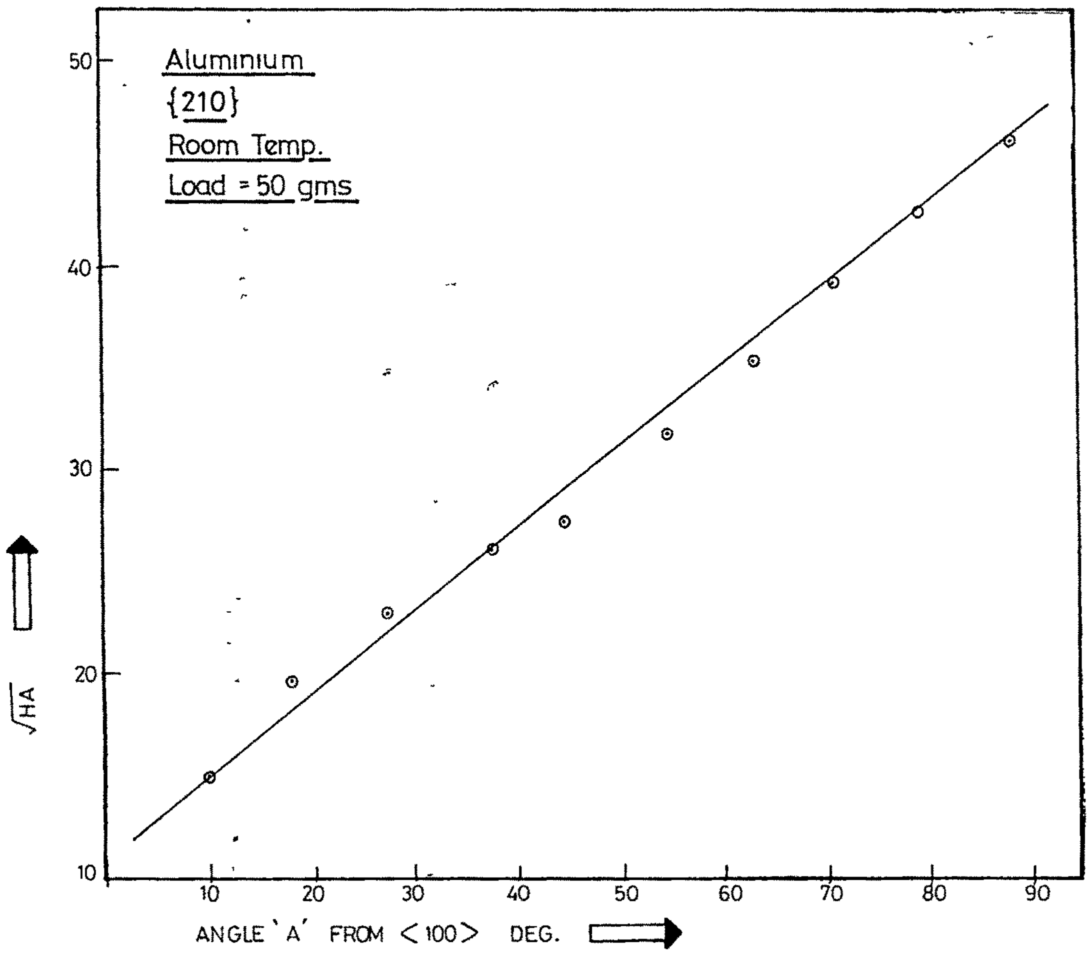
PLOT OF '√H·A' vs 'A'.

Fig.: 6·3B(I)



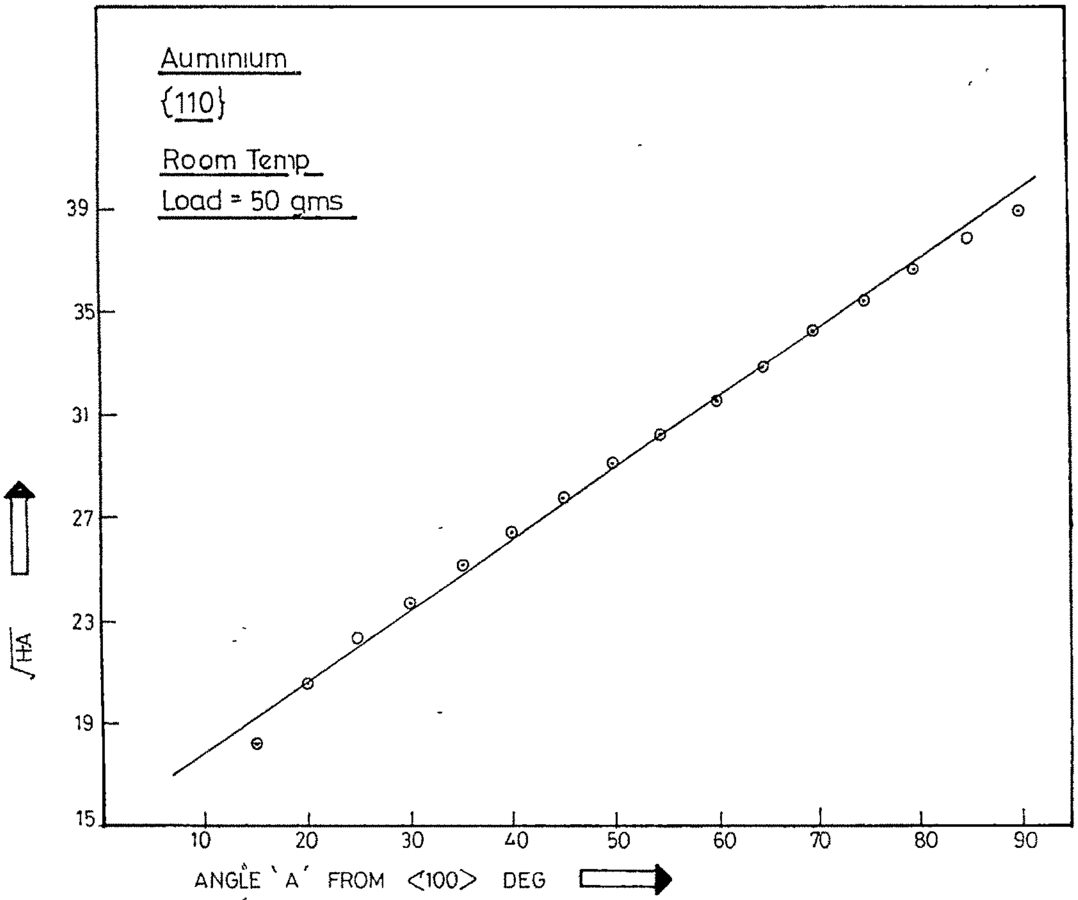
PLOT OF '$\sqrt{H \cdot A}$' vs 'A'

Fig.: 6·3B(II)



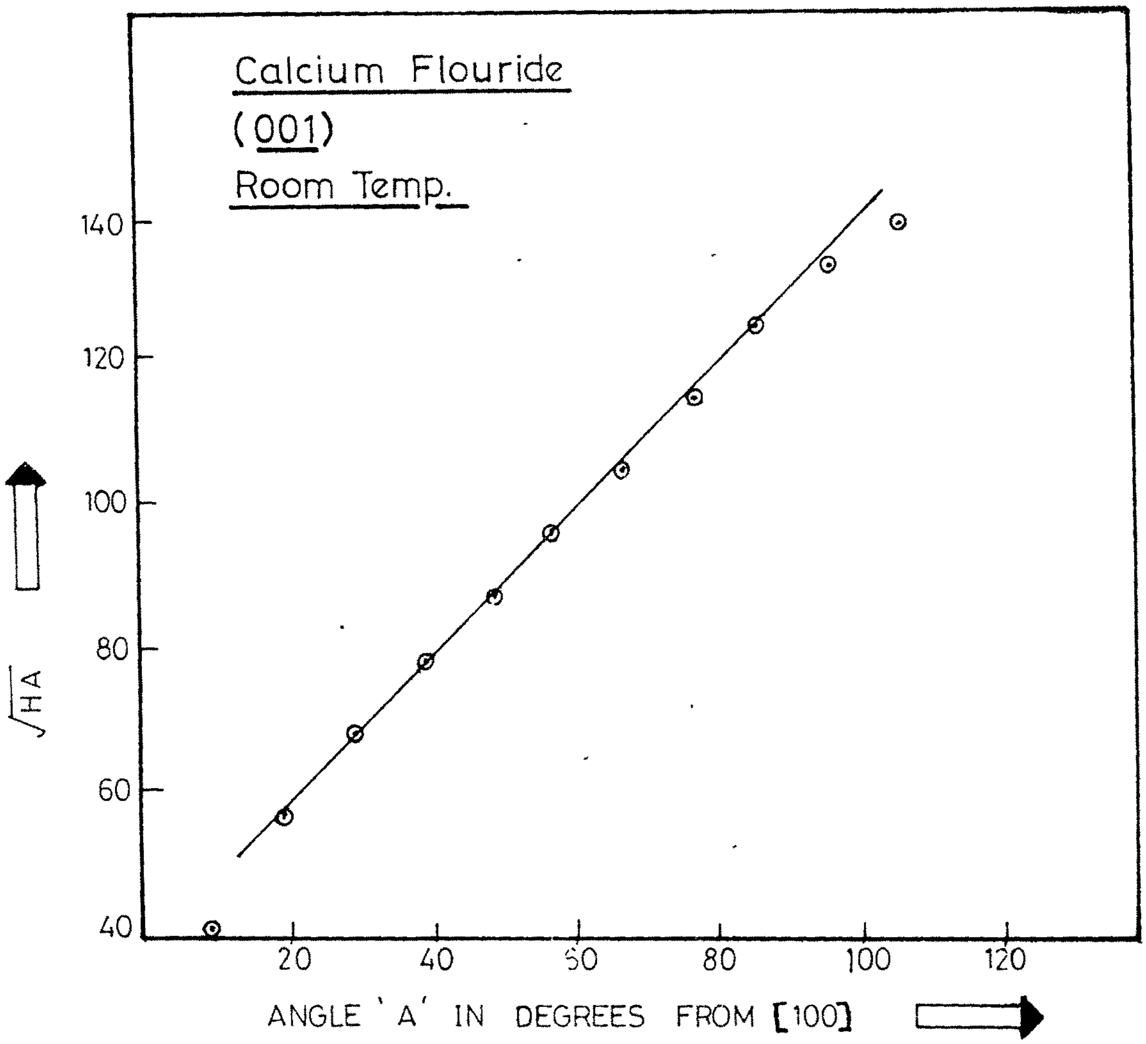
PLOT OF '√H·A' vs 'A'.

Fig.: 6·3B (III)



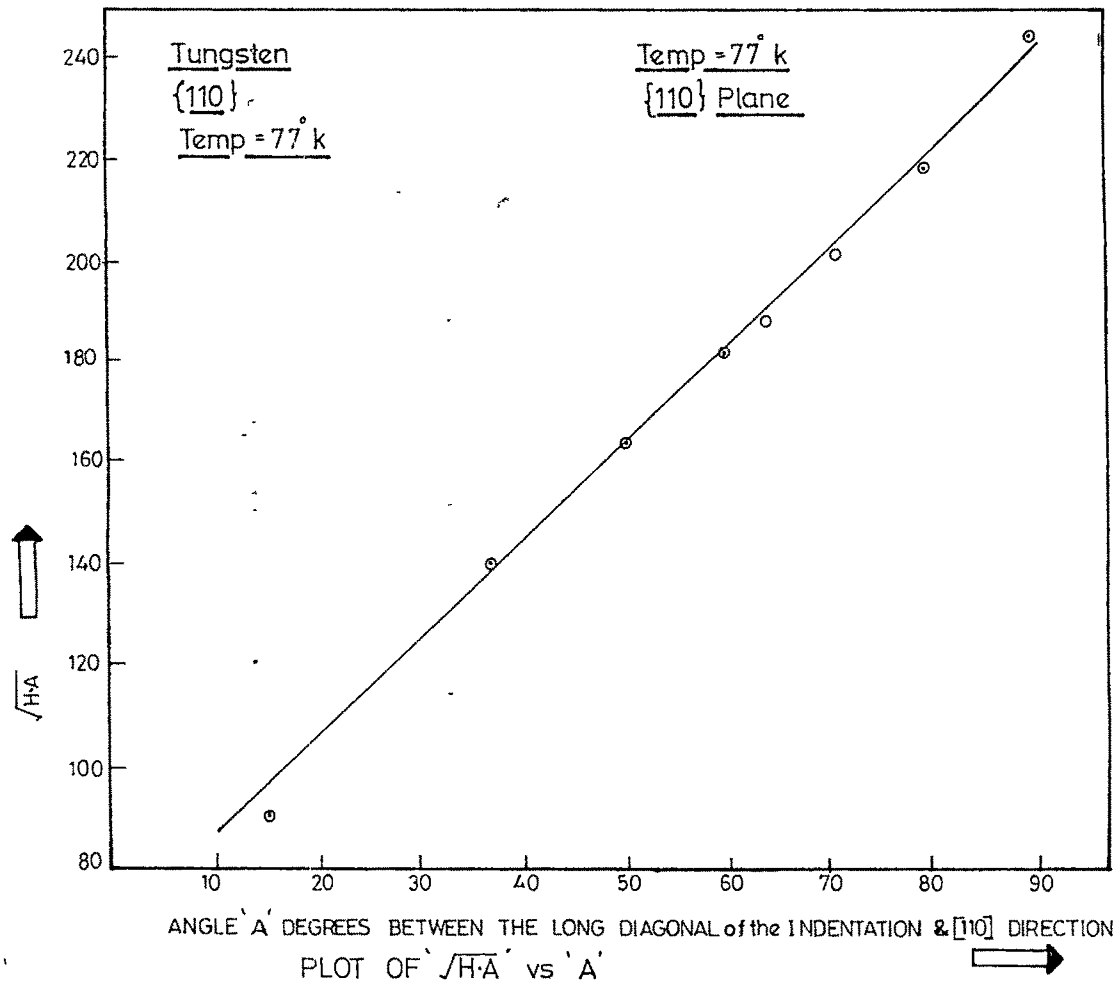
PLOT OF '√HA' vs 'A'.

Fig : 6·3 B(IV)



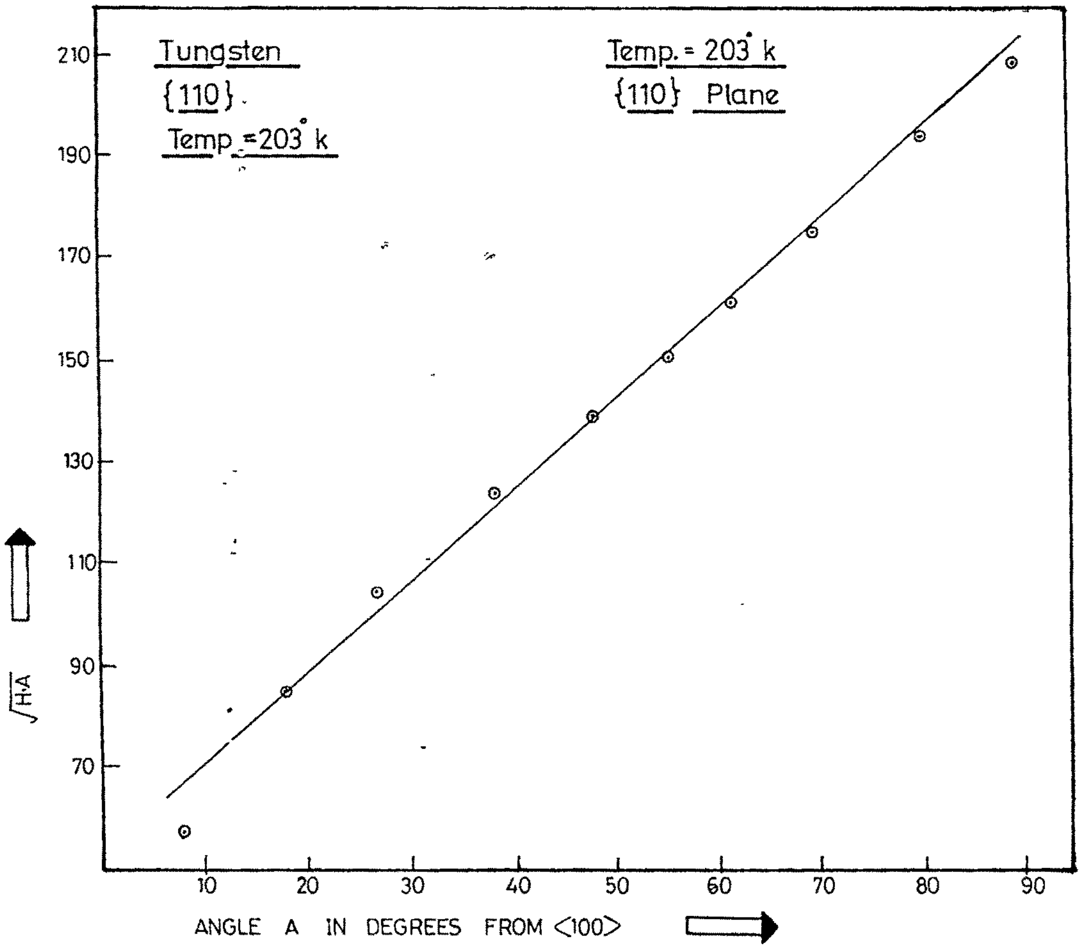
PLOT OF '√H·A' vs 'A'

Fig.:6·3B(V)



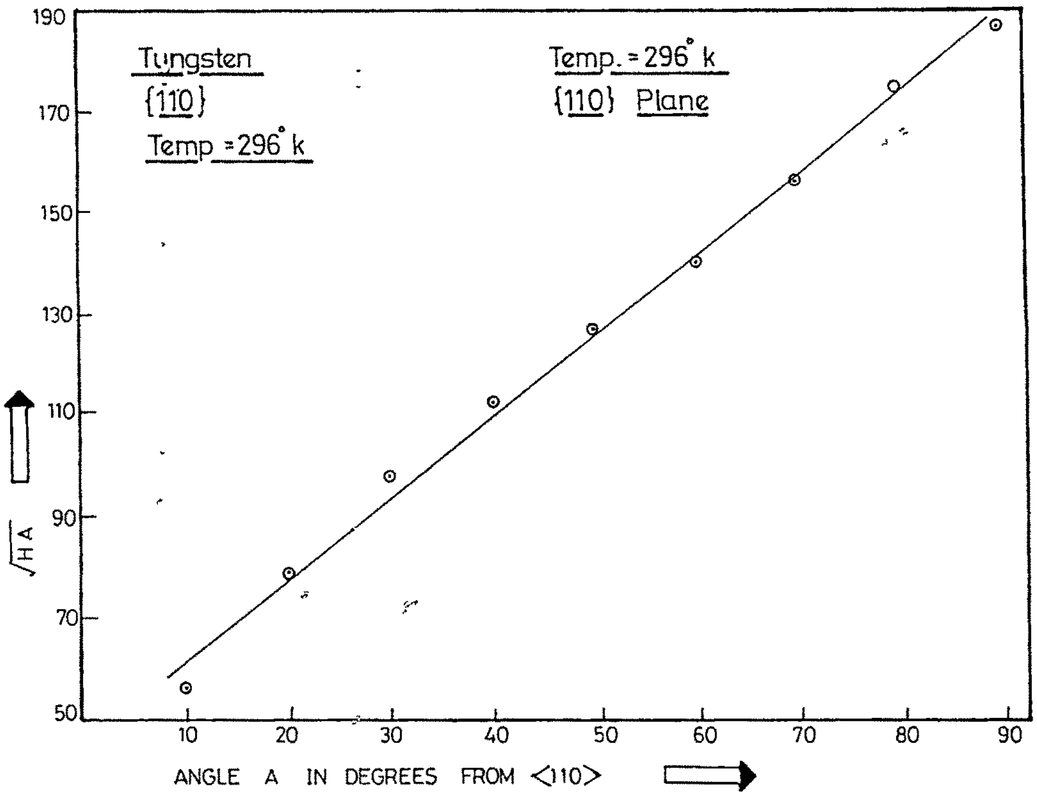
PLOT OF '√H·A' vs 'A'

Fig. 6·3·B(VI)



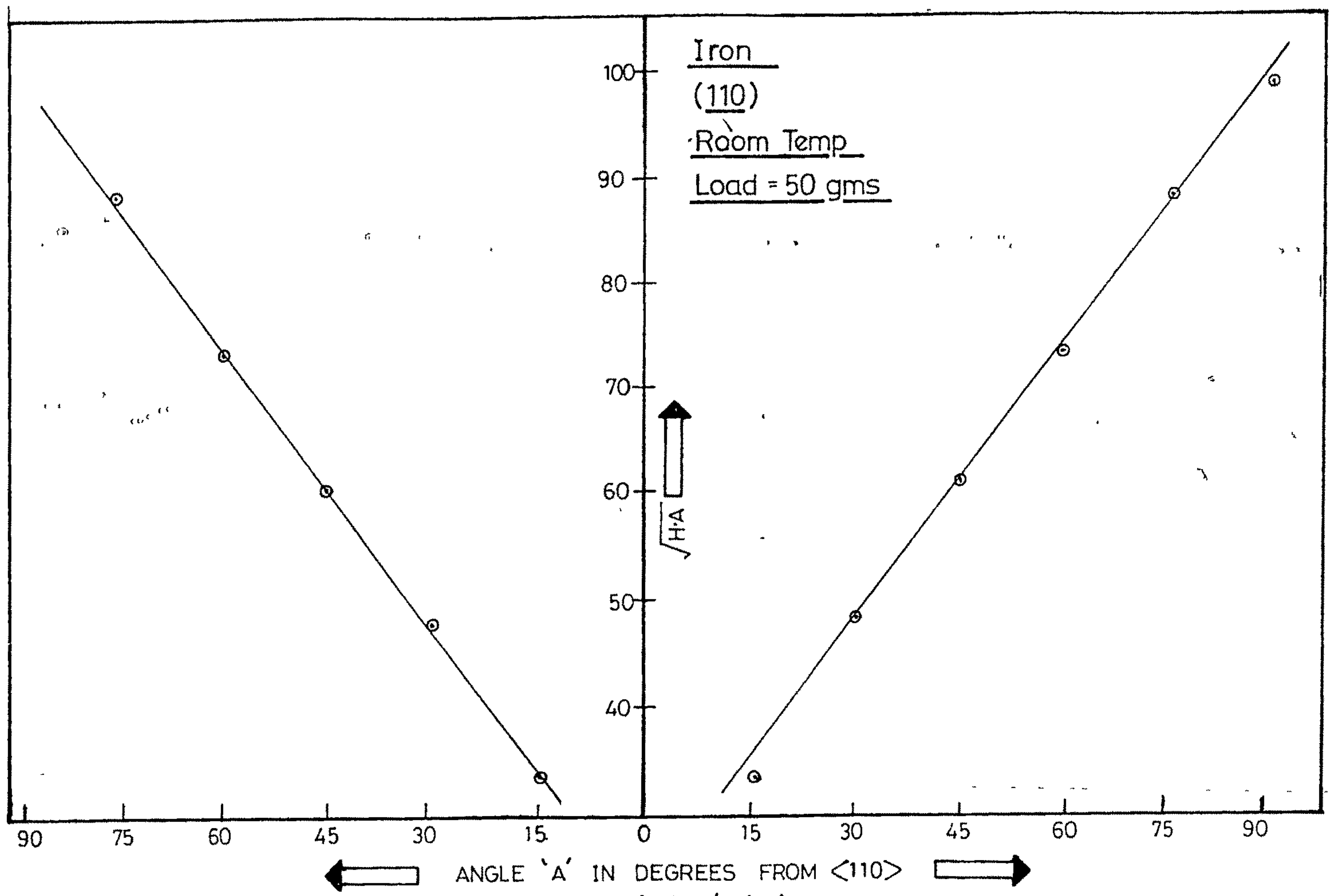
PLOT OF '√HA' vs 'A'

Fig. 6·3B(VII)



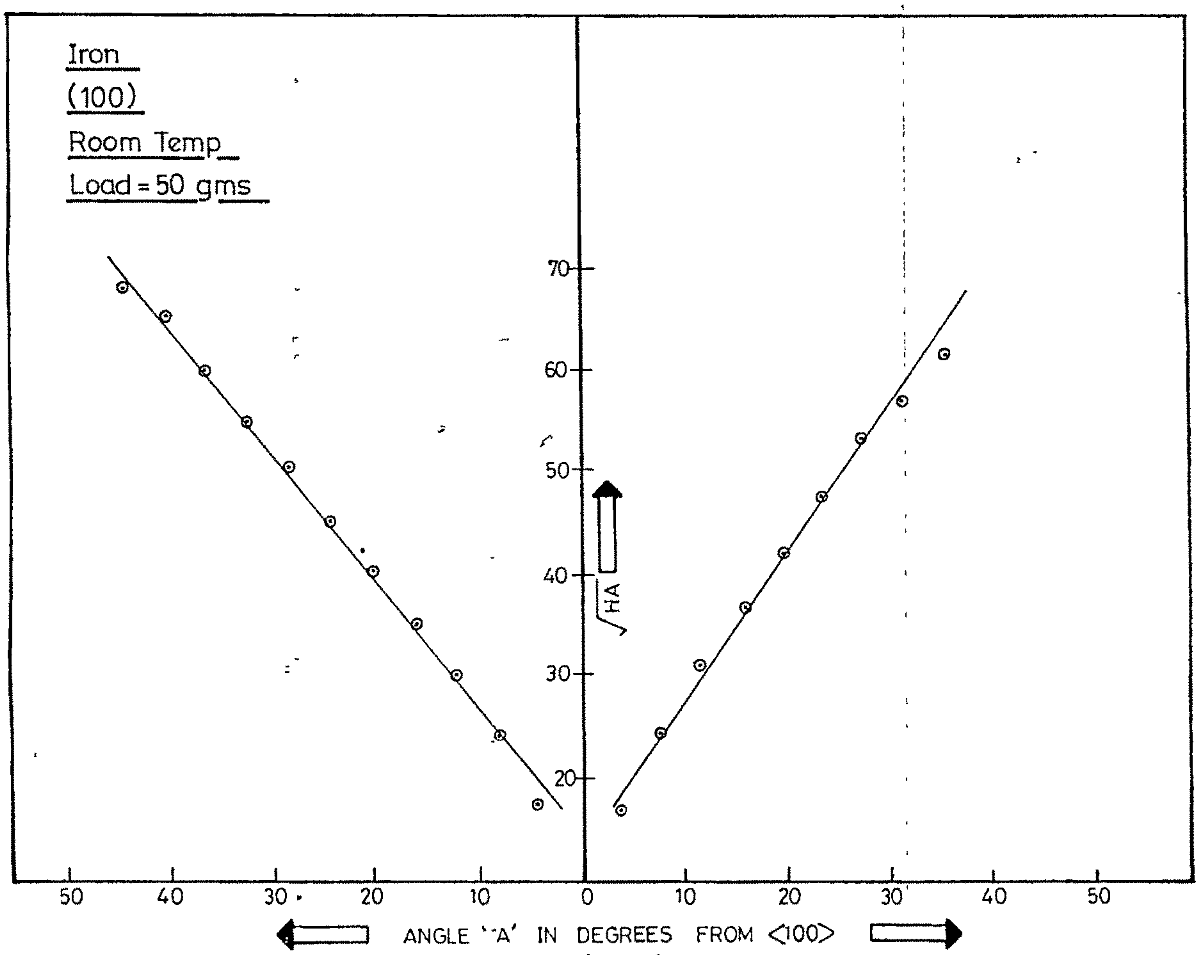
PLOT OF '√HA' vs 'A'

Fig. : 6·3 B(VIII)



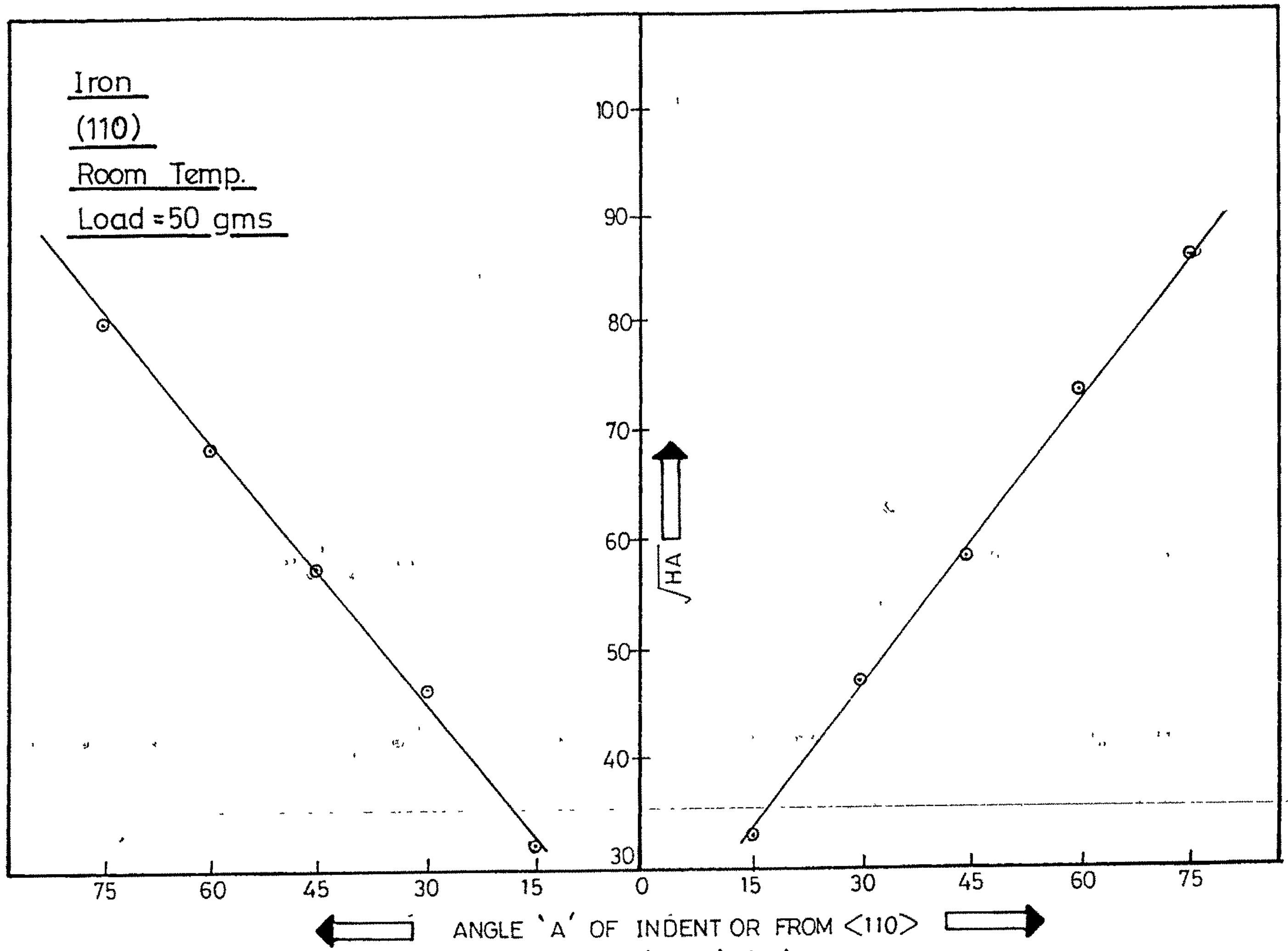
PLOT OF $\sqrt{H \cdot A}$ vs 'A'

Fig.: 6·3B (IX)



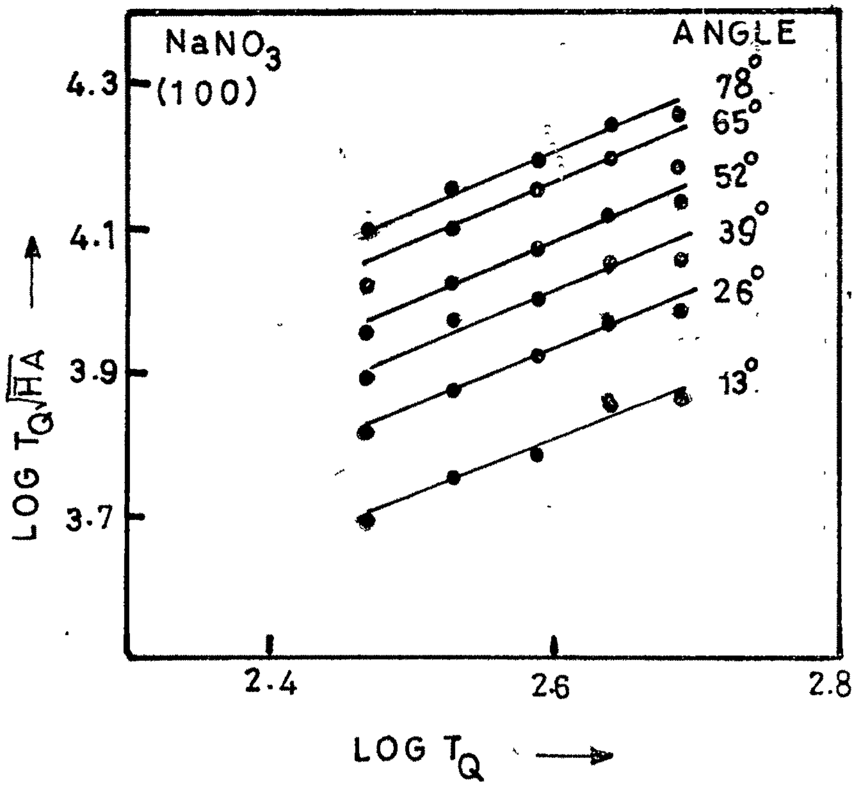
PLOT OF '√HA' vs 'A'.

Fig.: 6·3 B (X)



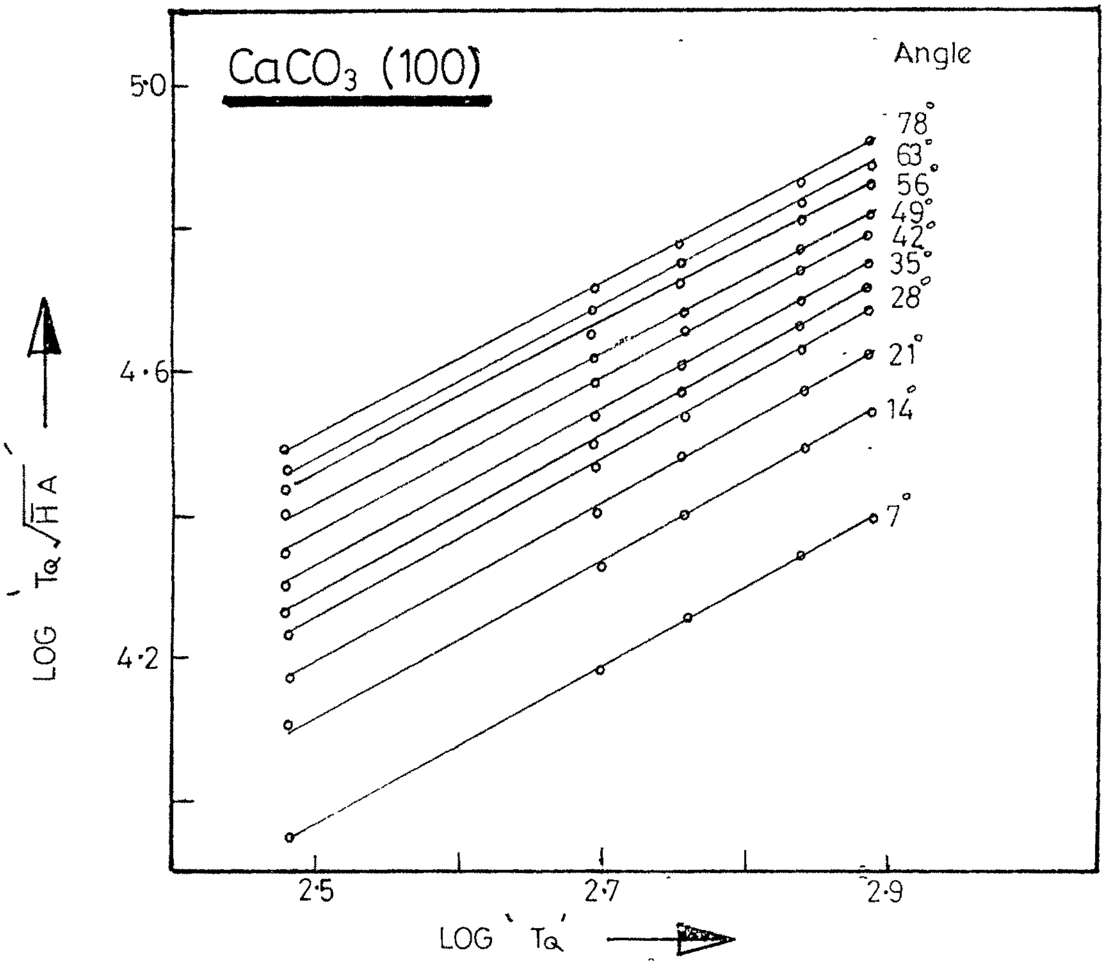
PLOT OF '√HA' vs 'A'

Fig.: 6·3 B (XI)



PLOT OF LOG $T_Q\sqrt{HA}$ Vs LOG T_Q

FIG : 6.4A



Plot of Log 'TQ √HA' vs Log 'TQ'

Fig. 6·4 (B.)

REFERENCES

1. Westbrook, J.H. and Conrad, H. Quoted in "The Science of Hardness Testing and its Research Applications" American Society for Metals, Metal Park, Ohio (1973).