CHAPTER 5

STOCHASTIC INVENTORY MODEL UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENT FOR TWO SUPPLIERS

CHAPTER 5

5.1. INTRODUCTION:

In this chapter we have introduced the effect of inflation and time value of money was investigated under given sets of inflation and discount rates.

5.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under inflation and permissible delay in payment is developed on the basis of the following assumptions.

(a) r_1 = discount rate representing the time value of money.

(b) f = inflation rate

(c) $R = f - r_1$ = present value of the nominal inflation rate.

(d) t_1 = time period with inflation

(e) c_0 = present value of the inflated price of an item Rs. /unit = $ce^{(f-r_1)t_1} = ce^{Rt_1}$

(f) $Ie(1i)$ =Interest earned over period (0 to T_{0i}) $= dce^{Rt_1} T_{00}\, T_{0i}\, ie_i$

(g) $Ie(2i)$ =Interest earned over period (T_{0i} to T_{00}) upon interest earned ($Ie(1i)$) previously.

$Ie(2i) = \left(dce^{Rt_1} T_{00} + Ie(1i)\right)(T_{00} - T_{0i}) ie_i$

(h) Interest charged by the i^{th} supplier clearly ($ic_i > ie_i$) $\quad i=1, 2$

$$Ic_i = \alpha i\, dce^{Rt_1} ic_i (T_{00} - T_{0i})$$

$A(q_i, r, \theta)$ =(cost of ordering)+(cost of holding inventory)+ (cost of item that deteriorate during a single interval that starts with an inventory of (q_i+r) units and ends with r units with inflation rate);

$$A(q_i, r, \theta) == k + \frac{1}{2} \cdot \frac{hq_i^2 e^{Rt_1}}{(d+\theta)} + \frac{hrq_i e^{Rt_1}}{(d+\theta)} + \frac{\theta c q_i e^{Rt_1}}{(d+\theta)} \qquad i=0, 1, 2$$

C_{00}=E (cost per cycle); and T_{00}=E (length of a cycle);

$P_{ij}(t)$=P (Being in state j at time t/starting in state i at time 0), i, j=0, 1, 2, 3 ;

p_i=long run probabilities, i=0, 1, 2, 3

5.3. OPTIMAL POLICY DECISION FOR THE MODEL:

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The lemma which is used to obtain the transition probabilities is same as discussed in chapter 4, (lemma (4.3.1)) hence we omit it here.

Define C_{i0}=E (cost incurred to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1 or 2)

Lemma 5.3.1: C_{i0} is given by

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right) A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right) \left[A(q_i, r, \theta) + C_{j0}\right] \quad i=0, 1, 2 \tag{5.3.1}$$

$$C_{30} = \overline{C} + \sum_{i=1}^{2} \rho_i C_{i0} \tag{5.3.2}$$

Where $\rho_i = \frac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}} e^{Rt_1}}{\delta^2} \left[he^{\frac{\delta r}{(d+\theta)}} (\delta r - (d+\theta)) + (\pi \delta d + h(d+\theta) + \hat{\pi}) - \theta c \delta \right] + \frac{\theta c e^{Rt_1}}{\delta} \tag{5.3.3}$$

Proof: First consider i=0. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d+\theta}\right)A(q_0,r,\theta) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d+\theta}\right)\left[A(q_0,r,\theta) + C_{j0}\right] \qquad (5.3.4)$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities

$P_{00}\left(\frac{q_0}{d+\theta}\right), P_{01}\left(\frac{q_0}{d+\theta}\right), P_{02}\left(\frac{q_0}{d+\theta}\right)$ and $P_{03}\left(\frac{q_0}{d+\theta}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0,r,\theta)$. On the other hand, if state j=1, 2, 3 is observed when inventory drops to r, then the expected cost will be $A(q_0,r,\theta) + C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d+\theta}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = \left[\overline{C} + C_{10}\right]\frac{\mu_1}{\mu_1 + \mu_2} + \left[\overline{C} + C_{20}\right]\frac{\mu_2}{\mu_1 + \mu_2} \qquad (5.3.5)$$

Here, $\overline{C}$ is defined as the expected cost from the time inventory drops to r until either supplier 1 or 2 becomes available and it is computed as follows:

Now referring to Fig 4.1., note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2e^{Rt_1}(d+\theta) + hye^{Rt_1}\left[r - y(d+\theta)\right] + \theta ce^{Rt_1}y \qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\frac{hr^2e^{Rt_1}}{(d+\theta)} + \pi e^{Rt_1}\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}e^{Rt_1}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cre^{Rt_1}}{(d+\theta)} \qquad y \geq \frac{r}{d+\theta}$$

Hence,

$$\overline{C} = \int_{0}^{r/(d+\theta)} \left\{ \frac{1}{2} h y^2 (d+\theta) e^{R t_1} + h e^{R t_1} y (r - y(d+\theta)) + \theta c e^{R t_1} \right\} \delta e^{-\delta y} dy$$

$$+ \int_{r/(d+\theta)}^{\infty} \left\{ \frac{1}{2} \frac{h r^2 e^{R t_1}}{(d+\theta)} + \pi e^{R t_1} \left[y - \frac{r}{(d+\theta)} \right] d + \frac{\hat{\pi} e^{R t_1}}{2} \left[y - \frac{r}{(d+\theta)} \right]^2 + \frac{\theta c r e^{R t_1}}{(d+\theta)} \right\} \delta e^{-\delta y} dy$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}} e^{R t_1}}{\delta^2} \left[h e^{\frac{\delta r}{(d+\theta)}} (\delta r - (d+\theta)) + (\pi \delta d + h(d+\theta) + \hat{\pi}) - \theta c \delta \right] + \frac{\theta c e^{R t_1}}{\delta}$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\overline{C}$. If the process makes a transition to state 1, the total expected cost would then be $\overline{C} + C_{10}$. The probability of a transition from state 3 to state 1 is

$$P(Y_1 < Y_2) = \int_{0}^{\infty} P(Y_1 < Y_2 / Y_2 = t) \mu_2 e^{-\mu_2 t} dt = \frac{\mu_1}{\mu_1 + \mu_2}$$

Multiplying this probability with the expected cost term above gives the first term of (5.3.5). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r, \theta)$, i=0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d+\theta}\right)$ i, j=0, 1, 2, 3.

Lemma 5.3.2: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01} C_{10} + P_{02} C_{20} + P_{03} \left(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20} \right) \qquad (5.3.6)$$

where the pair [C_{10}, C_{20}] solves the system

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & 1-P_{22}-P_{23}\rho_2 \end{bmatrix} \begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix} = \begin{bmatrix} A_1 + P_{13}\overline{C} \\ A_2 + P_{23}\overline{C} \end{bmatrix} \qquad (5.3.7)$$

Proof: Rearranging the linear system of four equations in lemma (5.3.1) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1-P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1-P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \overline{C} \end{bmatrix} \qquad (5.3.8)$$

We have $C_{30} = \overline{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (5.3.7), with (C_{10}, C_{20}).

From the first row of (5.3.8) we obtain $C_{00} = A_0 + \sum_{j=1}^{3} P_{0j} C_{j0}$.

Hence above lemma is proved.

The lemma (4.3.4) and (4.3.5) are same as discussed in chapter 4, hence we omit it here.

Proposition 5.3.1: The Average cost objective function for two suppliers when inflation and delay in payment is considered is given by

$$AC = \frac{C_{00}}{T_{00}} = \frac{\begin{array}{c} A(q_0, r, \theta) + P_{01}(C_{10} - (Ie(11) + Ie(21) + Ic_1) + P_{02}(C_{20} - (Ie(12) + Ie(22) + Ic_2) \\ + P_{03}(\overline{C} + \rho_1(C_{10} - (Ie(11) + Ie(21) + Ic_1) + \rho_2(C_{20} - (Ie(12) + Ie(22)) + Ic_2)) \end{array}}{\dfrac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

5.4. NUMERICAL EXAMPLE:

In this section we verify the results by a numerical example. We assume that

(i) k =Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$=Rs.25/unit/time, ic_1=0.11, ie_1=0.02, ic_2=0.13, ie_2=0.04, R=0.05, t_1=6, T_{01}=0.6, T_{02}=0.8, (α_1=1 and α_2=1) i.e. businessmen do not settle the account at the respective credit time given by both the suppliers, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222. The optimal solution is obtained as

q_0=3.506669, q_1=30.128739, q_2=29.56780, r=0.81358 and AC=$\frac{C_{00}}{T_{00}}$ = 8.1358.

(ii) Keeping other parameters as it is, we consider (α_1=0 and α_2=0) i.e. businessmen settle the account at the respective credit time given by both the suppliers.

The optimal solution is obtained as q_0=6.106844, q_1=33.97769, q_2=33.8575, r=1.026170 and AC=$\frac{C_{00}}{T_{00}}$ = 7.750814.

(iii) Keeping other parameters as it is, we consider (α_1=1 and α_2=0) i.e. businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 4.384248, q_1= 31.17163, q_2= 30.78434 r= 0.95295 and AC=$\frac{C_{00}}{T_{00}}$ = 7.935795.

(iv) Keeping other parameters as it is, we consider (α_1=0 and α_2=1) i.e. when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 4.12906, q_1= 30.80062, q_2= 30.3622, r= 0.925938 and AC=$\frac{C_{00}}{T_{00}}$ = 7.9908.

Conclusion:

From the above numerical example, we conclude that the cost is minimum when account is settled at the credit time given by the i^{th} supplier. Comparing the above results with that of chapter 4 we observe that cost is more here due to inflation. So in this situation also businessmen are advised to settle the account at the credit time given by the respective suppliers.

5.5. SENSITIVITY ANALYSIS:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=1$. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.1
Sensitivity Analysis Table by varying the parameter values of R
($\alpha_1=1$ and $\alpha_2=1$)

R	q_0	q_1	q_2	r	AC
0.05	3.50667	30.1287	29.5678	0.81888	8.1358
0.08	2.97984	29.0031	28.2686	0.7625	9.48985
0.1	2.69918	28.3961	27.5549	0.72583	10.5174
0.12	2.4587	27.8708	26.9295	0.69054	11.6591
0.15	2.15475	27.1996	26.1194	0.64053	13.6164

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.2
Sensitivity Analysis Table by varying the parameter values of R
(α_1=0 and α_2=1)

R	q_0	q_1	q_2	r	AC
0.05	6.10684	33.9777	33.8578	1.02617	7.75081
0.08	4.46811	30.6747	30.2617	1.01785	9.10662
0.1	3.80797	29.4459	28.8683	0.97442	10.1314
0.12	3.32578	28.5693	27.8497	0.92544	11.2683
0.15	2.79119	27.6076	26.7053	0.85213	13.2162

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where α_1=1 and α_2=0. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.3
Sensitivity Analysis Table by varying the parameter values of R
(α_1=1 and α_2=0)

R	q_0	q_1	q_2	r	AC
0.05	4.12907	30.8006	30.3623	0.92594	7.9908
0.08	3.39002	29.3407	28.6986	0.86341	9.34255
0.1	3.02527	28.6195	27.8568	0.82053	10.3676
0.12	2.72496	28.0219	27.1484	0.7788	11.5063
0.15	2.35853	27.286	26.2617	0.71948	13.4583

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of inflation rate R keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. Inflation rate R is assumed to take values 0.05, 0.08, 0.1, 0.12, 0.15. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC.

Table 5.5.4
Sensitivity Analysis Table by varying the parameter values of R
(α_1=0 and α_2=1)

R	q_0	q_1	q_2	r	AC
0.05	4.384242	31.17162	30.78432	0.95295	7.935795
0.08	3.57047	29.5611	28.9627	0.8954	9.28645
0.1	3.17314	28.7834	28.0601	0.85285	10.3108
0.12	2.84871	28.1486	27.3108	0.8104	11.4489
0.15	2.45606	27.3774	26.3844	0.74903	13.4005

We see that as inflation rate R increases values of q_0, q_1, q_2 and value of reorder quantity r decreases and hence average cost increases.

5.6. CONCLUSION:

From the above sensitivity analysis, in all the various situations of settling the account we conclude that the cost is minimum when account is settled at credit time given by the i^{th} supplier where i=1, 2. Comparing the above results with that of chapter 4 we observe that cost is more here due to inflation. So in this situation also businessmen are advised to settle the account at the credit time given by respective suppliers.

Comparing the above results with that of chapter 2 we observe the following:

Here, the long run probability of non-availability of both suppliers case is 0.02222 and in a single supplier case is 0.091. In this favorable condition of reduced probability we find that average cost is lower here than that obtained in chapter 2 i.e. in case of single supplier case. The moral that follows is that it is always advisable to go for two suppliers or multiple suppliers for reduced average cost.