Chapter 2

Generation Scheduling

2.1 Introduction

Generation scheduling is now well understood and various methods have been successfully developed for the same. Some of the well known methods are Lagrangian multiplier or incremental fuel cost, first order gradient, second order gradient and dynamic programming [6,7,8,22,59] . Other researchers have solved classical dispatch problem using Linear programming and Quadratic programming, technique. Most of the load dispatch programmes developed, use set of B-coefficients or calculate incremental transmission losses from classical load flows. Single area dispatch consists of plants with major constraints of equality and inequality constraints. Equality constraint refers to balance between demand and generation. Inequality constraint refers to minimum and maximum limit of generation of each unit. Normally, there are number of units at a plant and an estimated power at a plant is to be distributed among units participating in generation scheduling. Conventionally, incremental fuel cost method is used to estimate generation on each unit. Concept of Multiarea dispatch is presented by Shoults et al [63] and classical method is developed by Happ [131]. Recently Ramaraj has also developed a method for interchange evaluation using equivalent cost function technique [21]. In this Chapter recursive technique of dynamic programming is used for basic formulation for generation allocation and for cost evaluation. The main thurst is to represent a set of units by an equivalent cost function. Development of formulation is stagewise, where stages refer to units. At the end of each stage of inclusion of units, two expressions are formed. The first expression provides generation on the units and the second expression gives equivalent cost function of units so far included. The concept is further extended to single area dispatch, multiarea dispatch, as well as to estimation of generation for units with multiple fuel options. Conventional method of equal incremental fuel cost for multiarea dispatch is conceptually very simple and straight forward but is very tedious for large number of units. However, dynamic programming method developed for the multiarea dispatch is also simple and

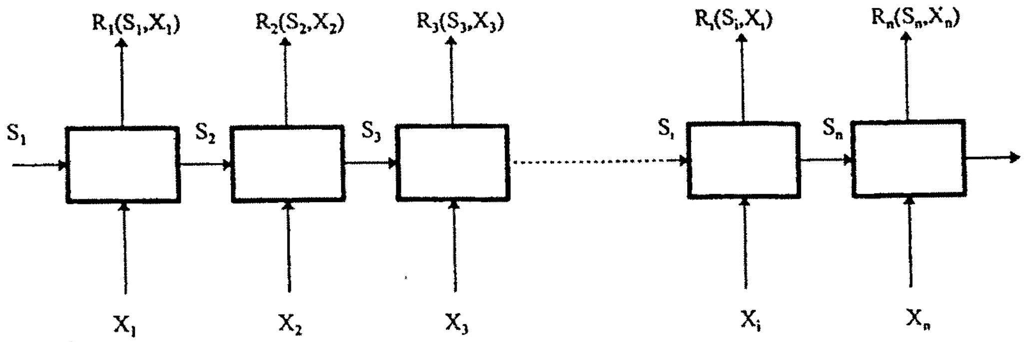

Figure 2.1: Stage Representation

straight forward but is iterative. Iterations are required to include effects of bounds. However, formulation is generalized and avoids complexity of creating λ tabulation for each load level as presented by Shoults et al [63]. Further, Equivalent cost method presented by Ramaraj et al [17] is also very complex, as compared to dynamic programming technique. For generation scheduling with multiple fuel options, Lin & Viviani [25] starts with initial estimation of λ and consequently generation on each unit. Thus, optimum selection of fuel number of each unit is iteratively estimated. But, in the technique developed using DP, first stage refers to first fuel number assigned to each unit and fuel numbers are increased stagewise optimally and the range of operation is estimated simultaneously . The process is continued till all fuels of all units are included. The tabular result can be repeatedly used for generation calculation. Hence for every demand rough estimation of generation allocation is avoided and thus the method is noniterative.

2.2 Basic Formulation

Conventionally, the Dynamic Programming (DP) is defined as a multistage, multistate, multi-decision process. There are number of practical problems which come under this category. Many Power system problems too are solved using dynamic programming by many researchers. Basically, dynamic programming technique decomposes a multistage decision process into a serial single stage process, wherein each stage called subproblem is solved successively such that optimal solution of the original multidimensional problem can be obtained from optimal solution of single stage subproblems. For example, a serial multistage problem can be represented as shown in Figure 2.1.

where,

x_i are decision variables at each stage,
s_i input variable at each stage i, and
R_i return function due to decision at i^{th} stage and input s_{i-1}

It is clear from the above illustration that at the end of every stage, there is a return

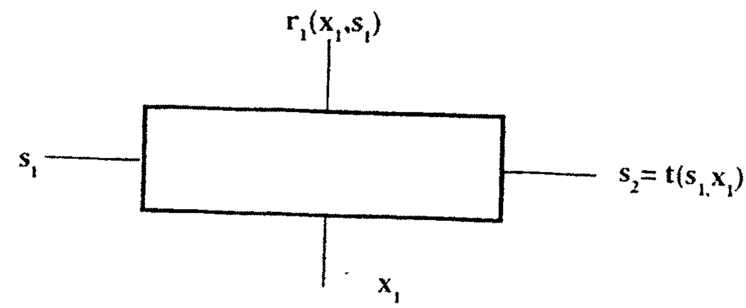

Figure 2.2: Single Stage Representation

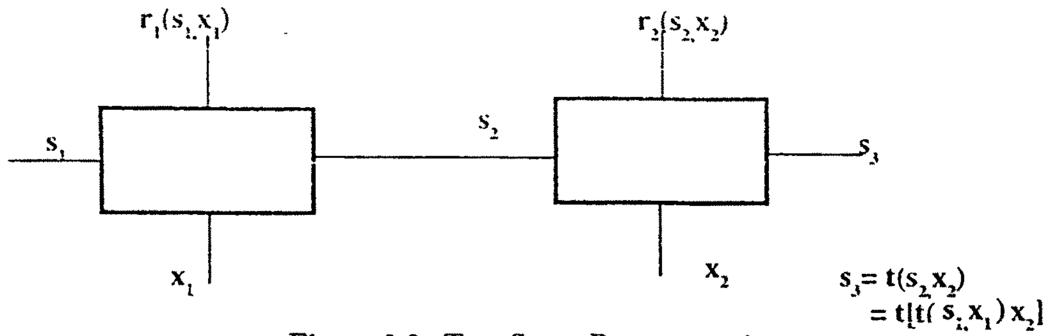

Figure 2.3: Two Stage Representation

function R_i and the DP is used to obtain

$$opt[F] = opt[\sum_i R_i] \tag{2.1}$$

or

$$opt[F] = opt[stackrel\pi i R_i] \tag{2.2}$$

For economic operation of power system, summation of total cost is required. Hence, equation (2.1) will be used to start with the first stage represented (Fig 2.2) as

$$s_2 = t[s_1, x_1] \tag{2.3}$$

The block in Fig 2.2 represents s_2 as an output due to s_1 and x_1 and return function due to decision x_1 and input s_1 from previous stage. The second stage is represented as shown as Fig 2.3 Thus, the solution of the two stage is dependent on previous decision as well as current decision; and since output s_3 and return function R_2 represent effect two stages,the two stages are represented (Fig 2.4) by the total return function as

$$R_1 + R_2 = f_1(s_1, x_2) + f_2(s_2, x_2) \tag{2.4}$$

Since the first stage is optimized, the total return function to be minimized by taking decision on second stage is

$$opt[F_2(s_2)] = opt[R_2(x_2, s_2) + F_1(s_2)] \tag{2.5}$$

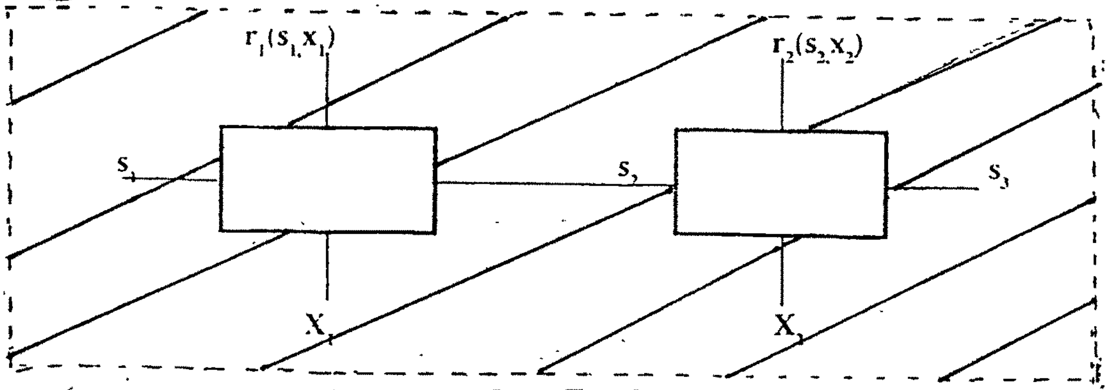

Figure 2.4: Joint Two Stage Representation

In general for n stages, the total return is

$$opt[F_n(s_n)] = opt[R_n(x_n, s_n) + F_{n-1}(s_{n-1})] \tag{2.6}$$

This is a recursive relation which is used in generation scheduling. Since the main task of generation scheduling is to obtain minimum cost of operation, the recursive relation takes the form

$$Min[F_n(D)] = Min[f_n(u) + F_{n-1}(D-u)] \tag{2.7}$$

where,
$F_n(D)$ is cost of generation of D MW on n units,
$f_n(u)$ is cost of generation of u MW on n^{th} unit, and
$F_{n-1}(D-u)$ is minimum cost of generation of $(D-u)$ MW from remaining units.

Thus, dynamic programming can be applied for generation scheduling by defining the problem of generation scheduling as a multistage, multistate, multi-decision process using equation (2.7) to obtain the minimum total cost of generation. Here, stages are referred as units, block of generation as states and allocation on each unit as decision variable.

2.3 Derivation of Equivalent Cost Function

For development of the expression, the following assumptions are made

(1) Cost functions are quadratic, and

(2) Units at each plant are available as per unit commitment.

For N unit system, a unit cost function is expressed as

$$F_j(p_j) = a_j p_j^2 + b_j p_j + c_j \qquad Rs/MW/hr \tag{2.8}$$

where,
$F_j(p_j)$ is cost of generation due to p_j MW on unit j
a_j, b_j, c_j are cost coefficients of unit j.

To obtain the minimum cost of generation of D MW from N units, stagewise calculation can be performed starting with equation (2.7). The first unit is selected as a base unit whose cost function is

$$F_1(p_1) = A_1p_1^2 + B_1p_1 + C_1 \qquad Rs/MW/hr \tag{2.9}$$

where,
$A_1 = a_1$; $B_1 = b_1$; $C_1 = c_1$

Then, the second unit from the list is selected and combination of two units for block generation of D_2 MW can be expressed using equation (2.7) as

$$Min[F_2(D_2)] = Min[f_2(p_2) + F_1(D_2 - p_2)] \tag{2.10}$$

Since, the cost functions are quadratic and differentiable, it is posible to obtain a unique value of p_2 to get minimum cost of generation. Assuming the unique value of p_2 as u_2,

$$Min[F_2] = Min[a_2u_2^2 + b_2u_2 + c_2 + A_1(D_2 - u_2)^2 + B_1(D_2 - u_2) + C_1] \tag{2.11}$$

Differentiating above equation $w.r.t.$ u_2 and equating the result to zero.

$$Min[F_2(D_2)] = \frac{d}{du_2}[a_2u_2^2 + b_2u_2 + c_2 + A_1(D_2 - u_2)^2 + B_1(d_2 - u_2) + C_1] \tag{2.12}$$

$$u_2 = A_1\frac{a_1}{(a_1 + a_2)} + \frac{(B_1 - b_2)}{2(A_1 + a_2)} \tag{2.13}$$

$$u_2 = X_2D_2 + Y_2 \tag{2.14}$$

where,

$$X_2 = \frac{A_1}{(A_1 + a_2)};\ Y_2 = \frac{(B_1 - b_2)}{2(A_1 + a_2)} \tag{2.15}$$

Substituting in equation (2.11), the value of u_2 obtained from equation (2.14),

$$Min[F_2(D_2)] = a_2[X_2D_2 + Y_2]^2 + b_2[X_2D_2 + Y_2] + c_2 \tag{2.16}$$
$$+A_1[D_2 - (X_2D_2 + Y_2)]^2 + B_1[D_2 - (X_2D_2 + Y_2)] + C_1$$

$$F_2(D_2) = A_2D_2^2 + B_2D_2 + C_2 \qquad Rs/MW/hr \tag{2.17}$$

where,

$$A_2 = X_2a_2 \tag{2.18}$$

$$B_2 = b_2 + 2Y_2a_2 \tag{2.19}$$

$$C_2 = C_1 + c_2 - 0.5Y_2(B_1 - b_2) \tag{2.20}$$

Equation (2.17) represents minimum cost of generation of D_2 MW due to two units selected from the list. This is a simple solution with optimal decision. Based on this solution, next unit from the list is selected for optimization and cost functions similar to equation (2.17) are obtained. Subsequently all units are selected one by one and cost functions are obtained up to the last unit. Hence, generalized expressions as shown below are obtained for combinations any number of units for allocating D_j MW among j units. That is,

$$F_j(D_j) = A_jD_j^2 + B_jD_j + C_j \qquad Rs/MW/hr \tag{2.21}$$

$$u_j = X_jD_j + Y_j \tag{2.22}$$

where

$$X_j = \frac{A_{j-1}}{(A_{j-1} + a_j)} \tag{2.23}$$

$$Y_j = \frac{B_{j-1} - b_j}{2(A_{j-1} + a_j)} \tag{2.24}$$

$$A_j = X_ja_j \tag{2.25}$$

$$B_j = b_j + 2Y_ja_j \tag{2.26}$$

$$C_j = C_{j-1} + c_j - 0.5Y_j(B_{j-1} - b_j) \tag{2.27}$$

and for total N unit system

$$F_N(D_N) = A_ND_N^2 + B_ND_N + C_N \tag{2.28}$$

where,

D_N is total generation to be allocated on N units. Now distribution of total generation (MW) on each unit can be calculated as

$$\begin{aligned} lu_N &= X_ND_N + Y_N \\ D_{N-1} &= D_N - u_N \\ u_{N-1} &= X_{N-1}D_{N-1} + Y_{N-1} \\ &\vdots \\ D_2 &= D_3 - u_3 \\ u_2 &= X_2D_2 + Y_2 \\ D - 1 &= D_2 - u_2 \\ u_1 &= D_1 \end{aligned} \tag{2.29}$$

The following observations can be made from these equations (2.14) to (2.20).

1) From equation (2.15), the critical value of D_j can be predicted which may violate the bounds $p_{j_{min}}$ or $p_{j_{max}}$. If these bounds are violated, the units are fixed at their boundary values with a correction on demand as follows.

   $D_{j-1} = D_j - \ (p_{j_{min}} \text{ or } p_{j_{max}})$

2) Equation (2.28) represents total cost due to optimal generation on all units of the system, and there is no need to calculate individual cost due to allocation of MWs on each unit.

3) Equation (2.28) can be used to represent a plant for a single area dispatch or a multiarea dispatch.

4) Set of equation (2.28) can be used to find generation on each unit.

5) At a plant, once a set of X_j and Y_j are estimated, they need not be calculated repeatedly for various values of demand variable, if the units do not violate the bounds.

6) Equation (2.21) may be written for j and $(j-1)$ units as

$$F_{j-1}(D) = A_{j-1}D^2 + B_{j-1}D + C_{j-1} \tag{2.30}$$

and

$$F_j(D) = A_jD^2 + B_jD + C_j \tag{2.31}$$

From these two equations, the critical value of D can be obtained above which the combination of j units will be economical as compared to $(j-1)$ units. This idea is being used for unit commitment and also for generation scheduling with multiple fuel options.

2.4 Equivalent Cost Function under Constraint Violation

In generation scheduling, the two main constraints observed are

(1) Balance between demand and generation, and

(2) Loading of units up to their minimum and maximum generation capacity.

These constraints accordingly are called equality and inequality constraints. During scheduling, it is natural that some units may violate inequality constraint. On violation of this constraint, units are fixed at their limiting values. For single area and multi-area dispatch, equivalent cost functions are required. Whenever, the units violate the constraint, correction is required of original cost function. Assuming that m units are violating the following inequality constraints.

$$p_j = p_{j_{max}} \quad ; \quad j=1,m \tag{2.32}$$

$$p_j = p_{j_{min}} \quad ; \quad j=1,m \tag{2.33}$$

let

$$D_m = \sum_j p_j \quad and \tag{2.34}$$

$F_m(D_m) = \sum_j F_j(p_j) = cc$

Let D be the original demand, then the cost of generation of remaining demand from rest of the units is

$$F_{N-m}(D - D_m) = A_{N-m}(D - D_m)^2 + B_{N-m}(D - D_m) + C_{N-m} \tag{2.35}$$

or

$$F_{N-m}(D - D_m) = A_{N-m}D^2 + D[B_{N-m} - 2A_{N-m}D_m] + C_{N-m} + A_{N-m}D_m^2 - B_{N-m}D_m \tag{2.36}$$

The total cost of generation is

$$F_N(D) = F_{N-m}(D - D_m) + F_m(D_m) \tag{2.37}$$

that is

$$F_N(D) = A_{N-m}D^2+D[B_{N-m}-2A_{N-m}D_m]+C_{N-m}+A_{N-m}D_m^2-B_{N-m}D_m+cc \tag{2.38}$$

or

$$F_N(D) = A_N D^2 + B_N D + C_N \tag{2.39}$$

where,

$A_N = A_{N-m}$

$B_N = B_{N-m} - 2A_{N-m}D_m$ where cc is the total cost of Generation

$C_N = C_{N-m} + A_{N-m}D_m^2 - B_{N-m}D_m + cc$

of bound violating units.

Using the above equations, the equivalent cost functions can be corrected.

2.5 Single Area Dispatch

The aim of load dispatch is to obtain optimum generation scheduling of committed units to minimize the cost of generation subject to relevant constraints. Numerous optimization techniques have been developed, namely, classical λ method (equal incremental fuel cost method), co-ordination method using penalty factors, Linear programming, quadratic programming, dynamic programming etc. Very few researchers have used dynamic programming [22] technique for estimation of generation scheduling including transmission losses. The method based on co-ordination equation is simple and fast. In co-ordination technique, generation scheduling is estimated by solution of a set of equations. These set of equations are formed using penalty factors. Penalty factors are computed by using transmission loss formula through B-coefficients or by using classical load flow analysis and estimating transmission losses and penalty factors. In classical method after having known the value of Lagrangian multiplier, generation of each unit or plant is calculated using Gauss-Seidal method[6]. Both the methods are iterative in nature. At the end of every iteration, a new set of units' generation values emerge and are compared with old set of values. If necessary, the new value of λ is estimated and procedure is repeated until convergence is obtained. A well simplified method for single area dispatch is presented by Shaults et.al.[63]. This method is again based on λ. In this method, minimum and maximum values of λ pertaining to each generating units in an area are calculated and a table of λ in ascending order is prepared which simultaneously provides information on different modes of operation of each unit. (that is, whether a unit is in co-ordination mode, shutdown, minimum or maximum mode) and also capability of power generation of system. For any load units' mode is determined from the table and using linear interpolation method, system λ is calculated. The procedure is further modified to include transmission losses. To estimate λ under this condition, equivalent cost coefficients are found and as described above, unit generation is calculated. In this work, an attempt is made to use Dynamic Programming (DP) for single area dispatch. Using DP, a procedure is developed which combines the two methods, namely co-ordination method using penalty factors and the technique adopted by Shaults et al [63]. However, this method avoids preparation of λ table and estimation of system λ as discussed in [6] and penalty factors are used to correct cost coefficients of each plant. Moreover, though a unit may be in maximum mode or minimum mode, it is not deleted from list; and on violation of constraints by the unit, plant cost function is recalculated to include this effect. This procedure is also iterative in nature and at the end of each iteration, a new set of penalty factors is used to correct the cost coefficients. This method is also very simple and provides very accurate results.

A single area consists of a number of plants and each plant is having a set of units. Initially, plant cost function is estimated from committed units at each plant. Using these cost coefficients (equal number of plants), plantwise generation is estimated, on the basis of which generation share is allocated to each unit of each plant. On violation of constraint on one or more units, plant cost coefficients are corrected and procedure is repeated until convergence is achieved.

2.5.1 Problem Formulation

In an area at each plant, a set of units are represented by cost functions as follows.

$$f_{ij}(p_{ij}) = a_{ij}p_{ij}^2 + b_{ij}p_{ij} + c_{ij} \qquad Rs/MW/hr \qquad j = 1, m_i \text{ i=1,NP} \tag{2.40}$$

Using plantwise equations, equivalent cost function are estimated as follows.

$$F_i(p_i) = A_i P_i^2 + B_i P_i + C_i \qquad Rs/MW/hr \tag{2.41}$$

For N plants in an area, total generation must balance demand plus transmission losses. Thus,

$$\sum_{i}^{N} P_i = D + P_L \tag{2.42}$$

and

$$P_i = \sum_{i}^{m_i} p_{ij} \tag{2.43}$$

at each plant, subject to minimum and maximum limits of generation of each unit. Hence, $pmin_{ij} \leq p_{ij} \leq pmax_{ij}$
where,
D is area of the demand,
P_L is transmission losses,
N is number of plants,
m_i is units at a plant,
$pmin_{ij}$ is the minimum generation limit of j^{th} unit at i^{th} plant,
$pmax_{ij}$ is the maximum generation limit of $j^t h$ unit at i^{th} plant,
P_i is the generation at i^{th} plant,
p_{ij} is the generation of j^{th} unit at i^{th} plant,
i is the index of plant,
j is the index of the unit,
$f_{ij}(p_{ij})$ is the cost function of j^{th} unit at i^{th} plant, and
$F_i(P_i)$ is the cost function of i^{th} plant.

2.5.2 Method of Solution

Conventionally, coordination equation is expressed as

$$\frac{d(F_i)}{dP_i}(pf_i) = \lambda \tag{2.44}$$

$$pf_i = \frac{1}{(1 - \frac{d(PL)}{dP_i})} \tag{2.45}$$

where,
$\frac{d}{dP_i}(F_i)$ is incremental fuel cost at plant i,
pf_i is the penalty factor at plant i, and
$\frac{d(PL)}{dP_i}$ is the incremental transmission losses at plant i.
Penalty factors may be calculated either by set of B-coefficients or from load flow. Now using only quadratic components of B-coefficients

$$\frac{dPL}{dP_i} = 2\sum_{k=1}^{n} B_{ik}P_k \tag{2.46}$$

This equation for a plant can be written as

$$\frac{d}{dP_i}[A_iP_i^2 + B_iP_i + C_i] * pf_i = \lambda \tag{2.47}$$

that is

$$(2A_iP_i + B_i) * pf_i = \lambda \tag{2.48}$$

that is

$$2(A_i * pf_i) * P_i + (B_i * pf_i) = \lambda \tag{2.49}$$

that is

$$2A_i'P_i + B_i' = \lambda \tag{2.50}$$

where,

$$A_i' = A_i * pf_i$$

$$B_i' = B_i * pf_i$$

In generation scheduling only A and B coefficients are corrected as shown above and then denoted by A' and B' . Plant generation is then estimated using equation (2.29) and eventually units' share is allocated. On violation of generation limits, generation is fixed at units'limiting value and cost function of the unit is corrected using equation(2.38) and the process is repeated till convergence is obtained. The entire procedure can be summarized by the following algorithm.

1) From the set of committed units at each plant, form the equivalent cost function of each plant. There will be as many cost functions as the number of plants.

2) Calculate generation scheduling at each plant for given demand. Set iteration count to 1.

3) Calculate transmission losses and penalty factors at each plant.

4) Find total generation required as
$P_{tg} = Demand + losses$

5) Using penalty factors, correct coefficients of cost function of each plant.

6) Calculate generation scheduling at each plant

7) Using plant generation and using equations (2.29), calculate generation allocation on each plant.

8) Check lower and upper limits of units at each plant. On violation of these limits, set unit generation at their limits and correct A and B coefficients of respective plants.

9) Calculate transmission losses and penalty factors at each plant, correct total generation required as $P_{tg}^r = demand + losses$
and check for convergence, that is $(P_{tg}^r - P_{tg}^{r-1}) \leq \epsilon$; r is an iteration count

10) If convergence is obtained, calculate total cost of generation and stop; otherwise go to step 5 and continue.

The methodology presented above is very simple and straight forward. The remarkable point to be noted is that penalty factors (p.f.) are used only once in each iteration whereas in afore-mentioned methods, p.f.'s are used twice in each iteration. Moreover, very few researchers have suggested correction of cost functions due to violation of units' constraints. The methodology is also versatile to include the effect of violation of bounds by a unit or units at a plant.

2.6 Multiarea Dispatch

Multiarea dispatch is practiced in many countries to derive well known advantages such as economy, reliability and security of supply system. In many systems, due to unavoidable reasons, shortage of power is met by importing power from an area or areas having surplus capacity, which may be more economical than to generate at its end. Mostly, all systems form a pool so as to operate economically, meeting all constraints and maintaining needful import and export of power among areas. In addition to above main factor of economy, there are other reasons for multiarea operation as follows,

(1) Capacity interchange.

(2) Diversity interchange.

(3) Energy banking.

(4) Emergency power exchange.

(5) Inadvertant power exchange.

Capacity interchange is mainly required to cover reserve margin due to unit outage. Diversity concept is mainly required to peak demand of areas. If Peak of each area occurs at different times, system may interchange power among areas on economic ground. Energy banking is mainly concerned with Hydro-thermal systems and during need of power, exchange of power is practised on interchange accounting. Emergency power exchange is practised due to failure of units of system. Areas have an agreement for exchange of power during emergency. Inadvertant power exchange is mainly related to imperfect AGC devices operation. However, ultimately economics plays the dominant rôle.

Concept of Multiarea dispatch is very well discussed in [63] and it is also developed by Happ [128]. Both pioneers are using conventional λ method. Ramaraj [21] has recently presented a method using area cost function. But this method is again based on λ. In this work, the concept of multiarea dispatch is presented in most simplified manner using dynamic programming. Conventionally, multiarea dispatch involves estimation of each area generation subject to inequality constraints of all committed units and area equality & inequality constraints. The method involves [63] estimation of λ for each area from minimum to maximum generation capacity for every load level, and arranging all λ's in tabular form and ascending order, then system λ is estimated using interpolation technique. Though, this method is simple and straightforward, it involves preparation of λ table which may have to be corrected for each load variation. Further, the method seems to be simple for few units of all areas but it becomes very tedious for large number of units. The same problem can be solved efficiently using dynamic programming. The method does not require preparation of long tables of λ's for every load level. The method begins with formation of equivalent cost function of each area, formation of import/export limit constraints and as usual using equations (2.29) generation allocation is estimated. On violation of constraints, area equivalent cost functions are corrected and calculations are repeated. Thus the problem is of iterative nature which converges very fast, usually within three to four iterations.

2.6.1 Problem Formulation

In multi-area operation, main constraints involved are area interchange capacity, that is, import and export limits and pool generation must balance pool load. For simplicity and for conceptual presentation, transmission losses are not considered. Normally there are (N+1) constraints for N area system.

For example consider three-area system interconnected by tie lines and each area has its load and generation.

For the three-area system, the four constraints are:

$$\sum_i P^a_{g1} + \sum_i P^a_{g2} + \sum_i P^a_{g3} = L_1 + L_2 + L_3 \quad (2.51)$$

$$L_1 - I_1 \leq \sum_i P^a_{g1} \leq L_1 + E_1 \quad (2.52)$$

$$L_2 - I_2 \leq \sum_i P^a_{g2} \leq L_1 + E_2 \quad (2.53)$$

$$L_3 - I_3 \leq \sum_i P^a_{g3} \leq L_3 + E_3 \quad (2.54)$$

where,
P^a_{gk} is net area generation,
L_k is area load,
I_k is import limit of area k, and
E_k is export limit of area k.
The objective of the problem is to estimate minimum cost of pool generation subject to above constraints. The objective function can be stated as

$$F_t = Min[\sum_k^{NA} F_k] \quad (2.55)$$

where F_t is total cost of generation with a committed set of units along with their limits.

Hence,

$$F_t = \sum_{k=1}^{NA} \sum_{j=1}^{m_k} [a_{kj} p_{kj}{}^2 + b_{kj} p_{kj} + c_{kj}] \quad (2.56)$$

subject to,

$$\sum_{k=1}^{NA} \sum_{j=1}^{m_k} pg_{kj} = \sum_{k=1}^{NA} L_k \quad (2.57)$$

and

$$L_k - I_k \leq \sum_{j=1}^{m_k} pg_{kj} \leq L_k + E_k \tag{2.58}$$

$p_{min_{kj}} \leq pg_{kj} \leq p_{max_{kj}}$ where,
NA are number of areas,
m_k are number of units in area k,
pg_{kj} is generation of j^{th} unit in area k,
$p_{min_{kj}}$ is minimum generation limit of j^{th} unit in area k, and
$p_{max_{ij}}$ is maximum generation limit of j^{th} unit in area k.

2.6.2 Method of Solution

As mentioned earlier, conventional method requires tabular form of λ whereas this method does not require such tables and solution is initiated with equivalent cost function of each area from committed units and using set of equations (2.22 to 2.29) and subsequently, if required, using equations (2.38) to satisfy constraints. Area generation and ultimately unit generation of the system are thus estimated. The algorithm is summarized in following steps.

(1) On the basis of committed units form the equivalent cost function of each area ignoring individual unit's limits. Set iteration count equal to 1.

(2) For a particular load, using equations (2.22 to 2.29), allocate generation to each area, say D_k^a.

(3) Allocate generation on each unit from area generation D_k^a and calculate total cost.

(4) If no unit of all areas violate bounds, then stop at this stage. Otherwise, correct area cost function using equation (2.38).

(5) Compare cost of generation with previous iteration. If convergence is obtained, stop here; otherwise, go to step 2.

Two sample examples are given in Table 2.19 and 2.23. The first example illustrates the usefulness of algorithm without considering constraints of import and export. The second example involves constraints due to import and export capability of each area.

2.7 Generation Scheduling with Multiple Fuel Options

In many utilities, generation cost functions for fossil fired units have piecewise quadratic cost functions. The reasons for segmental cost function are varied. One of the reasons for segmental cost function is the multiple fuel for each generating unit. Presence of such units in a plant poses problem of optimal selection of type of fuel for every unit for a particular demand. This problem results due to intersecting curves for each type of fuel of each unit. The curves imply that it may be more economical and efficient to burn one type of fuel (say oil) for some MW and other type (say gas) for the higher MWs. Another reason for segmental cost curve is derated conditions due to partial mechanical failure of unit. A unit may be compelled to operate on lower cost curve up to certain MW and after repairing the unit, it may be shifted to to next segment.

The approach for such a problem is reported by Lin & Viviani [25] and by N.Ramaraj and R. Rajaram et al [26]. In Lin and Viviani's method, approach is hierarchical. Units are arranged in groups called subsystem or areas. Using conventional Lagrangian multiplier method with binary search, for a particular demand, status of units and its fuel is decided. Ramaraj and Rajaram used the same method with a slight modification.

2.7.1 Problem Formulation

For n units, the cost curves are

$$F_{il}(P_{il}) = a_{il}p^2_{il} + b_{il}p_{il} + c_{il} \qquad Rs/MW/hr \tag{2.59}$$

where
a_{il},b_{il},c_{il}, are cost coefficients.
l is a fuel variable, and
i is an unit variable.
For each type of fuel on each unit, the bounds are
$(p_{min} - p_1)$ for first fuel,
$(p_1 - p_2)$ for second fuel, and
$(p_2 - p_{max})$ for third fuel.
The objective is to operate all committed units at a particular load at a minimum cost with optimum number of fuel of units, that is,

$$Min[F_n(D)] = Min[F_{il}(p_{il})] \tag{2.60}$$

subject to $D = \sum_i p_{il(*)}$
and
$\underline{p_{il(*)}} \leq p_{il} \leq \overline{p_{il(*)}}$
where, $l(*)$ represents optimal fuel selection and D is demand in MW. Lower and upper bar denotes minimum and maximum generation capacity for a selected fuel.

2.7.2 Method of Solution

It is possible to form composite equations for number of units using equations (2.25) to (2.27) as developed earlier. Hence, for different combinations of fuels and units at any stage, composite cost function can be formed. In conventional method of generation scheduling, Lagrangian multiplier is calculated which itself depends on demand. The search for optimal selection of fuel and satisfying constraints lead the methodology to be iterative. In present methodology, using set of equations (2.30 and 2.31), it is possible to find critical power (MW) above which a particular policy will be economical. Hence, the methodology aims at successive development of critical policy of optimal switching on a particular fuel along with certain range of operation. The method is continued till all fuels of all units are included. The procedure can be summarized in the following steps.

(1) Initially at first stage, combination equation of all units with first fuel is formed.

(2) In the next stage, next fuel is included as a state variable successively and for N states, N equations are formed.

These two stages can be represented as

$$F_N^{(1)}(D) = A_N^{(1)}(D)^2 + B_N^{(1)}D + C_N^{(1)} \tag{2.61}$$

$$F_n^{(2)}(D) = A_n^{(2)}(D)^2 + B_n^{(2)}D + C_n^{(2)} \tag{2.62}$$

where superscript (1) represents initial stage with first fuel on each unit and superscript (2) represents second stage with latest fuel status of a unit, which is selected as a state variable. Equation (2.58) is a set of N equations which are formed by taking one unit at a time as a state variable. As an example for three unit system, with first fuel on each unit, combination equations in terms of equation (2.58) are

$$F_1^1 = a_{11}(p_1)^2 + b_{11}p_1 + c_{11} \tag{2.63}$$

$$F_2^1 = a_{21}(p_2)^2 + b_{21}p_2 + c_{21} \tag{2.64}$$

$$F_3^1 = a_{31}(p_3)^2 + b_{31}p_3 + c_{31} \tag{2.65}$$

In the subscript ij, i represents unit variable and j represents fuel number of one particular unit. In above equations (2.60-2.63) on each unit, first fuel is assigned. Now the combination equation of these units can be formed and expressed as

$$F_{31}^{(1)} = A_{31}^{(1)}(D)^2 + B_{31}^{(1)}D + C_{31}^{(1)} \tag{2.66}$$

where subscript 31 represents combination of three units with first fuel on each unit. Now assume that fuel number on unit 1 is changed to 2 then the combination equation similar to equation (2.63) is expressed as

$$F_{31}^{(2)} = A_{31}^{(2)}(D)^2 + B_{31}^{(2)}D + C_{31}^{(2)} \tag{2.67}$$

On similar lines, if fuel number on other two units are sequentially changed then the correspondingly two combination equations formed are

$$F_{32}^{(2)} = A_{32}^{(2)} D^2 + B_{32}^{(2)} D + C_{32}^{(2)} \tag{2.68}$$

$$F_{33}^{(2)} = A_{32}^{(2)} D^2 + B_{33}^{(2)} D + C_{32}^{(2)} \tag{2.69}$$

Now a critical Dct_i can be calculated by combining equations (2.64, 2.65, 2.66) with first stage equation (2.58). The value of Dct_i will denote a critical level of load above which recent combination will be economical. Corresponding to three states shown above, there will be three values of Dct_i. In general for n units, there will be N Dct_i's. Since inclusion of next fuel on a particular unit is associated with rise in MW level compared to previous load level, Dct_i will always provide rise in MWs. However, selection of particular Dct_i is decided by a simple logic as follws
$[D_i]^* = Min[Dct_i]; i = 1, N$
However, due to similarity of units, Dct_i's may be equal for such units. The selection of Dct_i is done for such cases on priority basis. Other difficulty which may arise is due to violation of bounds by Dct_i's. These bounds are

(1) $[Dct_i]$ may be less than $\sum_i [Pmin]_i$ or

(2) $[Dct_i]$ may be greater than $\sum_i [Pmax]_i$

where $[Pmin]_i$ and $[Pmax]_i$ indicate summation of lower and upper bounds. Then, to arrive at accurate decision, a generalized approach is adopted. The approach is based on average full load cost. For every state and for each Dct_i, average full load cost is calculated. From these set of average full load costs, decision is made for a state which gives minimum cost. That is,
$Prmx(s) = Min[Dct_i]^*$; s is a stage variable
subject to above statements
$Prmx(s)$ gives optimal fuel status of each unit at stage s.
For the next stage, that is (s+1), base equation will be a previous combination equation at stage (s)

$$F_{il}(D) = A_{il}^s D^2 + B_{il}^s D + C_{il}^s \tag{2.70}$$

where, j represents fuel number on i^{th} unit.
This procedure is repeated for the next optimal selection of fuel number of a unit along with estimation of $Prmx(s+1)$. From these consecutive two stages, a range of operation is obtained as shown below.
$Prmn(s+1) = Prmx(s) + 1.0$ MW, and

$Prmx(s+1)$ as obtained in recent stage. Hence range of operation is
$Prmn(s+1) \rightarrow Prmx(s+1)$
where $Prmn(s+1)$ is minimum possible generation of (s+1) stage and for $Prmx(s+1)$ is

the maximum possible generation, along with status of fuel of each units. Above procedure is repeated till all fuels of all units are sequentially included and simultaneously range of operation is formed. For example, for three units for three stages, the result is shown below.

stage	gen. No.	Range of operation In MW
k	$1(2^*), 2(1^*), 3(3^*)$	400.0–500.0
k+1	$1(2^*), 2(2^*), 3(3^*)$	501.0–600.0
k+2	$1(2^*), 2(3^*), 3(3^*)$	601.0–700.0

where l^* represents fuel number. Hence, at stage s, generator 1 will have fuel number 2, generator 2 will have fuel number 1 and generator 3 will have fuel number 3 with a range of operation $400 to 500 MW$. Once a table as shown above is formed, the next step is to find generation scheduling or allocation of generation on each unit. For any demand, a search is made of all ranges of operation in lookup table. The stage or a range is selected in which the demand will appear. The selection of load range simultaneously provides status of fuel number of each unit. For example, for 550 MW, stage number is $(K + 1)$ in above lookup table; and status of fuel of each unit is 2, 2, 3. Now, after determining this stage and corresponding set of equations which are all formed and stored, they can be used to calculate generation on each unit.

2.8 System Studies and Results

To establish the effectiveness of the formulations developed in this work, sample problems are solved, as discussed in following sections.

2.8.1 Equivalent Cost Function

Three Unit system is selected to form equivalent cost function and for a load demand from $300 MW to 1200 MW$ in steps of $50 MW$ are assumed and corresponding costs are evaluated and compared with costs evaluated by earlier researchers. Table 2.1 is the input data; Table 2.2 shows the formation of equivalent cost functions in the order of unit combinations and Table 2.3 is the comparison of costs estimated using DP with costs estimated by others [27].

2.8.2 Correction of Equivalent Cost Function Under Constraint Violation

To illustrate method of correction of equivalent cost function under constraint violation, an exmaple is solved and result is tabulated. Table 2.4 is the input data and Table 2.5 represents formation of equivalent cost function. Table 2.6 is the result table, with no constraint, showing generation allocation on each unit, total cost estimated from individual cost of generation of each unit and total cost calculated from equivalent cost function. Table 2.7 shows the formation of equivalent cost function with constraint imposed on second unit, whereas Table 2.8 is the result showing generation allocation on each unit, total cost estimated from individual cost of generation and cost calculated from equivalent cost function. Further, to illustrate usefulness of equivalent cost function and generation allocation, a multiplant system is solved. Table 2.9 is the system comprising three plant with three units on each plant. Table 2.10 shows the detailed result. Plant costs and total cost are calculated by equivalent cost functions. Table 2.11 is a comparison of the result obtained with that of others [30].

2.8.3 Single Area Dispatch

For single area dispatch two examples are illustrated. First example assumes only plant cost functions and generation on each plant is calculated including transmission losses using B-coefficients. Table 2.12 is the plant cost functions and Table 2.13 is the B-coefficients. Table 2.14 shows the result obtained by proposed method which is compared with results as per reference [19]. Further, a sample problem is solved assuming a multiplant system with multiple units on each plant. Table 2.15 and Table 2.16 is the input data and Table 2.17 represents detailed result.

2.8.4 Multiarea Dispatch

For Multiarea dispatch, two examples are solved. First example is for two area system to confirm the validity of the method. Tables 2.18 and 2.19 are input data and Table 2.20 illustrates generation allocation without interchange evaluation, whereas Table 2.21 shows result with interchange evaluation which is compared with the reffered [21] result. Second example is the generalization of interchange evaluation in multiarea system. Tables 2.22 and 2.23 are the input data and Table 2.24 shows the generation on each unit of each area. Table 2.25 shows area generation, resulting import/export and corresponding cost.

2.8.5 Multifuel System

An example is illustrated for a dispatch of units with multiple fuel options on each unit. Fuel numbers are indicated and each fuel has range of operation. This example is taken from reference [25] for ten generating units. Table 2.26 is the input data. Table 2.27

Table 2.1: Equivalent Cost Function

Sr. No	Cost Coefficients		
	a_j	b_j	c_j
1	0.001562	7.92	561.0
2	0.002716	10.99	434.0
3	0.00723	11.955	117

Table 2.2: Composite Cost Functions

Unit combinations	X_i	Y_i	Cost Coefficients		
			A_i	B_i	C_i
1 ,2	.365123	-358.8125	9.9167648E-4	9.040930	444.222
1,2,3	.1206173	-177.2187	8.7206313E-4	9.392417	303.008

is the formation of a lookup table which indicate stagewise status of fuels of each unit along with range of operation and capacity bounds. Table 2.28 illustrate detailed calculation indicating generation of each unit along with their fuel numbers. Table 2.29 is the comparison of result with referred [25] results. As Per equation No. composite cost functions

Table 2.3: Cost Comparision

Plant Load	Total Cost by Proposed Method	Total Cost by [27]	Total Cost by [8]
300.0	3199.21	3199.22	4137.69
350.0	3697.18	3697.18	4507.243
400.0	4199.50	4199.51	4897.378
450.0	4706.19	4706.19	5308.097
500.0	5217.23	5217.23	5739.400
550.0	5732.63	5732.64	6191.287
600.0	6252.40	6252.40	6663.759
650.0	6776.52	6776.53	7156.813
700.0	7305.01	7305.01	7670.452
750.0	7837.85	7837.86	8204.675
800.0	8375.06	8375.06	8759.482
850.0	8916.63	8916.63	9334.873
900.0	9462.55	9462,56	9972.320
950.0	10012.84	10012.84	10593.610
1000.0	10567.48	10567.49	11235.750
1050.0	11126.49	11126.51	11898.725
1100.0	11689.86	11689.86	12582.540
1150.0	12257.59	12257.59	13287.195
1200.0	12829.68	12829.88	14012.690

Table 2.4: Generation Allocation and Cost Evaluation

Unit No	Cost Coefficients			P_{min_j}	P_{max_j}
	a_j	b_j	c_j	MW	MW
1	0.001562	7.92	561.0	150.0	850.0
2	0.00194	7.85	310.0	100.0	400.0
3	0.00578	9.56	93.6	10.0	200.0

Table 2.5: Generation Allocation and Cost Evaluation

Unit Combinations	X_i	Y_i	A_i	B_i	C_i
1,2	0.446030	9.99428	8.652998E-4	7.888777	870.6501
1,2,3	0.130212	-125.74467	7.526271E-4	8.1063915	859.17655

Table 2.6: Generation Allocation and Cost Evaluation with No Constraint

Demand MW	P_1 MW	P_2 MW	P_3 MW	$\sum F_i$ Rs.	F_t Rs.	Difference Rs.
1050	565.59	473.43	10.97	10200.65	10200.65	0.0
1100	589.68	492.82	17.48	10686.88	10686.88	0.0
1150	613.77	512.22	23.99	11176.87	11176.87	0.0
1200	637.86	531.62	30.51	11670.62	11670.62	0.0
1250	661.95	551.02	37.02	12168.14	12168.62	0.0
1300	686.05	570.41	43.53	12669.42	12669.42	0.0

Table 2.7: Generation Allocation and Cost with Constraint On Second Unit

Unit Combinations	X_i	Y_i	A_i	B_i	C_i
1,3	0.212	-111.68	1.2297E-3	8.2689	563.017
1,3,2	—	—	1.2296E-3	7.2851	1212.60

Table 2.8: Generation Allocation and Cost with Constraint On Second Unit

Demand MW.	P_1 MW	P_2 MW	P_3 MW	$\sum F_i$ Rs.	F_t Rs.	Difference Rs.
1050	623.39	400.00	26.60	10217.74	10217.74	0.0
1100	662.76	400.00	37.23	10714.19	10714.19	0.0
1150	702.12	400.00	47.87	11216.79	11216.79	0.0
1200	741.48	400.00	58.51	11725.54	11725.54	0.0
1250	780.85	400.00	69.15	12240.43	12240.43	0.0
1300	820.22	400.00	79.78	12761.48	12761.48	0.0

Table 2.9: Input Data Three Plant System

Plant No.	Unit No.	Cost Coefficients			Bounds	
		a_{ij}	b_{ij}	c_{ij}	$P_{min_{ij}}$	$P_{max_{ij}}$
1	1	0 .015	1.950	60.0	10.0	102.0
	2	0 .020	2.050	80.0	10.0	73.0
	3	0 .025	2.109	100.0	5.0	50.0
2	1	0 .020	1.450	85.0	10.0	100.0
	2	0 .030	1.400	100.0	5.0	75.0
	3	0 .035	1.900	120.0	10.0	75.0
3	1	0 .009	2.000	80.0	10.0	200.0
	2	0 .015	2.400	105.0	1.0	100.0
	3	0.01000	2.2000	95.0	10.0	150.0

Table 2.10: Optimum Generation Scheduling in a Multiplant System

Demand MW	Plant Gen	Unit Generation MW			Unit Generation cost Rs			Plant cost Rs	Total Cost Rs
200.0	55.563	26.069	17.045	12.456	121.00	120.75	130.15	371.91	
—	66.126	32.041	22.197	11.883	152.00	145.85	147.52	445.38	
—	78.310	40 657	11.061	26.591	176.19	133.38	160.57	470.15	1287.44
300.0	84 238	38.263	26.197	19.777	156.57	147.43	151.49	455.50	
—	86.608	41.197	28.298	17.112	178.68	163.64	162.76	505.09	
—	129.152	60.998	44.894	23.260	235.47	168.94	213.92	618.35	1578.93
400.0	112.913	50 465	35.349	27.099	196.60	177.45	175.51	549.58	
—	107.090	50.349	34.399	22.342	208.70	183.65	179.92	572.29	
—	179.995	81.331	35.465	63.19	302.19	208.98	273.97	785.16	1907.02
500 0	141.589	62 667	44 500	34 420	241.11	210 83	202.21	654.16	
—	127 573	59.50	40.500	27 571	242.08	205.90	198.99	646.99	
—	230 837	101 668	47 667	81.501	376.36	253 48	340.72	970.58	2271.72

Table 2.11: Cost Comparison

Demand	Plant Cost		Total Cost	
MW	By Proposed Method	By Rajaram Method	By Proposed Method	By Rajaram Method
200.0	470.15	470.15	1287.44	1287.44
	445.38	445.38		
	371.91	371.91		
300.0	618.35	618.35	1578.93	1578.93
	505.09	505.09		
	455.50	455.50		
400.0	785.16	785.16	1907.02	1907.02
	572.29	572.29		
	549.58	549.58		
500.0	970.58	970.58	2271.72	2271.72
	646.99	646.99		
	654.16	654.16		
600.0	1174.61	1174.61	2673.03	2673.03
	729.18	729.18		
	769.23	769.23		
700.0	1397.26	1397.26	3110.94	3110.52
	818.88	818.88		
	894.80	894.80		

Table 2.12: Economic Generation Scheduling Including Transmission Losses

Unit No	Cost Coefficients			Bounds	
	a_i	b_i	c_i	$Pmin_i$ MW	$Pmax_i$ MW
1	.0050	2.00	100.00	5.00	250.00
2	.0100	2.000	200.0000	5.00	250.00
3	.0200	2.000	300.0000	5.00	250.00
4	.00300	1.95000	80.000	5.00	250.00
5	.01500	1.45000	100.000	5.00	250.00
6	.01000	.9500	120.000	5.00	250.00

Table 2.13: Transmission Loss Coefficients

.000200	.000010	.000015	.000005	.00000	-.000030
.000010	.000300	-.000020	.000001	.000012	.00001
.000015	-.000020	.000100	-.000010	.000010	.000008
.000005	.000001	-.000010	.000150	.000006	.000050
.000000	.000012	.000010	.000006	.000250	.000020
-.000030	.000010	.000008	.000050	.000020	.000210

Table 2.14: Comparisom of Result

System Demand (MW)	Method	P_1 MW	P_2 MW	P_3 MW	P_4 MW	P_5 MW	P_6 MW	Loss MW	Fuel cost \$/hr	cost Daviation
200	Accurate method	30.95	15.25	8.15	54.54	28.01	65.29	2.19	1289.47	
	Proposed method	30.95	15.24	8.15	54.54	28.01	65.29	2.19	1289.47	0.0
	Method from ref.[*]	31.22	15 32	8.17	54.30	28.01	65.17	2.19	1289.47	0.0
300	Accurate method	55.26	27.59	14.85	93.06	36.41	77.36	4.53	1536.15	
	Proposed method	55.26	27.59	14.84	93.06	36.40	77.35	4.52	1536.15	0.0
	Method from ref.[*]	55.64	27.72	14.88	92.64	36.43	77.21	4.52	1536.16	0.0007
400	Accurate method	79.81	40.11	21.73	131.70	44.97	89.58	7.90	1810.37	
	Proposed method	79.81	40.10	21.71	131.69	44.96	89.57	7.88	1810.37	0.0
	Method from ref.[*]	80.32	40.30	21.79	131.02	45.03	89.40	7.87	1810.36	-0.0006
500	Accurate method	104.60	52.79	28.80	170.44	53.69	101.97	12.29	2112.98	
	Proposed method	104.60	52.79	28.79	170.43	53.68	101.96	12.28	2112.98	0.0
	Method from ref.[*]	105.27	53.06	28.90	169.45	53.80	101.77	12.25	2112.97	-0.0005
600	Accurate method	129.64	65.65	36.08	209.29	62.57	114.53	17.76	2444.86	
	Proposed method	129.66	65 65	36.06	209.28	62.57	114.52	17.76	2444.86	0.0
	Method from ref.[*]	130 50	66.03	36.23	207.89	62.75	114.30	17.71	2444 80	-0.0008

Table 2.15: Single Area Dispatch Including Transmission Losses

Plant No.	Unit No.	Cost Coefficients			Bounds	
		a_{ij}	b_{ij}	c_{ij}	$P_{min_{ij}}$	$P_{max_{ij}}$
1	1	0 .015	1.950	60.0	10.0	102.0
	2	0 .020	2.050	80.0	10.0	73.0
	3	0 .025	2.109	100.0	5.0	50.0
2	1	0 .020	1.450	85.0	10.0	100.0
	2	0 .030	1.400	100.0	5.0	75.0
	3	0 .035	1.900	120.0	10.0	75.0
3	1	0 .009	2.000	80.0	10.0	200.0
	2	0 .015	2.400	105.0	1.0	100.0
	3	0.01000	2.2000	95.0	10.0	150.0

Table 2.16: B-Coefficient Matrix

0.0003	0.00015	0.00024
0.00015	0.0004	0.000025
0.00024	0.000025	0.00002

Table 2.17: Result of Single Area Dispatch

Demand	Plant No.	Total Genera tion	plant Genera tion	Transmission loss	unit generation P_{ij}	unit generation P_{ij}	unit generation P_{ij}	total cost of generation
MW		MW	MW	MW	MW	MW	MW	
200.00		205.722		5.72				
	1		57.204		26.75	17.56	12.87	
	2		67.298		32.569	22.546	12.185	
	3		81.215		41.820	11.759	27.639	1126.57
300.00		313.51		13.510				
	1		88.112		39.911	27.433	20.767	
	2		89.375		42.433	29.122	17.819	
	3		136.021		63.741	24.911	47.367	1621.12
400.00		424.943		24.943				
	1		120.06640		53.509	37.631	28.925	
	2		112.199802		52.631	35.921	23.646	
	3		192.67690		86.404	38.509	67.763	1994.56
650.00		722.278		72.277				
	1		205.320		90.187	65.140	50.000	
	2		173.100		79.842	54.061	39.195	
	3		343.840		146.810	74.790	122.185	3213.49
700.00		785.498		85.498				
	1		223.45630		100.546	72.909	50.000	
	2		186.04970		85.628	57.919	42.502	
	3		375.99230		159.730	82.504	133.757	3513.8

Table 2.18: Multiarea Dispatch - Area 1 Input Data

	cost coeffecients			Boumds	
Unit no	a_i	b_i	c_i	$Pmin_i$	$Pmax_i$
1	.003124	15.8400	1122.00	150.000	600.000
2	.00388	15.700	620.00	100.00	400.000
3	.00964	15.9400	156.0	50.000	200.000

Table 2.19: Multiarea Dispatch - Area 2 Input Data

Unit No	Cost Coeffecients			Boumds	
	a_i	b_i	c_i	P_{min_i}	P_{max_i}
1	.0026410	13.4140	950.000	140.00	590.000
2	.0034960	14.1740	560.5000	110.00	440.000
3	.0034960	14.1740	560.5000	110.000	440.000

Demand of Area 1 is 700 MW.
Demand of Area 2 is 1100 MW.
Generation for both areas is operating independently.

Table 2.20: Result of Multiarea Dispatch

Plant No	Interchange Evaluation By Proposed Technique		Interchange Evaluation By Rajaram Method	
	Area 1	Area 2	Area 1	Area 2
1	322.722	524.675	322.7	524.7
2	277.882	287.662	277.9	287.7
3	99.396	287.662	99.4	287.7

Table 2.21: Result of Multiarea Dispatch (Cost)

Cost of Area 1 is 13677.25	Cost of Area 1 is 13677.21
Cost of Area 2 is 18569.26	Cost of Area 2 is 18569.23
Total Cost = 32246.50	Total Cost = 32246.44

Table 2.22: Result of Multiarea Dispatch (Comparison)

Plant No	Interchange Evaluation By Proposed Technique		Interchange Evaluation By Rajaram Method	
	Area 1	Area 2	Area 1	Area 2
1	183.995	590.000	144.0	590.0
2	166.186	402.689	166.2	402.7
3	54.440	402.689	54.4	402.7
Cost of Area 1 is 8530.86 Cost of Area 2 is 23453.84 Total Cost of generation is 31984.71			Cost of Area 1 is 8530.93 Cost of Area 2 is 23453.89 Total Operating Cost = 31984.82	
Net saving in cost = 261.79			Net saving in cost = 261.79	
Power Imported in Area 1 is 295.37 MW Power Exported from Area 2 is 295.37 MW			Power imported in Area 1 is 295.4 Power exported from Area 2 is 295.4	

Table 2.23: Multi Area Generation Scheduling for Three Area System

Area	Unit No	Cost Coefficients			Bounds		Area Capacity	
		Ar_{ij}	Br_{ij}	Cr_{ij}	P_{min} MW	P_{max} MW	Import MW	Export MW
1	1	.003124	15.84	1122.00	150.00	600.00	300.0	300.0
	2	.00388	15.70	620.00	100.00	400.00		
	3	.0034960	14.174	560.00	110.00	440.000		
2	1	.003124	15.84	1122.00	150.00	600.000	300.00	300.0
	2	.00388	15.700	620.00	100.00	400.000		
	3	.009640	15.940	156.00	50.00	200.000		
3	1	.002641	13.414	950.00	140.00	590.000	300.0	300.0
	2	.0034960	14.174	560.50	110.00	440.000		
	3	.0096400	15.940	156.00	50.00	200.000		

Table 2.24: Areawise Demand (Three Areas)

Area	1	2	3
Demand	900.0	800.0	700.0
Total Demand 2400.0 MW			

Table 2.25: Areawise Generation Details

Unit No	Generation in MW		
	Area 1	Area 2	Area 3
1	236.150	236.150	590.0
2	208.179	208.179	360.0
3	440.000	71.342	50.0

Table 2.26: Result of Multiarea Dispatch

Area	Demand MW	Generation MW	Import MW	Export MW	Cost of Generation Rs
1	900.0	884.3292	15.678	00.00	16566.78
2	800.0 MW	515.6708	284.3293	00.00	10435.65
3	700.0 MW	1000.000	0.0	300.00	16876.91

Total Cost Of Generation is Rs 43879.40

Table 2.27: Multifuel Dispatch

Unit No.	Fuel No.	Cost Coefficients			Bounds	
		a_{il}	b_{il}	c_{il}	$P_{min_{il}}$	$P_{max_{il}}$
1	1	.002176	-.397500	26.97000	100.00	196.00
	2	.0018610	-.3059000	21.13000	197.00	250.00
2	1	.0011380	-.0398800	1.86500	50.00	114.00
	2	.0016200	-.1980000	13.65000	115.00	157.00
	3	.0041940	-1.2690000	118.40000	158.00	230.00
3	1	.0014570	-.3116000	39.79000	200.00	332.00
	2	.0008035	.0338900	-2.87600	332.00	388.00
	3	.0000118	.4864000	-59.14000	389.00	500.00
4	1	.0010490	-.0311400	1.98300	99.00	138.00
	2	.0027580	-.6348000	52.85000	139.00	200.00
	3	.0059350	-2.3380000	266.80000	201.00	265.00
5	1	.0010660	-.0873300	13.92000	190.00	338.00
	2	.0015970	-.5206000	99.76000	339.00	407.00
	3	.0001498	.4462000	-53.99000	408.00	490.00
6	1	.0010490	-.0311400	1.98300	85.00	138.00
	2	.0027580	-.6348000	52.85000	139.00	200.00
	3	.0059350	-2.3380000	266.80000	201.00	265.00
7	1	.0011070	-.1325000	18.93000	200.00	331.00
	2	.0011650	-.2267000	43.77000	332.00	391.00
	3	.0002454	.3559000	-43.35000	392.00	500.00
8	1	.0010490	-.0311300	1.98300	99.00	138.00
	2	.0027580	-.6348000	52.85000	139.00	200.00
	3	.0059350	-2.3380000	266.80000	201.00	265.00
9	1	.0006121	-.0181700	14.23000	130.00	213.00
	2	.0015540	-.5675000	88.53000	214.00	370.00
	3	.0006121	-.0181700	14.23000	371.00	440.00
10	1	.0011020	-.0993800	13.97000	200.00	362.00
	2	.0011370	-.2024000	46.71000	363.00	407.00
	3	.0000416	.5084000	-61.13000	408.00	490.00

Table 2.28: Multifuel Generation Scheduling

Unit Fuel Status											Range of Operation		Capacity	
Unit No.	1	2	3	4	5	6	7	8	9	10				
Stage No.											Lower MW	Upper MW	$\sum P_{min_{il}}$	$\sum P_{max_{il}}$
1	1	1	1	1	1	1	1	1	1	1	1353.00	1417.23	1353	2300
2	1	1	1	1	1	1	1	2	1	1	1418.23	1455.35	1393	2362
3	1	1	1	2	1	1	1	2	1	1	1456.35	1570.35	1433	2424
4	1	2	1	2	1	1	1	2	1	1	1571.35	1728.35	1498	2467
5	1	3	1	2	1	1	1	2	1	1	1729.35	1867.35	1541	2540
6	1	3	1	2	1	2	1	2	1	1	1868.35	1911.90	1595	2602
7	1	3	1	2	1	2	1	3	1	1	1912.90	1964.64	1657	2667
8	1	3	1	2	1	3	1	3	1	1	1965.64	2017.39	1719	2732
9	1	3	1	3	1	3	1	3	1	1	2018.39	2231.39	1781	2797
10	1	3	1	3	1	3	1	3	2	1	2232.39	2428.39	1865	2954
11	2	3	1	3	1	3	1	3	2	1	2429.39	2808.72	1962	3008
12	2	3	1	3	1	3	2	3	2	1	2809.72	3068.00	2094	3068
13	2	3	2	3	1	3	2	3	2	1	3069.00	3124.00	2226	3124
14	2	3	3	3	1	3	2	3	2	1	3125.00	3236.00	2283	3236
15	2	3	3	3	1	3	2	3	3	1	3237.00	3306.00	2440	3306
16	2	3	3	3	1	3	3	3	3	1	3307.00	3415.00	2500	3415
17	2	3	3	3	2	3	3	3	3	1	3416.00	3484.00	2649	3484
18	2	3	3	3	3	3	3	3	3	1	3485.00	3567.00	2718	3567
19	2	3	3	3	3	3	3	3	3	2	3568.00	3612.00	2881	3612
20	2	3	3	3	3	3	3	3	3	3	3613.00	3695.00	2926	3695

2.9 Conclusion

Recursive Technique of Dynamic Programming is used to obtain equivalent cost function of a thermal generating units. The formulation is very simple and in the process of formulation of equivalent cost function, expression for generation allocation is also formed. Due to possible violation of bounds by a unit, plant cost function is required to be corrected. The same is also developed. The equivalent cost function can be formed to represent a group of units, a plant comprising number of units, or an area comprising of number of plants. Further algorithm is developed to estimate generation scheduling for a single area dispatch with and without losses. Further a successful attempt is made to estimate interchange evaluation among areas of a multiarea system. The proposdd technique is also being applied to estimate generation scheduling for multifuel units. The work is tested by solving illusrative examples and it is revealed that

(1) Approach for forming equivalent cost function is correct and gives accurate results. [27].

(2) Generation allocation by proposed method gives accurate results. The results are compared with the work already reported [17, 30]. The results are almost matching.

(3) On violation of bounds by a unit, plant functions are corrected. Using this corrected function, total cost and generation scheduling is found to be accurate.

(4) Conventionally, in a system comprising of plants with multiple generating units, scheduling is calculated using penalty factors. Penalty factors are to be used as many times as number of units, whereas in the proposed method number of times penalty factors to be used is limited to number of plants.

(5) Allocation of generation on each unit of a plant can be easily calculated from plant generation. In conventional method of Lagrangian multiplier, once a unit violates a bound, it is assigned with the bound value and deleted from list for further calculation (except for calculation of transmission losses) whereas in the proposed method it is shown that having corrected the cost function, plant generation can be calculated without deleting that unit from the list.

(6) Representation of a system by an equivalent cost function is usefull in interchange evaluation. The results obtained for a sample example matches with the result [t:2.21] already published.

(7) The methodology developed is also useful for estimating generation allocation for units with multiple fuels. The methods proposed by Lin[25] and Rajaram[26] are basically developed using Lagrangian multiplier and the same is different for different

Table 2.29: Generation Scheduling for Multitfuel system

Load demand in MW											
2400.0			2500.0			2600.0			2700.0		
Unit No	Fuel . No	Generation MW	Unit No	Fuel No	Generation MW	Unit No	Fuel No	Generation MW	Unit No	Fuel No	Generation MW
1	1	189.740	1	2	206.519	1	2	216.544	1	2	226.5692
2	3	202.342	2	3	206.457	2	3	210.905	2	3	215.3542
3	1	253.895	3	1	265.739	3	1	278.543	3	1	291.3489
4	3	233.045	4	3	235.953	4	3	239.096	4	3	242.2402
5	1	241.829	5	1	258.017	5	1	275.519	5	1	293.0211
6	3	233.045	6	3	235.953	6	3	239.096	6	3	242.2402
7	1	253.274	7	1	268.863	7	1	285.717	7	1	302.5705
8	3	233.045	8	3	235.953	8	3	239.096	8	3	242.2402
9	2	320.383	9	2	331.4878	9	2	343.4934	9	2	355.4991
10	10	239.397	10	1	255.056	10	1	271.986	10	1	288.9162

Table 2.30: Generation Scheduling Cost Comparison

Demand	Cost by Proposed Method	Cost by Method as per Ref[*]	Cost by Method as per Ref[*]
2400.0 Mw	481.72	488.46	487.91
2500.0 MW	526.24	526.16	525.69
2600.0 MW	574.38	573.32	574.28
2700.0 MW	626.25	625.22	623.80

demands. whereas in the proposed method a look up table is formed, which provides information regarding units, their fuels status for various load ranges. Hence for any load observing the appropriate range units' fuel can be easily decided.