OPTIMAL GROUNDWATER EXTRACTION

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OPTIMAL GROUNDWATER EXTRACTION IN STOCHASTIC GROUNDWATER SYSTEM

8.1 Over view

Farmers of the watershed seemed to have preference for cotton and cotton based cropping systems. The previous section established a conclusive evidence of a strong relationship of these systems with the annual groundwater extraction. It was suggested that in a partial shifting of area under crop from water intensive cotton to other less water intensive crops would be desirable from water productivity view point, leading to saving in groundwater use and could help attain sustainability of the resource in the long run. It was also observed that with increased groundwater recharge, as a result of high rainfall, and therefore, availability of more groundwater, farmers increased more number of crops in the cotton based cropping system. This also resulted in more groundwater being extracted in the watershed. However, higher extraction of groundwater could affect adversely the buffer role of groundwater stock in the event of the following year being rainfall deficit. In addition, the prominent cotton based cropping systems were reported to have significant relationship with average depth to water table, implying, thereby, that the existing cropping systems being practiced would lower the water table affecting the groundwater use prospects of small and medium farmers, who have limited capacity to reach the lower groundwater strata. Hence, there is a vital need to determine the response of the groundwater system with respect to groundwater extraction and the future events, in terms of its sustainability over a longer period of time. This entails examining the effect of extraction, in a dynamic model, to move the system over time from a less desirable to a more desirable position.

It is, therefore, imperative to study the existing groundwater system serving the watershed and examine the rates of groundwater extraction in terms of their effect on desired and optimal rates of increase in depth to groundwater table in the watershed.

8.2 Review of literature

The theory of optimal control has been applied to macro economic problems such as the choice of the best policy to regulate and stabilize the economy. These policy problems include how and when taxes should be raised or lowered, at what rate money supply should grow, how fast the apex bank should change credit conditions etc.

In the optimal control literature scholars have demonstrated the potential application of optimal control theory to economic stabilizations (Chow, 1966; Sengupta, 1970; Theil, 1964). Similarly, some scholars demonstrated its application to examine long terms economic growth and development (Dobell and Ho, 1967; Fox et al., 1966; Kendrick and Taylor, 1969, 1970; Shell, 1967).

Pindyck (1972) used the optimal control problem to study the choice of policy to best regulate and stabilize the economy. In his study the optimal control problem was defined as a dual discrete-time tracking problem where nominal state and nominal policy trajectories were tracked for a linear time invariant system with a quadratic cost functional. Philips (1954) through tracking problem study showed that in a multiplier- accelerator macro economic model, application of certain stabilization policies might result into fluctuations. Collings et al., (1996) used the information-state approach to obtain solutions to risk-sensitive quadratic control problems. They considered the case of tracking a desired trajectory. Results were presented for linear discrete-time models with Gaussian noise, and also for finite-discrete state, discrete-time hidden Markov models with continuous-range observations. Using such methods the tracking solution was obtained without appealing to a certainty equivalence principle. The results demonstrated the link to standard linear quadratic Gaussian control and achieving zero steady state error with risk-sensitive control policies. Kendrick, (1979, 1982) used control technique following the linear tracking approach to a small macro economic model of the U. S. economy. He used both monetary and fiscal policy variables to study probing and active learning in tracking the macro economic parameters of the economy. Coomes (1988) used stochastic control method following to solve the agricultural policy problem. He constructed a small model of corn market with two state variables (acreage and price) and two USDA controls (acreage set aside requirements and acreage diversion payment). Martens and Pindyck (1975) used quadratic linear optimal control to long term multi sectoral planning in Tunisia.

8.3 Deterministic control approach

A deterministic dynamic system can be formulated and the desirable path of various policy interventions solved as a quadratic linear problem (QLP). A deterministic problem is a control problem in which there is no uncertainty. Deterministic problems fall into two types; quadratic problems and general non-linear problems. We have used quadratic linear problem in this study.

8.4 Quadratic linear problem (QLP)

These are problems in which the criterion function is quadratic and the system equations are linear. A discrete time problem has been formulated here (Kendrick, 1981, 2002). In discrete time problem the criterion is a summation over time and the system equations are difference equations.

8.4.1 Problem formulation

Find $(q_t)_{t=0}^{N-1}$ the set of control vector from the period zero through period *N-1*, $(q_0, q_1, \ldots, q_{N-1})$. Period N is the terminal period of the model.

To minimize the criterion

$$J = \frac{1}{2} h_N' W_N h_N + w_N' h_N + \sum_{t=0}^{N-1} (\frac{1}{2} h_t' W_t h_t + w_t' h_t + h_t' F_t q_t - \frac{1}{2} q_t' \Lambda_t q_t + \lambda_t' q_t) \quad ...(1)$$

subject to the system equations

$$h_{t+1} = A_t h_t + B_t q_t + c_t \text{ for } t = 0,1,...,N-1 \quad ...(2)$$

and the initial condition

$$h_0 \text{ given} \quad ...(3)$$

where

h_t = state (groundwater depth) vector for the period t (n-element),

q_t = control (groundwater extraction) vector for period t (m-element),

W_t = positive definite symmetric matrix (n x n – element),

w_t = n – element vector,

F_t = n x m – matrix,

Λ_t = m x m – positive definite symmetric matrix,

λ_t = m – element vector,

A_t = n x n matrix,

B_t = n x m matrix,

c_t = n – element vector.

The problem is to find the time paths for the control variable, groundwater extraction, for the time periods from *0* to *N-1* to minimize the quadratic equation (1) with respect to the condition (2), while starting at the initial conditions (3). This formulation is used as the basis for formulating the tracking problem as under.

8.4.2 Quadratic linear tracking problem

For the tracking problem, the criterion function is given as,

$$J = \frac{1}{2}[h_N - h_N^{\#}]' W_N^{\#}[h_N - h_N^{\#}] + \frac{1}{2}\sum_{t=0}^{N-1}([h_t - h_t^{\#}]' W_t^{\#}[h_t - h_t^{\#}] + [q_t - q_t^{\#}]' \Lambda_t^{\#}[q_t - q_t^{\#}]) \ldots(4)$$

where,

$h_t^{\#}$ = desired vector for state (groundwater depth) variable in period *t*,

$q_t^{\#}$ = desired vector for control (groundwater extraction) in period *t*,

$W_t^{\#}$ = positive definite symmetric penalty matrix on deviations of state (groundwater depth) variable from the desired path (diagonal matrix),

$\Lambda_t^{\#}$ = positive definite symmetric penalty matrix on control (groundwater extraction) variable for deviation from desired paths (diagonal matrix).

Normally $W_t^{\#}$ and $\Lambda_t^{\#}$ are diagonal matrices. Equation (4) is equivalent to equation (1) in formulation.

8.4.3 Solution method

The solution method (Kendrick, 1981, 2002) for such quadratic linear control tracking problem lies in dynamic programming, the crucial notion of which is that of the optimal cost-to-go. The basic idea of dynamic programming is that from a given point, the path the rest of the way to finish would be the same no matter how one happened to get to that point. Since the path is the same from that point the rest of the way to finish, the cost-to-go from that point to the finish is the same no matter how one arrived at that point. It is called optimal cost-to-go since it is the minimum-cost path for rest of the route. It is written in symbols as $J^*(h_t)$, where h_t is vector giving initial point of departure of the system and $J^*(h_t)$ is the cost of moving the system from point h_t to point of finish.

Continuing the argument further, one can associate with every point h_t a minimum cost path to the finish and an associated optimal cost-to-go. If this information was available, one could strive to arrive at the finish point with minimum cost. This idea gives rise to the notation of a feedback rule of the form,

$$q_t = G_t h_t + g_t$$

...(5)

Where,

h_t = state vector giving the location of initial point of departure at time t

q_t = control vector at time t

G_t = matrix of coefficients

g_t = vector of coefficients

The feedback rule (5) says that given the groundwater system is in a state h_t at time t, the best policy to take is the set of policies in the vector q_t, the groundwater extraction at time t. In quadratic linear problems framework, this feed back rule is linear. Also the cost-to-go is a quadratic function of the state of the system at time t,

$$J^*(h_t) = J^*(t) = \frac{1}{2}h_t^{/}K_t h_t + \mathrm{P}_t^{/}h_t + \nu_t \qquad ...(6)$$

Where,

K_t = an n X n matrix, which is called the Riccati matrix

P_t = an n-element vector

ν_t = a scalar term

Equation (6) says that when the system is in the state h_t at time t, the optimal cost-to-go is a quadratic function of that state.

To derive the optimal feedback rule, the solution is to begin at the terminal time and work backwards toward the initial time. Hence, if the optimal cost-to-go at time t is defined by equation (6), the optimal cost-to-go at terminal time T can be written as,

$$J^*(x_N) = J^*(N) = \frac{1}{2}h_N^{/}K_N h_N + \mathrm{P}_N^{/}h_N + \nu_N \qquad ...(7)$$

From equation (1), the cost which are incurred in the terminal period N are given as

$$\frac{1}{2}h_N^{/}W_N h_N + w_N^{/}h_N \qquad ...(8)$$

By comparison of equations (7) and (8) we obtain

$$K_N = W_N \qquad ...(9)$$

$$\mathrm{P}_N = w_N \qquad ...(10)$$

$$\nu_N = 0 \qquad ...(11)$$

Equations (9) and (10) provide the terminal values for a set of difference equations which are used to determine K_N and P_N for all time periods. In fact the information in K_N and P_N is like price information in that W_N and w_N provide the information about the value of having the groundwater system in state h_N at time N. The difference equations in K and P are called Riccati equations and these transmit this price information from the last period backward in time to the initial period. K_t's and P_t's, in turn, are used to compute the G_t and g_t components of the feedback rule of equation (5).

The optimal cost-to-go for the terminal period N, given in equation (7) can be used to work backward in time to get the optimal cost-to-go in period $N-1$,

$$J^{*}(N-1) = \min_{q_{N-1}} \{J^{*}(N) + L_{N-1}(h_{N-1}, q_{N-1})\}$$

...(12)

Where L_{N-1} is the cost-function term in equation (1) for period $N-1$, which can be written as,

$$L_{N-1}(h_{N-1}, q_{N-1}) = \frac{1}{2}h'_{N-1}W_{N-1}h_{N-1} + w'_{N-1}h_{N-1} + h'_{N-1}F_{N-1}q_{N-1} + \frac{1}{2}q'_{N-1}\Lambda_{N-1}q_{N-1} + \lambda'_{N-1}q_{N-1}$$

...(13)

Equation (12) says that the optimal cost-to-go at time $N-1$ would be the minimum over the control at time $N-1$ of the optimal cost-to-go at state h_N in time N and the cost incurred in time $N-1$, that is L_{N-1}.

Substitution of equation (7) and (13) into equation (12) results into,

$$J^{*}(N-1)=\min_{q_{N-1}}(\frac{1}{2}h_N'K_Nh_N+\mathrm{P}_N'h_N+\frac{1}{2}h_{N-1}'W_{N-1}h_{N-1}+w_{N-1}'h_{N-1}+h_{N-1}'F_{N-1}q_{N-1}+\frac{1}{2}q_{N-1}'\Lambda_{N-1}q_{N-1}+\lambda_{N-1}'q_{N-1}) \quad ...(14)$$

Further, h_N in equation (14) can be written in terms of lagged values of h_{N-1} and q_{N-1} by using the system equation (2) as,

$$h_N=A_{N-1}h_{N-1}+B_{N-1}q_{N-1}+c_{N-1} \quad ...(15)$$

Then substituting of equation (15) into (14) and collection of like terms results into,

$$J^{*}(N-1)=\min_{q_{N-1}}(\frac{1}{2}h_{N-1}'\phi_{N-1}h_{N-1}+\frac{1}{2}q_{N-1}'\Theta_{N-1}q_{N-1}+h_{N-1}'\varphi_{N-1}q_{N-1}+\Phi_{N-1}'h_{N-1}+\theta_{N-1}'q_{N-1}+\eta_{N-1}) \quad ...(16)$$

Where,

$$\left.\begin{aligned}
\phi_{N-1}&=A_{N-1}'K_NA_{N-1}+W_{N-1}\\
\Theta_{N-1}&=B_{N-1}'K_NB_{N-1}+\Lambda_{N-1}\\
\varphi_{N-1}&=A_{N-1}'K_NB_{N-1}+F_{N-1}\\
\theta_{N-1}&=A_{N-1}'(K_N'c_{N-1}+\mathrm{P}_N)+w_{N-1}
\end{aligned}\right\} \quad ...(17)$$

$$\eta_{N-1}=\frac{1}{2}c_{N-1}'K_Nc_{N-1}+\mathrm{P}_N'c_{N-1}$$

The minimization for q_{N-1} in equation (16) is performed to give the first order condition,

$$q_{N-1}'\Theta_{N-1}+h_{N-1}'\varphi_{N-1}+\theta_{N-1}'=0 \quad ...(18)$$

This is written as,

$$q_{N-1}\Theta'_{N-1} + h_{N-1}\varphi'_{N-1} + \theta_{N-1} = 0$$

This can be solved for q_{N-1} in terms of h_{N-1} to obtain the feedback rule for the period $N-1$.

$$q_{N-1}\Theta'_{N-1} = -h_{N-1}\varphi'_{N-1} - \theta_{N-1}$$

$$q_{N-1} = -(\Theta'_{N-1})^{-1}\varphi'_{N-1}h_{N-1} - (\Theta'_{N-1})^{-1}\theta_{N-1}$$

$$q_{N-1} = G_{N-1}h_{N-1} + g_{N-1} \qquad ...(19)$$

Where,

$$G_{N-1} = -(\Theta'_{N-1})^{-1}\varphi'_{N-1} \qquad ...(20)$$

$$g_{N-1} = -(\Theta'_{N-1})^{-1}\theta_{N-1} \qquad ...(21)$$

This is the feedback rule for period $N-1$. This procedure is followed to obtain the feedback rule for the period $N-2$ as,

$$J^*(N-2) = \min_{q_{N-2}}\{j^*(N-1) + L_{N-1}(h_{N-2}, q_{N-2})\}$$

This after solution yields,

$$q_{N-2} = G_{N-2}h_{N-2} + g_{N-2}$$

Where,

$$G_{N-2} = -(\Theta'_{N-2})^{-1}\varphi'_{N-2}$$

$$g_{N-2} = -(\Theta'_{N-2})^{-1}\theta_{N-2}$$

and

$$\phi_{N-2} = A'_{N-2}K_{N-1}A_{N-2} + W_{N-2}$$

$$\Theta_{N-2} = B'_{N-2}K_{N-1}B_{N-2} + \Lambda_{N-2}$$

$$\varphi_{N-2} = A'_{N-2}K_{N-1}B_{N-2} + F_{N-2}$$

$$\theta_{N-2} = A'_{N-2}(K'_{N-1}c_{N-2} + \mathrm{P}_N) + w_{N-2}$$

$$\eta_{N-2} = \frac{1}{2}c'_{N-2}K_{N-1}c_{N-2} + \mathrm{P}'_{N-1}c_{N-2}$$

The feedback rules (19) and (20) generalize the rule as

$$q_t = G_t h_t + g_t \qquad \text{...(22)}$$

Further, if the optimal cost-to-go is stated as a function of the state h_{N-1} in equation (16), then q_{N-1} must be substituted out. This is accomplished by substituting this from equation (19)into equation (16) and collecting the like terms,

$$J^*(N-1) = \frac{1}{2}h'_{N-1}K_{N-1}h_{N-1} + \mathrm{P}'_{N-1}h_{N-1} + \nu_{N-1}$$

Where,

$$K_{N-1} = \phi_{N-1} + G'_{N-1}\Theta_{N-1}G_{N-1} + 2\varphi_{N-1}G_{N-1}$$

$$\mathrm{P}_{N-1} = (\varphi_{N-1} + G'_{N-1}\Theta_{N-1})g_{N-1} + G'_{N-1}\theta_{N-1} + \theta_{N-1}$$

$$\nu_{N-1} = -\frac{1}{2}\theta'_{N-1}(\Theta'_{N-1})^{-1}\theta_{N-1} + \eta_{N-1}$$

A similar computation for period $N-2$ yields these equations as,

$$K_{N-2} = \phi_{N-2} + G'_{N-2}\Theta_{N-2}G_{N-2} + 2\varphi_{N-2}G_{N-2}$$

$$\mathrm{P}_{N-2} = (\varphi_{N-2} + G'_{N-2}\Theta_{N-2})g_{N-2} + G'_{N-2}\theta_{N-2} + \theta_{N-2}$$

$$\nu_{N-2} = -\frac{1}{2}\theta'_{N-2}(\Theta'_{N-2})^{-1}\theta_{N-2} + \eta_{N-2}$$

These on further generalization give the Riccati equation for the problem as,

$$K_t = \phi_t + G'_t\Theta_t G_t + 2\varphi_t G_t$$

...(23)

$$\mathrm{P}_t = (\varphi_t + G'_t\Theta_t)g_t + G'_t\theta_t + \theta_t$$

...(24)

$$\nu_t = -\frac{1}{2}\theta'_t(\Theta'_t)^{-1}\theta_t + \eta_t$$

Thus, the optimal control problem given by equations (1) to (3) is solved by beginning with the terminal conditions (9) and (10) on K_N and P_N, then integrating the Riccati equations (23) and (24) backward in time. With K_t and P_t computed for all time periods, G_t and g_t for each time period can be calculated with (20) and (21). These in turn are used in the feedback rule (22) to compute q_o. Then, q_o and h_o are used in the system equation (2) to calculate h_1. Then h_1 is used in the feedback rule to compute q_1. The computations are done until all the h_t's and q_t's have been obtained.

8.5 Data set for the QLP problem

Data on depth to groundwater table and groundwater extraction for the watershed is being collected regularly at the Central Soil & Water Conservation Research & Training Institute, Research Centre, Vasad. These data sets were matched for the irrigation events. Thus, the data set included the depth to water table and the groundwater extraction in the current period for all the irrigation events for which consistent data set was available. The available data points for the period 2002-03 through 2005-06 were used.

8.6 Results and discussion

8.6.1 Groundwater state function

A simple groundwater model with one control and one state variable was formulated for the optimal control solution of the problem.

A linear function of the following form was considered to define fluctuation in groundwater depth,

$$h_t = f(q_t, h_{t-1})$$

Where,

h_t = Groundwater depth in period't'

h_{t-1} = Groundwater depth in the period't-1'

q_t = Groundwater extraction in period't'

It was hypothesized that groundwater depth in the current period will depend upon the groundwater extraction in the current period and the depth of groundwater table in the previous period. The effect of rain fall and the resulting groundwater recharge in the previous period would be captured by the groundwater depth in that period.

Function of the following form was estimated for the watershed.

$$h_t = c + A\, h_{t-1} - B\, q_t + u_t$$

Where,

h_t = Groundwater depth in period't'

h_{t-1} = Groundwater depth in the period't-1'

q_t = Groundwater extraction in period't'

c = Intercept

A, B = Coefficients

The groundwater depth for the period *t* was regrssed over the groundwater depth in period *t-1* and the groundwater extraction in period *t*.

To test auto regression in the function, the test suggested by Durrbin was conducted.

The usual Durbin-Watson d statistic may not be used to detect the first order serial correlation in the autoregressive models, because there is a built-in bias against discovering the first order serial correlation (Gujarati, 1988). Durbin (1970) himself has proposed a large-sample test of first-order serial correlation in autoregressive models (cf. Gujarati, 1988). This test is called the h statistic and is given as,

$$h = \hat{\rho}\sqrt{\frac{N}{1 - N[\mathrm{var}(B)]}}$$

Where,

N = sample size,

$\mathrm{var}(B)$ = variance of the coefficient of the lagged variable

$\hat{\rho}$ = estimate of the first order serial correlation ρ

$\hat{\rho}$ can be approximated from the estimated d as,

$$\hat{\rho} = 1 - \frac{1}{2}d$$

Where, d is the usual Durbin-Watson statistic. Therefore, h statistic is given by,

$$h = (1 - \frac{1}{2}d)\sqrt{\frac{N}{1 - N[\text{var}(B)]}}$$

8.6.2 Model estimation

The OLS estimate of the function was computed and the results are given in Tables 8.1 and 8.2.

The Durbin's h-statistic (-0.7915) computed from the data ranged between -1.95 and 1.96, which suggested that the null hypothesis, that there was no first-order (positive or negative) autocorrelation, was not rejected with a probability of 95 percent (Gujarati, 1988; pp.527). The absence of autocorrelation could be explained in terms of the nature of the data. The groundwater aquifer as suggested by the geo-morphological tests (Sharda et al., 2005) is leaky semi-confined in nature, therefore there are movements of water into the aquifer and from the aquifer outside, as a result of the extraction activities occurring elsewhere. Such types of aquifers are quite uncertain in nature in this sense and can not be predicted precisely.

The ANOVA revealed that the model was a good fit, given the significance level of the 'F' statistics (Table 8.1). The model fit ($\bar{R}^2$ = 0.96) suggested that 96 percent variation in the dependent variable was explained in terms of the independent variables. The coefficients turned out to be significant. The signs of the variables groundwater extraction and water depth in the previous period were as desired. The groundwater extraction had negative relationship with groundwater depth.

The final fitted model was,

$$h_t = 1.474 + 0.958\ h_{t-1} - 0.0125\ q_t \qquad \bar{R}^2 = 0.96 \quad n = 276$$

(0.362) (0.013) (0.002)

Durbin's h-statistic = -0.7915

These variables were used in the quadratic linear problem model for analysis.

QLP programme, version 2 in MS - DOS (Kendrick, 1989) was used to find the solution.

Initial groundwater depth to water table was taken as 20 m and volume of groundwater extraction, 16.60 ha-cm, respectively. The minimum depth of motor in the tube well observed in the watershed was the reason for considering this initial depth. However, other scenarios were also examined. The rate of increase in desired path of groundwater extraction was proposed 0.5 per cent in each time period for the bases scenario. A base scenario is one where desired path as well as the optimal path trajectory of groundwater table was same. The penalty matrix for deviation from desired path was taken as unit value of 1 for both state as well as control variables. The analysis was done for 24 periods of time, though this could be extended to longer period of horizon without affecting the model results.

The desired/ optimal trajectories for the state and control variables for the initial run are given in table 8.3.

At this rate of increase in groundwater extraction, the desired rate of increase in depth to groundwater level is also the optimal rate of increase (Figure (8.1). To examine the stability of the model, the model parameters were changed and the groundwater depth was tracked with respect to change in groundwater extraction (Table 8.4).

(a) Increase in control (groundwater extraction) parameter

Keeping the parameters of state and intercept of the model the same, an increase up to 8 per cent in parameter value of control (-0.0115) from the original value (-0.0125) does not result into change in the behaviour of the groundwater system of base scenario. At that rate and beyond, the optimal path of state trajectory changes from the desired path. In fact, the optimal trajectory lies higher than the desired trajectory (Figure 8.2), the increase declining from 2.25 per cent in the beginning to 0.6 per cent at the end of the horizon.

(b) Decrease in control parameter value

It was revealed that a 16 per cent decrease in the parameter value changed the desired and optimal path trajectory of the groundwater table (Figure 8.3). Up to that level a change in the parameter value does not change the two path trajectories. The optimal path trajectory lies below the desired path trajectory. The percentage change in trajectory growth declines over the period (Table 8.5) from 1.84 per cent in the initial period to as low as 0.46 per cent at the end of the period.

(c) Increase in state parameter value

The parameter value of state variable was increased, keeping the other two variables at the same level and the behaviour of the groundwater system was examined. It was revealed that a 0.21 % increase in parameter value (0.958 to 0.960) alone was enough to change the behaviour of the groundwater system (Figure 8.4). The optimal path of groundwater table became higher than that of the desired path. This indicated sensitivity of the model to increase in the value of state parameter. The percent increase in optimal path of state variable changed from 2.4 per cent to 0.6 per cent over the horizon.

(d) Decrease in state parameter value

In the event of a decrease in the parameter value of the state variable, the behaviour changes in the other way. A 0.84% decrease in state parameter value lowered the optimal path (Figure 8.5). The optimal path of water table, in this eventuality, changed from 1.4 per cent in the beginning of the period to 0.3 per cent at the end of the period.

(e) Increase in the intercept value

The increase in constant value of the groundwater system model, up to a level of 1.1 per cent, while keeping the other parameter values same, does not change the behaviour of the model. At this and beyond this increase in rate, the optimal path of groundwater table changes from the desired path. The optimal path goes slightly higher than the desired path (Figure 8.6).

(f) Decrease in the intercept value

A decrease in constant (intercept) parameter value, similarly lowers the optimal path of groundwater extraction below the desired path. The percentage decrease in optimal path of groundwater table decreases over the period (from 2.00 per cent to 0.52 per cent) (Figure 8.7).

At a depth of 20m, the trajectory of groundwater table is rising, indicating an increasing in the water level in response to the groundwater extraction. This behaviour is explained in terms of the nature of the aquifer at this depth. Between 13.7m to 24.4m depth lies the leaky semi-confined aquifer with predominant lithology of fractured amygdaloidal basalt (Sharda et al., 2006). As extraction increases at a lower level of 0.5per cent, because of movement of water into the aquifer, the water table rises. If groundwater extraction further continues at comparatively higher rate, the behaviour of groundwater changes. To study this behaviour different extraction rate scenarios were examined to draw a caution about the groundwater mining as happened in Gujarat. In addition, three different depth of groundwater levels were analyzed based on the geological feature of the under ground rock formation.

(A) Groundwater extraction scenarios at 20m depth

A growth rate of 1% in groundwater extraction over the period would result in an almost similar path of groundwater trajectory. Up to 1 per cent increase in groundwater extraction, the groundwater system is quite stable as the desired path of groundwater table is also the optimal path (Figure 8.8). The water table trajectory grows at a declining rate. As the groundwater extraction rate increased to 5 per cent, the behaviour of the groundwater system changed. The per cent change in growth of optimal path water table started declining as compared to the desired path and became negative after 18th period (table 8.9). Beyond this period, if groundwater extraction continued, the water table started declining at a slightly higher percentage point with increasing time period (0.10 per cent in 18th period to 0.64 per cent in 24th period). A further increase in groundwater extraction rate to 10 per cent worsened the situation. The water table decline occurred early, at 11th period (Figure 8.10). Up to 10th period, growth in optimal path of water table declined sharp from 2.1 per cent in the 1st period to 0.13 per cent in 10th period (Table 8.6). At a still higher rate of 15% of groundwater extraction, the groundwater table declined still earlier (at 8th period) and continued declining beyond that period (Figure 8.11).

This depth of water table coincides with a leaky aquifer as described earlier and is a better transmitting zone. But a higher rate of groundwater extraction beyond 5 per cent could worsen the water table situation in the watershed. A higher extraction rate would lead to decline in water table faster. The continued extraction had its consequence in the past when many well became defunct due to lowering of water table. Those capable of deepening the wells did either deepened it further or dug new wells at deeper depths in the watershed.

(B) Groundwater extraction scenarios at 35m depth

Another scenario with a depth of 35m was examined. At this initial depth of water table the water table declines continuously even at a lower extraction rate of 0.5 per cent

(Figure 8.12). In the beginning of the period, the optimal path of water table declined by 0.56 per cent, which towards the end of the period was 0.28 per cent (Table 8.7). The situation was almost similar even at 1 per cent extraction rate (Figure 8.13). But at a higher 5 per cent, the optimal water table path declined at a higher rate at each time period (0.6 per cent at the beginning to 1.3 per cent at the end) (Figure 8.14). Further at 10 per cent and 15 per cent, the decline became steep (Figures 8.15 and 8.16). At 10 per cent extraction rate, percentage decline in optimal water table increases from 0.6 per cent to 6 per cent at the end. This decline was much faster at 15 per cent extraction rate, declining from 0.6 per cent to 74 per cent.

This water table depth coincided with another water bearing strata, inter-trappean semi-confined to confined aquifer. The behaviour of this aquifer is quite uncertain in terms of water supply. The analysis revealed that the water table continuously declined as a result wells penetrated to this depth could not be depended for regular groundwater supply.

(C Groundwater extraction scenarios at 50m depth

At the depth of 50m groundwater table, the base scenario was modeled at 0.5 per cent groundwater extraction rate (Figure 8.17). The water declined over the period, the percentage decline varying from 1.66 per cent in the beginning to 0.86 per cent at the end. This scenario was similar at 1 per cent extraction rate also (Figure 8.18). The rate of increase in groundwater extraction was changed from 5% to 15% to examine the behaviour of the water table movement (Figures 8.19 trough 8.21). At the different rates analyzed, the optimal growth path of groundwater table declined below the desired path. The decline initially was around 1.7% and toward the period end decline to 1.8 %. At the extraction rate of 10 per cent increase, the decline was still faster, from 1.7% to 6.0%. Similarly, at 15% increase in extraction rate, the groundwater table declined at much faster rate, from 1.7% in initial period to 40% at the end of the studied period. A higher depth of 70m behaved in a similar way (Figure 8.22).

This water table depth is another water bearing zone, an aquifer with intermittent mud layers. The water is stored in and moves through open fractures, the size, number and distribution and their interconnection being highly variable (Sharda et al., 2006). These tend to decrease in size and number with depth. Thus, overall capacity of water storage of these fractures is small and tends to decrease with depth. Wells penetrating to this aquifer do not yield dependable supplies of water up to a sizable increase in groundwater extraction rate.

This showed that at deeper depth the nature of aquifer does not permit higher extraction of groundwater as the groundwater depth declines at faster rate. This has implications for the sustainability of the resource in the watershed. A majority of the wells in the watershed extract groundwater from this aquifer (60m to 70m depth). With higher acreage under water intensive crops, the groundwater extraction would increase and an increase in rate of groundwater extraction would lower the groundwater table further.

8.7 Summary and conclusion

The groundwater system was examined in quadratic linear problem framework and the behaviour of groundwater table was tracked in response to the different groundwater extraction scenarios. The geomorphology of the underlying rocks in the watershed demarcates the water bearing strata into different kinds of aquifers. Accordingly, the groundwater system was examined for different water table depths. The main conclusions of the analysis are as under,

1) Up to 1 per cent increase in groundwater extraction, the groundwater system is quite stable as the desired path of groundwater table is also the optimal path. As the groundwater extraction rate increased to 5 per cent, optimal path water table started declining as compared to the desired path. A further increase in

groundwater extraction rate to 10 per cent or more worsened the situation. This depth of water table coincides with a leaky aquifer as described earlier and is a better transmitting zone. At lower rate of extraction (less than 5 per cent), the water table does not drop sharply. A higher extraction rate (beyond 5 per cent) would lead to decline in water table faster.

2) At 35m depth, the water table declines continuously even at a lower extraction rate of 0.5 per cent. At 10 per cent and 15 per cent, the decline became steeper. This water table depth coincided with another water bearing strata, inter-trappean semi-confined to confined aquifer. The behaviour of this aquifer is quite uncertain in terms of water supply. The analysis revealed that the water table continuously declined as a result wells penetrated to this depth could not be depended for regular groundwater supply.

3) At the groundwater table depth of 50m, the optimal and desired path of groundwater table increase was the same, if the growth rate in groundwater table was kept at 1 per cent or below. At the growth rate higher than 1% in the groundwater extraction, the optimal path drifted from the desired path, the percentage rate of decline in groundwater table increasing with each time period. The depth of groundwater table falls in the lower water bearing strata, that is aquifer with intermittent mud layer. Wells penetrating to this aquifer do not yield dependable supplies of water up to a sizable increase in groundwater extraction rate. Hence, higher extraction from this depth would drastically affect the groundwater depth.

2) A majority of the wells (60m to 70m) in the watershed extract water from this aquifer and, therefore, dependability of groundwater supply, in the long run, may not be quite reliable. The previous sections proved a close relationship between

cropping systems and groundwater extraction as well as the groundwater depth. This has implications for sustainable use of the resource in future.

8.8 Limitations of the study

The present study limits itself to examination of response of the groundwater system to different groundwater extraction rates. The uncertain nature of the groundwater movement below the ground surface makes it difficult to arrive at specific rates of extraction. This would vary with season as the quantity and distribution of rainfall and recharge change, and therefore, warrant study of a hydrologic-economic model. This was not attempted in the study and hence, leaves the scope for further work in this area.

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Table 8.1 Analysis of variance results for the fitted model

	df	SS	MS	F	Significance F
Regression	2	8698.65	4349.32	320010.3	0
Residual	273	3.71	0.01		
Total	275	8702.37			

Table 8.2 Coefficients of the fitted variable in the model

	Coefficients	Standard Error	t Stat	Significance level	R^2
Intercept	1.4741	0.36184	4.0741	0.00	
Groundwater depth in period 't-1'	0.95841	0.012832	74.6889	0.00	0.96
Groundwater extraction in period 't'	-0.01259	0.00259	-4.851	0.00	

R-Squared	0.96051	R-Bar-Squared	0.96022
S. E. of Regression	1.1234	F-stat F(, 272)	3307.5 (0.00)
Mean of dependent variable	24.39	S. D. of Dependent Variable	5.632
Equation Log-likelihood	-420.70	Akaike Info. Criterion	-423.71
Schwarz Bayesian Criterion	-429.13	DW-statistic	2.0933
Durbin's h-statistic	-0.7915 (0.429)		

Diagnostic Tests

	Test Statistics	LM Version	F Version
A:	Serial correlation	CHSQ(1) = 0.6415 (0.423)	F (1, 271) = 0.63364 (0.427)
B:	Functional form	CHSQ(1) = 0.2217 (0.638)	F (1, 271) = 0.2186 (0.640)
C:	Normality	CHSQ(2) = 302327.7 (0.00)	Not applicable
D:	Heteroscedasticity	CHSQ(1) = 2.0049 (0.57)	F (1, 273) = 2.0050 (0.158)

A: Lagrange multiplier test of residual serial correlation
B: Ramsey's RESET test using the square of the fitted values
C: Based on a test of skewness and kurtosis of residuals
D: Based on the regression of squared residuals on squared fitted values

Table 8.3: *Desired/ optimal state (groundwater depth) and control (groundwater extraction) trajectories of the groundwater table*

Initial groundwater table 55m, Initial groundwater extraction 44.10 ha-cm

Time period	State variable (Depth to water table, m)	Control variable (Groundwater extraction, ha-cm)
1	53.735	44.307
2	52.499	44.516
3	51.292	44.725
4	50.112	44.935
5	48.959	45.146
6	47.833	45.358
7	46.733	45.572
8	45.658	45.786
9	44.608	46.001
10	43.582	46.217
11	42.580	46.434
12	41.600	46.653
13	40.644	46.872
14	39.709	47.092
15	38.795	47.313
16	37.903	47.536
17	37.031	47.759
18	36.180	47.984
19	35.348	48.209
20	34.535	48.436
21	33.740	48.663
22	32.964	48.892
23	32.206	49.122
24	31.465	49.353

Table 8.4: *Sensitivity analysis of the variables of the fitted model*

S. No.	Variable	Original value of the parameter of fitted model	New value at which optimal rate differs from desired rate	Per cent change*
1	Control (groundwater extraction)	-0.0125	-0.0175	(-) 40.00
2	Control (groundwater extraction)	-0.0125	-0.0118	(+) 5.00
3	State (groundwater level)	0.9580	0.9599	(+) 0.20
4	State (groundwater level)	0.9580	0.9522	(-) 0.60
5	Intercept (constant parameter)	1.4740	1.4887	(+) 1.00
6	Intercept (constant parameter)	1.4740	1.4592	(-) 1.00

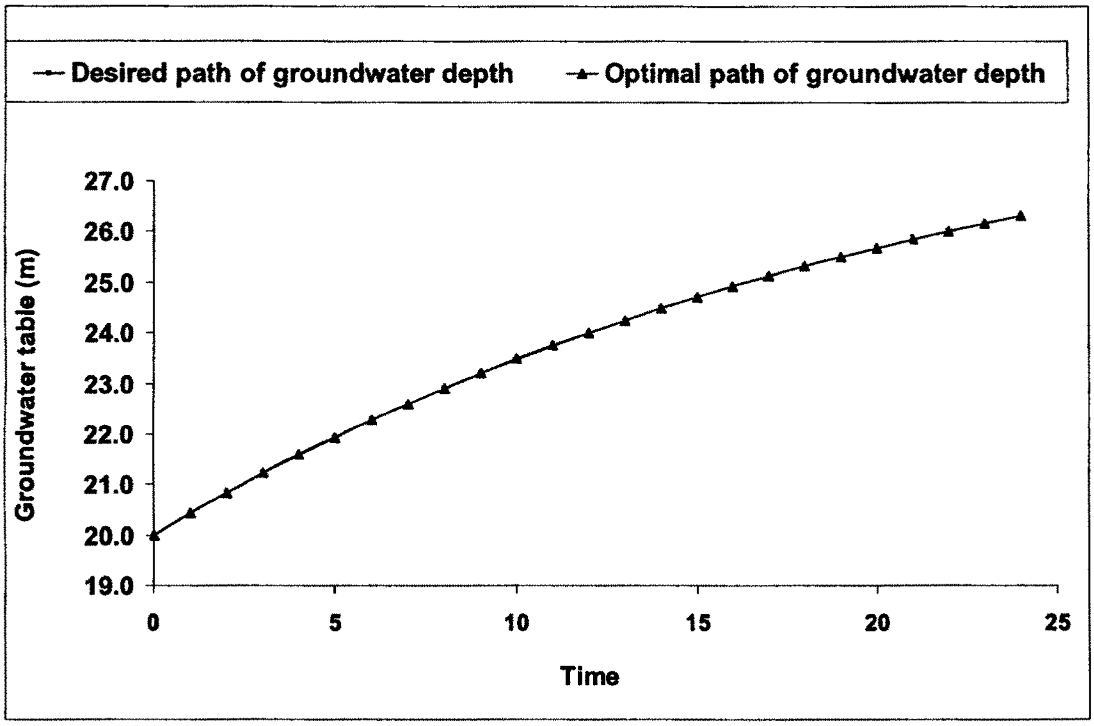
* The minus and plus sign in parentheses indicate the decrease and increase, respectively, in the value of the parameters.

Table 8.5. *Change (%) in Optimal path of water table trajectories, Initial groundwater depth 55m, Initial groundwater extraction 44.10 ha-cm, Sensitivity analysis*

Time period	Control (extraction) parameter value decreased		Control (extraction) parameter value increased		State (water table) parameter value decreased		State (water table) parameter value increased		Intercept value decreased		Intercept value increased	
	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)
0	55.000		55.000		55.000		55.000		55.000		55.000	
1	53.417	-2.88	53.675	-2.41	53.306	-3.08	53.686	-2.39	53.617	-2.51	53.645	-2.46
2	51.898	-2.84	52.404	-2.37	51.691	-3.03	52.423	-2.35	52.289	-2.48	52.345	-2.42
3	50.439	-2.81	51.184	-2.33	50.151	-2.98	51.209	-2.32	51.015	-2.44	51.097	-2.38
4	49.038	-2.78	50.012	-2.29	48.683	-2.93	50.043	-2.28	49.792	-2.40	49.899	-2.34
5	47.692	-2.74	48.888	-2.25	47.282	-2.88	48.922	-2.24	48.617	-2.36	48.748	-2.31
6	46.398	-2.71	47.808	-2.21	45.947	-2.82	47.844	-2.20	47.489	-2.32	47.643	-2.27
7	45.156	-2.68	46.771	-2.17	44.672	-2.77	46.807	-2.17	46.406	-2.28	46.582	-2.23
8	43.961	-2.65	45.776	-2.13	43.456	-2.72	45.811	-2.13	45.366	-2.24	45.563	-2.19
9	42.813	-2.61	44.819	-2.09	42.296	-2.67	44.852	-2.09	44.367	-2.20	44.584	-2.15
10	41.710	-2.58	43.901	-2.05	41.189	-2.62	43.931	-2.05	43.407	-2.16	43.643	-2.11
11	40.648	-2.55	43.018	-2.01	40.132	-2.57	43.044	-2.02	42.484	-2.13	42.739	-2.07
12	39.627	-2.51	42.171	-1.97	39.123	-2.51	42.191	-1.98	41.598	-2.09	41.871	-2.03
13	38.645	-2.48	41.356	-1.93	38.159	-2.46	41.371	-1.94	40.746	-2.05	41.036	-1.99
14	37.701	-2.44	40.573	-1.89	37.239	-2.41	40.582	-1.91	39.928	-2.01	40.234	-1.95
15	36.791	-2.41	39.821	-1.85	36.360	-2.36	39.822	-1.87	39.141	-1.97	39.463	-1.92
16	35.916	-2.38	39.098	-1.82	35.521	-2.31	39.091	-1.84	38.384	-1.93	38.721	-1.88
17	35.073	-2.35	38.403	-1.78	34.719	-2.26	38.387	-1.80	37.657	-1.89	38.008	-1.84
18	34.261	-2.32	37.734	-1.74	33.952	-2.21	37.709	-1.77	36.957	-1.86	37.323	-1.80
19	33.479	-2.28	37.092	-1.70	33.219	-2.16	37.057	-1.73	36.285	-1.82	36.663	-1.77
20	32.726	-2.25	36.474	-1.67	32.518	-2.11	36.429	-1.69	35.638	-1.78	36.029	-1.73
21	32.000	-2.22	35.879	-1.63	31.849	-2.06	35.824	-1.66	35.015	-1.75	35.419	-1.69
22	31.300	-2.19	35.308	-1.59	31.208	-2.01	35.242	-1.62	34.416	-1.71	34.832	-1.66
23	30.625	-2.16	34.758	-1.56	30.595	-1.96	34.681	-1.59	33.840	-1.67	34.267	-1.62
24	29.975	-2.12	34.229	-1.52	30.008	-1.92	34.140	-1.56	33.285	-1.64	33.723	-1.59

Table 8.6. *Optimal path of water table trajectories at different groundwater extraction rates, Initial groundwater depth 55m, Initial groundwater extraction 44.1 ha-cm*

Time period	Groundwater increase 1.0%		Groundwater increase 5.0%		Groundwater increase 10%		Groundwater increase 15%		Groundwater increase 20%	
	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)	(m)	Change (%)
0	55.000		55.000		55.000		55.000		55.000	
1	53.629	-2.49	54.100	-1.64	54.098	-1.64	54.097	-1.64	54.098	-1.64
2	52.311	-2.46	53.234	-1.60	53.228	-1.61	53.223	-1.62	53.221	-1.62
3	51.042	-2.43	52.402	-1.56	52.388	-1.58	52.375	-1.59	52.366	-1.61
4	49.822	-2.39	51.601	-1.53	51.575	-1.55	51.550	-1.58	51.529	-1.60
5	48.647	-2.36	50.830	-1.49	50.789	-1.52	50.746	-1.56	50.704	-1.60
6	47.515	-2.33	50.089	-1.46	50.028	-1.50	49.960	-1.55	49.889	-1.61
7	46.426	-2.29	49.375	-1.43	49.289	-1.48	49.188	-1.55	49.076	-1.63
8	45.376	-2.26	48.687	-1.39	48.570	-1.46	48.428	-1.55	48.259	-1.66
9	44.365	-2.23	48.024	-1.36	47.871	-1.44	47.676	-1.55	47.432	-1.71
10	43.391	-2.20	47.384	-1.33	47.188	-1.43	46.927	-1.57	46.585	-1.79
11	42.451	-2.17	46.768	-1.30	46.521	-1.41	46.178	-1.60	45.709	-1.88
12	41.545	-2.13	46.173	-1.27	45.867	-1.41	45.424	-1.63	44.791	-2.01
13	40.671	-2.10	45.599	-1.24	45.224	-1.40	44.659	-1.68	43.818	-2.17
14	39.828	-2.07	45.044	-1.22	44.590	-1.40	43.878	-1.75	42.773	-2.38
15	39.014	-2.04	44.507	-1.19	43.962	-1.41	43.074	-1.83	41.637	-2.66
16	38.228	-2.01	43.988	-1.17	43.340	-1.41	42.240	-1.94	40.386	-3.00
17	37.469	-1.99	43.486	-1.14	42.720	-1.43	41.367	-2.07	38.992	-3.45
18	36.735	-1.96	42.999	-1.12	42.100	-1.45	40.446	-2.23	37.422	-4.03
19	36.026	-1.93	42.527	-1.10	41.477	-1.48	39.465	-2.43	35.636	-4.77
20	35.341	-1.90	42.069	-1.08	40.849	-1.51	38.414	-2.66	33.588	-5.75
21	34.678	-1.88	41.624	-1.06	40.212	-1.56	37.277	-2.96	31.220	-7.05
22	34.036	-1.85	41.192	-1.04	39.563	-1.61	36.039	-3.32	28.464	-8.83
23	33.415	-1.82	40.771	-1.02	38.900	-1.68	34.681	-3.77	25.239	-11.33
24	32.813	-1.80	40.360	-1.01	38.218	-1.75	33.184	-4.32	21.448	-15.02



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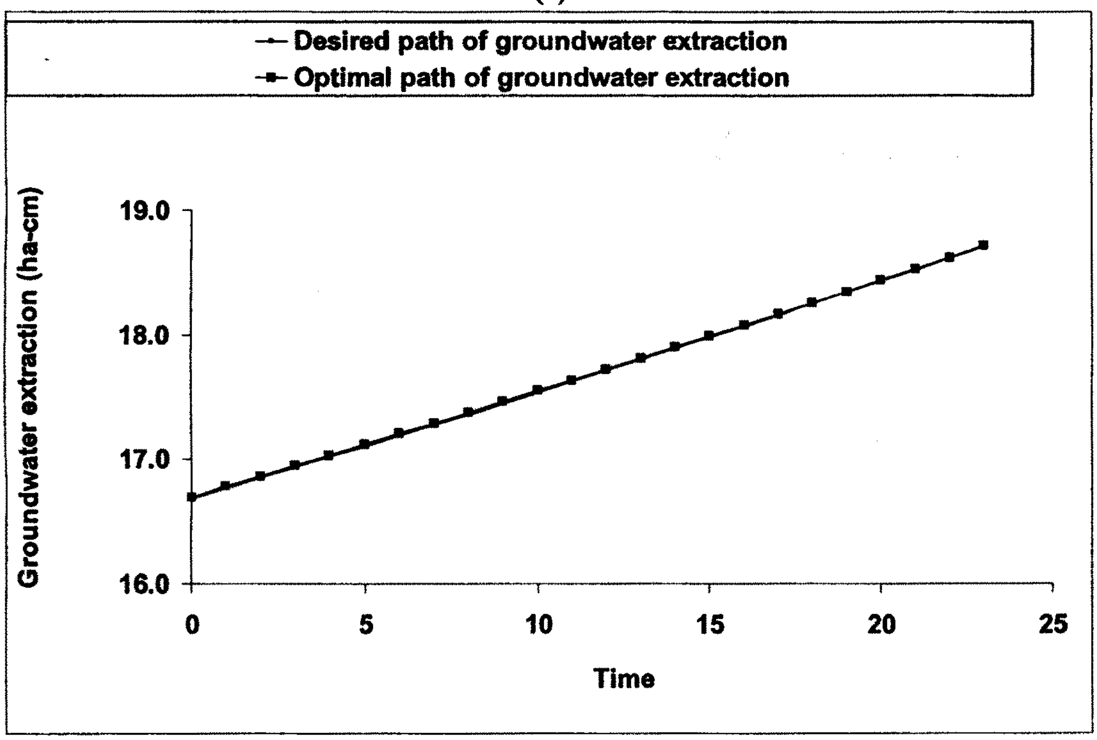
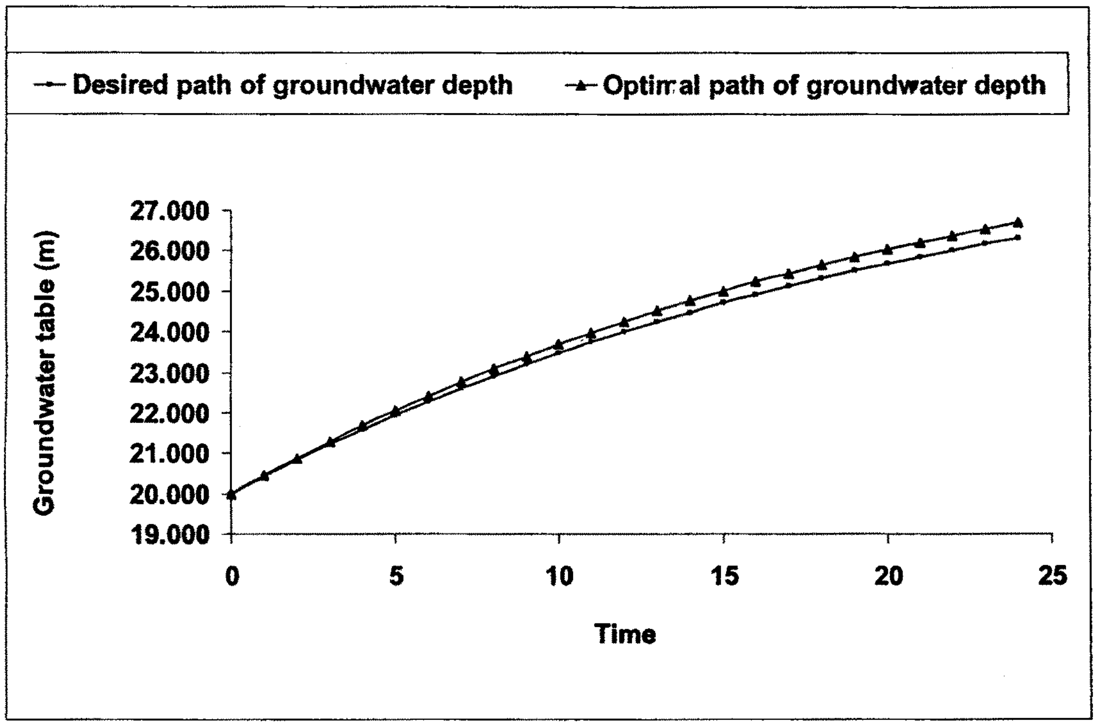


(b)

Figure 8.1: *Desired and optimal paths of groundwater trajectories, (Initial depth to groundwater table, 20m) (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*



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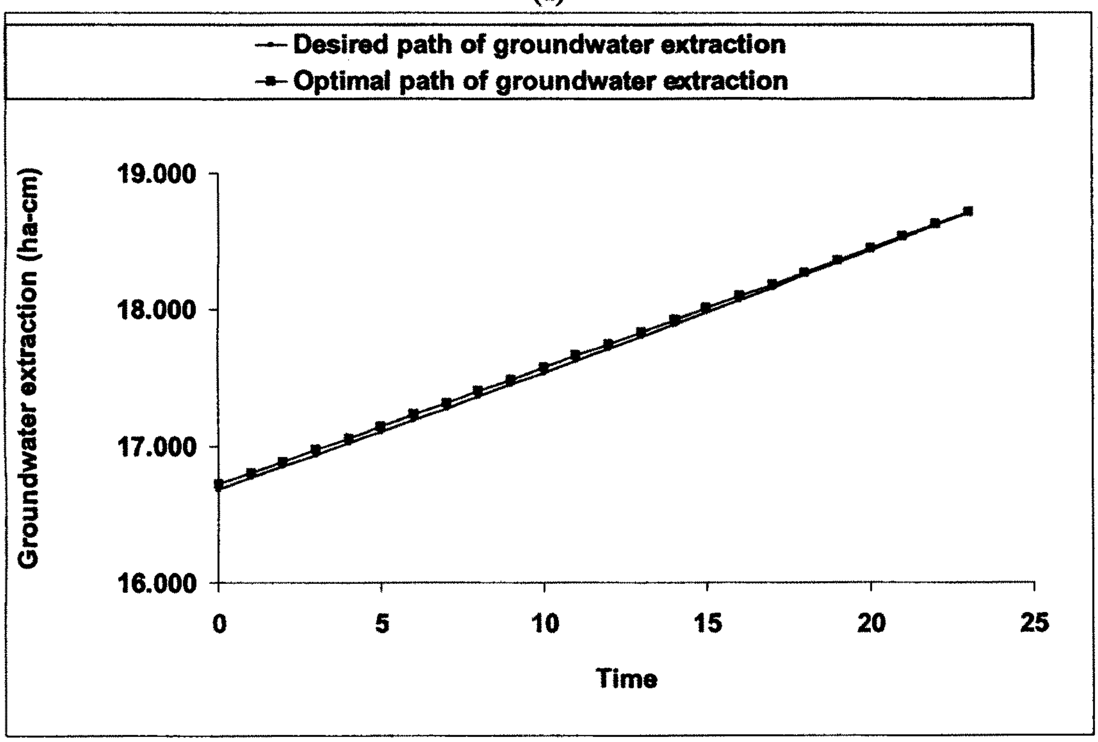
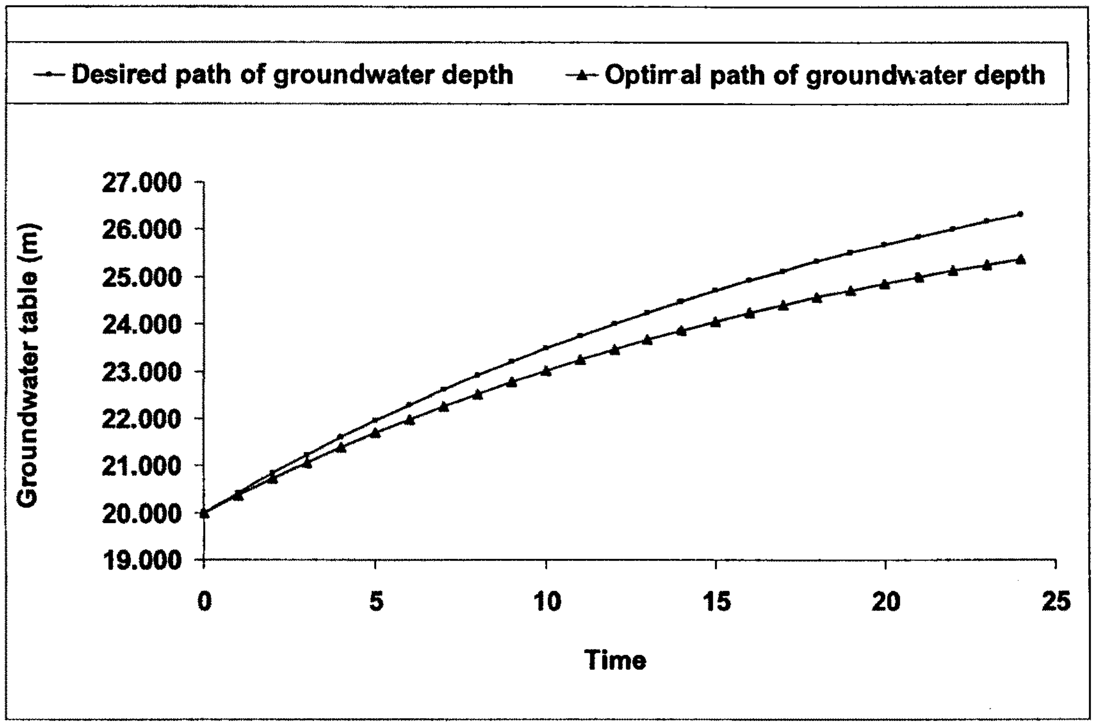


(b)

Figure 8.2: *Desired and optimal paths of groundwater trajectories, control parameter value increased by 8%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*



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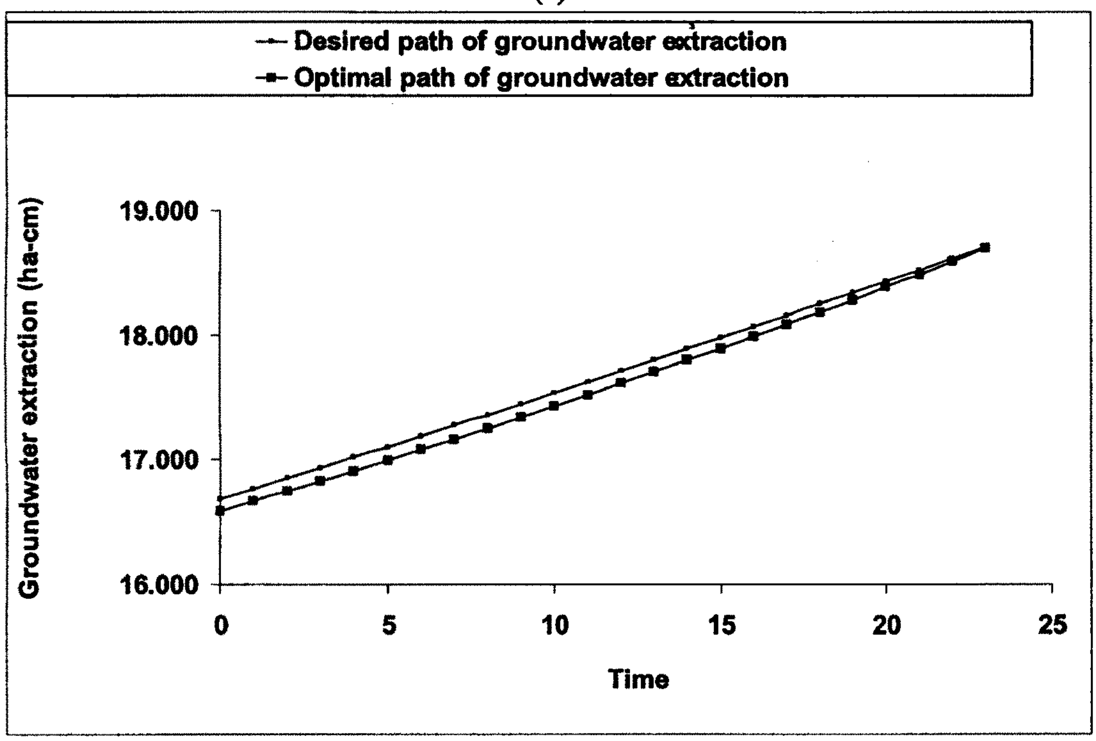
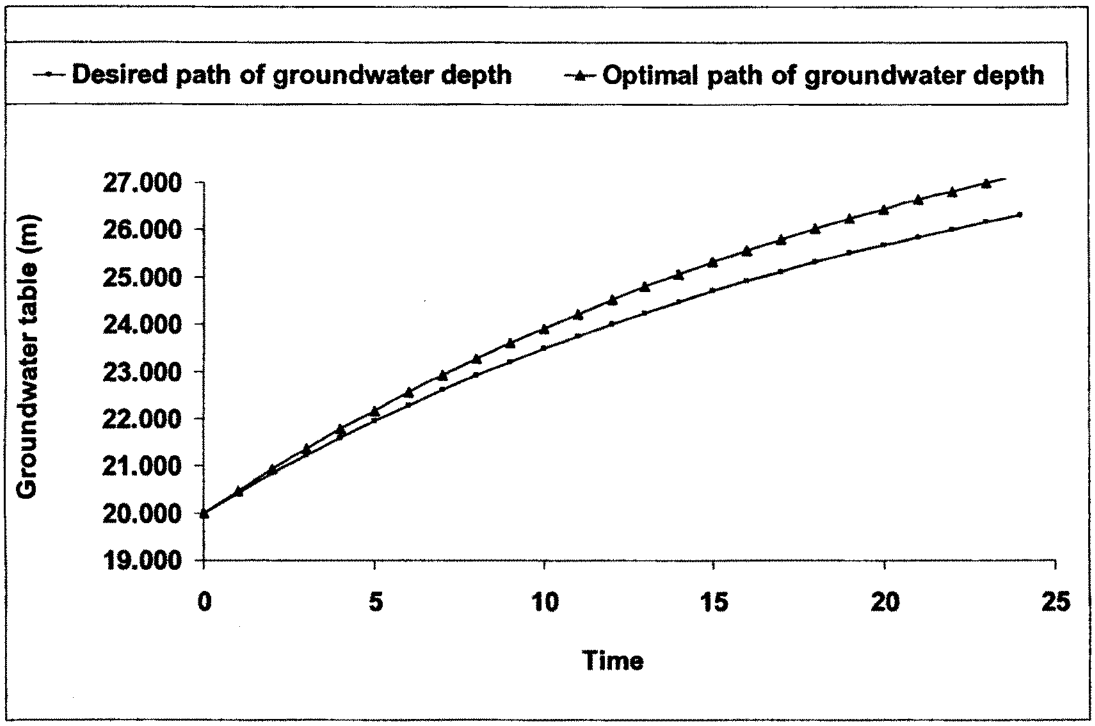
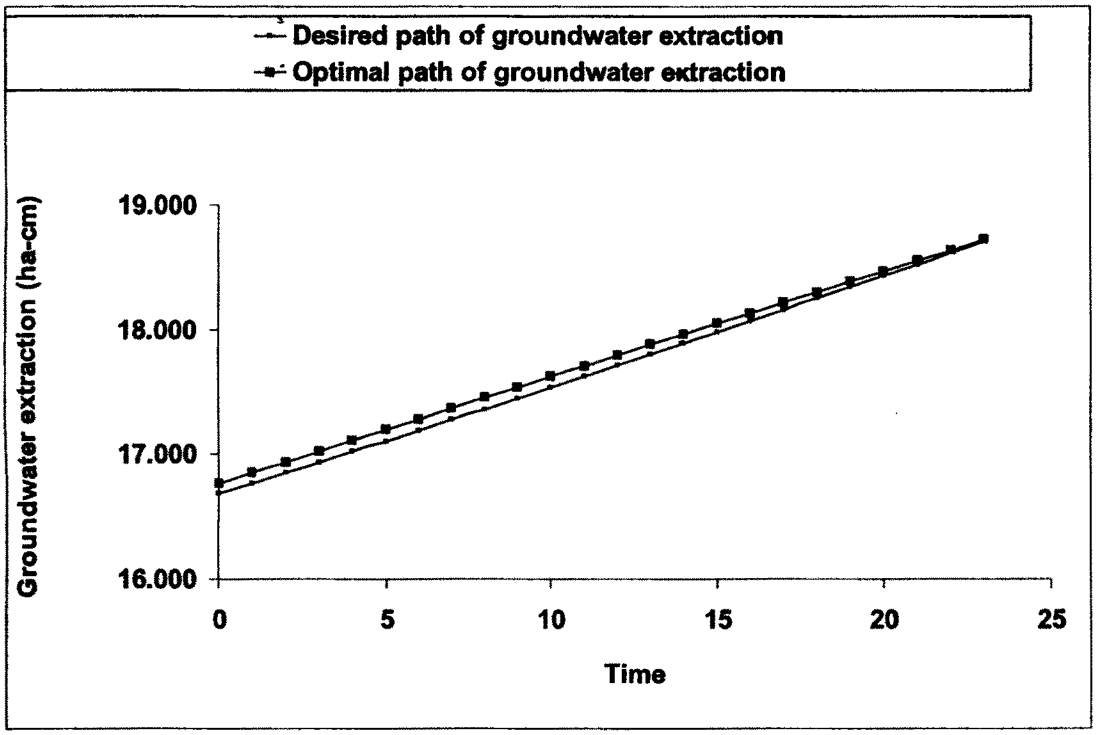


(b)

Figure 8.3: *Desired and optimal paths of groundwater trajectories, control parameter value decreased by 16%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*

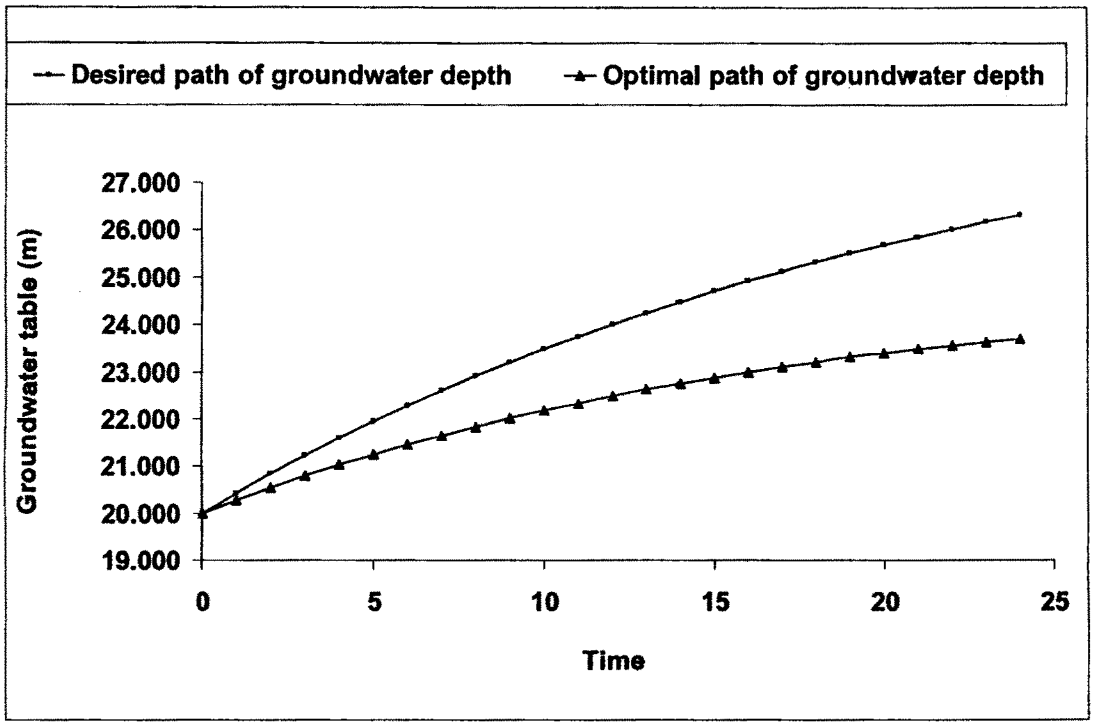


(a)



(b)

Figure 8.4: *Desired and optimal paths of groundwater trajectories, state parameter value increased by 0.2%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*



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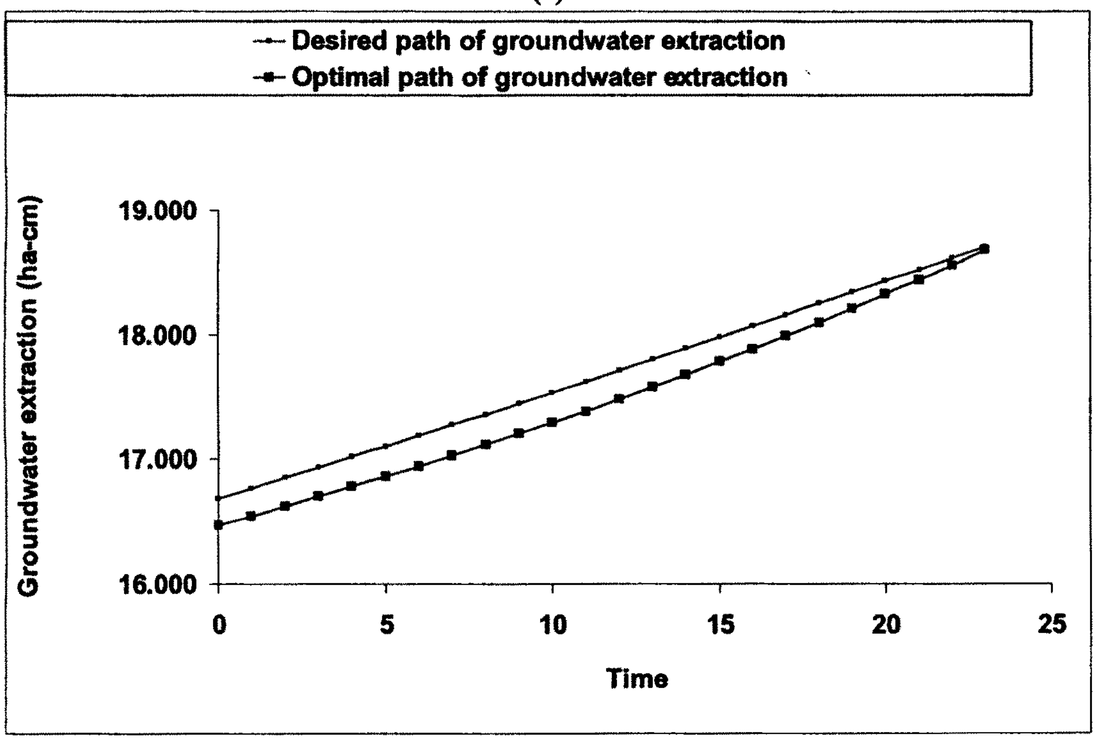
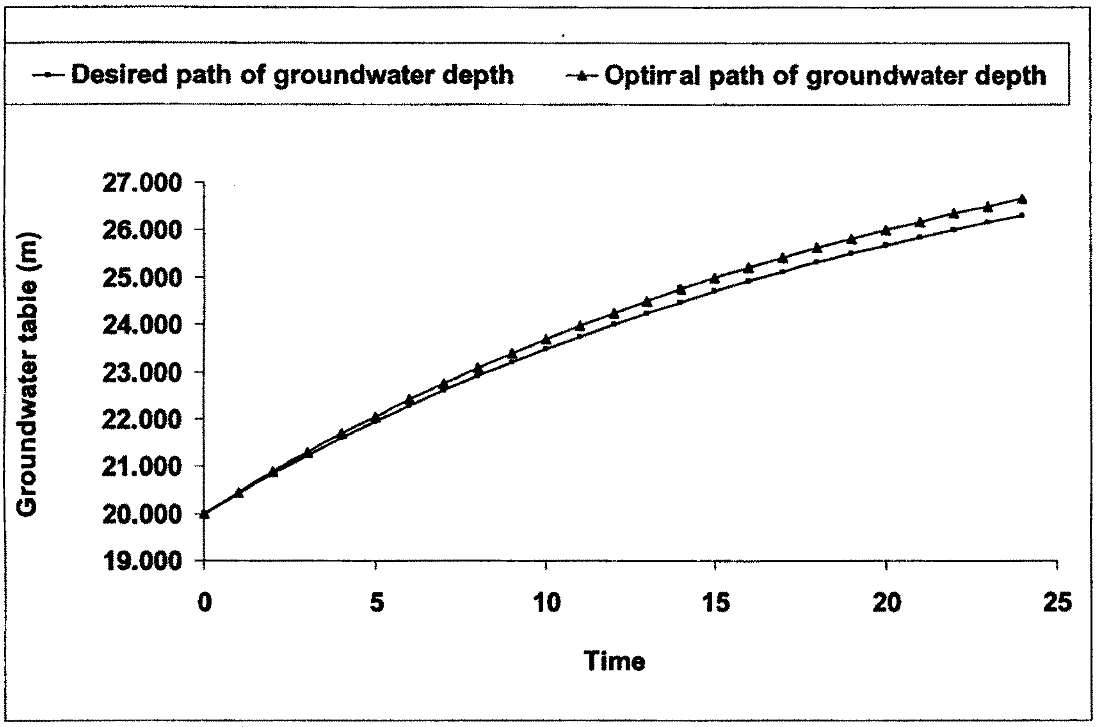
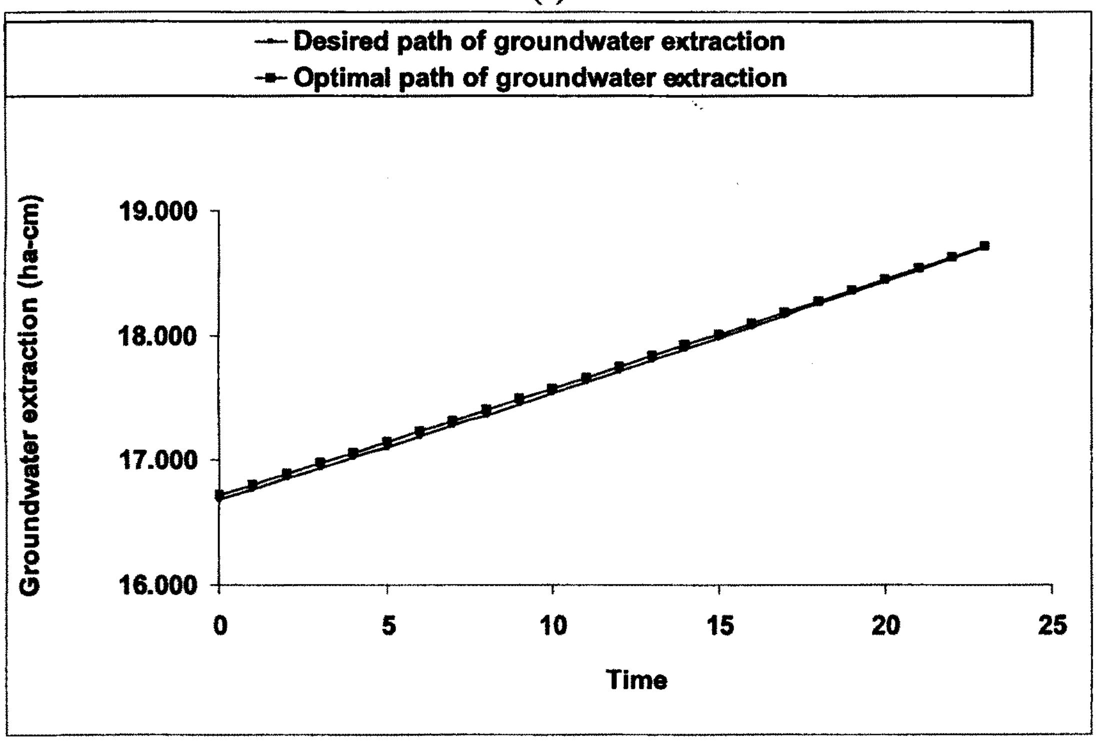


(b)

Figure 8.5: *Desired and optimal paths of groundwater trajectories, state parameter value decreased by 0.84%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*

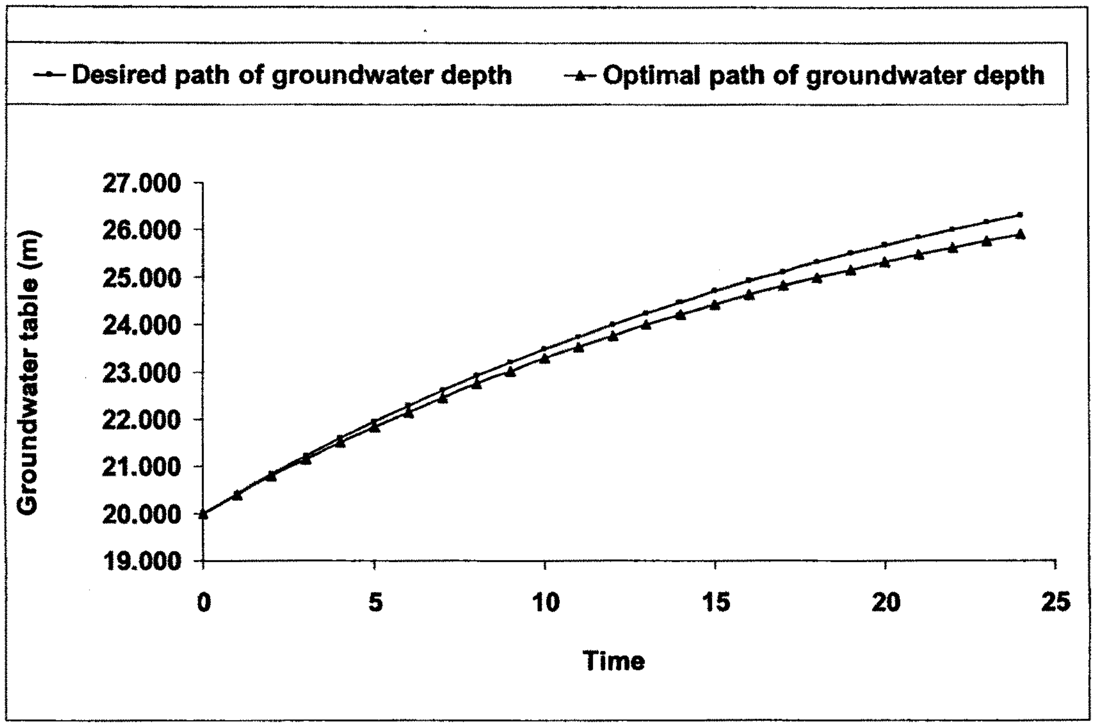


(a)

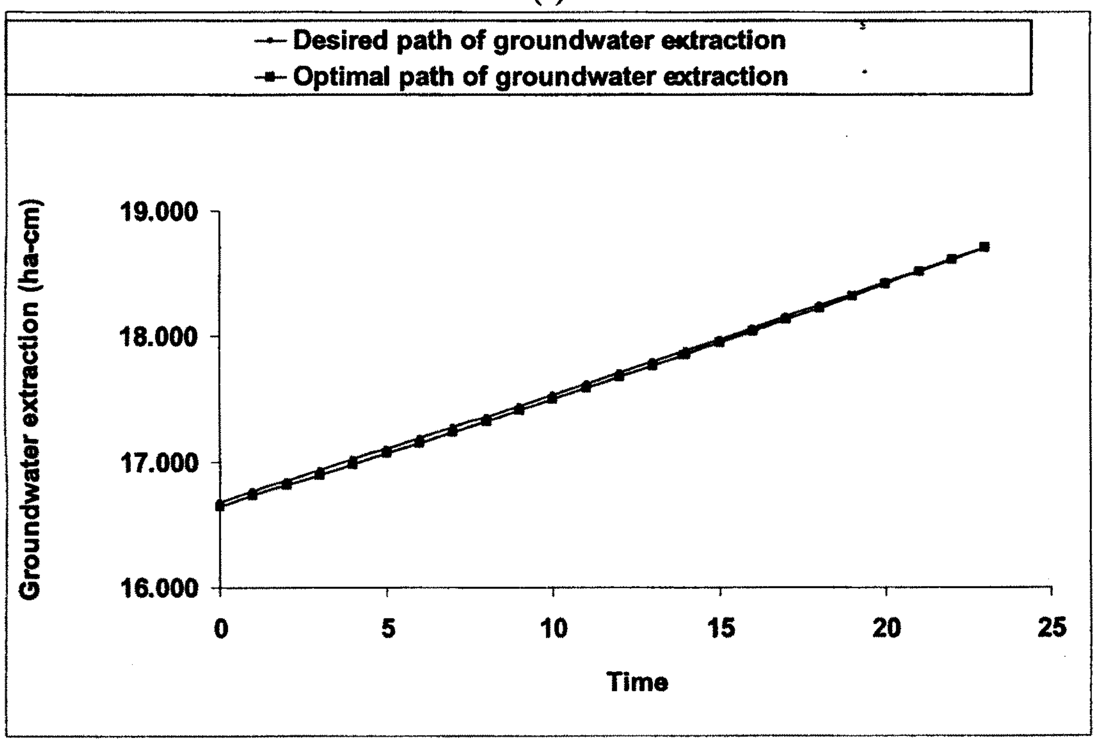


(b)

Figure 8.6: *Desired and optimal paths of groundwater trajectories, intercept parameter value increased by 1.09%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*

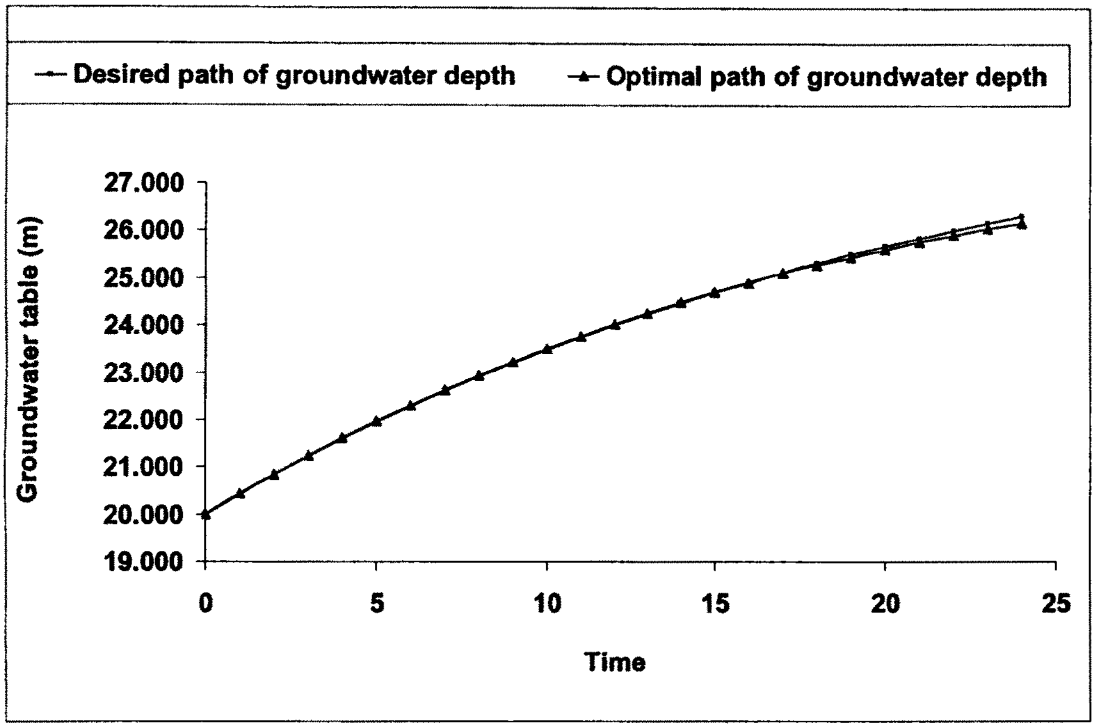


(a)

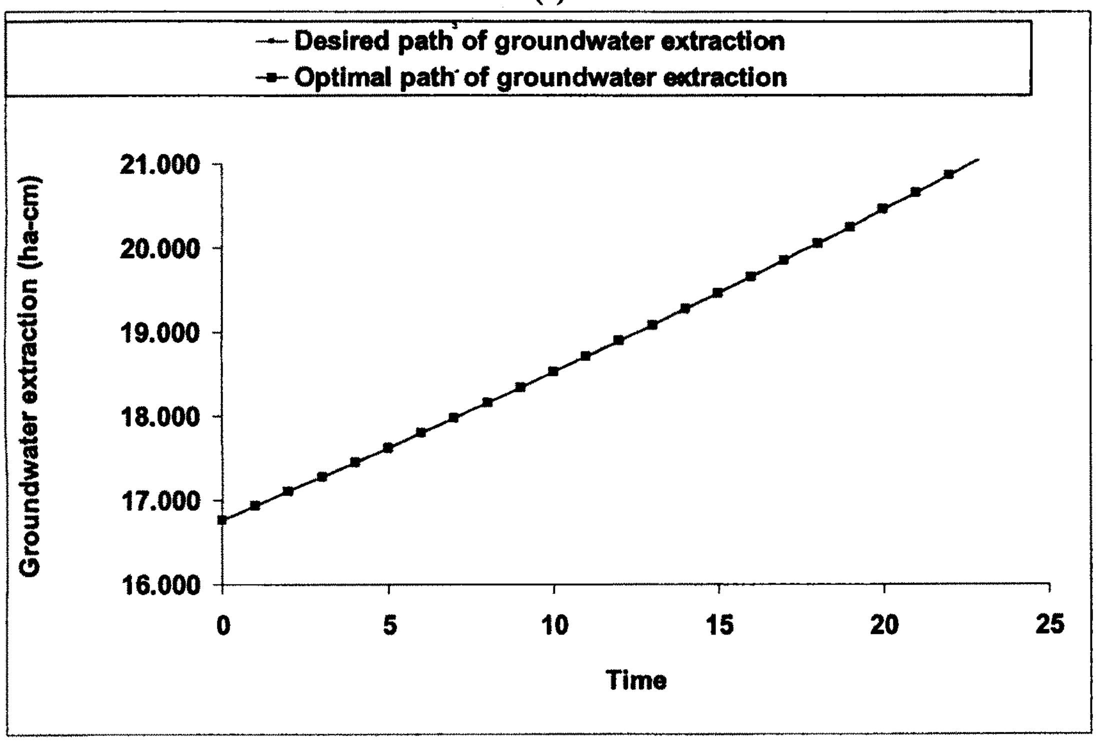


(b)

Figure 8.7: *Desired and optimal paths of groundwater trajectories, intercept parameter value decreased by 2.31%, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*

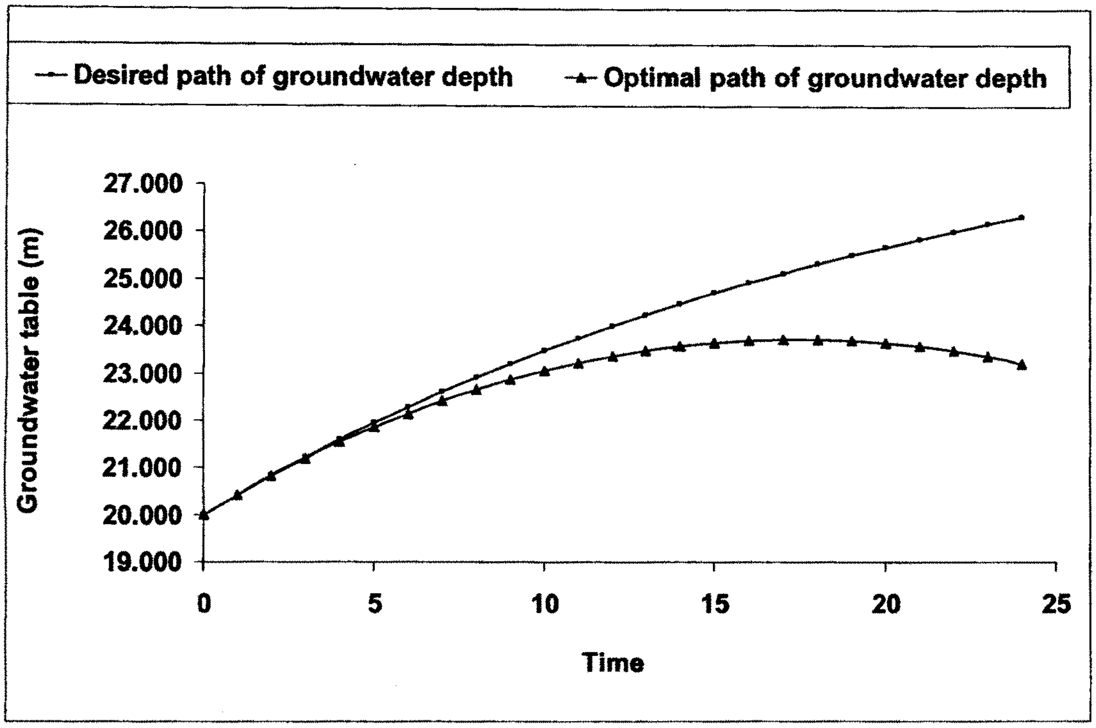


(a)

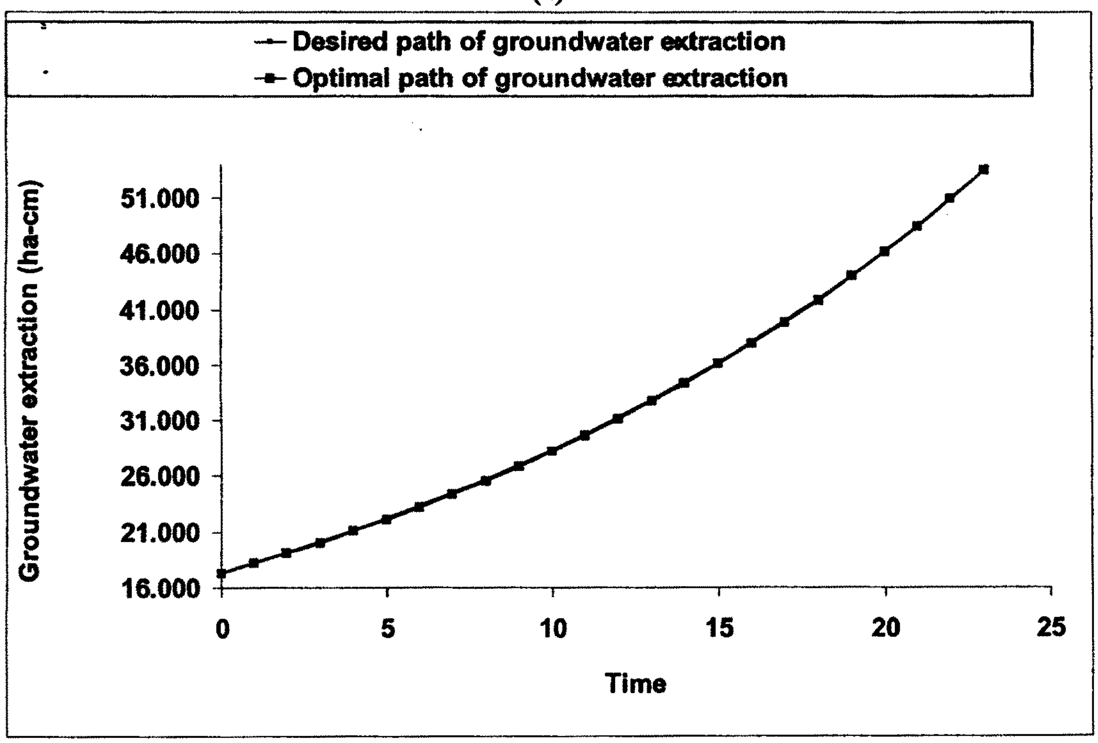


(b)

Figure 8.8: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 20m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 1%*

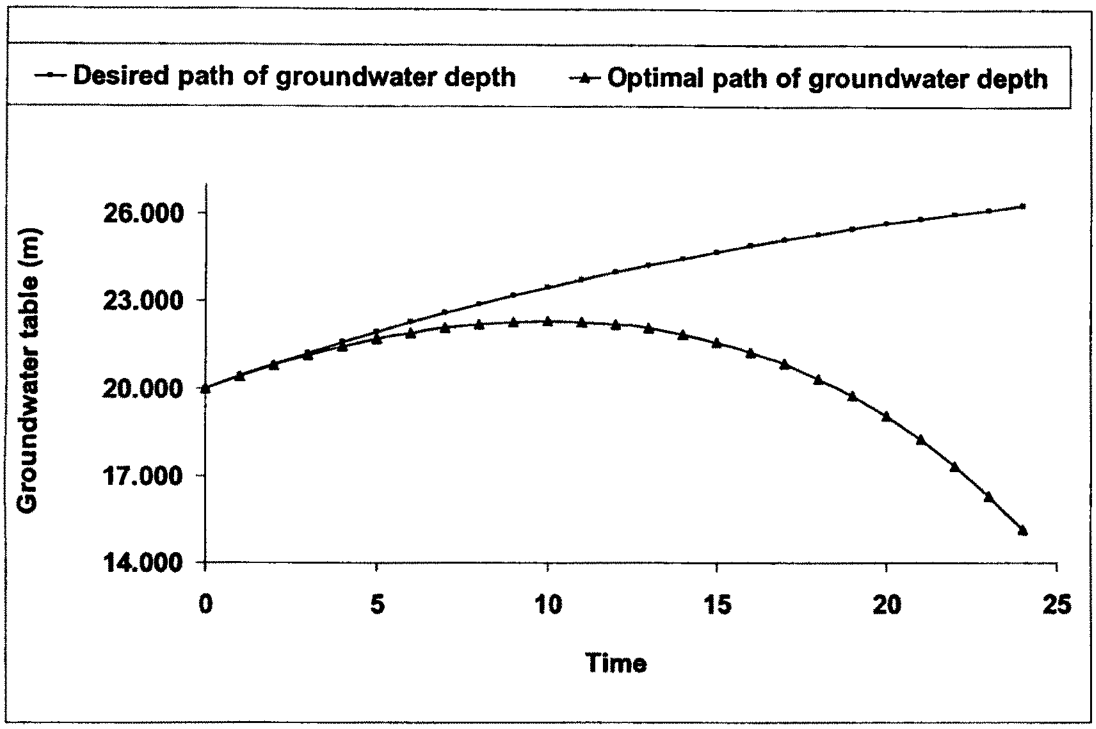


(a)



(b)

Figure 8.9: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 20m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 5%*



(a)

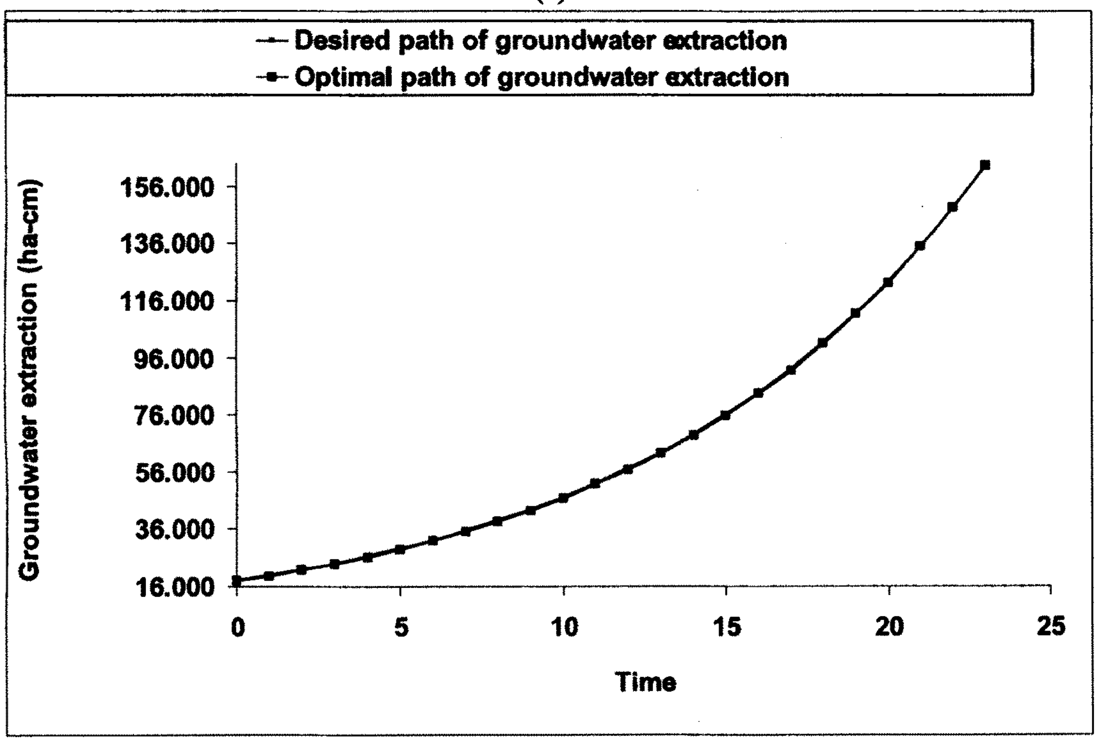
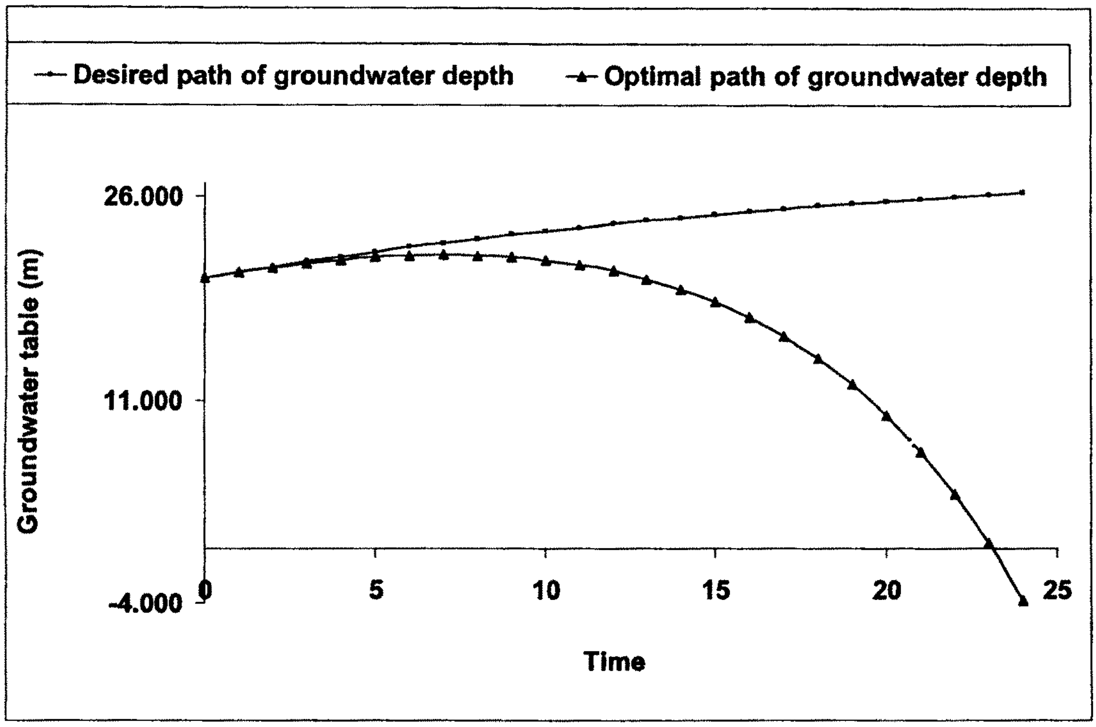
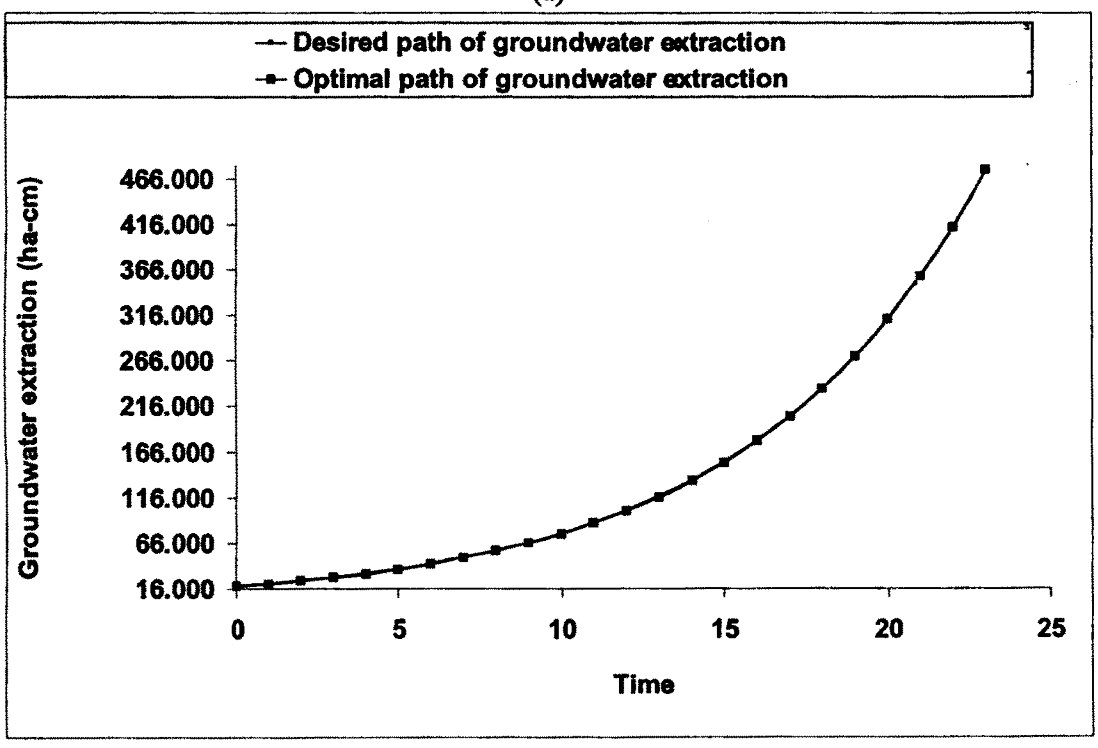


Figure 8.10: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 20m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 10%*

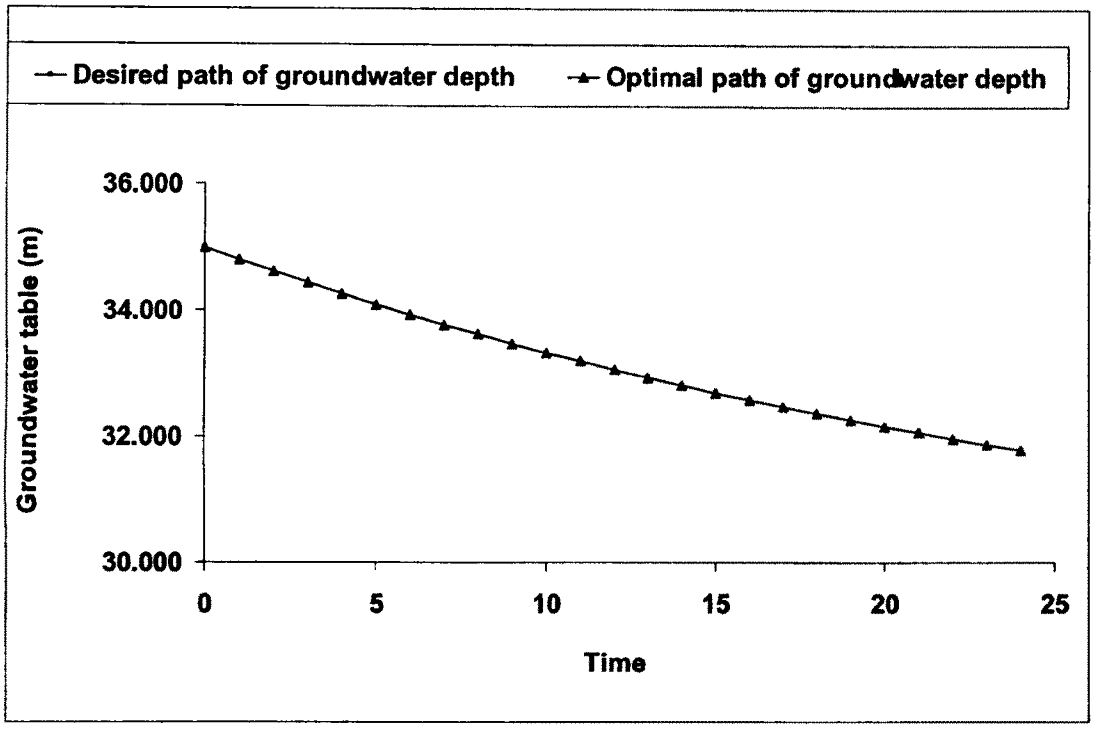


(a)

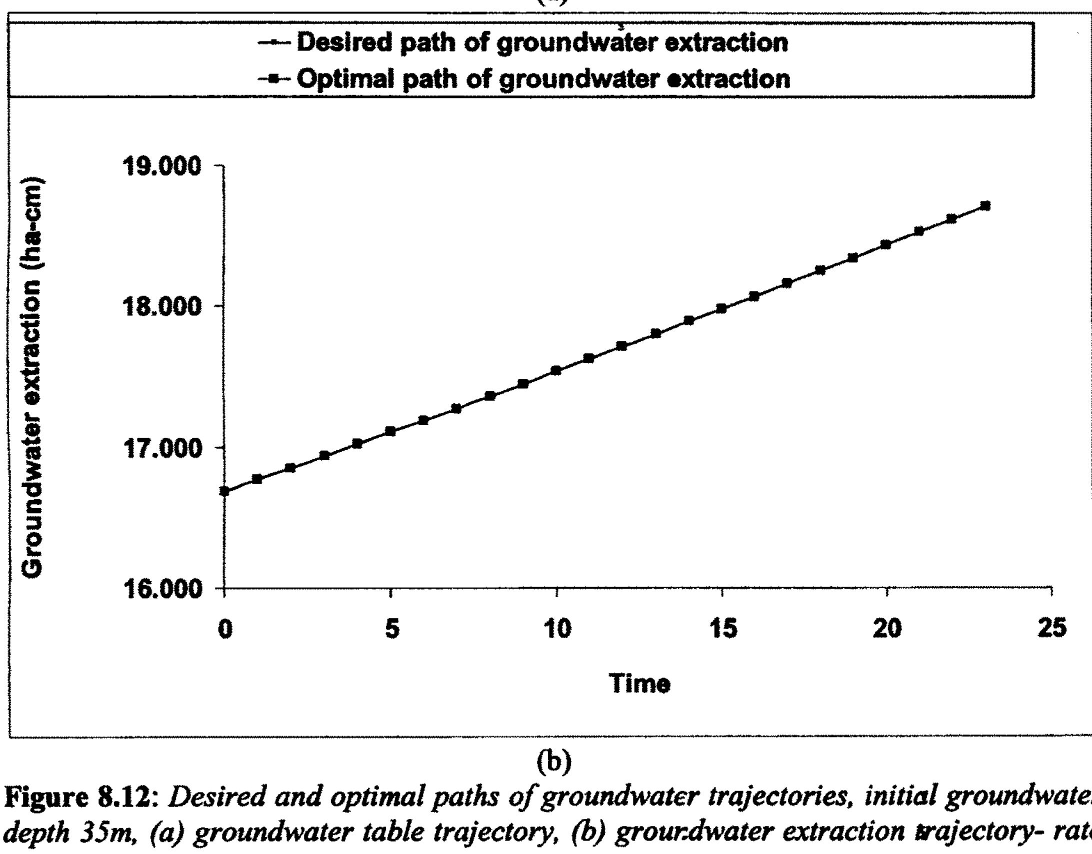


(b)

Figure 8.11: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 20m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 15%*

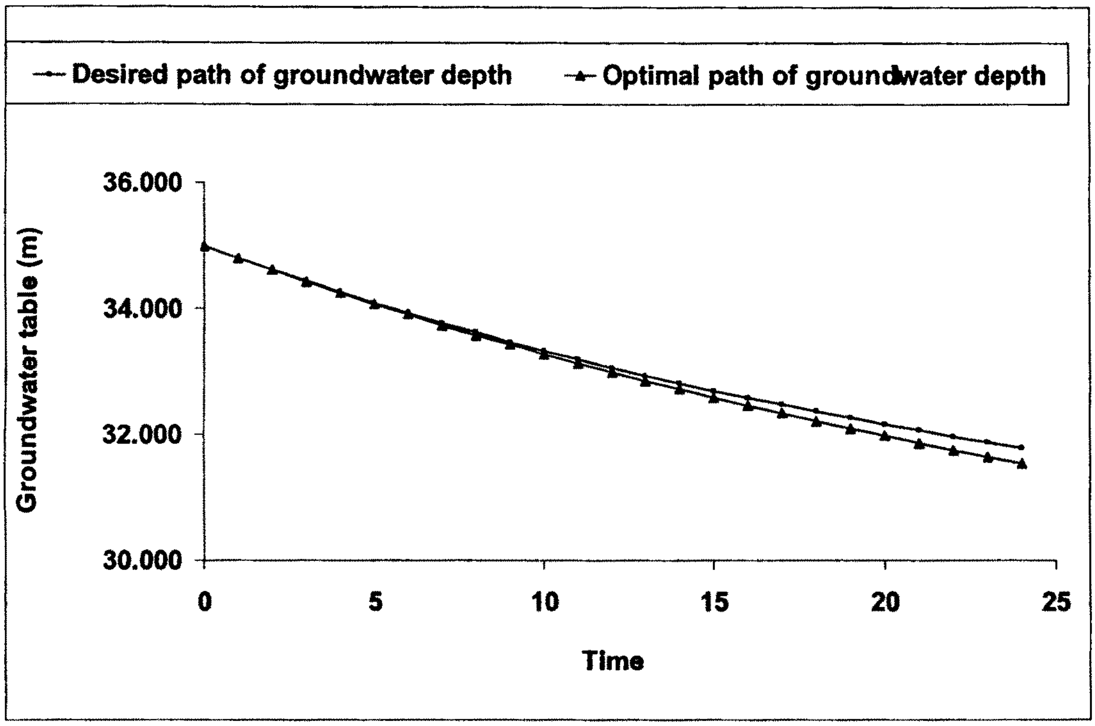


(a)

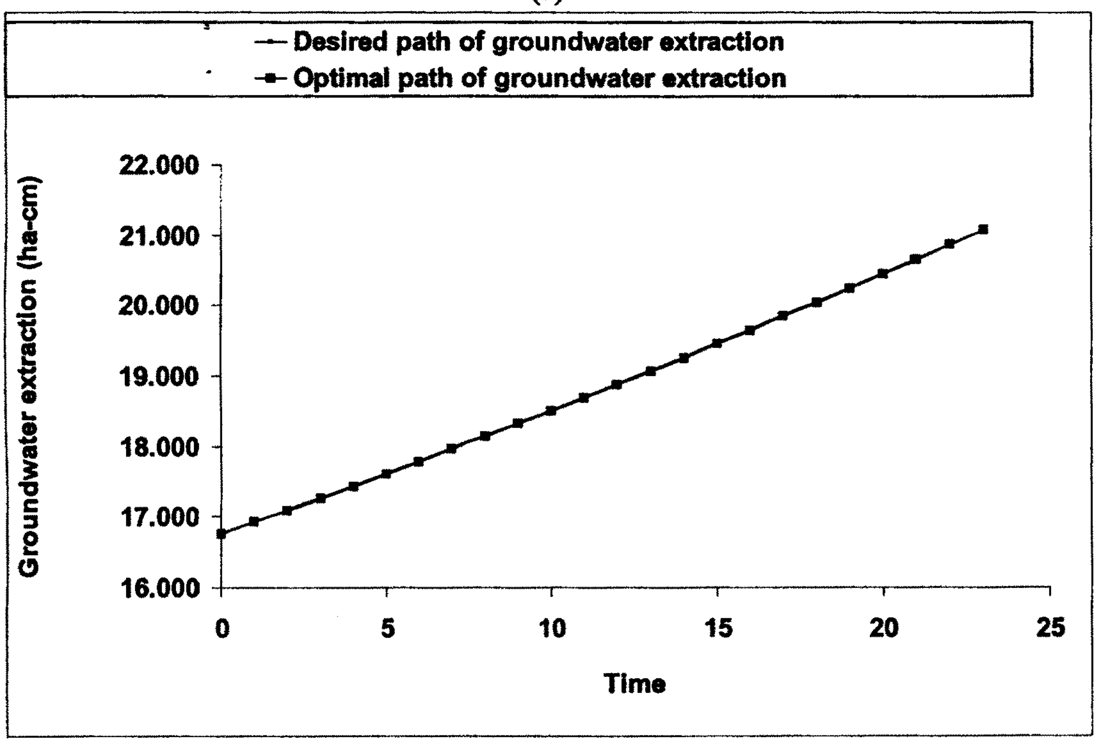


(b)

Figure 8.12: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 35m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*

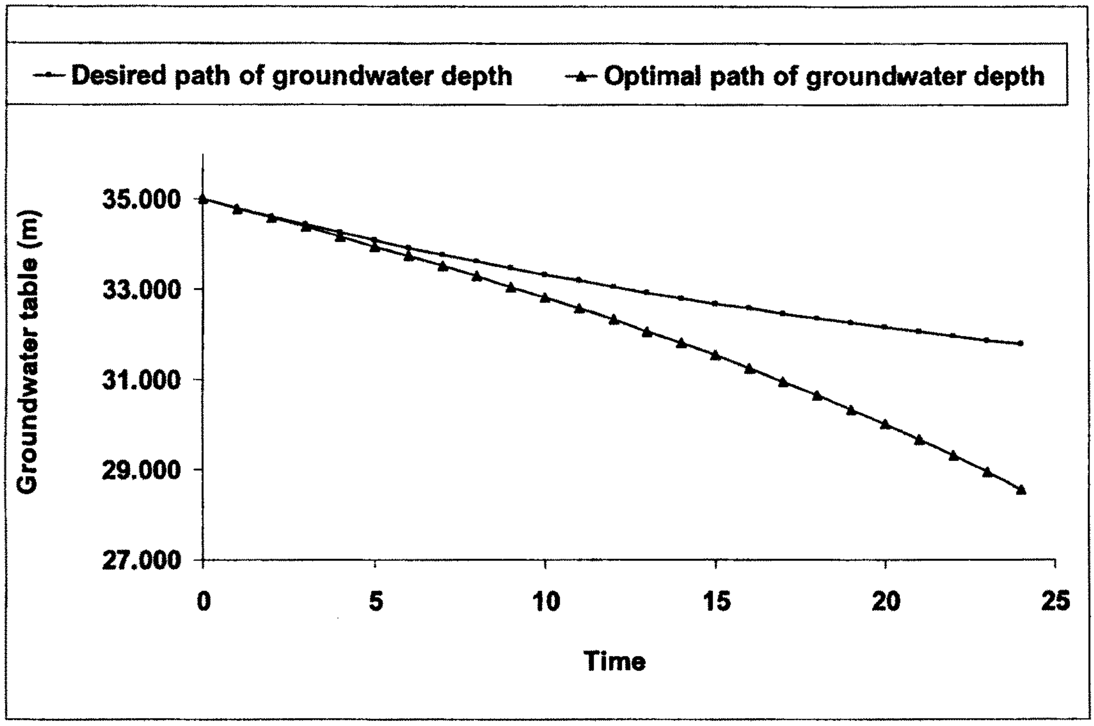


(a)

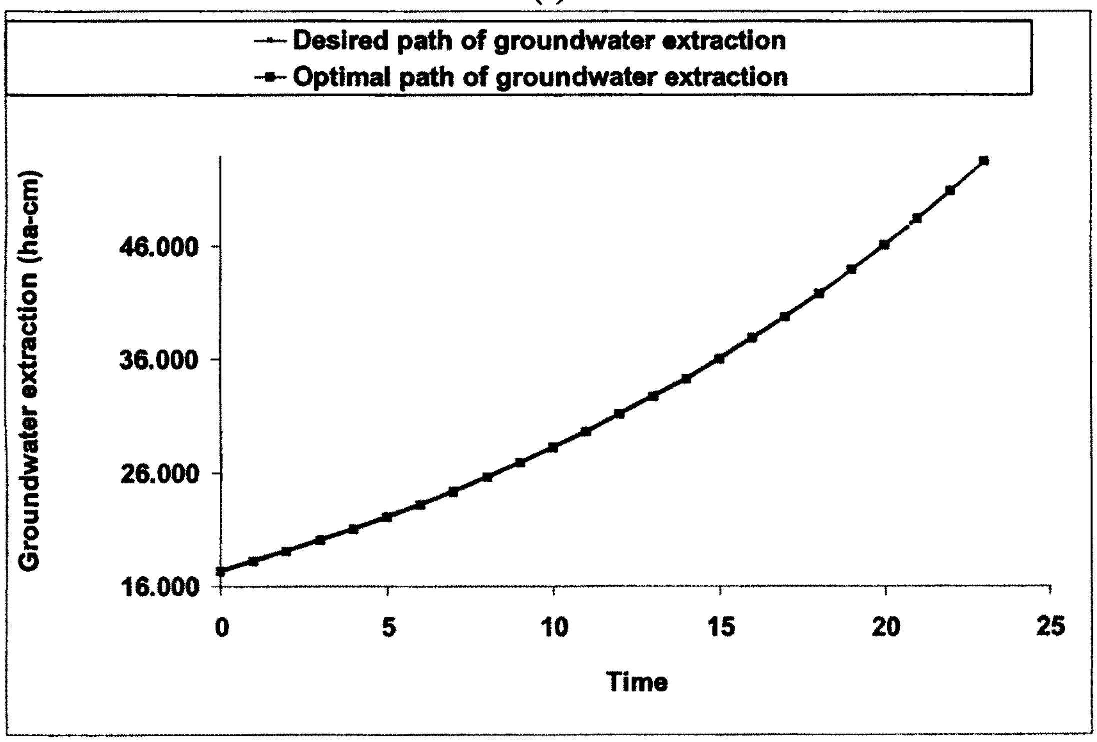


(b)

Figure 8.13: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 35m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 1%*

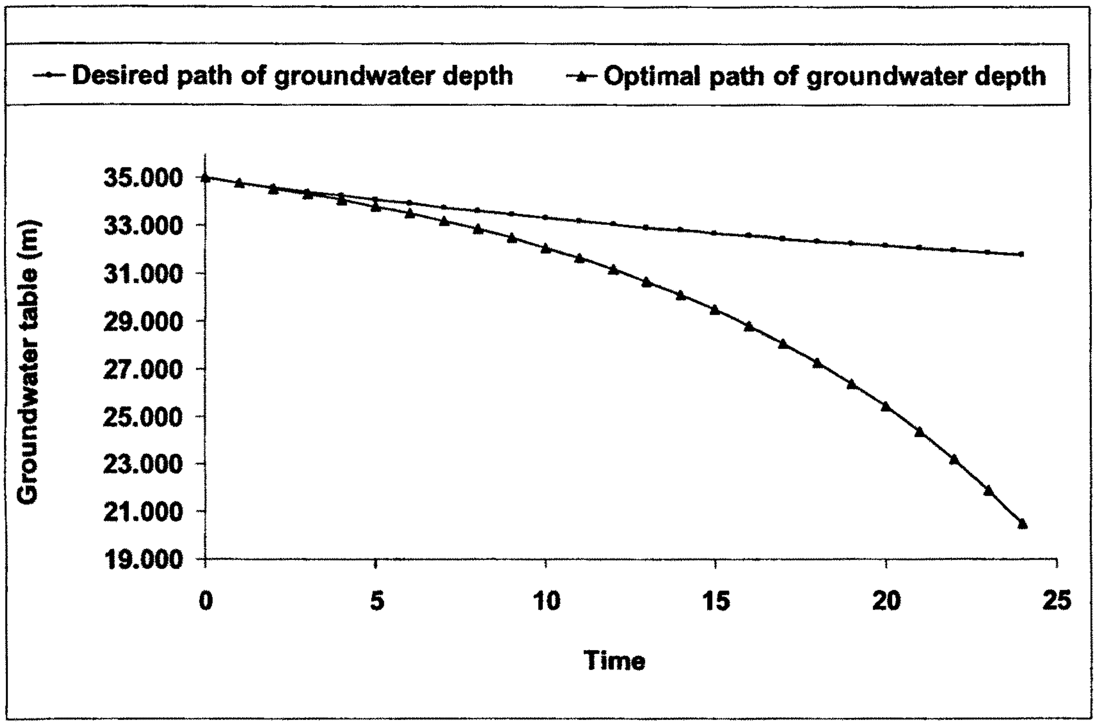


(a)

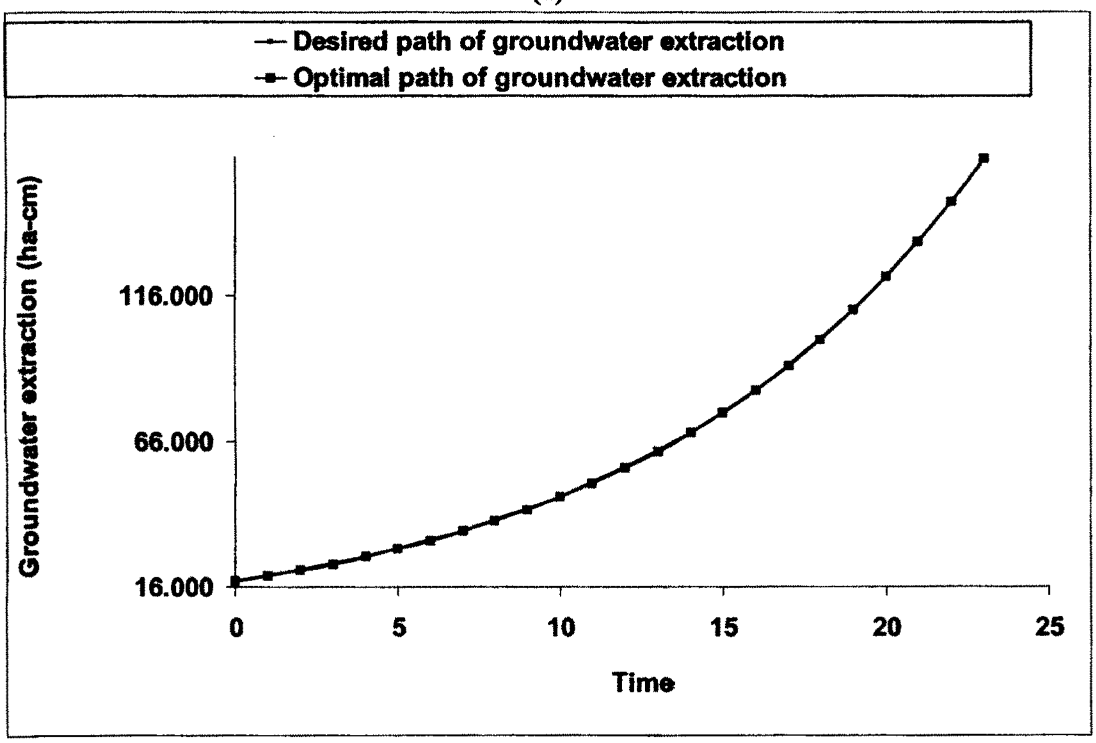


(b)

Figure 8.14: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 35m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 5%*

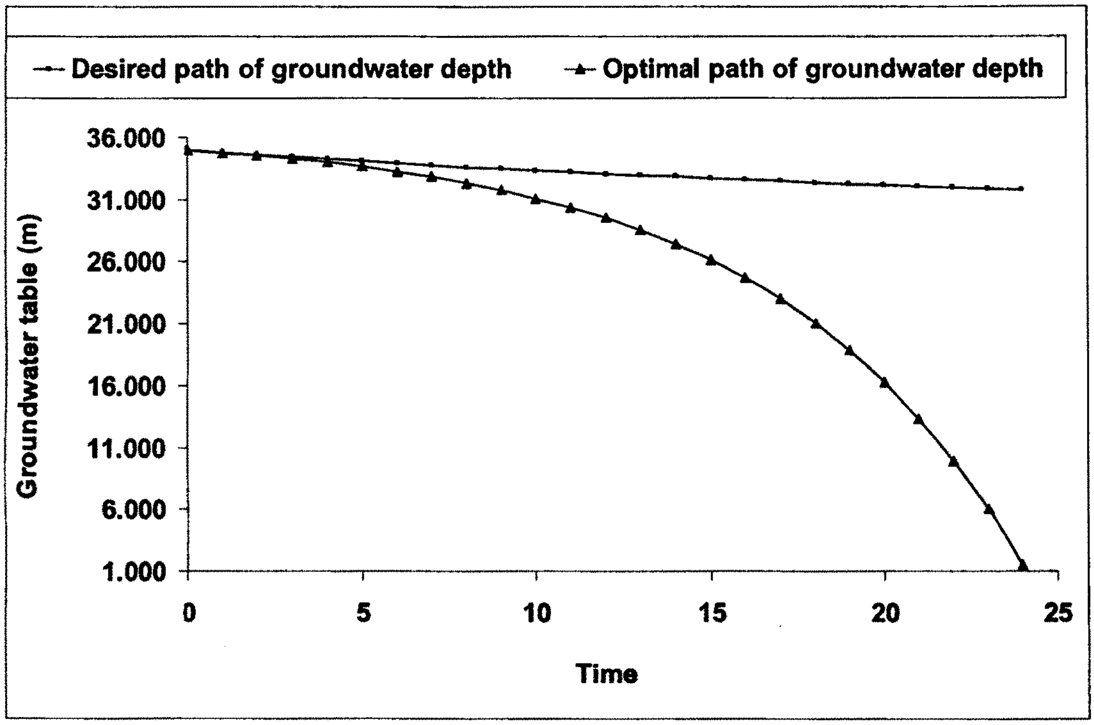


(a)

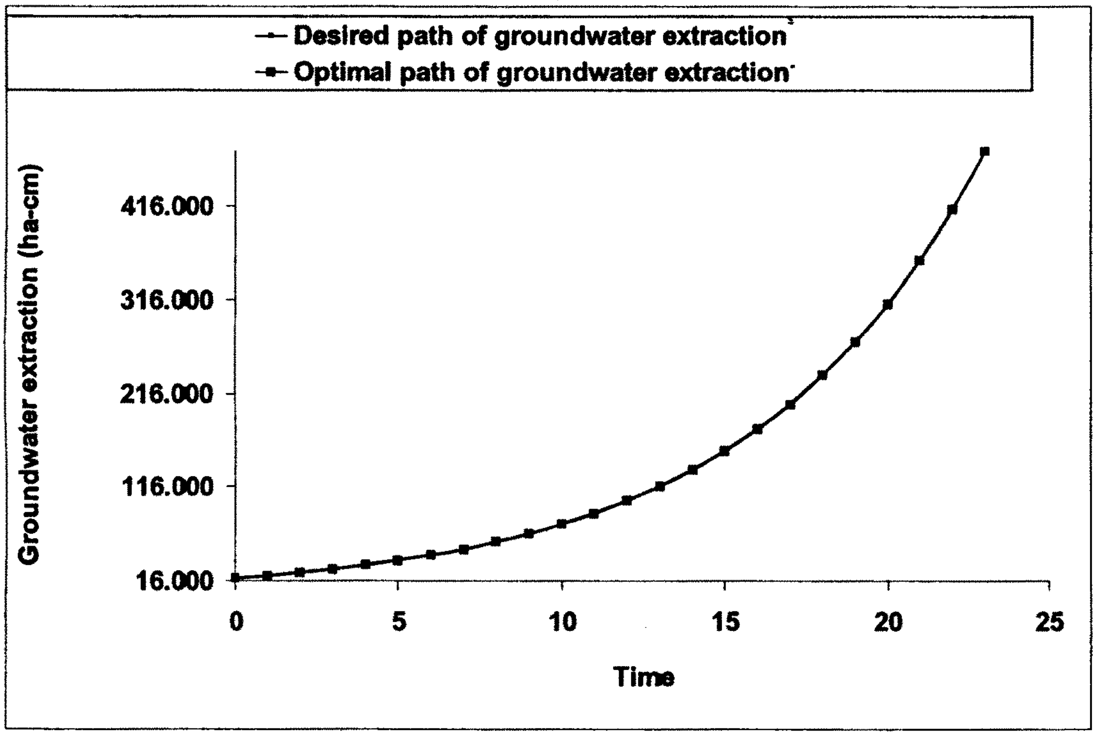


(b)

Figure 8.15: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 35m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 10%*

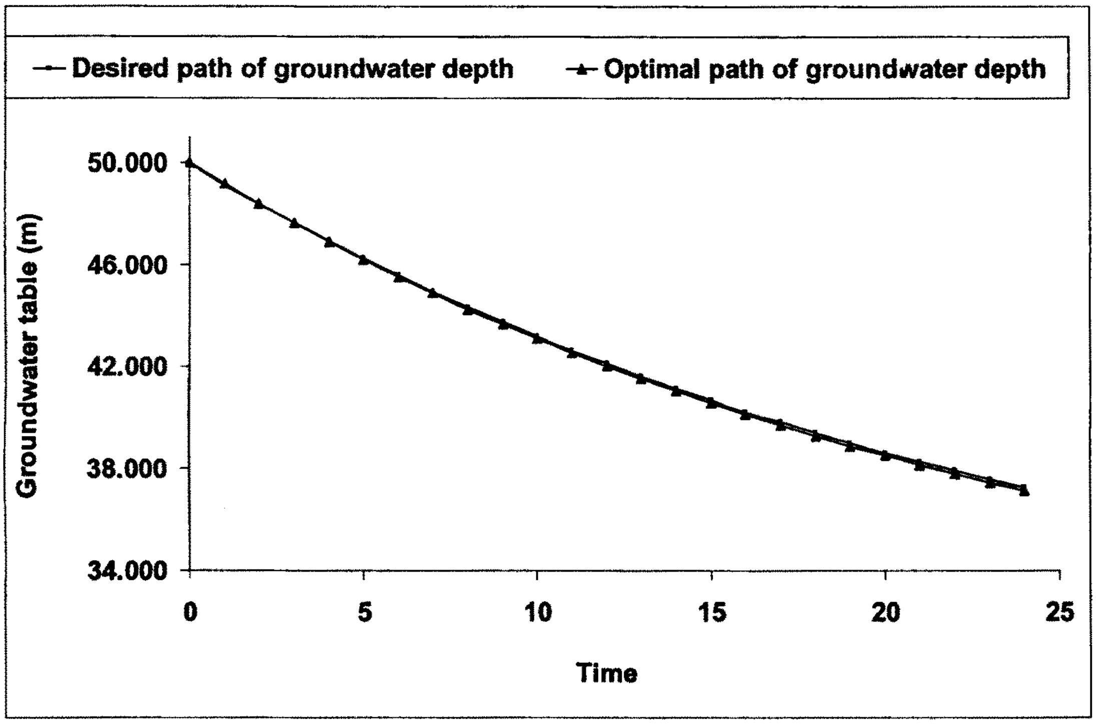


(a)

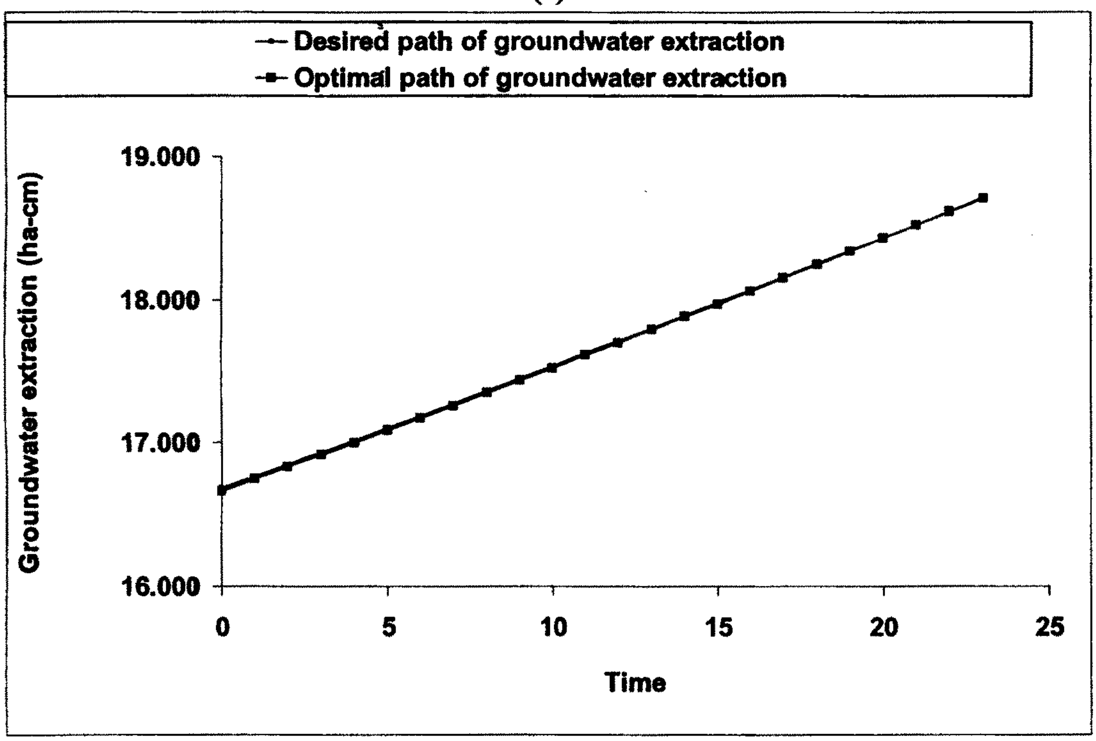


(b)

Figure 8.16: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 35m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 15%*

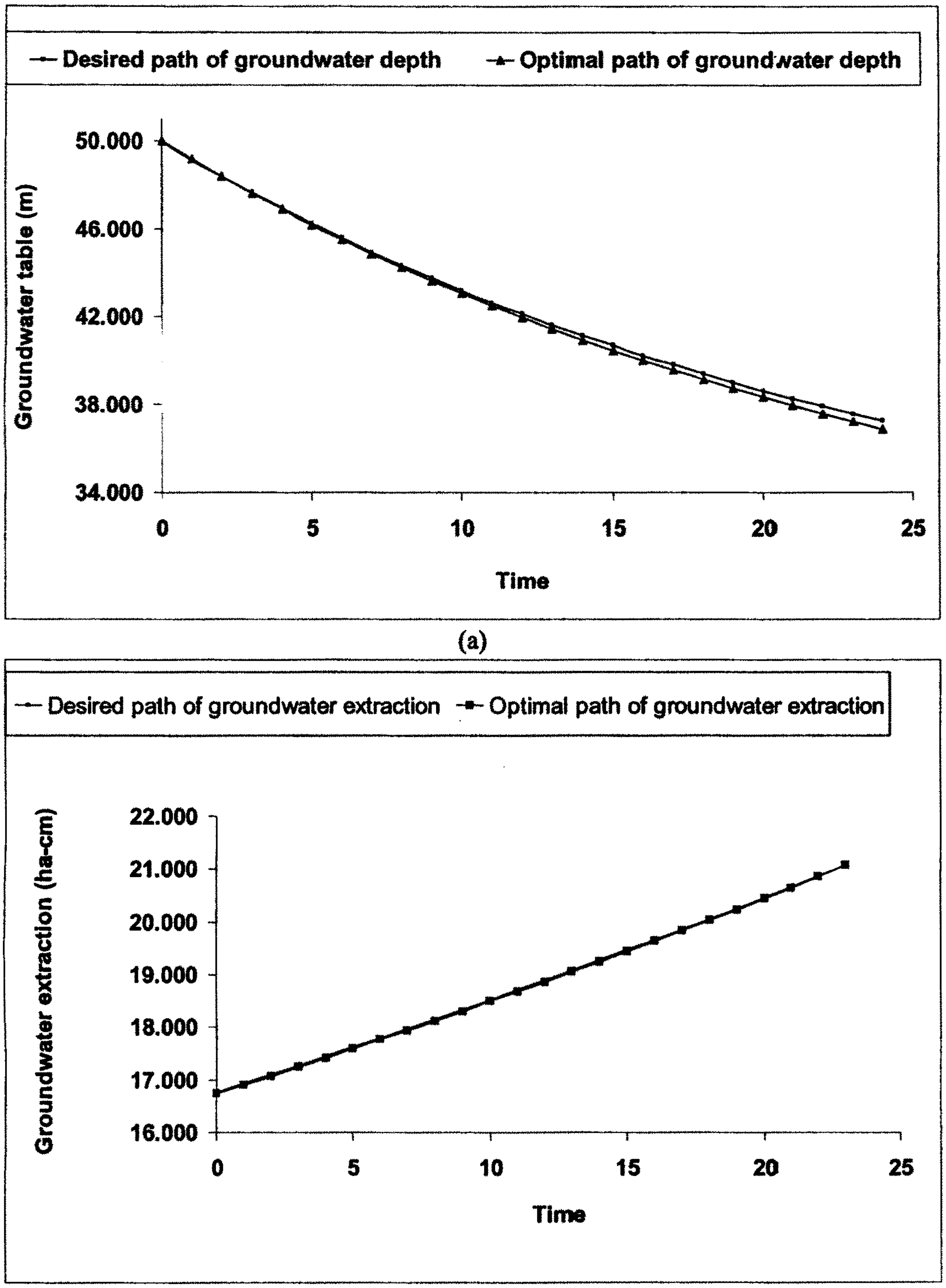


(a)



(b)

Figure 8.17: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 50m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*



(a)

(b)

Figure 8.18: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 50m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 1%*

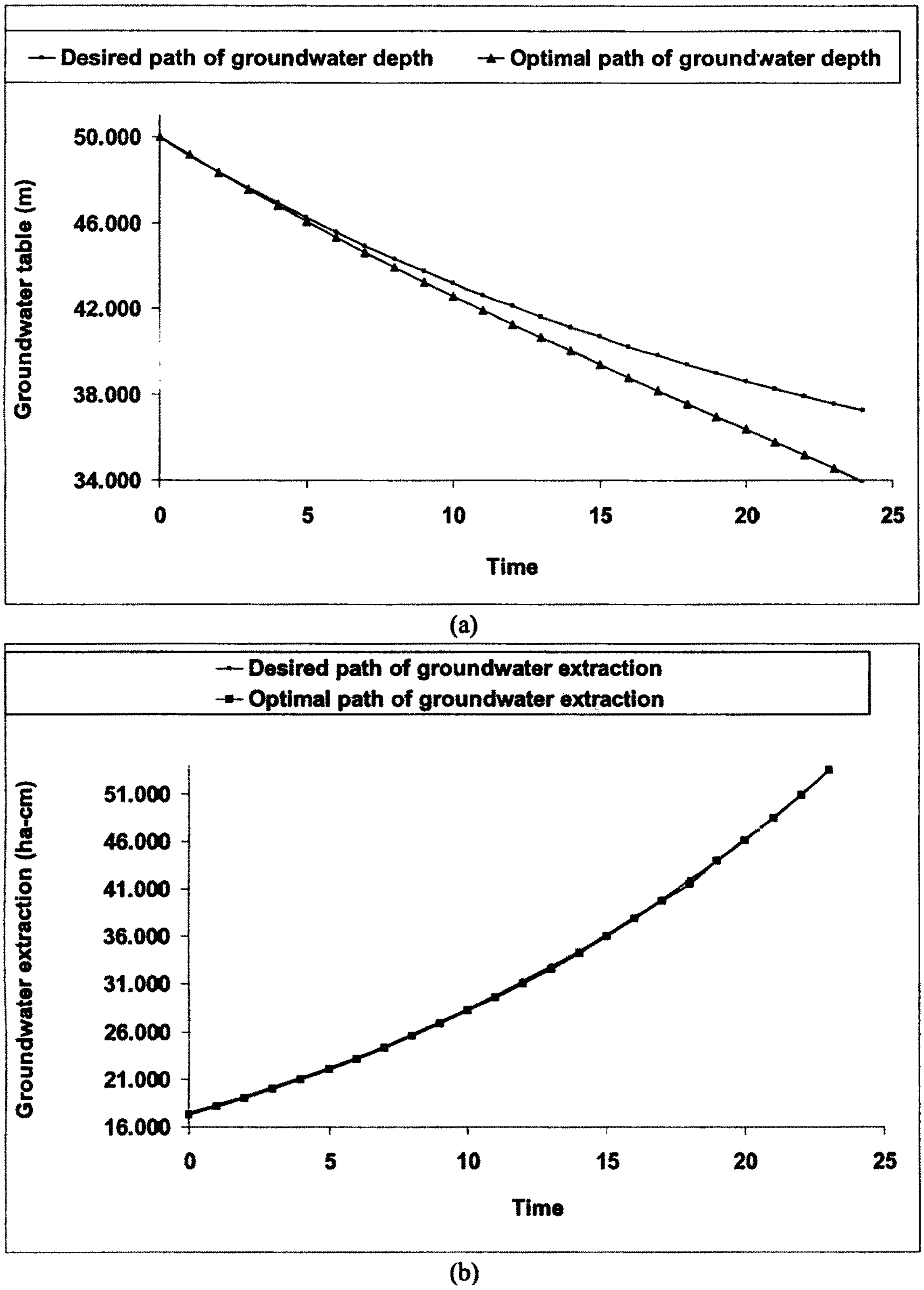
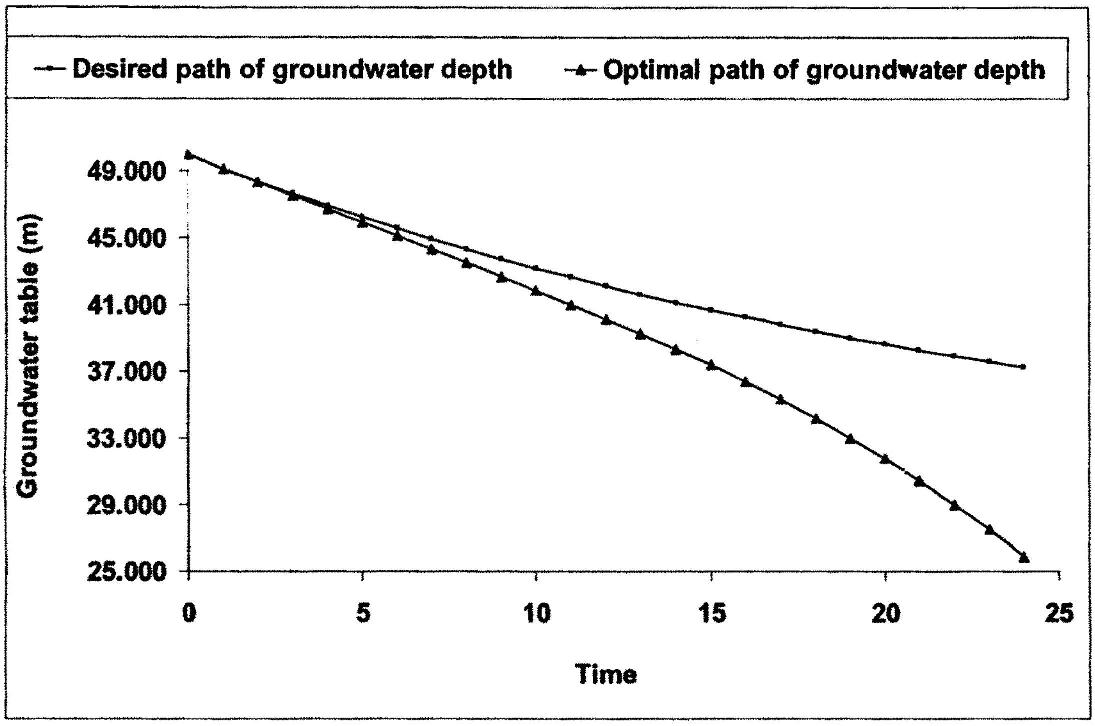
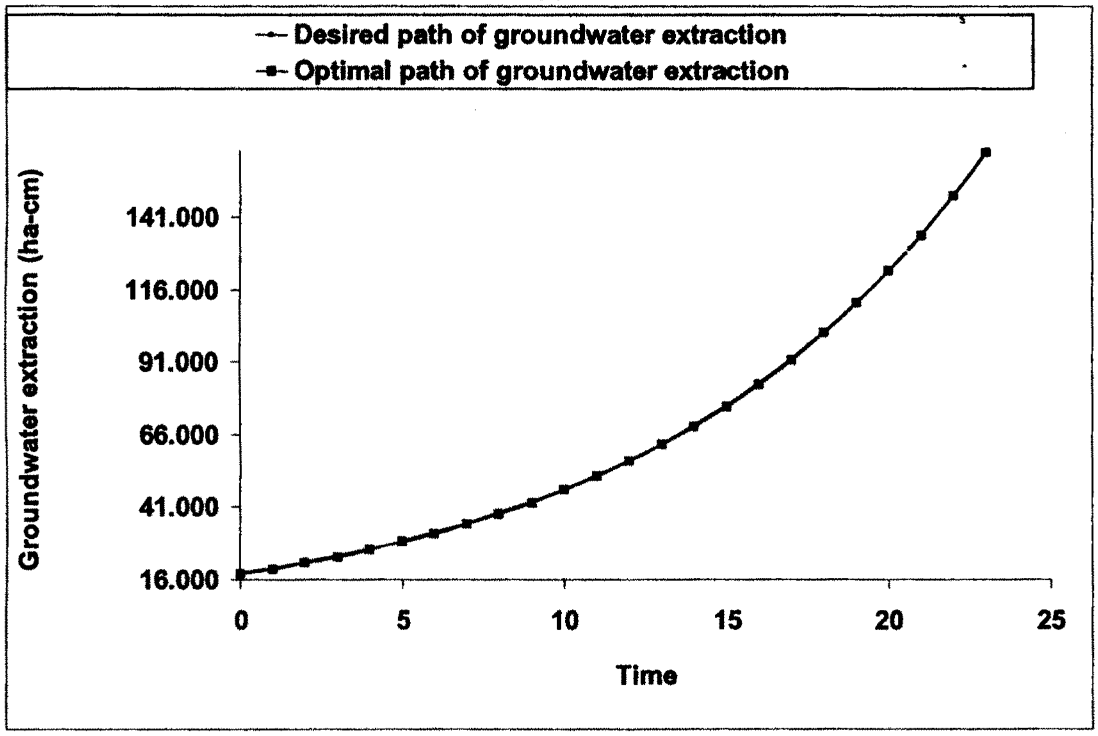


Figure 8.19: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 50m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 5%*



(a)



(b)

Figure 8.20: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 50m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 10%*

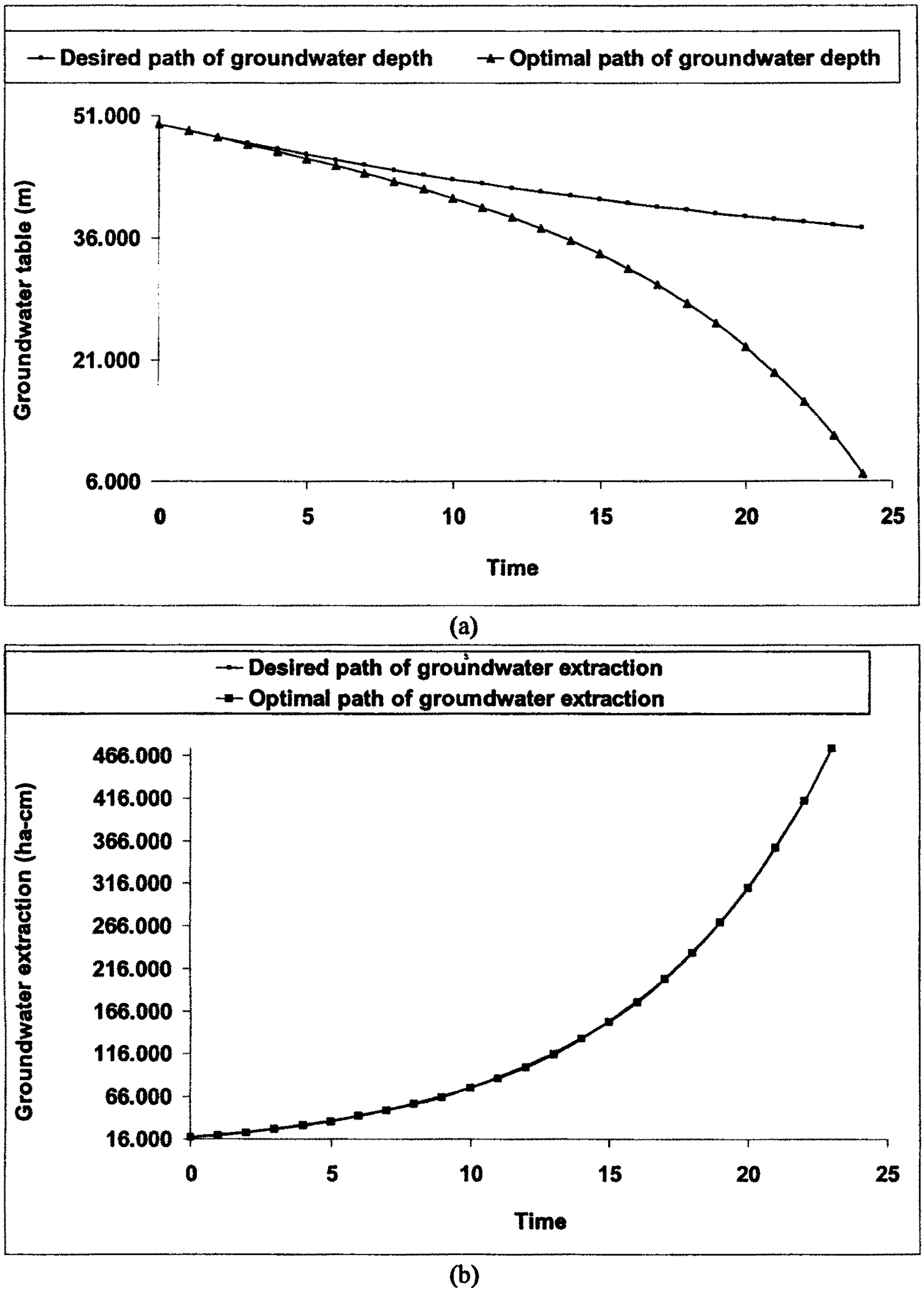
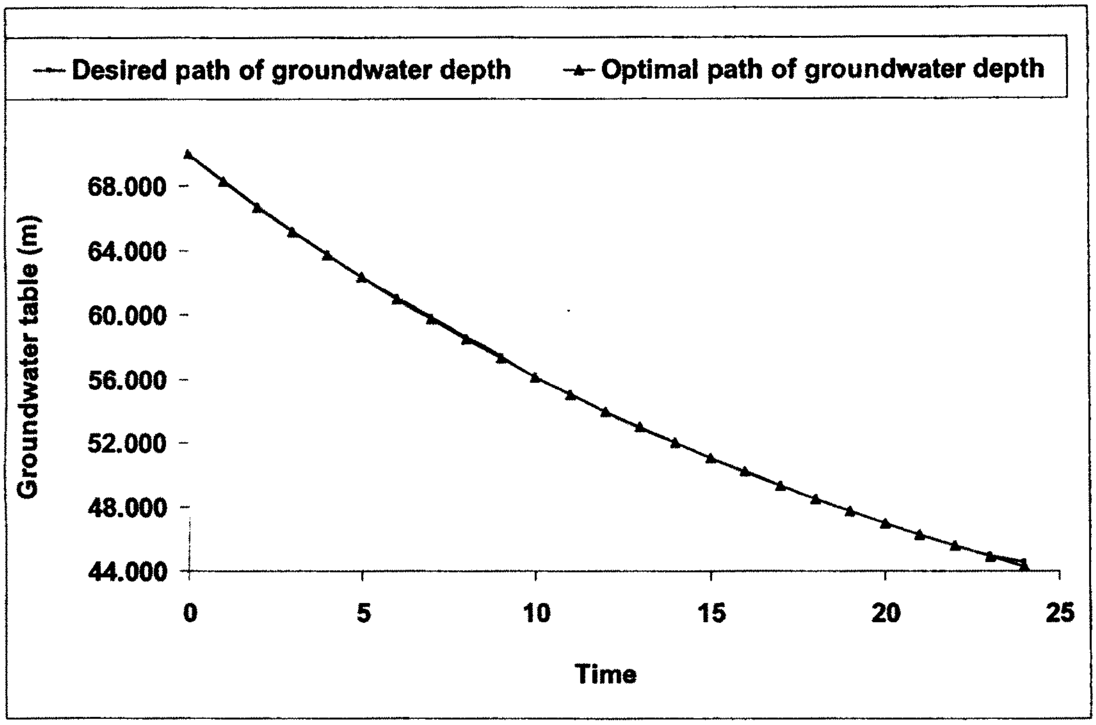
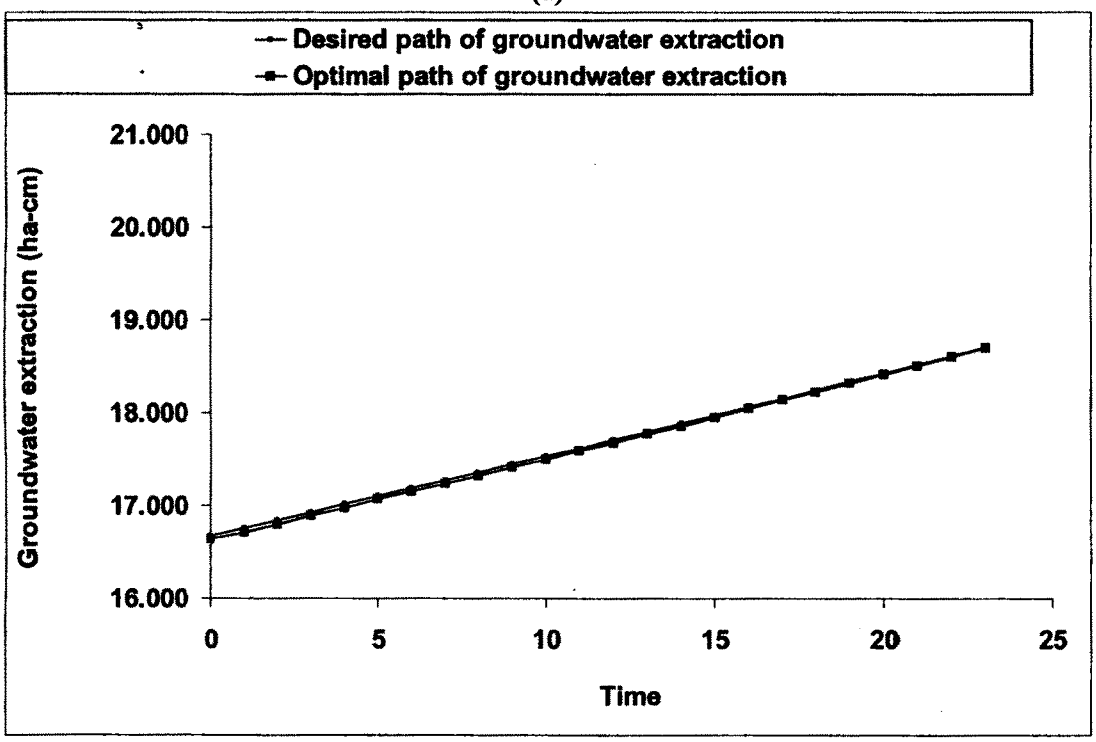


Figure 8.21: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 50m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 15%*



(a)



(b)

Figure 8.22: *Desired and optimal paths of groundwater trajectories, initial groundwater depth 70m, (a) groundwater table trajectory, (b) groundwater extraction trajectory- rate of extraction 0.5%*