

Chapter 4

Optimum Priority Ordering of Units

4.1 Introduction

Unit commitment is the most important aspect in power system planning. For determining unit commitments, one of the aspect is the preparation of priority ordering of units. The aim of priority ordering is to commit or decommit units commensurate with increase or decrease in demand and to economically schedule the same. Normally, along with priority ordering, order of combination of units is also estimated along with respective ranges of operation. Methods employed for ordering of units are dynamic programming and average full load cost(AFLC). Ayoub et al.[69] have developed unit commitment (UC) table by Dynamic Programming (DP) using discrete step size. The table provides the order of units, the order of combination of units and range of operation of each combination. Process of combination starts with most economical unit and units are added stagewise to obtain optimum UC table. Based on this table, a method is presented to commit units as per load. The complete unit commitment is worked out through three steps. In the first step, priority ordering is obtained and units are committed as per this priority. In the second step, commitment is corrected applying security criteria and in the final step commitment is corrected to include startup aspect of units. The methodology seems to be very simple for fewer number of units but the same becomes complicated and cumbersome for large number of units. The AFLC method is very simple in which the AFLC of units are calculated and units are arranged in ascending order of AFLC, called as priority ordering. For unit commitment, other techniques are also developed such as Dynamic Programming Sequential Combination (DPSC) [70] and Dynamic Programming Truncated Combination (DPTC) [71] which are also based on priority list. There is another method called Dynamic Programming Sequential/ Truncated Combination (DP-STC) which is a combination of DP-SC and DP-TC methods. The aim of the methods are to develop UC programme for large number of units. However, none of these methods take into account physical location of units. In reality, units may be in large number but all are not located at the place hence, if physical location is taken into account, units are

naturally grouped plantwise and if at a plant optimum ordering is estimated, then final commitment may prove to be more appropriate. Hence, priority ordering is attempted in this work using recursive technique of Dynamic Programming. However, instead of using discrete steps, direct method is adopted using equivalent cost function criteria. The method includes consideration for single fuel system as well as multifuel system. So far no exact method is reported to include multifuel units in forming UC table. The aim of the work is to obtain unit priority ordering, order of combination of units and corresponding range of operation. Since number of units at a plant is not too large hence priority ordering is naturally simple. The application of this ordering is made to obtain the unit commitment of the system. For single fuel system, four methods are developed and for multifuel system, one method is developed.

4.2 Unit Ordering Of Single Fuel System

In this section, four methods are developed using DP. In all these methods equivalent cost functions are used to form the unit commitment tables. The first method is based on average full load cost(AFLC). The second method is based on heuristic search of economical combination. The third method is a combination of the above two methods in forward direction; The procedure starts from first unit and terminates covering all the units. The fourth method is similar to the third one but starts in backward direction. Advantage of this method is that the method terminates itself at the most economical unit.

4.2.1 Average Full Load Cost (AFLC) Method

The AFLC criteria is used by many researchers to commit the units appropriately as per demand. Priority ordering is simply made by calculating AFLC of every unit and arranging the same in ascending order. Hence, priority is decided by AFLC but criteria for making combination at different load level is achieved in discrete steps. However, in this work along with AFLC, equivalent cost function criteria is used to find order of combination along with the ranges of operation. The method is very simple and straight forward. The method is initiated by first calculating cost per MW of each unit by assigning unit maximum capacity, and ordering is completed by arranging cost per MW in ascending order. In the next step, combination order is obtained by sequentially adding the unit as per this order and simultaneously range of operation is estimated. The strategy is described as follows:

- As per the AFLC, units are numbered from 1 to n
- The first stage estimation is assessed simply by declaring

$$A_1 = a_1 \quad ; \quad B_1 = b_1 \quad ; \quad C_1 = c_1 \quad ;$$

where,

a_1, b_1, c_1 are cost coefficients of the first unit;

A_1, B_1, C_1 are coefficients of equivalent cost functions of first stage that is when $i = 1$; and

A_i, B_i, C_i represent cost coefficients representing i number of units combined at i^{th} stage.

- Second stage estimation proceeds by forming combination equation of first two units and critical load level $D_{(critical)_1}$ is obtained by equating cost functions representing one and two units, that is,

$$A_1 D^2 + B_1 D + C_1 = A_2 D^2 + B_2 D + C_2 \quad (4.1)$$

where D represents load level. Hence

$$(A_1 - A_2)D^2 + (B_1 - B_2)D + (C_1 - C_2) = 0 \quad (4.2)$$

Solving this quadratic equation for D , a unique value of D is estimated and this represents a critical load level $D_{(critical)_1}$. Above this critical load level, combination of two units will be economical as compared to first unit or in other words, first unit can supply load up to $D_{(critical)_1}$ economically and above this load level, second unit should be committed along with first to obtain economic schedule. The $D_{(critical)_1}$ obtained in second stage will provide upper value of 'range' of operation of first stage. Lower value of first stage is naturally minimum generation capacity of first unit. Lower value of 'range' of current stage is obtained by adding 1.0 MW to $D_{(critical)_1}$. Hence result can be tabulated as shown below, where D_1 represents $D_{(critical)_1}$ obtained at second stage.

Stage No	Combination	Lower Range	Upper Range
1	1	P_{min_1}	D_1
2	1,2	$D_1 + 1.0$	-

- In the third stage, third unit is combined with the previous combination of two units. Hence combination equation is formed representing three units and $D_{(critical)_2}$ is obtained as follows.

$$A_2 D^2 + B_2 D + C_2 = A_3 D^2 + B_3 D + C_3 \quad (4.3)$$

Solving this equation for D , a limiting value is obtained say, D_2 which will indicate that combination of three units will be economical above previous stage. Hence, the range of operation of second stage and the lower value of current stage are as follows.

Stage No	Combination	Lower Range	Upper Range
1	1	$Pmin_1$	D_1
2	1,2	$D_1 + 1.0$	D_2
3	1,2 3	$D_2 + 1.0$	-

Therefore, it can be concluded that in each stage of calculation, upper value of range of operation of previous stage and lower value of the range of operation of current stage is obtained simply by adding 1.0 MW in the critical value so obtained. The procedure is continued till all units are included. At final stage, say n , $D_{(critical)_n}$, will be upper value of $(n - 1)^{th}$ stage; and upper value this stage is simply the sum of maximum capacities of all units of the system. As an illustration of four unit system, combination order table will appear as shown in the following Table.

Stage No	Combination	Bounds	
		Lower Range MW	Upper Range MW
1	1 0 0 0	$Pmin_1$	D_1
2	1 1 0 0	$D_1 + 1.0$	D_2
3	1 1 1 0	$D_2 + 1.0$	D_3
4	1 1 1 1	$D_3 + 1.0$	$\sum Pmax_i$

Where, 1 and 0 represent on and off status of units. The table obtained is still required to be corrected. The correction is required in third column due to possible violation of last two columns. On violation of these bounds, particularly upper bound, D_i 's are replaced by corresponding upper bounds. Finally, the Table so obtained can be used to commit the units and schedule the same as per load level. At any time, search is made of range in which the load of the system may fit. Once the range is selected, the number of units to be committed can be simultaneously selected as per load.

The method described above is very simple. However, it is observed that the same is not appropriate in the sense that it does not take into account the possibility of combinations due to all remaining units at any stage, which may perhaps be more economical. The following three methods have been developed taking this aspect into account.

4.2.2 Direct Forward Tracking Approach

This approach is heuristic in nature and checks every unit as a probable candidate to be searched as a most economical unit. Here, the stage corresponds sequential order of combination and the states correspond to checking process of every unit as a probable candidate to be included to form the combination. The main objective of this method is to obtain optimal combination of two units, and then to find the most economical unit among these two units. Based on equivalent cost function of these two units, the search

is then made to obtain optimum combination of three units in the following stage. The process is repeated till all the units are included and finally Combination Table (CT) is obtained similar to that shown in the earlier method. The method is initiated by selecting randomly any unit as a first unit and then equivalent cost function of two units combination is formed by choosing each unit sequentially, and then combined with the first unit selected randomly. Next a critical load level is obtained by equating cost function of first unit and the cost functions thus formed. The procedure is repeated selecting every unit as first unit. Thus, at first stage the total number of states to be checked will be $n(n-1)$ and from these states, decision is made by the following criteria. First, the i^{th} unit is chosen as the first unit, a set of $D_{(critical),i}$ is obtained forming combinations of two units and taking every unit sequentially, the load level is $[D_{(critical),ij}]$; where i is first unit and $j = 1, n, i \neq j$

The best combination, based on the i^{th} unit is

$$\text{Min}[D_{(critical),ij}] ; j = 1, n, i \neq j$$

The reason being that i^{th} unit as economical unit will continue to be economical till load level crosses the $D_{(critical)}$. However, the combination which overcomes this i^{th} unit earlier is naturally the best combination. Therefore, if j^{th} is the particular unit selected then

$$\text{Min}[D_{(critical),ij}] = [D_{(critical),ij}^*] \quad ; \quad i \neq j \quad (4.4)$$

In the next step i is also varied, that is next unit from the list is selected as a base unit and procedure is repeated to find minimum of critical load level. In this manner a set of $[D_{(critical),j^*}]$ will be obtained if all units are tested as the first unit, the best combination among the set is $\text{Max}[D_{(critical),ij}^*], i = 1, n$.

For a particular i^* , the order of two units is (i^*j^*) , that is i^* is the first unit and j^* is the second unit. The above logic is justified due to the fact that, among i^* and j^* so selected, i^* is more economical as compared to j^* . The first stage and second stage estimation are over by selecting i^* as first unit at first stage and i^*j^* combination at second stage. Simultaneously, the range of operation is estimated exactly the same way as explained in previous section. The third stage is initiated by taking equivalent cost function of second stage representing two units, as a base equation. The selection of third unit is again a multistate process, in which the states are formed by merging all the remaining units, considering one at a time with previous two units combination. The unit selection is done by finding a set

$$D_{(critical),k}, k = 1, n, K \neq i^* ; K \neq j^*$$

by equating cost function representing two units with cost function of three units. Thus, a quadratic cost function similar to eq (4.3) results and selection of third unit is obtained simply by searching as

$$\text{Min}[D_{(critical),k}], k = 1, n, k \neq i^* \text{ and } k \neq j^*$$

thus priority ordering is continued till last unit and simultaneously the range of operation is obtained using the same criteria as used in the previous method.

4.2.3 Unit Ordering by AFLC and Heuristic Forward Tracking Approach

On applying AFLC approach to number of systems, it is observed that the same is not appropriate and fails to provide accurate commitment order. The inaccuracy of the method is demonstrated by an example in this section. Hence, method based on AFLC is corrected by combining AFLC and the method developed in previous section. The method is initiated to select or search a unit which is most economical. The criteria for the same is minimum of a set of cost/MW calculated for all units by assigning unit's maximum capacity. However, next unit selection is estimated by checking every unit as a probable candidate for next position. The entire procedure is adopted through following steps.

- For initialization, AFLC(cost/MW) is calculated as described in the previous section. All cost/MWs are arranged in ascending order. The unit having minimum cost/MW is selected as the most economical unit and treated as a base unit for estimating merit ordering of units.
- The unit selected as base unit is declared as the first unit and corresponding cost coefficients that is base equivalent cost coefficients are defined as
 $A_1 = a_i^*$; $B_1^* = b_i$; $C_1^* = c_i$
 Where, i^* is the particular unit selected.
- In the next stage, for selection of second unit, a set of combination equation of two units is formed by combining every unit sequentially with base unit. In essence, for finalizing decision of second stage, checking of every unit is a state policy and number of $[D_{critical}]$ or load levels is a decision vector. The optimum decision is the minimum of these critical values. Mathematically, the same can be represented as

$$F_1(D) = A_1 D^2 + B_1 D + C_1 \quad (4.5)$$

and for second stage

$$F_2(D) = A_{2(i)} D^2 + B_{2(i)} D + C_{2(i)} \quad (4.6)$$

Where,

$A_{2(i)}$, $B_{2(i)}$, $C_{2(i)}$ are cost coefficients representing the combination of two units, 'i' indicates that i^{th} unit is being tested. $i = 1, 2, \dots, (n-1)$ are possible states at second stage. Naturally for $D_{critical}$, there will be $(n-1)$ values. The critical values can be calculated as done previously by equating above two equations. Each $D_{critical}$, so calculated will indicate the load above which combination of two units will be economical, however, the best $D_{critical}$ will naturally be the minimum of these values. Thus at second stage combination of two units can be decided as

$$Min[D_{critical}, i] ; i = 1, (n-1)$$

The above logic is justified by the fact that the first unit will continue to be economical till load level of stage crosses the $D_{critical}$ of different combinations and which ever combination overcomes the first unit earlier, will naturally be the best combination. Hence the first range of operation is

stage No	Combination	Lower Range	Upper Range
1	1	P_{min_1}	$D_{critical_{m^*}}$

and lower range of operation for combination of two units will be

stage No	Combination	Lower Range
2	$1, m_*$	$D_{critical_m^*} + 1.0$

Where m^* is the particular unit which is making the best combination with the first unit.

- For the third stage, base combination equation is second stage equation representing two units selected in previous stage. In the third stage, the procedure for the selection of optimal third unit is the same as that in second stage. The $D_{(critical)}$ so selected becomes the upper value of range of operation of previous stage. And the lower value of current stage is simply $D_{(critical)_2} + 1.0$. Hence the range of operation of previous stage and lower value of the third stage is

stage No	Combination	Lower Range	Upper Range
2	$1, m^*$	$D_{(critical)_1} + 1.0$	D_2
3	$1, m^*, k^*$	$D_2 + 1.0$	-

Where k^* is the particular unit selected as third unit. Since the procedure is recursive by nature, the same is continued till all units are included. At the final stage, the $D_{critical}$ will provide upper value of the last but one stage and upper value of final stage is the sum of maximum capacity of each unit.

4.2.4 Unit Ordering by Back Tracking

Merit ordering of unit can be obtained by back tracking. Exactly the same order is obtained by this method as obtained by the previous method. Advantage of this method is that there is no need to search for the first economical unit by back tracking, the procedure automatically terminates at the first unit which is most economical. In back tracking, instead of adding of units stagewise, uneconomical units are dropped. the procedure begins as if all units are committed and upper value of the final stage is maximum capacity of the system. The aim of the method is to search for a most uneconomical unit to be dropped at the preceding stage. For this purpose, $D_{critical}$ is required to be estimated below which a current stage is economical. Hence, the method initiates with final stage

and terminates at the first stage. The stage variable is units which are reduced stagewise and state variables are combination of units. Suppose there is a system of five units, then possible combination of four units at fourth stage will be five, that is the number of states will be five to search the most uneconomical unit. As stages or units are reduced, combination states are also reduced by the same order. $D_{critical}$ at any state is obtained as

$$A_{(n-1),i}D^2 + B_{(n-1),i}D + C_{(n-1)-i} = A_nD^2 + B_nD + C_n \quad (4.7)$$

where n is final stage and i is possible number of states

Solving for D yields an estimate of a critical D . For ' i ' combinations, there will be set of ' i ' $D_{critical}$ values; and the selection of most uneconomical unit is simply dictated by following logic

$$Max[D_{critical}], i = 1, n$$

This is because unit is to be declared uneconomical and so the previous stage must have large value of upper limit of range. Repeating this procedure, finally a table is formed with unit combination order and respective range of operation, exactly similar way as that described in previous section.

4.2.5 Correction of Combination Order Table

During decision process of forming combination table, following hurdles are observed.

- Lower value and upper value ranges may violate sum of minimum capacities and sum of maximum capacities of units at a stage.
- Even if at any stage constraint is not violated, the same range may occur due to similar units.
- Even though units may not be similar, the same range may occur for different combinations.

To arrive at an accurate decision for selection of optimum combination of units, following remedies are adopted for forward tracking method.

- On violation of bounds at any stage, respective bound values are affixed.
- For the same range of operation for similar units are assigned priority, and as per this priority unit is selected.
- For the same range due to similar units, a test dispatch is performed taking demand as average of lower and upper bound, and thus cost/MW for different combination is estimated. The unit and corresponding minimum cost/MW is selected in that stage.

As a general policy, criteria of cost/MW at every stage is adopted to include the facts mentioned above. In case of back tracking, the same hurdles as mentioned above are encountered and the same possible remedies are adopted with slight modification for the same range of operation. Here instead of finding minimum cost / MW criteria, a unit dropping criteria of unit having maximum (cost/MW) is adopted and maximum (cost/MW) is also adopted as a general policy to obtain best combination satisfying lower and upper constraints as well as to overcome the difficulties mentioned above.

4.3 Unit Commitment Procedure

Formulation of merit ordering is the first step of unit commitment. The next task is to commit the units as per load and schedule the same economically. A set of demand is available say for 24 hours. The subinterval may be an hour or less (may be 15 minutes). However approach in this work is static hence large sub interval of (1) is assumed. At any hour and corresponding demand, search is made for range and the range in which demand fits, is selected. The same range provides order of combination. Once units are selected they are economically dispatched for the load. The procedure is repeated till last hour. Finally unit commitment table is formed which provides information of unit status at every hour as per load, share(MW) of unit, and finally cost of dispatching them. The formation of the above table is not yet complete. In the next step, unit status is corrected taking into account security and startup cost criteria.

4.3.1 Assessment of Security and Startup Cost

Besides economy aspects, reliability of electric supply in respect of its availability is also expected by the consumer. Therefore, economy and quality of electric supply must be co-ordinated to estimate unit commitment. Similarly, units take certain time to start which requires additional cost. Sometimes it is not economical to stop unit if the same is to be restarted after few hours. Therefore to arrive at final decision of UC table, these aspects have to be included. In this subsection, the afore-mentioned aspects are covered. These aspects are not new and are practiced by many users. However, to complete the final task of making UC table the same methods are employed.

4.3.1.1 Security Assessment

A breach of system security is defined as some intolerable or undesirable condition. The breach of security specifically considered here is insufficient generation capacity. The Patton's security function, which quantitatively estimates the probability that the available generation capacity(sum of capacities of units committed) at a particular hour is less than system load at that time, is defined as [7,68]

$$S = \sum_{i=1}^n (P_i)(r_i) \quad (4.8)$$

where,

P_i = probability of system being in that state 'i'

r_i = probability of system state 'i' that causes breach of system security. When system load is deterministic, $r = 0$, if committed capacity is greater than the load, and

$r = 1$, if committed capacity is less than the load.

Though theoretically above equation suggests that the summation is to be carried over all possible states (which may be very large otherwise), the sum need to be [7,68] carried out over states reflecting small number of units on outage. For example for a set of ten units, more than 5 units are neglected as the probability of failure of 50% of capacity is very low. For any load, units are selected by heuristic search and tested for security(S) using the above equation. To test the same, certain Maximum Tolerable Insecurity Level(MTIL) is compared with calculated value of 'S'. MTIL for a given system is dictated by past experience. If the value of 'S' exceeds MTIL, the UC table is modified by including the next most economical unit in uc table. The 'S' is again calculated and checked. The process is continued till for certain combination so arrived at, 'S' is less than MTIL.

4.3.2 Startup Cost Criteria

The startup criteria is included in final step. In load cycle, it is possible that certain units might be stopped for few hours and are restarted. In that case, these units are checked for economy. At certain hour whenever such check is required, the same is done in two steps. In the first step, such units are assumed as 'stopped' for certain load level and cost is calculated for remaining units, and in the next step units are assumed 'not stopped' and cost is calculated. Naturally, state which gives minimum cost is finally included in that hour as a best combination.

Therefore, in summary unit commitment is to be done in three steps. In step one, unit commitment table is formed as per load cycle from combination order table. In next step, UC table is corrected due to security aspects and in the last step, UC table is corrected considering start up cost of units.

4.4 Merit Ordering of Units With Multifuel Options

Now a days ceratin boilers are designed for multiple fuel options. Each fuel may be used singly or in conjunction with others. The fuel used singly may have certain range of operation. For example, if a boiler is designed for three fuels, then unit will possess three ranges of operation correspondingly. Hence, if a system possesses N units each having

three fuels, then system will function as if it has three units. Moreover, such units may have conflicting cost curves. Hence, preparation of merit order and corresponding order of combination along with range of operation is a very complex process. So far no work is reported on this aspect. In this work an attempt is made to prepare merit ordering of multi fuel units. Based on previous method on single fuel system, the method developed is Based on AFLC by considering forward tracking. Data of cost curves is taken from [25] published in IEEE. After preparing combination order, schedules are calculated and compared with that of published work.

4.4.1 Merit Ordering By Forward Tracking Based on AFLC

The method is basically same as described in section 4.2.3. The unit selection at first place of merit order is processed by calculating AFLC by assigning maximum capacity of first fuel of every unit. The unit possessing minimum cost per MW with first fuel is treated as base unit along with its cost function as base function of first stage. Now total number of stages are

$$N_g = \sum N_{ij} \quad (4.9)$$

where

i is the index of unit, and

j is the index of fuel on an unit i

For example, if for a 2 unit system, the first unit has two fuels and the second has three fuels then total stages will be five (3+2), each stage possessing its own range of operation. For the next stage assessment, the probable candidate may be the second fuel of the same unit or the first fuel of all the remaining units. However, number of states to be checked will be equal to number of units. As usual, cost functions are formed and ($D_{critical}$) values are calculated and selection is made using the logic

$$\text{Min}[D_{critical}] \quad ; \quad i = 1, n$$

As discussed in earlier section, the same problems, namely, possible violation of bounds on the same value of $D_{critical}$ for similar and dissimilar units may occur. The same remedies as discussed earlier are applied. On violation of bounds, limiting values are assigned and as per general policy at every stage, using ' $D_{critical}$,' AFLC is calculated and second stage assessment is completed. The process is continued till last stage. Thus, merit order table is formed providing stagewise information of status of units and their fuel number, range of operation, minimum capacity and maximum capacity of the stage. Based on this table, generation allocation can be estimated as per load. For any load, units along with their fuel can be selected by searching appropriate range of operation. Once units and their corresponding fuels are selected then using corresponding cost functions, generation allocation can be estimated by the method developed in second Chapter. For increasing load, units are loaded in the sequence of their fuel number. Similarly, for decreasing load, units are decommitted in the order of fuels, that is, unit is decommitted when its first

fuel is not required. A program for this method is developed and tested on IEEE DATA referring [25].

4.5 System Studies and Results

In this Chapter, four methods are developed to obtain optimal combination order of thermal units. Based on this order, unit commitment table is prepared for a given system. First method is based on Average Full Load Cost (AFLC). For this purpose, a sample example is attempted. Table 4.1 is the input data. Table 4.2 is ALFC table, based on which sequence order of units is fixed and then combination order along with the range of operation of each combination is obtained. Table 4.3 shows this result and for a load 440 MW cost is estimated. Second method is based on forward Dynamic programming. For this case, a sample example is solved. Table 4.4 is the input data. Table 4.5 is the sequence order of units. Based on this sequence order, order of combination of units is obtained which is shown in Table 4.6. Table 4.7 shows unit commitment schedule for some selected loads. Next, a sample example is solved for method No. 3. Data used for method No. 1. is again is used for this method. Table 4.8 gives sequence order of units and Table 4.9 is the combination order of units. For the 440 MW load, the cost is computed and found to be less than that of method No. 1. An example for method No. 4 is solved. Table 4.10 is input data and Table 4.11 is the combination order. To prove usefulness of the method, sample scheduling calculations are shown with all units committed. Table 4.12 shows this result. Table 4.13 is unit commitment schedule as per sequence order. It can very easily be revealed that optimal combination gives less cost of power generation. Next two problems are attempted using data provided in ref[69] applying method No. 3 and 4. Table 4.14 is the input data. Table No 4.15 is a combination order obtained using method No. 3. Table 4.16 is the unit commitment schedule which shows the total cost incurred in 24 hours. Table 4.17 shows combination order obtained using method No. 4. It can be seen that this table is almost the same except for first two stages. The reason is being that at every stage, units were not selected by calculating cost/MW, But if units selected are based on cost/MW at every stage, the same table as 4.15 is obtained. Table 4.18 is the unit commitment schedule which shows the total cost incurred in 24 hours. The method adopted by Ayoub[69] and Kothari[24], that is, three-step method to estimate unit commitment schedule, is applied to estimate unit commitment for 24 hours. The data used is the same as given in Table 4.14. Naturally, order of combination and range of operation is the same as given in Table 4.15. Table 4.18 is a result of Unit Commitment Schedule for a load pattern for 24 hours shown as step 1. In step 2, Patton's security function is used to correct the result of step 1. Table 4.20 is the correction of Table 4.19; and in step 3, the same table is corrected applying startup cost criteria shown in Table 4.21. Finally, an attempt is made to find unit commitment schedule for a multi-fuel plant. Method No 3 is used to obtain order of combination and then as per load pattern, unit

Table 4.1: Input Data to Obtain Commitment Order based On(AFLC) Method No 1

Unit No.	Cost Coefficients			Bounds	
	a_i	b_i	c_i	$Pmin_i$	$Pmax_i$
1	0.01	.1	100.0	50.0	200.0
2	0.02	.1	120.0	30.0	150.0
3	0.01	.2	150.0	50.0	200.0
4	0.015	.2	170.0	30.0	150.0

Table 4.2: Merit Order

Unit No	1	2	3	4
AFLC	2.6	3.9	2.95	3.58
Unit Commitment Order	1	4	2	3

commitment schedule is estimated. Data for this problem is taken from Ref[25] and is given in Table 4.22. Table 4.23 is order of combination whereas for selected loads, unit commitment schedule is shown in Table 4.24.

Table 4.3: Order Of Combination

Sr No.	Order of combination	Range of Operation		capacity	
		Lower MW	Upper MW	$\sum Pmin_i$ MW	$\sum Pmax_i$ MW
1	1 0 0 0	50	178	50	200
2	1 0 1 0	178	373	100	400
3	1 0 1 1	374	441	130	550
4	1 1 1 1	442	700	160	700

Cost Of Generation for 440 MW is Rs. 1217.34

Table 4.4: Input Data for unit Commitment Order Based on Method No 2

unit No.	Cost Coefficients			Bounds	
	a_i	b_i	c_i	$Pmin_i$	$Pmax_i$
1	0.05	14.0	100.0	30.0	150.0
2	0.06	16.0	120.0	30.0	150.0
3	0.08	15.0	160.0	30.0	150.0
4	0.075	9.0	140.0	30.0	150.0
5	0.07	10.0	150.0	30.0	150.0

Table 4.5: Sequence Order

Sr. No.	Original Order	Sequence Order	Cost Coefficients		
			a_i	b_i	c_i
1	4	1	0.075	9.0	140.0
2	5	2	0.07	10.0	150.0
3	1	3	0.05	14.0	100.0
4	2	4	0.06	16.0	120.0
5	3	5	0.08	15.0	160.0

Table 4.6: Order of Combination

Sr. No.	Order of combination	Range of operation		Maximum Generation Capacity
1	1 0 0 0 0	30.0	69.0	150.0
2	1 1 0 0 0	70.0	142.0	300.0
3	1 1 1 0 0	143.0	257.0	450.0
4	1 1 1 1 0	258.0	328.0	600.0
5	1 1 1 1 1	329.0	750.0	750.0

Units are arranged as per new sequence order.

Table 4.7: Generation Scheduling as per Commitment Order

Sr. No.	Load MW	P_1 MW	P_2 MW	P_3 MW	P_4 MW	P_5 MW	Remark
1	50.0	50.0	-	-	-	-	Subscripts of P as per sequence Order Obtained
2	100.0	51.7	48.3	-	-	-	
3	200.0	72.0	70.0	58.0	-	-	
4	300.0	86.173	85.185	79.26	49.382	-	
5	400.0	96.02	95.736	95.03	61.700	52.514	

Table 4.8: Merit Order Based on Method No 3

Unit No	1	2	3	4
Unit Commitment Order	1	3	2	4

Table 4.9: Order Of combination

Sr No.	Order of combination	Range of Operation		capacity	
		Lower MW	Upper MW	$\sum P_{min_i}$ MW	$\sum P_{max_i}$ MW
1	1 0 0 0	50	178	50	200
2	1 0 1 0	179	341	100	400
3	1 1 1 0	342	456	130	550
4	1 1 1 1	457	700	160	700

Cost Of Generation for 440 MW is 1205.85

Table 4.10: Input data

Unit No.	Cost Coefficients			Bounds	
	a_i	b_i	c_i	$Pmin_i$	$Pmax_i$
1	.011	.12	150.00	10.00	350.00
2	.020	.11	150.00	30.00	200.00
3	.050	.14	120.00	10.00	150.00
4	.020	.22	160.00	30.00	200.00
5	.013	.12	130.00	10.00	200.00

Table 4.11: Optimal Combination Order

Stage No.	Combintion Order	Range Operation		$\Sigma Pmax_i$
		Min	Max	
1	1 0 0 0 0	10.0000	160.5775	350.000
2	1 0 0 0 1	161.5775	330.3371	550.000
3	1 1 0 0 1	331.3371	443.2248	750.000
4	1 1 0 1 1	444.2248	680.5350	950.000
5	1 1 1 1 1	681.5350	1100.0000	1100.000

Table 4.12: Generation Scheduling : All Units Committed

Demand	P_1	P_2	P_3	P_4	P_5	Total cost
150.0	43.333	30.000	10.000	30.000	36.666	810.0
410.0	130.268	71.897	28.459	69.147	110.227	1350.09
350.0	111.318	61.474	24.286	58.724	94.192	1183.45
200.00	63.941	35.418	13.867	32.668	54.104	876.27
750.00	238.125	131.218	52.187	128.468	200.000	2766.95

Table 4.13: Generation Scheduling as per Order of Combination

Demand	Stage No.	P_1	P_2	P_3	P_4	P_5	Total Cost
150.0	1	150.000	0.000	0.0	0.0	0.0	415.5
410.0	3	171.003	94.301	0.0	0.0	144.695	1249.95
350.00	3	145.963	80.529	0.0	0.0	123.507	1033.55
200.00	2	108.333	0.0	0.0	0.0	91.667	542.33
750.00	5	238.125	131.218	52.187	128.468	200.000	2766.95

Table 4.14: Input Data

Unit No.	Cost Coefficients			Bounds	
	a_i	b_i	c_i	$Pmin_i$	$Pmax_i$
1	.0051000	2.2034000	15.00000	15.00000	60.00000
2	.0039600	1.9161000	25.00000	20.00000	80.00000
3	.0039300	1.8518000	40.00000	30.00000	100.00000
4	.0038200	1.6966000	32.00000	25.00000	120.00000
5	.0021200	1.8015000	29.00000	50.00000	150.00000
6	.0026100	1.5354000	72.00000	75.00000	280.00000
7	.0028900	1.2643000	49.00000	120.00000	320.00000
8	.0014800	1.2136000	82.00000	125.00000	445.00000
9	.0012700	1.1954000	105.00000	250.00000	520.00000
10	.0013500	1.1285000	100.00000	250.00000	550.00000

Table 4.15: Unit Combination Order

Stage No.	Unit No.										Operation Range		ΣMin	ΣMax
	1	2	3	4	5	6	7	8	9	10	lower	upper		
1	0	0	0	0	0	0	0	0	0	1	250.00	388.35	250.00	550.00
2	0	0	0	0	0	0	0	1	0	1	389.35	663.81	375.00	995.00
3	0	0	0	0	0	0	0	1	1	1	664.81	986.61	625.00	1515.00
4	0	0	0	0	0	0	1	1	1	1	987.61	1467.67	745.00	1835.00
5	0	0	0	0	1	0	1	1	1	1	1468.67	1767.82	795.00	1985.00
6	0	0	0	0	1	1	1	1	1	1	1768.82	1887.12	870.00	2265.00
7	0	0	0	1	1	1	1	1	1	1	1888.12	2246.93	895.00	2385.00
8	0	1	0	1	1	1	1	1	1	1	2247.93	2465.00	915.00	2465.00
9	0	1	1	1	1	1	1	1	1	1	2466.00	2565.00	945.00	2565.00
10	1	1	1	1	1	1	1	1	1	1	2566.00	2625.00	960.00	2625.00

Table 4.16: Unit Commitment Table

Sr. No.	Demand	Unit Status									
		1	2	3	4	5	6	7	8	9	10
1	2000.00	0	0	0	1	1	1	1	1	1	1
2	1980.00	0	0	0	1	1	1	1	1	1	1
3	1940.00	0	0	0	1	1	1	1	1	1	1
4	1900.00	0	0	0	1	1	1	1	1	1	1
5	1840.00	0	0	0	0	1	1	1	1	1	1
6	1870.00	0	0	0	0	1	1	1	1	1	1
7	1820.00	0	0	0	0	1	1	1	1	1	1
8	1700.00	0	0	0	0	1	0	1	1	1	1
9	1510.00	0	0	0	0	1	0	1	1	1	1
10	1410.00	0	0	0	0	0	0	1	1	1	1
11	1320.00	0	0	0	0	0	0	1	1	1	1
12	1260.00	0	0	0	0	0	0	1	1	1	1
13	1200.00	0	0	0	0	0	0	1	1	1	1
14	1160.00	0	0	0	0	0	0	1	1	1	1
15	1140.00	0	0	0	0	0	0	1	1	1	1
16	1160.00	0	0	0	0	0	0	1	1	1	1
17	1260.00	0	0	0	0	0	0	1	1	1	1
18	1380.00	0	0	0	0	0	0	1	1	1	1
19	1560.00	0	0	0	0	1	0	1	1	1	1
20	1700.00	0	0	0	0	1	0	1	1	1	1
21	1820.00	0	0	0	0	1	1	1	1	1	1
22	1900.00	0	0	0	1	1	1	1	1	1	1
23	1950.00	0	0	0	1	1	1	1	1	1	1
24	1990.00	0	0	0	1	1	1	1	1	1	1

Total Cost Of Generation = \$ 78832.55

Table 4.17: Unit Commitment Table Using Backword Dynamic Programming

Stage No.	Unit No.										Operation Range		Capacity	
	1	2	3	4	5	6	7	8	9	10	Min	Max	$\sum Min$	$\sum Max$
1	0	0	0	0	0	0	0	0	0	1	250.00	413.30	250.00	520.00
2	0	0	0	0	0	0	0	0	1	1	414.30	677.99	500.00	1070.00
3	0	0	0	0	0	0	0	1	1	1	678.99	986.61	625.00	1515.00
4	0	0	0	0	0	0	1	1	1	1	987.61	1467.67	745.00	1835.00
5	0	0	0	0	1	0	1	1	1	1	1468.67	1767.82	795.00	1985.00
6	0	0	0	0	1	1	1	1	1	1	1768.82	1887.12	870.00	2265.00
7	0	0	0	1	1	1	1	1	1	1	1888.12	2246.93	895.00	2385.00
8	0	1	0	1	1	1	1	1	1	1	2247.93	2465.00	915.00	2465.00
9	0	1	1	1	1	1	1	1	1	1	2466.00	2565.00	945.00	2565.00
10	1	1	1	1	1	1	1	1	1	1	2566.00	2625.00	960.00	2625.00

Table 4.18: Unit Commitment Table as per Order of Combination

Sr. No.	Demand	Unit Status									
		1	2	3	4	5	6	7	8	9	10
1	2000.00	1	0	0	1	1	1	1	1	1	1
2	1980.00	1	0	0	1	1	1	1	1	1	1
3	1940.00	1	0	0	1	1	1	1	1	1	1
4	1900.00	1	0	0	1	1	1	1	1	1	1
5	1840.00	1	0	0	0	1	1	1	1	1	1
6	1870.00	1	0	0	0	1	1	1	1	1	1
7	1820.00	1	0	0	0	1	1	1	1	1	1
8	1700.00	1	0	0	0	1	0	1	1	1	1
9	1510.00	1	0	0	0	1	0	1	1	1	1
10	1410.00	1	0	0	0	0	0	1	1	1	1
11	1320.00	1	0	0	0	0	0	1	1	1	1
12	1260.00	1	0	0	0	0	0	1	1	1	1
13	1200.00	1	0	0	0	0	0	1	1	1	1
14	1160.00	0	0	0	0	0	0	1	1	1	1
15	1140.00	0	0	0	0	0	0	1	1	1	1
16	1160.00	0	0	0	0	0	0	1	1	1	1
17	1260.00	0	0	0	0	0	0	1	1	1	1
18	1380.00	0	0	0	0	0	0	1	1	1	1
19	1560.00	0	0	0	0	1	0	1	1	1	1
20	1700.00	0	0	0	0	1	0	1	1	1	1
21	1820.00	0	0	0	0	1	1	1	1	1	1
22	1900.00	0	0	0	1	1	1	1	1	1	1
23	1950.00	0	0	0	1	1	1	1	1	1	1
24	1990.00	0	0	0	1	1	1	1	1	1	1

Total cost of Generation = \$ 78832.55

Table 4.19: Three Steps Unit Commitment : Step 1

Sr. No.	Demand MW	Unit Status									
		1	2	3	4	5	6	7	8	9	10
1	2000.00	0	0	0	1	1	1	1	1	1	1
2	1980.00	0	0	0	1	1	1	1	1	1	1
3	1820.00	0	0	0	0	1	1	1	1	1	1
4	1870.00	0	0	0	0	1	1	1	1	1	1
5	1940.00	0	0	0	1	1	1	1	1	1	1
6	1870.00	0	0	0	0	1	1	1	1	1	1
7	1820.00	0	0	0	0	1	1	1	1	1	1
8	1700.00	0	0	0	0	1	0	1	1	1	1
9	1510.00	0	0	0	0	1	0	1	1	1	1
10	1410.00	0	0	0	0	0	0	1	1	1	1
11	1320.00	0	0	0	0	0	0	1	1	1	1
12	1260.00	0	0	0	0	0	0	1	1	1	1
13	1200.00	0	0	0	0	0	0	1	1	1	1
14	1160.00	0	0	0	0	0	0	1	1	1	1
15	1140.00	0	0	0	0	0	0	1	1	1	1
16	1160.00	0	0	0	0	0	0	1	1	1	1
17	1260.00	0	0	0	0	0	0	1	1	1	1
18	1380.00	0	0	0	0	0	0	1	1	1	1
19	1560.00	0	0	0	0	1	0	1	1	1	1
20	1700.00	0	0	0	0	1	0	1	1	1	1
21	1820.00	0	0	0	0	1	1	1	1	1	1
22	1900.00	0	0	0	1	1	1	1	1	1	1
23	1950.00	0	0	0	1	1	1	1	1	1	1
24	1990.00	0	0	0	1	1	1	1	1	1	1

Total cost is = \$ 78712.81

Table 4.20: Three Steps Unit Commitment : Step 2

[illegible]

Table 4.21: Three Steps Unit Commitment : Step 3

Sr. No.	Demand	Unit Status									
		1	2	3	4	5	6	7	8	9	10
1	2000.00	1	1	1	1	1	1	1	1	1	1
2	1980.00	1	1	1	1	1	1	1	1	1	1
3	1820.00	0	1	0	1	1	1	1	1	1	1
4	1870.00	0	1	0	1	1	1	1	1	1	1
5	1940.00	0	1	1	1	1	1	1	1	1	1
6	1870.00	0	1	0	1	1	1	1	1	1	1
7	1820.00	0	0	0	1	1	1	1	1	1	1
8	1700.00	0	0	0	0	1	1	1	1	1	1
9	1510.00	0	0	0	0	1	1	1	1	1	1
10	1410.00	0	0	0	0	1	0	1	1	1	1
11	1320.00	0	0	0	0	1	0	1	1	1	1
12	1260.00	0	0	0	0	0	0	1	1	1	1
13	1200.00	0	0	0	0	0	0	1	1	1	1
14	1160.00	0	0	0	0	0	0	1	1	1	1
15	1140.00	0	0	0	0	0	0	1	1	1	1
16	1160.00	0	0	0	0	0	0	1	1	1	1
17	1260.00	0	0	0	0	0	0	1	1	1	1
18	1380.00	0	0	0	0	1	0	1	1	1	1
19	1560.00	0	0	0	0	1	1	1	1	1	1
20	1700.00	0	0	0	0	1	1	1	1	1	1
21	1820.00	0	0	0	1	1	1	1	1	1	1
22	1900.00	0	1	0	1	1	1	1	1	1	1
23	1950.00	0	1	1	1	1	1	1	1	1	1
24	1990.00	1	1	1	1	1	1	1	1	1	1

Cost of Generation = \$ 79066.66
Statrtup Cost = \$ 878.583300
Total Operating Cost = \$ 79945.25

Table 4.22: Multi Fuel Unit Ordering-Input Data

Unit No.	Fuel No.	Cost Coefficients			Bounds	
		a_{ij}	b_{ij}	c_{ij}	$P_{min_{ij}}$ MW	$P_{max_{ij}}$ MW
1	1	.0021760	-.3975000	26.970	100.00	196.00
	2	.0018610	-.3059000	21.130	197.00	250.00
2	1	.0011380	-.0398800	1.865	50.00	114.00
	2	.0016200	-.198000	13.6500	115.00	157.00
	3	.0041940	-1.269000	118.40000	158.00	230.00
3	1	.0014570	-.311600	39.7900	200.00	332.00
	2	.0008035	.033890	-2.87600	332.00	388.00
	3	.0000118	.486400	-59.1400	389.00	500.00
4	1	.0010490	-.031140	1.9830	99.00	138.00
	2	.0027580	-.634800	52.8500	139.00	200.00
	3	.0059350	-2.338000	266.800	201.00	265.00
5	1	.0010660	-.087330	13.9200	190.00	338.00
	2	.0015970	-.520600	99.7600	339.00	407.00
	3	.0001498	.446200	-53.9900	408.00	490.00
6	1	.0010490	-.031140	1.9830	85.00	138.00
	2	.0027580	-.634800	52.8500	139.00	200.00
	3	.0059350	-2.338000	266.800	201.00	265.00
7	1	.0011070	-.132500	18.9300	200.00	331.00
	2	.0011650	-.226700	43.7700	332.00	391.00
	3	.0002454	.3559000	-43.3500	392.00	500.00
8	1	.0010490	-.031130	1.9830	99.00	138.00
	2	.0027580	-.634800	52.8500	139.00	200.00
	3	.0059350	-2.338000	266.800	201.00	265.00
9	1	.0006121	-.018170	14.2300	130.00	213.00
	2	.0015540	-.567500	88.5300	214.00	370.00
	3	.0006121	-.018170	14.2300	371.00	440.00
10	1	.0011020	-.0993800	13.9700	200.00	362.00
	2	.0011370	-.202400	46.7100	363.00	407.00
	3	.0000416	.508400	-61.1300	408.00	490.00

Table 4.23: Multi Fuel Unit Order of Combination

Stage No.	Unit & Units Fuel Status										Operation Range		Bounds	
	1	2	3	4	5	6	7	8	9	10	Lower	Upper	$\sum P_{mini j}$	$\sum P_{max ij}$
1	0	1	0	0	0	0	0	0	0	0	50.0000	114.0000	50.00	114.00
2	0	1	0	0	0	1	0	0	0	0	115.0000	234.0000	135.00	252.00
3	1	1	0	0	0	1	0	0	0	0	235.0000	333.0000	235.00	448.00
4	1	1	0	1	0	1	0	0	0	0	334.0000	432.0000	334.00	586.00
5	1	1	0	1	0	1	0	1	0	0	433.0000	615.6329	433.00	724.00
6	1	1	0	1	0	1	0	2	0	0	616.6329	653.7834	473.00	786.00
7	1	1	0	2	0	1	0	2	0	0	654.7834	662.9070	513.00	848.00
8	1	2	0	2	0	1	0	2	0	0	663.9070	714.1099	578.00	891.00
9	1	3	0	2	0	1	0	2	0	0	715.1099	754.1547	621.00	964.00
10	1	3	0	2	0	1	0	2	1	0	755.1547	944.9992	751.00	1177.00
11	1	3	0	2	0	2	0	2	1	0	945.9992	1143.1860	805.00	1239.00
12	1	3	0	2	0	2	0	3	1	0	1144.1860	1195.9320	867.00	1304.00
13	1	3	0	2	0	3	0	3	1	0	1196.9320	1248.6860	929.00	1369.00
14	1	3	0	3	0	3	0	3	1	0	1249.6860	1434.0000	991.00	1434.00
15	1	3	0	3	0	3	0	3	2	0	1435.0000	1591.0000	1075.00	1591.00
16	1	3	1	3	0	3	0	3	2	0	1592.0000	1791.0000	1275.00	1923.00
17	1	3	1	3	0	3	1	3	2	0	1792.0000	1981.0000	1475.00	2254.00
18	1	3	1	3	1	3	1	3	2	0	1982.0000	2181.0000	1665.00	2592.00
19	1	3	1	3	1	3	1	3	2	1	2182.0000	2378.0000	1865.00	2954.00
20	2	3	1	3	1	3	1	3	2	1	2379.0000	2808.7290	1962.00	3008.00
21	2	3	1	3	1	3	2	3	2	1	2809.7290	3068.0000	2094.00	3068.00
22	2	3	2	3	1	3	2	3	2	1	3069.0000	3124.0000	2226.00	3124.00
23	2	3	3	3	1	3	2	3	2	1	3125.0000	3236.0000	2283.00	3236.00
24	2	3	3	3	1	3	2	3	3	1	3237.0000	3306.0000	2440.00	3306.00
25	2	3	3	3	1	3	3	3	3	1	3307.0000	3415.0000	2500.00	3415.00
26	2	3	3	3	2	3	3	3	3	1	3416.0000	3484.0000	2649.00	3484.00
27	2	3	3	3	3	3	3	3	3	1	3485.0000	3567.0000	2718.00	3567.00
28	2	3	3	3	3	3	3	3	3	2	3568.0000	3612.0000	2881.00	3612.00
29	2	3	3	3	3	3	3	3	3	3	3613.0000	3695.0000	2926.00	3695.00

4.6 Conclusion

Based on formulation developed for equivalent cost function, an attempt is made to develop optimal order of combination of units. Methods reported by Ayoub[69] and Kothari[24] use discrete step size to form optimal order of combination of units along with the range of operation. However, in this work using equivalent cost function criteria, four methods are developed to form a table to represent sequential optimal order of units, optimal order combination of units and corresponding range of operation. The methods are direct and fast, and the process of discrete step size is eliminated. The methods developed are

- (1) Average Full Load Cost(AFLC)
- (2) Direct Forward Tracking Approach
- (3) Unit Ordering by AFLC and Heuristic Forward Tracking Approach.
- (4) Unit ordering by back Tracking.

After forming optimal order of combinations; the next step is to form unit commitment table. Three step strategy as reported by [69, 7] is adopted. Next, a successful attempt is made to form optimal order of combination of units and unit commitment table using method No.3 for a system having units of multiple fuel type. All the above methods are tested by solving sample examples. Result of these examples reveal following observations.

- (1) It is possible to form optimal order of combination of units by the methods described above.
- (2) Methods are direct and fast and process of step size is eliminated.
- (3) The method of AFLC is not always optimal as per result in Table 4.9. The same conclusion is made by Lee[65].
- (4) Rest of the Methods are accurate
- (5) Result in tables 4.16 and 4.18 are obtained using methods 3 and 4 . It can be realized from these results that both methods give the same cost for generation scheduling calculated for 24 hours. However, from the tables it can be seen that combination order of units is same except for first two stages; reason being that in method 4 while forming this Tables 4.15 and 4.17. AFLC criteria was not included in the stage decision. However, if the ALFC criteria is included the same results are obtained.
- (6) Three step procedure is adopted to finally form unit commitment table. The result is comparable with the results reported.

Table 4.24: Multi Fuel Dispatch with Units Status and Cost

Sr. No.	Demand MW	Stage No	Units' No	Units' Fuel Status												Total Cost in \$ /hr.
1	1800.0	17	8	1	3	1	3	0	3	1	3	2	0			326.60
2	2000.0	18	9	1	3	1	3	1	3	1	3	2	0			365.50
3	2200.0	19	10	1	3	1	3	1	3	1	3	2	1			403.64
4	2500.0	20	10	2	3	1	3	1	3	1	3	2	1			526.24
5	2800.0	20	10	2	3	1	3	1	3	1	3	2	1			681.85
6	3000.0	21	10	2	3	1	3	1	3	2	3	2	1			803.76

- (7) method 3 is successfully applied to form optimal combination order and unit commitment schedule is estimated for some selected loads. However, the result cannot be compared with any standard result because 'No ' attempt is so far reported on this aspect.