PART- II

PERISHABLE PRODUCTS STOCHASTIC INVENTORY MODELS FOR TWO SUPPLIERS

CHAPTER 4

STOCHASTIC INVENTORY MODEL UNDER PERMISSIBLE DELAY IN PAYMENT FOR TWO SUPPLIERS

CHAPTER 4

4.1. INTRODUCTION:

In the previous chapters we discussed monopolistic case. Here we consider a generalization of the model discussed in chapter 2 representing practical life situation by assuming that the supplier's market is not monopolistic as competitive spirit in the business is increased especially after induction of multinational companies. We undertake a duopolistic case which can be generalized further. In other words, it is assumed that the inventory manager may place his order with any one of two suppliers. This generalization results is a more difficult problem, however it makes the model more realistic when the manager may receive his supply from more than one source. Here we assume that the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 where as state 3 denotes the non availability of either of them. Status of both the suppliers is explained as below.

State	Status of supplier 1	Status of supplier 2
0	ON	ON
1	ON	OFF
2	OFF	ON
3	OFF	OFF

Here it is assumed that one may place order to either one of the two suppliers or partly to both when both suppliers are available (i.e. state 0 of the system).

In today's business transactions it is more and more common to see that the customer are allowed some grace period before settling the account with the supplier. This provides an advantage to the customers, due to the fact that they do not have to pay the supplier immediately after receiving the product but instead, can defer their payment until the end of the allowed period. The customer pays no interest during the fixed period, but if the payment is delayed beyond that period, interest will be charged. The customer can start to accumulate revenues on the sale or use of the product, and earn interest on that revenue. So it is to the advantage of the customer to defer the payment to the supplier until the end of the period. Shortages are very important, especially in a model that considers delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment.

4.2. NOTATIONS, ASSUMPTIONS AND MODEL:

The stochastic inventory model for two suppliers under permissible delay in payment is developed on the basis of the following assumptions.

(a) Demand rate d is deterministic and it is $d>1$.

(b) We define Xi and Yi to be the random variables corresponding to the length of ON and OFF period respectively for i^{th} supplier where i=1, 2. We specifically assume that Xi~ exp (λi) and Yi~ exp (μi). Further Xi and Yi are independently distributed.

(c) q_i = order up to level i=0, 1, 2

(d) r = reorder up to level ; q_i and r are decision variables.

(e) T_{0i} is a credit period allowed by i^{th} supplier where i=1, 2 which is a known constant.

(f) T_{00} is cycle period which is a decision variable.

(g) ie_i=Interest rate earned when purchase made from i^{th} supplier where i=1, 2

ic_i=Interest rate charged by i^{th} supplier where i=1, 2

(h) α_i= Indicator variable for i^{th} supplier where i=1, 2

$\alpha_i = 0$ if account is settled completely at T_{0i}

$= 1$ otherwise

(i) $Ie(1i)$=Interest earned over period (0 to T_{0i}) $= dcT_{00}T_{0i}ie_i$

(j) $Ie(2i)$=Interest earned over period (T_{0i} to T_{00}) upon interest earned ($Ie(1i)$) previously.

$$Ie(2i) = \left(dcT_{00} + Ie(1i)\right)(T_{00} - T_{0i})ie_i$$

(k) Interest charged by the i[th] supplier clearly ($ic_i > ie_i$) i=1, 2

$$Ic_i = \alpha_i dcic_i (T_{00} - T_{0i})$$

In this chapter we assume that

- A Supplier allows a fixed period 'T_{0i}' to settle the account. During this fixed period no interest is charged by the i[th] supplier but beyond this period, interest is charged by the i[th] supplier under the terms and conditions agreed upon.
- Interest charged is usually higher than interest earned.
- The account is settled completely either at the end of the credit period or at the end of the cycle.
- During the fixed credit period T_{0i}, revenue from sales is deposited in an interest bearing account.

The policy we have chosen is denoted by (q_0, q_1, q_2, r). An order is placed for q_i units i=0, 1, 2, whenever inventory drops to the reorder point r and the state found is i=0, 1, 2. When both suppliers are available, q_0 is the total ordered from either one or both suppliers. If the process is found in state 3 that is both the suppliers are not available nothing can be ordered in which case the buffer stock of r units is reduced. If the process stays in state 3 for longer time then the shortages start accumulating at rate of d units/time. When the process leaves state 3 and supplier becomes available, enough units are ordered to increase the inventory to q_i +r units where i=0, 1, 2.

$A(q_i, r, \theta)$=cost of ordering+ cost of holding inventory+ cost of items that deteriorate during a single interval that starts with an inventory of q_i units and ends with r units.

$$A(q_i, r, \theta) = k + \frac{1}{2}\frac{hq_i^2}{(d+\theta)} + \frac{hrq_i}{(d+\theta)} + \frac{\theta c q_i}{(d+\theta)} \qquad i=0, 1, 2$$

$P_{ij}(t)$=P (Being in state j at time t/starting in state i at time 0)   i, j=0, 1, 2, 3

p_i=long run probabilities   i=0, 1, 2, 3

4.3. OPTIMAL POLICY DECISION FOR THE MODEL:

For calculation of average cost objective function, we need to identify the cycles. Below given figure gives us the idea about cycles and their identification.

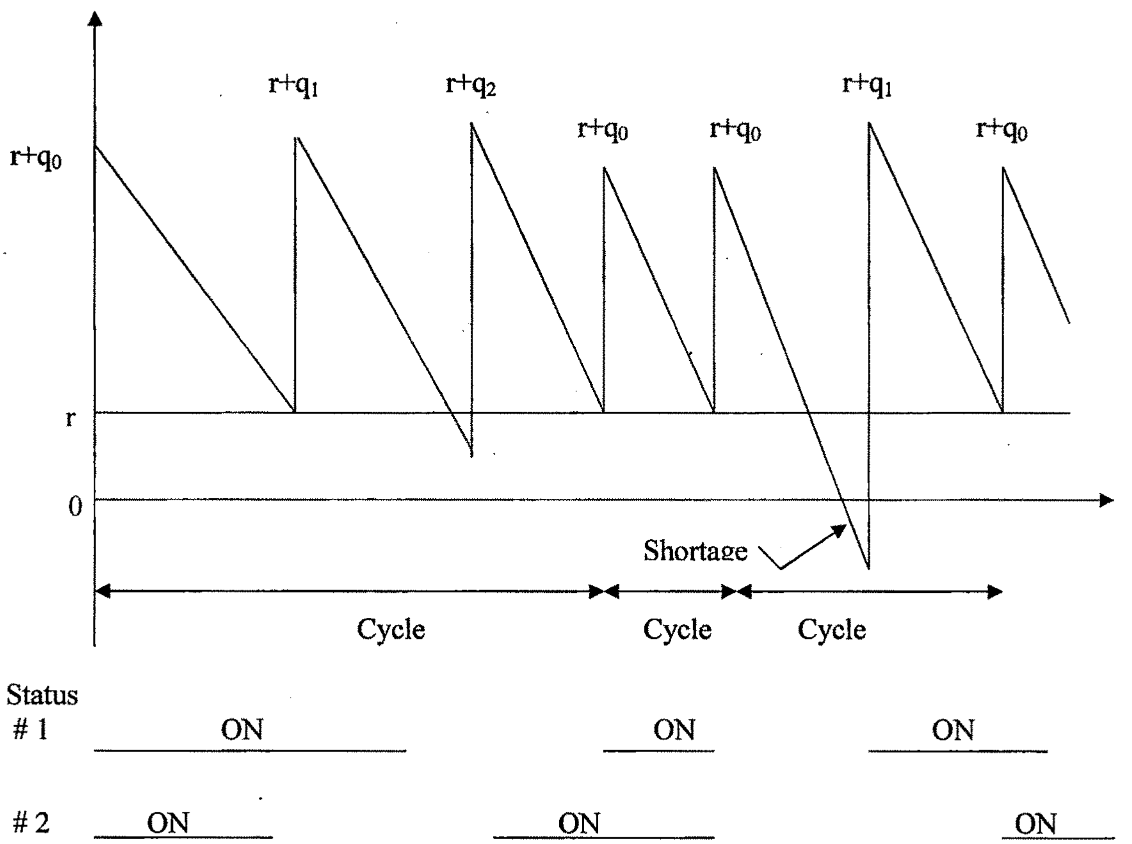

Fig. 4.1 Inventory level and status process with two suppliers

Referring to Fig.4.1, we see that the cycles of this process start when the inventory goes up to a level of q_0+r units. Once the cycle is identified, we construct the average cost objective function as a ratio of the expected cost per cycle to the expected cycle length.

i.e. $AC\ (q_0, q_1, q_2, r) = \dfrac{C_{00}}{T_{00}}$

where C_{00}=E (cost per cycle) and T_{00}=E (length of a cycle)

Analysis of the average cost function requires the exact determination of the transition probabilities $P_{ij}(t)$, i, j=0, 1, 2, 3 for the four state CTMC. The solution is provided in the following lemma.

Lemma 4.3.1: Let $P(t) = \lfloor P_{ij}(t) \rfloor$ t≥0, i, j=0, 1, 2, 3 be 4×4 matrix of transition functions for the CTMC. The exact transient solution is given as $P(t) = UD(t)U^{-1}$ .where

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

$$D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-(\lambda_1+\mu_1)t} & 0 & 0 \\ 0 & 0 & e^{-(\lambda_2+\mu_2)t} & 0 \\ 0 & 0 & 0 & e^{-(\lambda_1+\mu_1+\lambda_2+\mu_2)t} \end{bmatrix}$$

$$U^{-1} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Proof: We will provide a constructive proof and find the transition probabilities by solving the system of 16 ordinary linear differential equations i.e. the forward Kolmogorov equations. We will first describe the explicit derivation of the differential equations corresponding to $P_{00}(t)$ and then give the general result in matrix form.

Recall that in an infinitesimal time interval of length t we can only move from state 0 to state 0, 1 or 2. Therefore, we have

$$P_{00}(t+\Delta t) = [1-\lambda_1\Delta t + 0(\Delta t)][1-\lambda_2\Delta t + 0(\Delta t)]P_{00}(t) + [1-\lambda_1\Delta t + 0(\Delta t)][\lambda_2\Delta t + 0(\Delta t)]P_{10}(t)$$
$$+[\lambda_1\Delta t + 0(\Delta t)][1-\lambda_2\Delta t + 0(\Delta t)]P_{20}(t)$$

Subtracting $P_{00}(t)$ from both sides, dividing by Δt, and letting Δt→0 gives a differential equation as

$$P_{00}'(t) = -(\lambda_1+\lambda_2)P_{00}(t) + \lambda_2 P_{10}(t) + \lambda_1 P_{20}(t)$$

After generating a similar set of differential equations for the other states, the resulting 16 Kolmogorov equations can be put in a more convenient matrix form as $P'(t) = QP(t)$, $P(0) = I$ where

$$Q = \begin{bmatrix} -(\lambda_1+\lambda_2) & \lambda_2 & \lambda_1 & 0 \\ \mu_2 & -(\lambda_1+\mu_2) & 0 & \lambda_1 \\ \mu_1 & 0 & -(\lambda_2+\mu_1) & \lambda_2 \\ 0 & \mu_1 & \mu_2 & -(\mu_1+\mu_2) \end{bmatrix}$$

is the infinitesimal general of the stochastic process with states 0, 1, 2 and 3 and I is the identity matrix. We now solve this system using spectral analysis (Hilderbrand (1965), Bhat(1984)).

The solution to $P'(t) = QP(t)$, $P(0) = I$ can be written in the form $P(t) = e^{Qt}$, where

$$e^{Qt} = I + \sum_{n=1}^{\infty} \frac{Q^n t^n}{n!} \tag{4.3.1}$$

From the spectral theory of matrices (1965), we have $Q = UHU^{-1}$

Where U is a non-singular matrix formed with the right eigen vectors of Q and H is the diagonal matrix.

To find the right eigen vectors of Q, we first need to find the eigen values of Q that are obtained as the solution of the characteristic equation.

Let $(Q - wI) = 0$, solving gives $w_0=0$, $w_1=-(\lambda_1+\mu_1)$, $w_2=-(\lambda_2+\mu_2)$, $w_3=-(\lambda_1+\mu_1+\lambda_2+\mu_2)$ as the four distinct eigen values. Using the eigen values, we find the right eigen vectors and form the U matrix as

$$U = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -\mu_2/\lambda_2 & -\mu_2/\lambda_2 \\ 1 & -\mu_1/\lambda_1 & 1 & -\mu_1/\lambda_1 \\ 1 & -\mu_1/\lambda_1 & -\mu_2/\lambda_2 & \mu_1\mu_2/\lambda_1\lambda_2 \end{bmatrix}$$

If $Q = UHU^{-1}$ then $Q^n = UH^nU^{-1}$ and using (4.3.1) we get

$$P(t) = I + \sum_{n=1}^{\infty} \frac{UH^nU^{-1}t^n}{n!}$$

$$P(t) = UD(t)U^{-1} \tag{4.3.2}$$

where $D(t) = \begin{bmatrix} e^0 & 0 & 0 & 0 \\ 0 & e^{w_1t} & 0 & 0 \\ 0 & 0 & e^{w_2t} & 0 \\ 0 & 0 & 0 & e^{w_3t} \end{bmatrix}$

Because the inverse of U is formed as

$$U^{-1} = \frac{1}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \begin{bmatrix} \mu_1\mu_2 & \lambda_2\mu_1 & \lambda_1\mu_2 & \lambda_1\lambda_2 \\ \lambda_1\mu_2 & \lambda_1\lambda_2 & -\lambda_1\mu_2 & -\lambda_1\lambda_2 \\ \lambda_2\mu_1 & -\lambda_2\mu_1 & \lambda_1\lambda_2 & -\lambda_1\lambda_2 \\ \lambda_1\lambda_2 & -\lambda_1\lambda_2 & -\lambda_1\lambda_2 & \lambda_1\lambda_2 \end{bmatrix}$$

Hence above lemma is proved.

Using lemma (4.3.1) we obtain the following transition probabilities:

$$P_{01} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_2\mu_1 + \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} - \lambda_2\mu_1 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{02} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\mu_2 - \lambda_1\mu_2 e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{03} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\lambda_2 - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_0}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_0}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_0}{(d+\theta)}}\right]$$

$$P_{11} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_2\mu_1 + \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} + \mu_1\mu_2 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{12} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\mu_2 - \lambda_1\mu_2 e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} - \mu_2\lambda_1 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{13} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\lambda_2 - \lambda_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_1}{(d+\theta)}} + \mu_2\lambda_1 e^{\frac{-(\lambda_2+\mu_2)q_1}{(d+\theta)}} - \lambda_1\mu_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_1}{(d+\theta)}}\right]$$

$$P_{21} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_2\mu_1 - \mu_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} - \lambda_2\mu_1 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} + \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

$$P_{22} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\mu_2 + \mu_1\mu_2 e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} + \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} + \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

$$P_{23} = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)}\left[\lambda_1\lambda_2 + \mu_1\lambda_2 e^{\frac{-(\lambda_1+\mu_1)q_2}{(d+\theta)}} - \lambda_1\lambda_2 e^{\frac{-(\lambda_2+\mu_2)q_2}{(d+\theta)}} - \mu_1\lambda_2 e^{\frac{-(\lambda_1+\lambda_2+\mu_1+\mu_2)q_2}{(d+\theta)}}\right]$$

Corollary 4.3.1: The long run probabilities $P_j = \lim_{t\to\infty} P_{ij}(t)$ are

$$[p_0, p_1, p_2, p_3] = \frac{1}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)} [\mu_1\mu_2, \lambda_2\mu_1, \lambda_1\mu_2, \lambda_1\lambda_2]$$

Proof: As $t \to \infty$, we have

$$\lim_{t\to\infty} D(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence above corollary is proved.

Define C_{i0}=E (cost incurred to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1 or 2)

Lemma 4.3.2: C_{i0} is given by

$$C_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right) A(q_i, r, \theta) + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[A(q_i, r, \theta) + C_{j0}\right] \quad \text{i=0, 1, 2} \tag{4.3.3}$$

$$C_{30} = \overline{C} + \sum_{i=1}^{2} \rho_i C_{i0} \tag{4.3.4}$$

Where $\rho_i = \frac{\mu_i}{\delta}$ with $\delta = \mu_1 + \mu_2$ and

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[h e^{\frac{\delta r}{(d+\theta)}} (\delta r - (d+\theta)) + (\pi\delta d + h(d+\theta) + \hat{\pi}) - \theta c \delta \right] + \frac{\theta c}{\delta} \tag{4.3.5}$$

Proof: First consider i=0. Conditioning on the state of the supplier availability process when inventory drops to r, we obtain

$$C_{00} = P_{00}\left(\frac{q_0}{d+\theta}\right) A(q_0, r, \theta) + \sum_{j=1}^{3} P_{0j}\left(\frac{q_0}{d+\theta}\right)\left[A(q_0, r, \theta) + C_{j0}\right] \tag{4.3.6}$$

The equation follows because $q_0 + r$ being the initial inventory, when q_0 units are used up we either observe state 0, 1, 2 or 3 with probabilities

$P_{00}\left(\frac{q_0}{d+\theta}\right), P_{01}\left(\frac{q_0}{d+\theta}\right), P_{02}\left(\frac{q_0}{d+\theta}\right)$ and $P_{03}\left(\frac{q_0}{d+\theta}\right)$ respectively. If we are in state 0 when r is reached, we must have incurred a cost of $A(q_0, r, \theta)$. On the other hand, if state j=1, 2, 3 is observed when inventory drops to r, then the expected cost will be $A(q_0, r, \theta) + C_{j0}$ with probability $P_{0j}\left(\frac{q_0}{d+\theta}\right)$. The equation relating C_{10} and C_{20} are very similar but C_{30} is obtained as

$$C_{30} = \left[\overline{C} + C_{10}\right]\frac{\mu_1}{\mu_1 + \mu_2} + \left[\overline{C} + C_{20}\right]\frac{\mu_2}{\mu_1 + \mu_2}. \tag{4.3.7}$$

Here, $\overline{C}$ is defined as the expected cost from the time inventory drops to r until either supplier 1 or 2 becomes available, $\overline{C}$ is computed as follows:

Now referring to Fig 3.1., note that the cost incurred from the time when inventory drops to r and the state is OFF to the beginning of next cycle is equal to

$$\frac{1}{2}hy^2(d+\theta) + hy[r - y(d+\theta)] + \theta cy \qquad\qquad y < \frac{r}{d+\theta}$$

$$\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left(y - \frac{r}{(d+\theta)}\right)d + \frac{\hat{\pi}}{2}\left(y - \frac{r}{(d+\theta)}\right)^2 + \frac{\theta cr}{(d+\theta)} \qquad\qquad y \geq \frac{r}{d+\theta}$$

Hence,

$$\overline{C} = \int_0^{r/(d+\theta)} \left\{\frac{1}{2}hy^2(d+\theta) + hy(r - y(d+\theta) + \theta cy\right\}\delta e^{-\delta y}$$
$$+ \int_{r/(d+\theta)}^{\infty} \left\{\frac{1}{2}\frac{hr^2}{(d+\theta)} + \pi\left[y - \frac{r}{(d+\theta)}\right]d + \frac{\hat{\pi}}{2}\left[y - \frac{r}{(d+\theta)}\right]^2 + \frac{\theta cr}{(d+\theta)}\right\}\delta e^{-\delta y}$$

$$\overline{C} = \frac{e^{\frac{-\delta r}{(d+\theta)}}}{\delta^2}\left[he^{\frac{\delta r}{(d+\theta)}}(\delta r-(d+\theta))+(\pi\delta d+h(d+\theta)+\hat{\pi})-\theta c\delta \right]+\frac{\theta c}{\delta}$$

with $\delta = \mu_1 + \mu_2$ as the rate of departure from state 3. This follows because if supplier availability process is in state 3 (OFF for both suppliers) when inventory drops to r, then the expected holding and backorder costs are equal to $\overline{C}$. If the process makes a transition to state 1, the total expected cost would then be $\overline{C} + C_{10}$. The probability of a transition from state 3 to state 1 is

$$P(Y_1<Y_2)= \int_0^\infty P(Y_1 < Y_2 / Y_2 = t)\mu_2 e^{-\mu_2 t} dt = \frac{\mu_1}{\mu_1+\mu_2}$$

Multiplying this probability with the expected cost term above gives the first term of (4.3.7). The second term is obtained in a similar manner. Combining the results proves the lemma.

The following lemma provides a simpler means of expressing C_{00} in an exact manner. To simplify the notation, we let $A_i = A(q_i, r, \theta)$, i=0, 1, 2 and $P_{ij} = P_{ij}\left(\frac{q_i}{d+\theta}\right)$ i, j=0, 1, 2, 3.

Lemma 4.3.3: The exact expression for C_{00} is

$$C_{00} = A_0 + P_{01}C_{10} + P_{02}C_{20} + P_{03}\left(\overline{C} + \rho_1 C_{10} + \rho_2 C_{20}\right) \tag{4.3.8}$$

where the pair $[C_{10}, C_{20}]$ solves the system

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & 1-P_{22}-P_{23}\rho_2 \end{bmatrix}\begin{bmatrix} C_{10} \\ C_{20} \end{bmatrix}=\begin{bmatrix} A_1+P_{13}\overline{C} \\ A_2+P_{23}\overline{C} \end{bmatrix} \tag{4.3.9}$$

Proof: Rearranging the linear system of four equations in lemma (4.3.2) in matrix form gives

$$\begin{bmatrix} 1 & -P_{01} & -P_{02} & -P_{03} \\ 0 & 1-P_{11} & -P_{12} & -P_{13} \\ 0 & -P_{21} & 1-P_{22} & -P_{23} \\ 0 & -\rho_1 & -\rho_2 & 1 \end{bmatrix} \begin{bmatrix} C_{00} \\ C_{10} \\ C_{20} \\ C_{30} \end{bmatrix} = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \overline{C} \end{bmatrix} \tag{4.3.10}$$

We have $C_{30} = \overline{C} + \rho_1 C_{10} + \rho_2 C_{20}$ from the last row of the system. Substituting this result in rows two and three and rearranging gives the system in (4.3.9), with (C_{10}, C_{20}).

From the first row of (4.3.10) we obtain $C_{00} = A_0 + \sum_{j=1}^{3} P_{0j} C_{j0}$.

Hence above lemma is proved.

Define, T_{i0}=E [Time to the beginning of the next cycle from the time when inventory drops to r at state i=0, 1, 2, 3 and q_i units are ordered if i=0, 1, 2]

Lemma 4.3.4: Expected cycle length is given by

$$T_{i0} = P_{i0}\left(\frac{q_i}{d+\theta}\right)\frac{q_i}{d+\theta} + \sum_{j=1}^{3} P_{ij}\left(\frac{q_i}{d+\theta}\right)\left[\frac{q_i}{d+\theta} + T_{j0}\right] \qquad i = 0, 1, 2$$

$$T_{30} = \overline{T} + \sum_{j=1}^{2} \rho_i T_{i0}$$

where $\overline{T} = \dfrac{1}{\mu_1 + \mu_2}$ is the expected time from the time inventory drops to r until either supplier 1 or 2 becomes available.

Lemma 4.3.5: The exact expression for T_{00} is

$$T_{00} = \frac{q_0}{d+\theta} + P_{01}T_{10} + P_{02}T_{20} + P_{03}(\overline{T} + \rho_1 T_{10} + \rho_2 T_{20})$$

where the pair [T_{10}, T_{20}] solves the system.

$$\begin{bmatrix} 1-P_{11}-P_{13}\rho_1 & -(P_{12}+P_{13}\rho_2) \\ -(P_{21}+P_{23}\rho_1) & (1-P_{22}-P_{23}\rho_2) \end{bmatrix} \begin{bmatrix} T_{10} \\ T_{20} \end{bmatrix} = \begin{bmatrix} q_1 + P_{13}\overline{T} \\ q_2 + P_{23}\overline{T} \end{bmatrix}$$

The proof of the above two lemmas i.e. (4.3.4) and (4.3.5) are very similar to lemma (4.3.2) and (4.3.3)

Proposition 4.3.1: The Average cost objective function for two suppliers when delay in payment is considered is given by

$$AC=\frac{C_{00}}{T_{00}}=\frac{\begin{array}{c}A(q_0,r,\theta)+P_{01}(C_{10}-(Ie(11)+Ie(21)+Ic_1)+P_{02}(C_{20}-(Ie(12)+Ie(22)+Ic_2)\\ +P_{03}(\overline{C}+\rho_1(C_{10}-(Ie(11)+Ie(21)+Ic_1)+\rho_2(C_{20}-(Ie(12)+Ie(22))+Ic_2))\end{array}}{\frac{q_0}{d+\theta}+P_{01}T_{10}+P_{02}T_{20}+P_{03}(\overline{T}+\rho_1T_{10}+\rho_2T_{20})}$$

Proof: Proof follows using Renewal reward theorem (RRT). The optimal solution for q_0, q_1, q_2 and r is obtained by using Newton Rapson method in R programming.

4.4. NUMERICAL EXAMPLE:

In this section we verify the results by a numerical example. We assume that

(i) k =Rs. 5/order, c=Rs.1/unit, d=20/units, θ=4, h=Rs. 5/unit/time, π=Rs. 350/unit, $\hat{\pi}$=Rs.25/unit/time, ic_1=0.11, ie_1=0.02, ic_2=0.13, ie_2=0.04, T_{01}=0.6, T_{02}=0.8, (α_1=1 and α_2=1) i.e. businessmen do not settle the account at the respective credit time given by both the suppliers, λ_1=0.58, λ_2=0.45, μ_1=3.4, μ_2=2.5.

The last four parameters indicate that the expected lengths of the ON and OFF periods for first and second supplier are $1/\lambda_1$=1.72413794, $1/\lambda_2$=2.2222, $1/\mu_1$=.2941176 and $1/\mu_2$=.4 respectively. The long run probabilities are obtained as p_0=0.7239588, p_1=0.1303126, p_2 =0.1234989 and p_3=0.02222. The optimal solution is obtained as q_0=4.92015, q_1=33.130502, q_2=32.90077, r=0.8978675 and AC=$\frac{C_{00}}{T_{00}}$ = 6.291478.

(ii) Keeping other parameters as it is, we consider (α_1=0 and α_2=0) i.e. businessmen settle the account at the respective credit time given by both the suppliers.

The optimal solution is obtained as q_0=9.21634, q_1=41.82183, q_2=41.9396, r=0.76247 and AC=$\frac{C_{00}}{T_{00}}$ = 5.900553.

(iii) Keeping other parameters as it is, we consider (α_1=1 and α_2=0) i.e. businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 6.919021, q_1= 36.68451, q_2= 36.68383,

r= 0.925376 and AC=$\frac{C_{00}}{T_{00}}$ = 6.091088.

(iv) Keeping other parameters as it is, we consider (α_1=0 and α_2=1) i.e. when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

The optimal solution is obtained as q_0= 6.573681, q_1= 35.95015, q_2= 35.9173,

r= 0.938723 and AC=$\frac{C_{00}}{T_{00}}$ = 6.142968

Conclusion:

From the above numerical example, we conclude that the cost is minimum when businessmen settle the account at the respective credit time given by both the suppliers i.e. when (α_1=0 and α_2=0).

4.5. SENSITIVITY ANALYSIS:

To observe the effects of varying parameter values on the optimal solution we have conducted sensitivity analysis, by varying λ_1, λ_2, μ_1, μ_2, h, k.

4.5.1. Sensitivity Analysis for λ_1:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 that is length of ON period for 1st supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.1.

Table 4.5.1.1
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=1and α_2=1)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.21939	32.3257	32.1097	0.303045	6.346889
0.52	4.32861	32.41094	32.176	0.458388	6.336898
0.54	4.46853	32.54743	32.30188	0.609183	6.324458
0.56	4.65496	32.76605	32.5205	0.755719	6.30943
0.58	4.92015	33.1305	32.9007	0.897868	6.291478

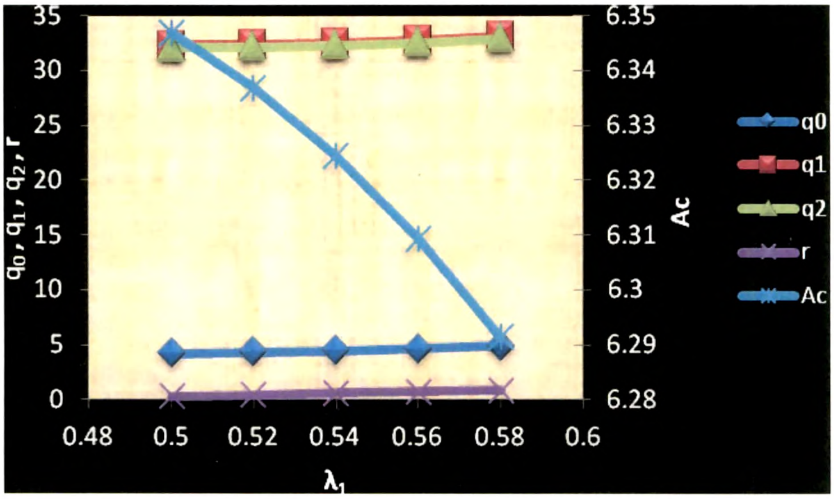

Fig. 4.5.1.1 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases but since $1/\lambda_2$ =1/0.45=2.2 that is expected length of ON period for 2nd supplier is fixed, which results in decrease in average cost. Decreasing the expected length of ON period for 1st supplier we see there is a decrease in average cost, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.2.

Table 4.5.1.2
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=0and α_2=0)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.21939	32.3257	32.1097	0.303045	6.346889
0.52	4.32861	32.41094	32.176	0.458388	6.336898
0.54	4.46853	32.54743	32.30188	0.609183	6.324458
0.56	4.65496	32.76605	32.5205	0.755719	6.30943
0.58	4.92015	33.1305	32.9007	0.897868	6.291478

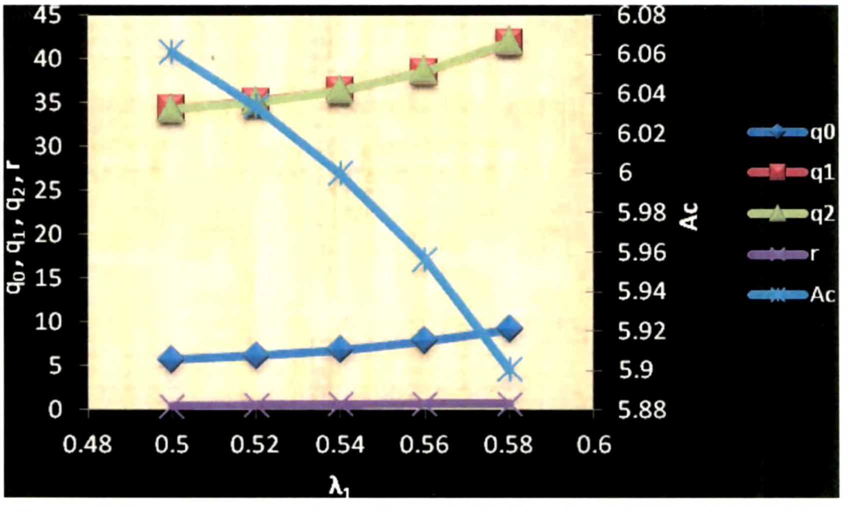

Fig. 4.5.1.2 Sensitivity analysis graph for λ_1

Increasing λ_1 i.e. decreasing expected length of ON period for 1st supplier results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.3.

Table 4.5.1.3
Sensitivity Analysis Table by varying the parameter values of λ_1
($\alpha_1=1$and $\alpha_2=0$)

λ_1	q_0	q_1	q_2	r	AC
0.5	4.875214	33.10239	32.96599	0.391809	6.202606
0.52	5.124579	33.43298	33.29805	0.548381	6.182258
0.54	5.472377	33.95825	33.84087	0.69646	6.157839
0.56	6.001515	34.8682	34.79183	0.82975	6.128266
0.58	6.919021	36.68451	36.68383	0.925376	6.091088

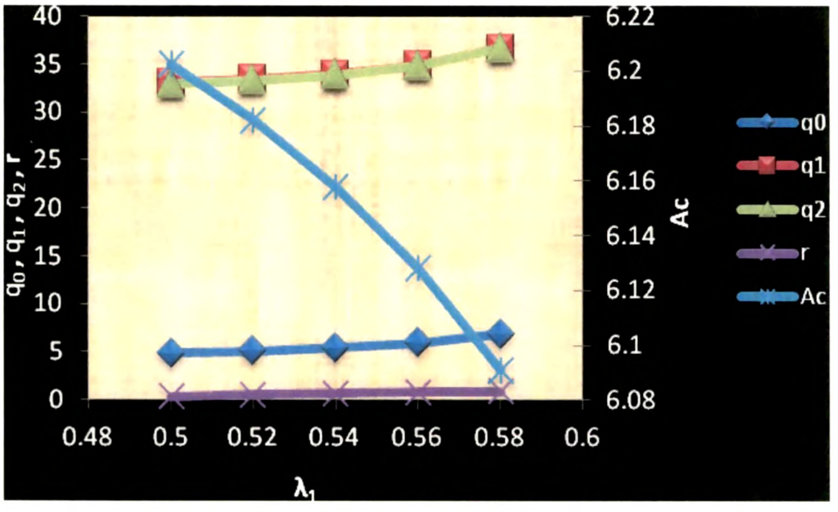

Fig.4.5.1.3 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases, average cost decreases when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_1 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.1.4.

Table 4.5.1.4
Sensitivity Analysis Table by varying the parameter values of λ_1
(α_1=0 and α_2=1)

λ_1	q_0	q_1	q_2	r	Ac
0.5	4.817789	32.99064	32.84593	0.388688	6.22261
0.52	5.028656	33.26011	33.11216	0.543481	6.20912
0.54	5.322365	33.68757	33.55083	0.691404	6.191908
0.56	5.770063	34.4272	34.32367	0.828704	6.170542
0.58	6.573681	35.95015	35.9173	0.938723	6.142968

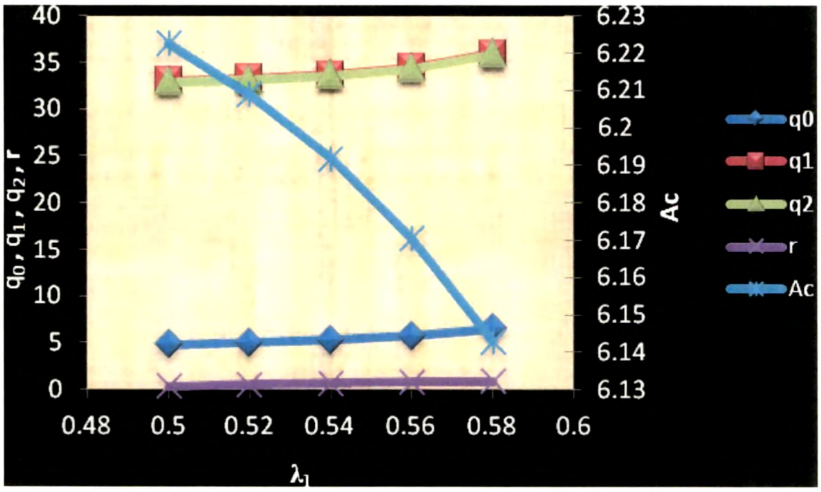

Fig. 4.5.1.4 Sensitivity analysis graph for λ_1

We see that as λ_1 increases i.e. expected length of ON period for 1st supplier decreases, average cost decreases when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.2. Sensitivity Analysis for λ_2:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 that is length of ON period for 2nd supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.1.

Table 4.5.2.1
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=1and α_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	4.51758	32.8025	32.3632	0.532947	6.301836
0.43	4.68582	32.90287	32.56939	0.71933	6.298501
0.45	4.92015	33.1305	32.90077	0.897868	6.291478
0.47	5.284412	33.61563	33.4926	1.06763	6.280406
0.49	6.03914	34.91542	34.91874	1.21790	6.263854

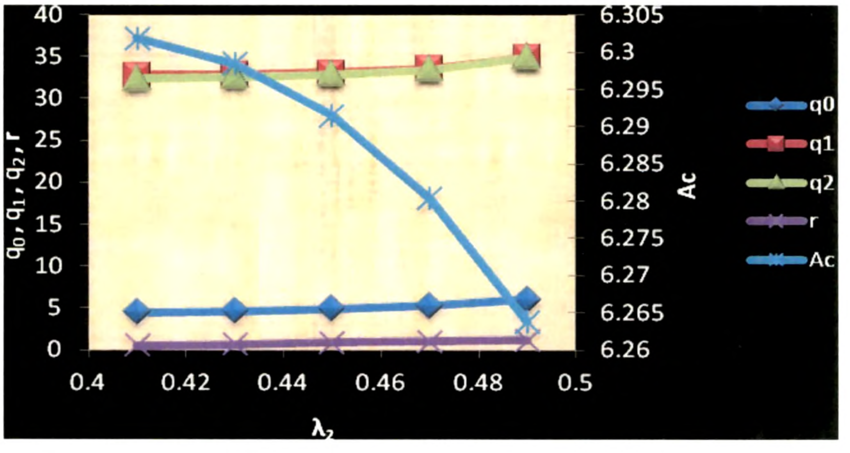

Fig. 4.5.2.1 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases but since $1/\lambda_1$ =1/0.58=1.72 that is expected length of ON period for 1st supplier is fixed, which results in decrease in average cost when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.2.

Table 4.5.2.2
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=0and α_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	6.97535	36.76276	36.67674	0.62583	5.9735
0.43	7.93252	38.75428	38.79006	0.73206	5.94342
0.45	9.21634	41.82184	41.93962	0.762478	5.90055
0.47	10.66972	45.76968	45.91412	0.720598	5.842058
0.49	12.13478	50.24337	50.37507	0.631777	5.766487

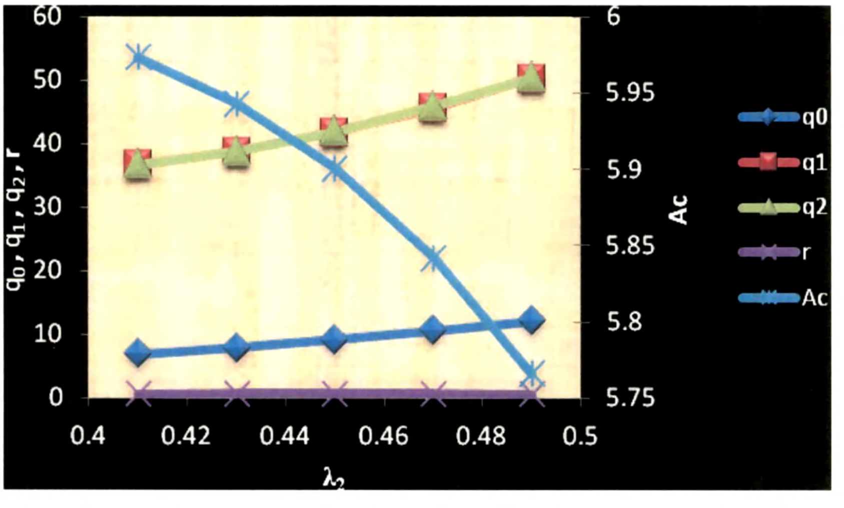

Fig. 4.5.2.2 Sensitivity analysis graph for λ_2

Increasing λ_2 i.e. decreasing expected length of ON period for 2nd supplier results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.3.

Table 4.5.2.3
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=1and α_2=0)

λ_2	q_0	q_1	q_2	r	AC
0.41	5.648678	34.42982	34.17833	0.621036	6.12024
0.43	6.122379	35.17788	35.04982	0.790781	6.1092
0.45	6.919021	36.68451	36.68383	0.925376	6.091088
0.47	8.37173	39.95611	40.06992	0.967174	6.061918
0.49	10.33746	45.19611	45.34853	0.871826	6.014996

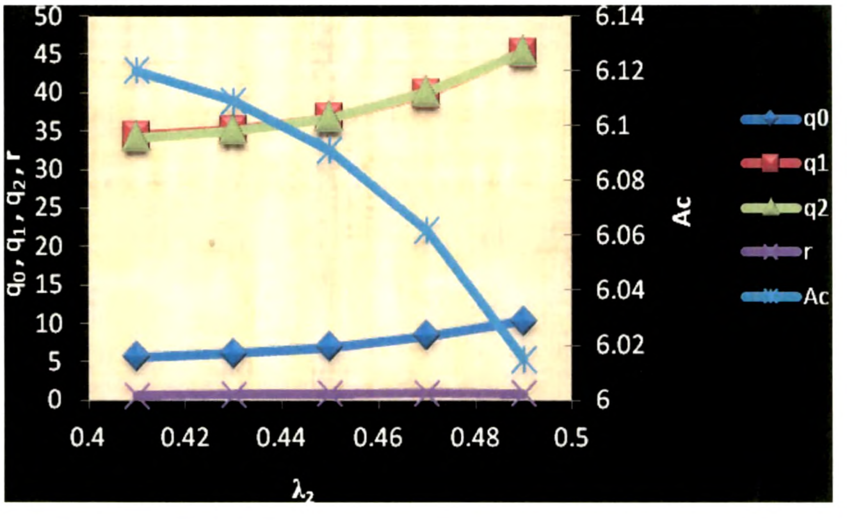

Fig. 4.5.2.3 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value λ_2 and keeping other parameter values fixed where α_1=0 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.2.4.

Table 4.5.2.4
Sensitivity Analysis Table by varying the parameter values of λ_2
(α_1=0 and α_2=1)

λ_2	q_0	q_1	q_2	r	AC
0.41	5.306492	33.85771	33.55111	0.607382	6.180515
0.43	5.761708	34.5091	34.33355	0.788504	6.16539
0.45	6.573681	35.95015	35.9173	0.938723	6.142968
0.47	8.30733	39.74837	39.8591	0.976085	6.107925
0.49	10.61271	45.88837	46.03926	0.840191	6.050721

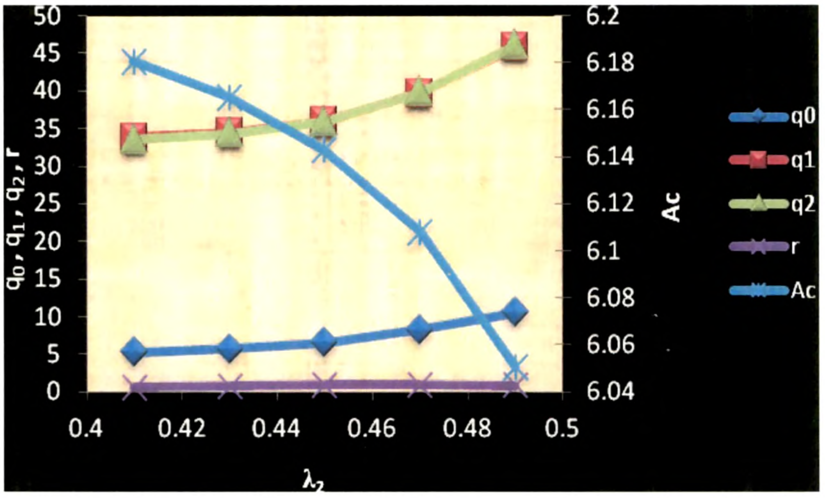

Fig. 4.5.2.4 Sensitivity analysis graph for λ_2

We see that as λ_2 increases i.e. expected length of ON period for 2nd supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.3. Sensitivity Analysis for μ_1:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 that is length of OFF period for 1st supplier and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.3.1.

Table 4.5.3.1
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=1and α_2=1)

μ_1	q_0	q_1	q_2	r	AC
3.4	4.92015	33.1305	32.90077	0.897868	6.291478
3.6	4.63996	32.37693	32.13742	0.75245	6.23645
3.8	4.47092	31.82851	31.59548	0.62584	6.184397
4	4.35657	31.3834	31.16678	0.514433	6.13567
4.2	4.274396	31.00246	30.80907	0.415555	6.090279

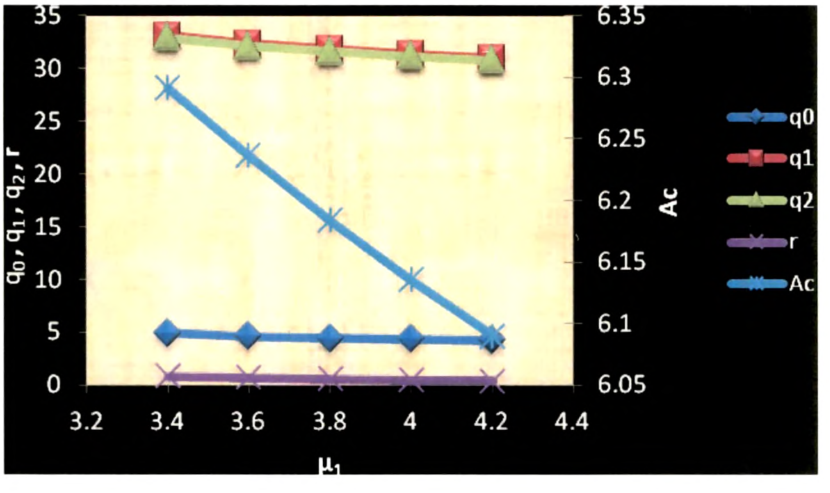

Fig. 4.5.3.1 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, average cost decreases when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig.4.5.3.2.

Table 4.5.3.2
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=0and α_2=0)

μ_1	q_0	q_1	q_2	r	AC
3.4	9.21634	41.82184	41.93962	0.762478	5.90055
3.6	7.38513	37.1336	37.20425	0.78833	5.8883
3.8	6.4575	34.90709	34.93798	0.71046	5.8604
4	5.93738	33.64032	33.65347	0.61157	5.82773
4.2	5.606021	32.78604	32.79784	0.513805	5.79421

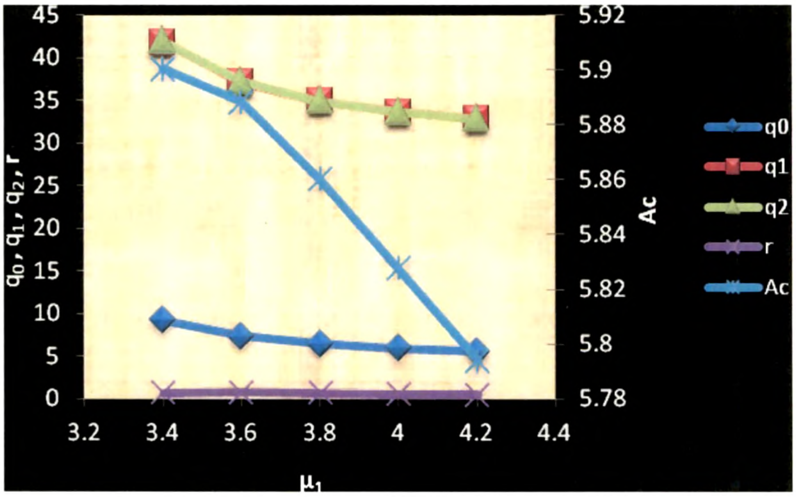

Fig. 4.5.3.2 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, which results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.3.3.

Table 4.5.3.3
Sensitivity Analysis Table by varying the parameter values of μ_1
(α_1=1and α_2=0)

μ_1	q_0	q_1	q_2	r	AC
3.4	6.919021	36.68451	36.68383	0.925376	6.091088
3.6	5.83538	34.22192	34.14971	0.817911	6.054619
3.8	5.353431	33.07427	32.97752	0.693801	6.013849
4	5.068892	32.32638	32.22671	0.578918	5.973334
4.2	4.877779	31.76316	31.67317	0.475274	5.934428

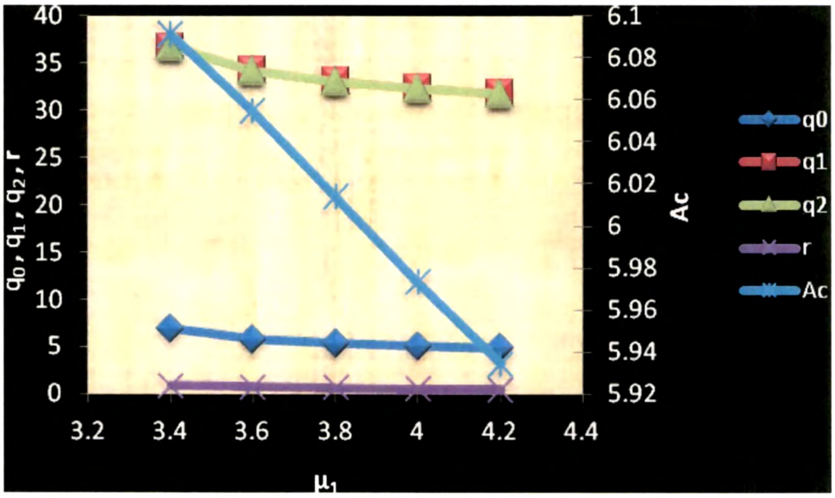

Fig. 4.5.3.3 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_1 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.3.4.

Table 4.5.3.4
Sensitivity Analysis Table by varying the parameter values of μ_1
($\alpha_1=0$ and $\alpha_2=1$)

μ_1	q_0	q_1	q_2	r	AC
3.4	6.573681	35.95015	35.9173	0.938723	6.142968
3.6	5.657289	33.87504	33.77881	0.817562	6.098598
3.8	5.250657	32.87008	32.75627	0.692375	6.052388
4	5.00937	32.19269	32.08098	0.578698	6.007538
4.2	4.847366	31.6709	31.57269	0.476728	5.964934

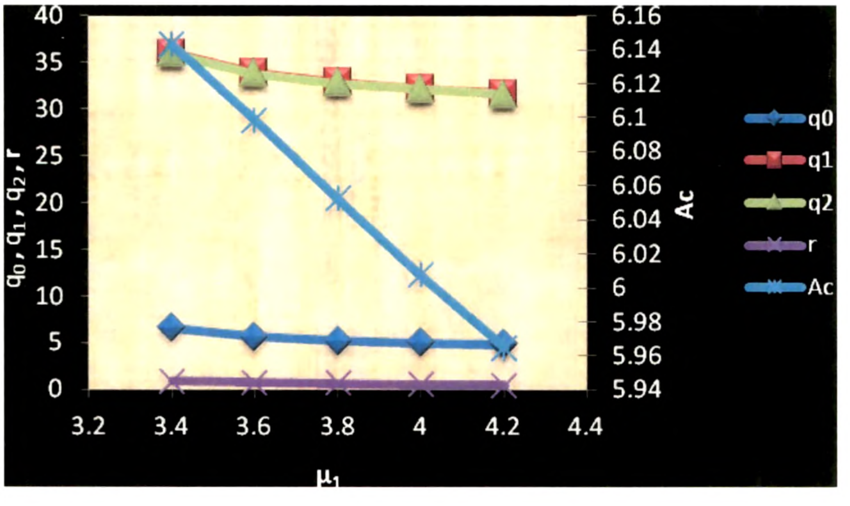

Fig. 4.5.3.4 Sensitivity analysis graph for μ_1

We see that as μ_1 increases i.e. expected length of OFF period for 1st supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.4. Sensitivity Analysis for μ_2:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 that is length of OFF period for 2nd supplier and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.1.

Table 4.5.4.1
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=1and α_2=1)

μ_2	q_0	q_1	q_2	r	AC
2.5	4.92015	33.1305	32.90077	0.897868	6.291478
2.7	4.65914	32.42439	32.18495	0.714823	6.22704
2.9	4.50296	31.89978	31.67263	0.558189	6.16661
3.1	4.40044	31.47235	31.26477	0.42312	6.11043
3.3	4.329983	31.10838	30.92135	0.305699	6.058361

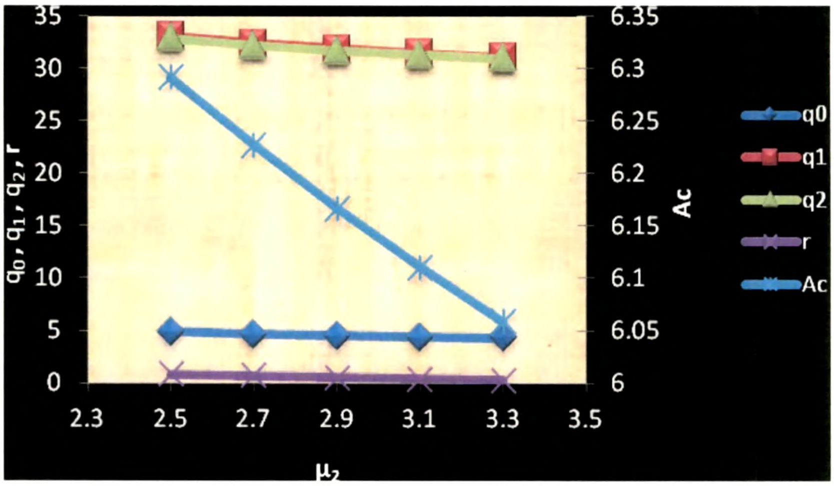

Fig. 4.5.4.1 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.2.

Table 4.5.4.2
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=0and α_2=0)

μ_2	q_0	q_1	q_2	r	AC
2.5	9.21634	41.8218	41.93962	0.76247	5.90055
2.7	7.60633	37.7112	37.72316	0.740037	5.87412
2.9	6.71613	35.5491	35.49144	0.63948	5.83621
3.1	6.19726	34.2676	34.17347	0.52256	5.79468
3.3	5.86514	33.3992	33.2874	0.40938	5.75344

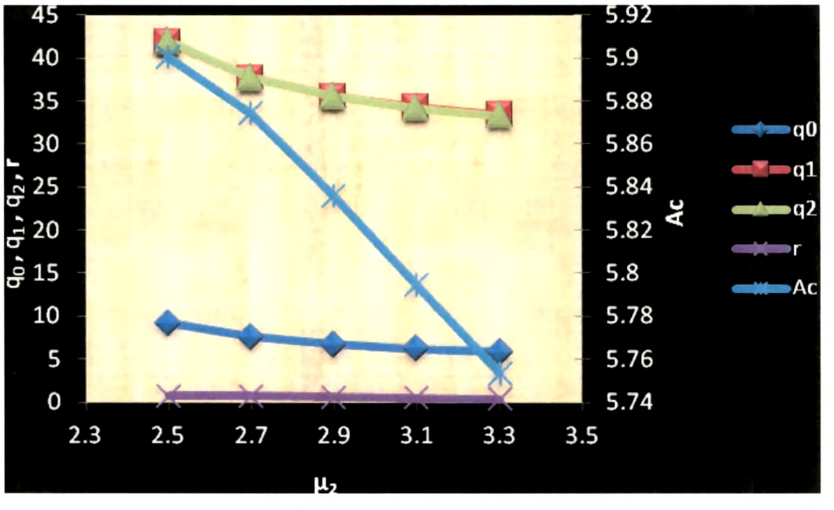

Fig. 4.5.4.2 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2[nd] supplier decreases, which results in decrease in average cost when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.3.

Table 4.5.4.3
Sensitivity Analysis Table by varying the parameter values of μ_2
(α_1=1and α_2=0)

μ_2	q_0	q_1	q_2	r	AC
2.5	6.919021	36.68451	36.68383	0.925376	6.091088
2.7	6.014855	34.59541	34.49889	0.782994	6.039226
2.9	5.579188	33.5183	33.38583	0.635233	5.986091
3.1	5.320454	32.79904	32.6555	0.501161	5.93477
3.3	5.149895	32.25601	32.11205	0.381983	5.886177

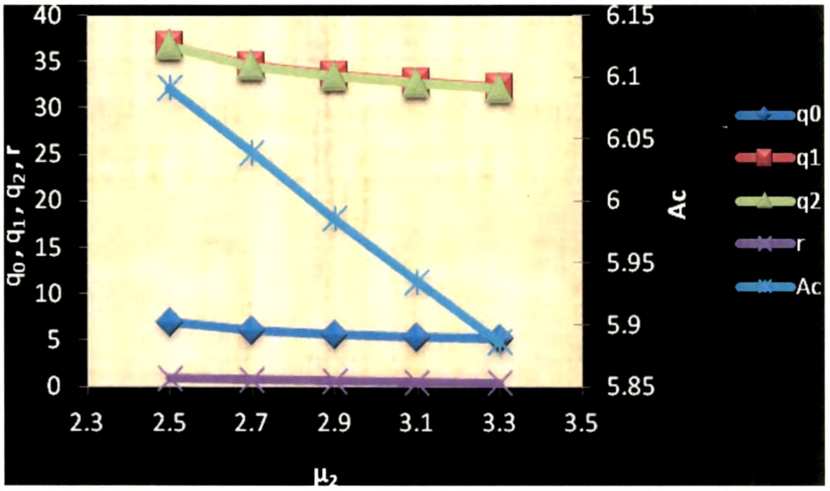

Fig. 4.5.4.3 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value μ_2 and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.4.4.

Table 4.5.4.4
Sensitivity Analysis Table by varying the parameter values of μ_2
($\alpha_1=0$ and $\alpha_2=1$)

μ_2	q_0	q_1	q_2	r	AC
2.5	6.573681	35.95015	35.9173	0.938723	6.142968
2.7	5.663003	33.94184	33.8112	0.779756	6.092003
2.9	5.250152	32.95139	32.79107	0.623286	6.039782
3.1	5.007076	32.28605	32.12098	0.484359	5.9895
3.3	4.846396	31.77769	31.61777	0.362246	5.942063

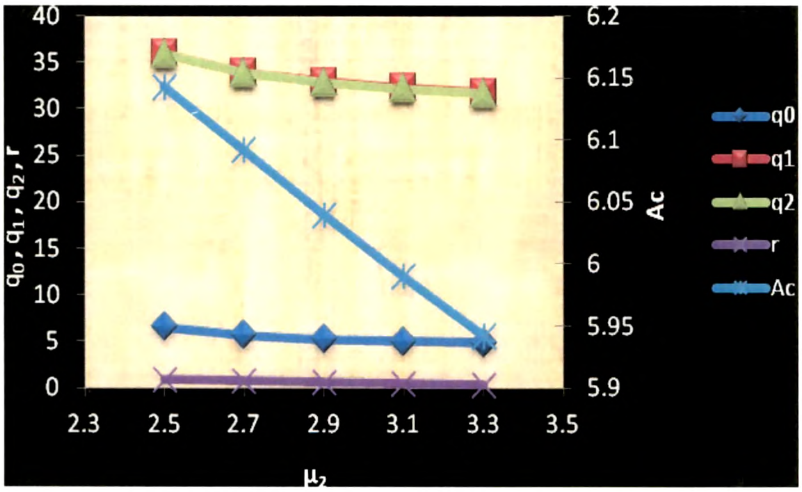

Fig. 4.5.4.4 Sensitivity analysis graph for μ_2

We see that as μ_2 increases i.e. expected length of OFF period for 2nd supplier decreases, which results in decrease in average cost when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.5. Sensitivity Analysis for h:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.1.

Table 4.5.5.1
Sensitivity Analysis Table by varying the parameter values of h
($\alpha_1=1$and $\alpha_2=1$)

h	q_0	q_1	q_2	r	AC
5	4.92015	33.1305	32.90077	0.897868	6.29147
5.2	4.51023	32.13459	31.81629	0.736602	6.44817
5.4	4.22727	31.4038	31.01222	0.582429	6.59807
5.6	4.01276	30.81565	30.36028	0.435484	6.74237
5.8	3.840995	30.31773	29.80502	0.295231	6.88182

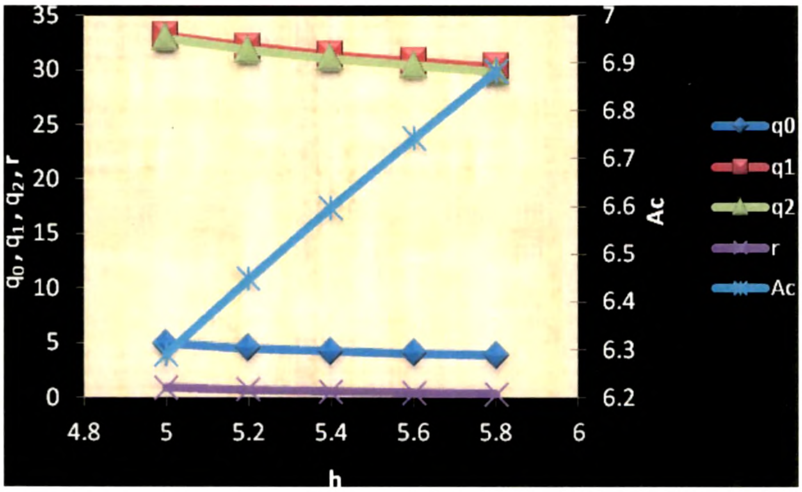

Fig. 4.5.5.1 Sensitivity analysis graph for h

We see that as holding cost h increases, average cost increases, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where α_1=0 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.2.

Table 4.5.5.2
Sensitivity Analysis Table by varying the parameter values of h
(α_1=0and α_2=0)

h	q_0	q_1	q_2	r	AC
5	9.21634	41.82184	41.93962	0.762478	5.90055
5.2	7.608994	37.48657	37.52147	0.775403	6.097604
5.4	6.458036	34.71559	34.63385	0.702893	6.275461
5.6	5.711362	33.02134	32.8299	0.58585	6.439946
5.8	5.204992	31.89054	31.60519	0.454638	6.594901

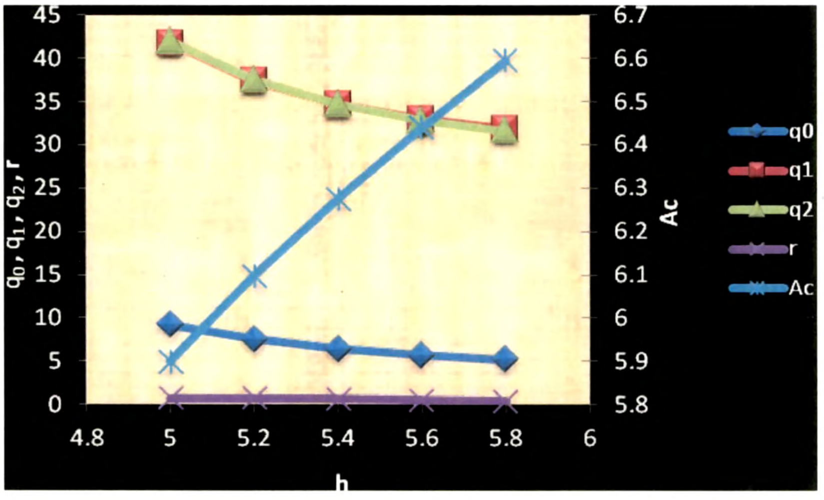

Fig. 4.5.5.2 Sensitivity analysis graph for h

We see that as holding cost h increases, average cost increases, when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where α_1=1 and α_2=0. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.3.

Table 4.5.5.3
Sensitivity Analysis Table by varying the parameter values of h
(α_1=1and α_2=0)

h	q_0	q_1	q_2	r	AC
5	6.919021	36.68451	36.68383	0.925376	6.091088
5.2	5.80463	34.07729	33.94275	0.816696	6.265027
5.4	5.181679	32.65309	32.41251	0.675464	6.426575
5.6	4.771607	31.69855	31.37156	0.530788	6.579658
5.8	4.472563	30.97895	30.57812	0.389227	6.726147

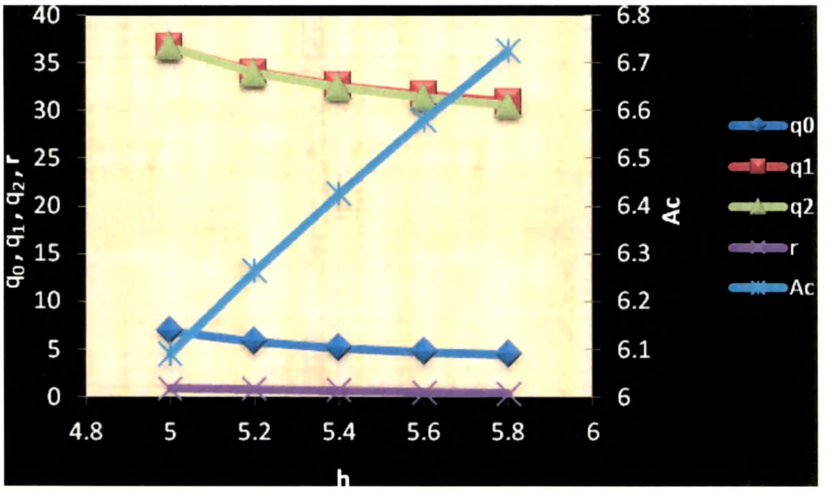

Fig. 4.5.5.3 Sensitivity analysis graph for h

Increasing the holding cost h, results in increase in average cost, when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of holding cost h and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.5.4.

Table 4.5.5.4
Sensitivity Analysis Table by varying the parameter values of h
(α_1=0 and α_2=1)

h	q_0	q_1	q_2	r	AC
5	6.573681	35.95015	35.9173	0.938723	6.142968
5.2	5.521374	33.5696	33.39709	0.812252	6.313326
5.4	4.958185	32.29829	32.02236	0.663447	6.472119
5.6	4.587592	31.43345	31.07425	0.51553	6.623014
5.8	4.31594	30.77104	30.34097	0.372561	6.767689

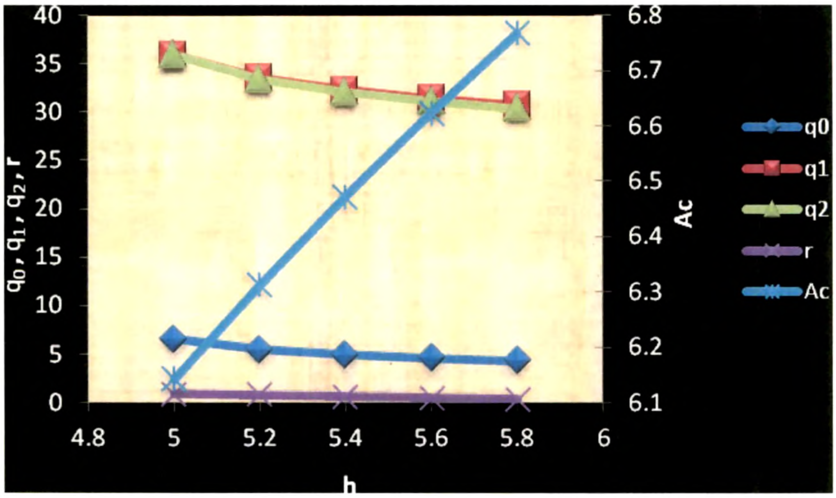

Fig. 4.5.5.4 Sensitivity analysis graph for h

Increasing the holding cost h, results in increase in average cost, when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.5.6. Sensitivity Analysis for k:

(i) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where α_1=1 and α_2=1. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.1.

Table 4.5.6.1
Sensitivity Analysis Table by varying the parameter values of k
(α_1=1and α_2=1)

k	q_0	q_1	q_2	r	AC
4.5	4.309901	31.82929	31.47698	0.875967	6.198703
5	4.92015	33.1305	32.90077	0.867868	6.291478
5.5	5.64599	34.71506	34.59938	0.842023	6.37395
6	6.494584	36.65105	36.63384	0.819756	6.44698
6.5	7.3987	38.84879	38.90353	0.82776	6.51175

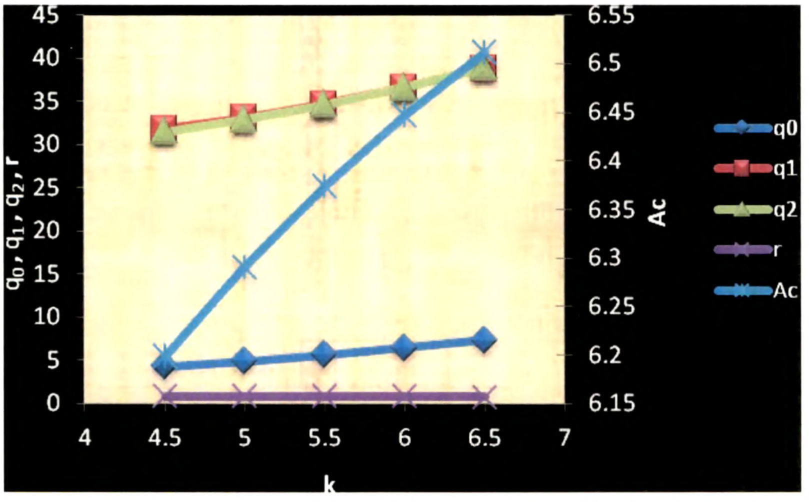

Fig. 4.5.6.1 Sensitivity analysis graph for k

We see that as ordering cost k increases, average cost increases, when the account is not settled at the respective credit time given by both the suppliers.

(ii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.2.

Table 4.5.6.2
Sensitivity Analysis Table by varying the parameter values of k
(α_1=0and α_2=0)

k	q_0	q_1	q_2	r	AC
4.5	8.3278	39.33008	39.41338	0.86655	5.845945
5	9.21634	41.82184	41.93962	0.762478	5.90055
5.5	9.88241	43.83543	43.96582	0.672766	5.950828
6	10.41764	45.54794	45.68173	0.594338	5.998093
6.5	10.86781	47.05556	47.18837	0.524441	6.04307

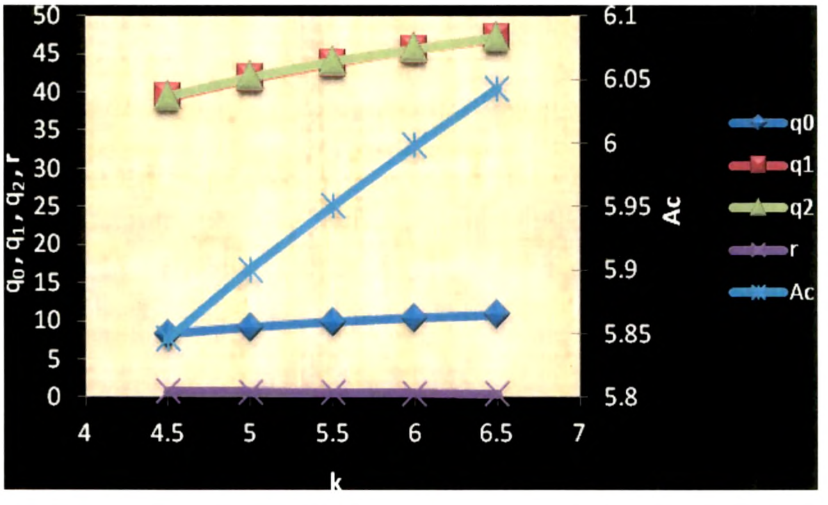

Fig. 4.5.6.2 Sensitivity analysis graph for k

We see that increasing the ordering cost k, results in increase in average cost, when businessmen settle the account at the respective credit time given by both the suppliers.

(iii) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where $\alpha_1=1$ and $\alpha_2=0$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.3.

Table 4.5.6.3
Sensitivity Analysis Table by varying the parameter values of k
(α_1=1and α_2=0)

k	q_0	q_1	q_2	r	AC
4.5	5.816673	34.16042	34.03268	0.97103	6.019221
5	6.919021	36.68451	36.68383	0.925376	6.091088
5.5	7.948582	39.27446	39.34869	0.845075	6.153469
6	8.783016	41.56245	41.6723	0.757834	6.209351
6.5	9.45033	43.5245	43.64968	0.675901	6.260824

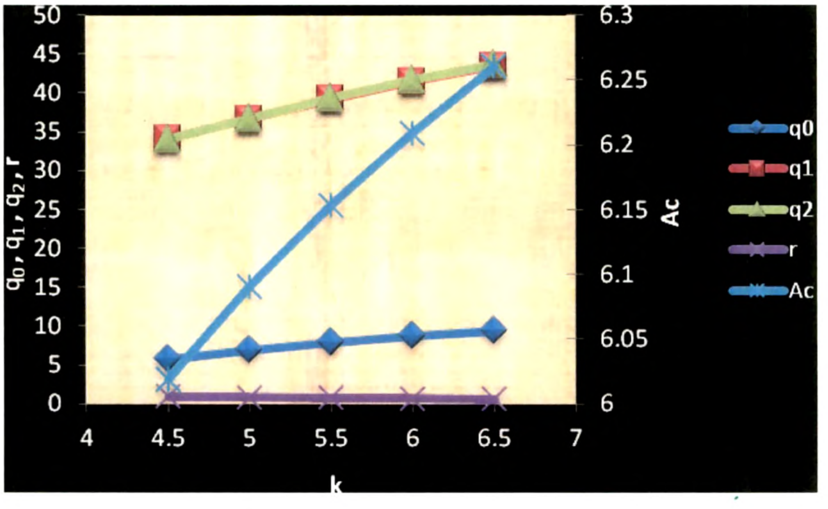

Fig. 4.5.6.3 Sensitivity analysis graph for k

We see that as ordering cost k increases, average cost increases, when businessmen do not settle the account at the credit time given by the 1st supplier but they settle the account at the credit time given by the 2nd supplier.

(iv) To observe the effect of varying parameter values on the optimal solution, we have conducted sensitivity analysis by varying the value of ordering cost k and keeping other parameter values fixed where $\alpha_1=0$ and $\alpha_2=1$. We resolve the problem to find optimal values of q_0, q_1, q_2, r and AC. The optimal values of q_0, q_1, q_2 and AC are plotted in Fig. 4.5.6.4.

Table 4.5.6.4
Sensitivity Analysis Table by varying the parameter values of k
($\alpha_1=0$ and $\alpha_2=1$)

k	q_0	q_1	q_2	r	AC
4.5	5.44694	33.48098	33.30266	0.965965	6.067404
5	6.573681	35.95015	35.9173	0.938723	6.142968
5.5	7.743695	38.77173	38.83411	0.861564	6.207476
6	8.698424	41.30941	41.41623	0.768249	6.264253
6.5	9.436885	43.43414	43.55884	0.68039	6.316019

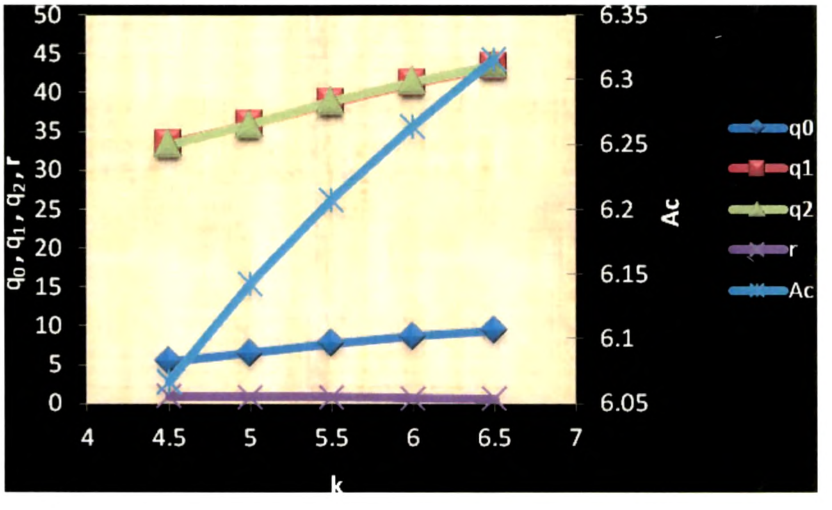

Fig. 4.5.6.4 Sensitivity analysis graph for k

Increasing the ordering cost k, results in increase in average cost, when the account is settled by businessmen at the credit time given by the 1st supplier but they do not settle the account at the credit time given by the 2nd supplier.

4.6. CONCLUSION:

From the above sensitivity analysis, we conclude that in all the situations cost is minimum when account is settled by businessman at the respective credit time given by both the suppliers i.e. when (α_1=0 and α_2=0). The privilege offered proves to be bliss for entrepreneurs, so they are in the business resulting keen competition due to increased number of entrepreneurs.