Chapter 6

Exterior first order resonance of Sun-Saturn system in ERTBP

6.1 Introduction

In this chapter, the first order exterior resonant periodic orbits in the Sun-Saturn system are computed using the method of PSS. Orbit-orbit resonances are mean motion resonances as they are generated due to numerical relation between mean motions of bodies. Consider two bodies A and B of arbitrary masses orbiting around a central mass with mean motions n_A and n_B, respectively. If there exist integers p and q such that the ratio of mean motions $n_A : n_B$ is of the form $p : s$, then the mean motion resonance exists between the two bodies. Here, p corresponds to mean motion of A and s corresponds to mean motion of B. Orbit-orbit resonances can be further classified into two types: exterior resonances and interior resonances. In exterior resonances, an integer ratio $p : s$ is such that $p < s$. In this case, the spacecraft spends majority of the time outside the vicinity of the second primary and has larger orbital period or semi-major axis compared to the second primary (Zimovan (2017)).

Periodic orbits are important for mission design. Further, these orbits are useful for understanding the behaviour of non-linear systems. The technique of PSS is very useful for finding the initial conditions of periodic orbits. Although the construction of PSS is tedious and time-consuming, it is very useful for the study of resonance.

The study of resonant periodic orbits in CRTBP for different resonant orders has been done by many researchers. In the Sun-Saturn CRTBP system, Patel et al. (2021) have computed first order interior as well as exterior resonant orbits by considering the

Saturn as an oblate spheroid. Also, with the help of non-linear multivariate regression analysis, Patel et al. (2022a) have generated regression model for getting initial position of the first order resonant orbits in the Saturn-Hyperion, Saturn-Titan and Earth-Moon systems. This regression model helps in getting initial conditions without plotting PSS. The stability of above resonant orbits has been studied by Patel et al. (2022b). In Kotoulas et al. (2022), the authors have studied the dynamics of interior mean motion resonance of first, second and third order with the Jupiter using CRTBP. Further, multiple families of resonant orbits in restricted-four-body problem has been obtained in Oshima (2022).

In this chapter, the first order exterior resonant orbits in the Sun-Saturn system are found by considering the Sun as a source of radiation in the framework of ERTBP. The effects of eccentricity of the orbit of the primaries, solar radiation pressure and Jacobi constant on parameters of these orbits are analyzed.

6.2 Computation of order of resonance

In this study, the notation $p : p + s$ is used to denote the resonance ratio which can be obtained using PSS and the two body approximation. For generating PSS, equations (1.49) are integrated using RKG method for $x \in [0.55, 1.00]$ with a fixed step size $h = 0.001$ and the solutions for which $y = 0$ and $y' > 0$ are plotted on the xx' plane. From the PSS, the centre of island is determined which corresponds to the location (x) of periodic orbits. Using the location of orbit and two-body approximation, with the Sun as the central body and the spacecraft moving around it, the velocity of the spacecraft can be obtained from Koon et al. (2011, p.8)

$$\mathbf{v} = x'\hat{\mathbf{i}} + (y' + x + \mu)\hat{\mathbf{j}}. \tag{6.1}$$

The magnitude of velocity vector (v), the angular momentum (h), eccentricity (e_s) and semi-major axis (a_s) of orbit of spacecraft are given by (Murray and Dermott (1999, p.424))

$$\begin{aligned} v =&\sqrt{x'^2+(y'+x+\mu)^2}, \\ h =&(x+\mu)(y'+x+\mu), \\ a_s =&\left[\frac{2}{r_1}-\frac{v^2}{(1-\mu)q}\right]^{-1}, \\ e_s =&\sqrt{1-\frac{h^2}{a_s(1-\mu)q}}. \end{aligned} \tag{6.2}$$

The ratio $p : p+s$ can be obtained from the relation (Murray and Dermott (1999))

$$\frac{p}{p+s}=\left(\frac{a}{a_s}\right)^{3/2}, \tag{6.3}$$

where a is the semi-major axis of the orbit of the primaries. In the ratio $p : p+s$, the number s gives the order of resonance. Also, the number of islands in a PSS corresponding to an orbit provides the order of resonance.

6.3 Results and Discussion

The mass factor of the Sun–Saturn system is $\mu = 0.0002857696$ and the eccentricity of the orbit of the Saturn around the Sun is $e = 0.052$ (`https://nssdc.gsfc.nasa.gov/planetary/factsheet/saturnfact.html`). The PSS for $e = 0.052, q = 0.99$ and $C = 2.8$ is shown in Fig. 6.1. The arcs in a PSS represent the islands containing the periodic orbits. The locations (x) of resonant periodic orbits are obtained from the PSS by identifying the centres of the islands. The periodic orbits corresponding to the islands of PSS in Fig. 6.1 are plotted in Fig. 6.2. The $1 : 2$ resonant orbit is the largest orbit and the spacecraft will be closest to the Saturn in this case.

PSS for different values of e in the range [0.0, 0.1], $q \in [0.98, 1.00]$ and $C \in [2.77, 2.85]$ are plotted for studying the effects of these parameters on resonant orbits. The numerical values of location (x), period (T), semi–major axis (a_s), the ratio $(a/a_s)^{3/2}$ and corresponding value of $p : p+s$ for $1 : 2, 2 : 3, 3 : 4, 4 : 5$ and $5 : 6$ resonant orbits for different values of C and q are given in Tables 6.1-6.9.

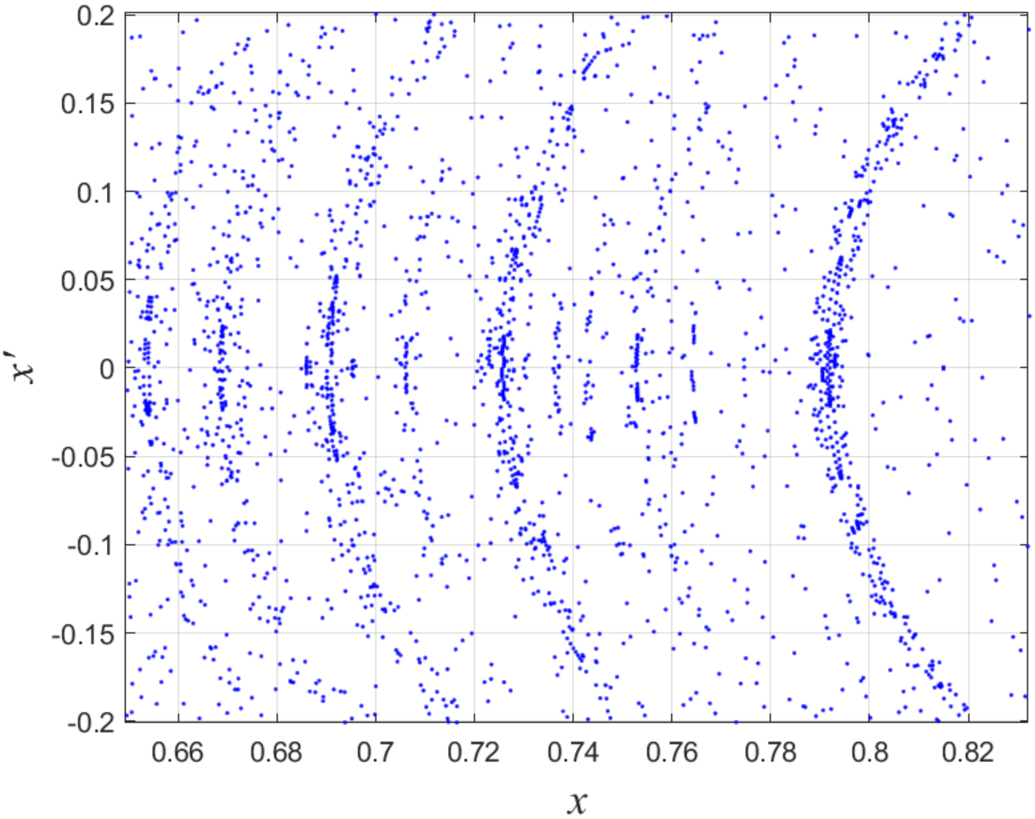


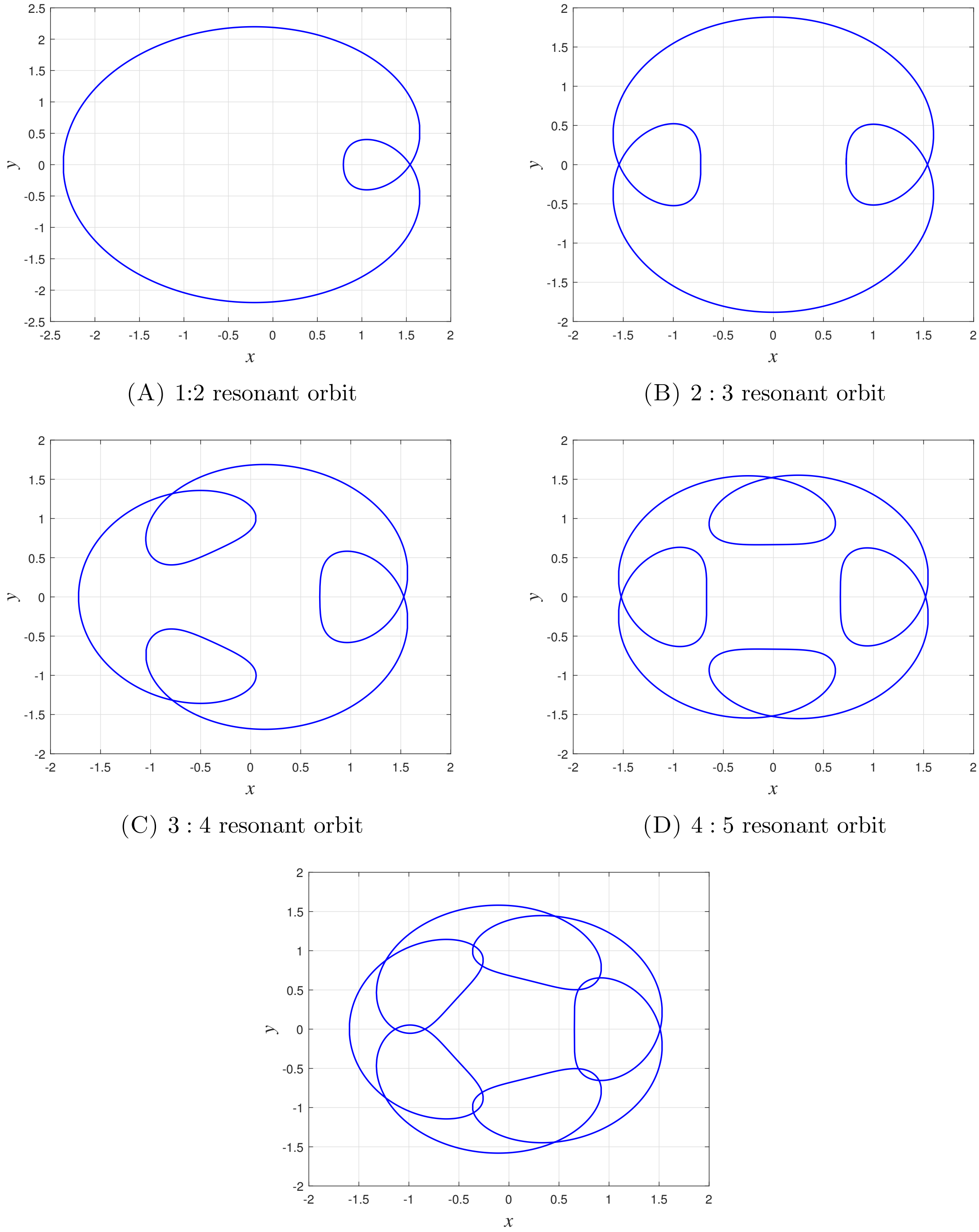
FIGURE 6.1: PSS for $e = 0.052$, $q = 0.99$ and $C = 2.8$

6.3.1 Effects of eccentricity of primaries' orbit

Seven different values of e in the range [0.0, 0.1] are taken into consideration for analyzing the variations in the parameters of the first order resonant orbits caused by variations in the eccentricity of the orbit of the primaries. Tables 6.1– 6.9 contain orbital parameters of periodic orbits for $e = 0, 0.03, 0.052$ and 0.09.

The variations in locations of $1 : 2$ resonant orbits for $C = 2.77$ corresponding to different values of $e \in [0.0, 0.1]$ are shown graphically in Fig. 6.3. Due to increase in the eccentricity of the orbit of the primaries, orbits shift towards the Sun. From Tables 6.1– 6.9, it can be observed that all $p : p + 1,\ p \in \{1, 2, 3, 4, 5\}$ resonant orbits shift towards the Sun.

Further, it can be observed that the periods of first order resonant orbits increase as the value of e increases. So, period of orbits is a monotonically increasing function of e. The periods of $1 : 2, 2 : 3, 3 : 4, 4 : 5$ and $5 : 6$ resonant orbits corresponding to $e = 0.00, 0.03, 0.052$ and 0.09 for different values of q and C are given in Tables 6.1–6.9.



(A) 1:2 resonant orbit

(B) 2 : 3 resonant orbit

(C) 3 : 4 resonant orbit

(D) 4 : 5 resonant orbit

(E) 5 : 6 resonant orbit

FIGURE 6.2: Periodic orbits corresponding to islands of Fig. 6.1

TABLE 6.1: Orbital parameters of spacecraft for $C = 2.77$ and $q = 1$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p + s$
	1	0.750937	12.5440	1.6048	0.5319	0.4975	1:2
	2	0.681265	18.8301	1.3188	0.4830	0.6678	2:3
0.000	3	0.645422	25.1167	1.2184	0.4700	0.7520	3:4
	4	0.623522	31.4025	1.1665	0.4652	0.8028	4:5
	5	0.608390	37.6931	1.1337	0.4631	0.8378	5:6
	1	0.748090	12.5586	1.5973	0.5314	0.5010	1:2
	2	0.678957	18.8490	1.3165	0.4840	0.6696	2:3
0.030	3	0.643300	25.1364	1.2172	0.4712	0.7532	3:4
	4	0.621503	31.4259	1.1656	0.4665	0.8037	4:5
	5	0.606413	37.7174	1.1331	0.4645	0.8385	5:6
	1	0.742370	12.5850	1.5826	0.5307	0.5080	1:2
	2	0.674287	18.8802	1.3120	0.4858	0.6730	2:3
0.052	3	0.639038	25.1748	1.2148	0.4737	0.75548	3:4
	4	0.617480	31.4740	1.1641	0.4693	0.8053	4:5
	5	0.602500	37.7713	1.1320	0.4675	0.8398	5:6
	1	0.725220	12.6716	1.5414	0.5293	0.5285	1:2
	2	0.660300	18.9734	1.2987	0.4913	0.6834	2:3
0.090	3	0.626270	25.2850	1.2076	0.4811	0.7621	3:4
	4	0.605340	31.6018	1.1594	0.4776	0.8102	4:5
	5	0.590700	37.9194	1.1285	0.4763	0.8437	5:6

Fig 6.4 shows the variation in the semi-major axis of $5 : 6$ resonant orbits for values of $e \in [0.0, 0.1]$. Here, $C = 2.85$ is considered. It can be observed from Fig. 6.4 that the semi-major axis of orbits decrease due to increase in the value of e. Also, the semi-major axis a_S is a non-linear monotonically decreasing function of e. Similar variations in a_s are observed for $1 : 2, 2 : 3, 3 : 4$ and $4 : 5$ resonant orbits (Tables 6.1–6.9). These values of a_s are obtained using (6.2).

In Fig. 6.5, variations in the eccentricity of resonant orbits for $C = 2.85$ are shown graphically. The values of eccentricity (e_s) are computed using equation (6.2). Fig. 6.5(A) shows that the value of e_s decreases as the value of e increases for $1 : 2$ resonant orbits while from Fig. 6.5(B), it can be observed that the value of e_s increases with the increase in the value of e for $4 : 5$ resonant orbits. From Tables 6.1–6.9, it can be observed that the eccentricity e_s is a monotonically increasing function of e for $2 : 3, 3 : 4$ and $5 : 6$ orbits as well.

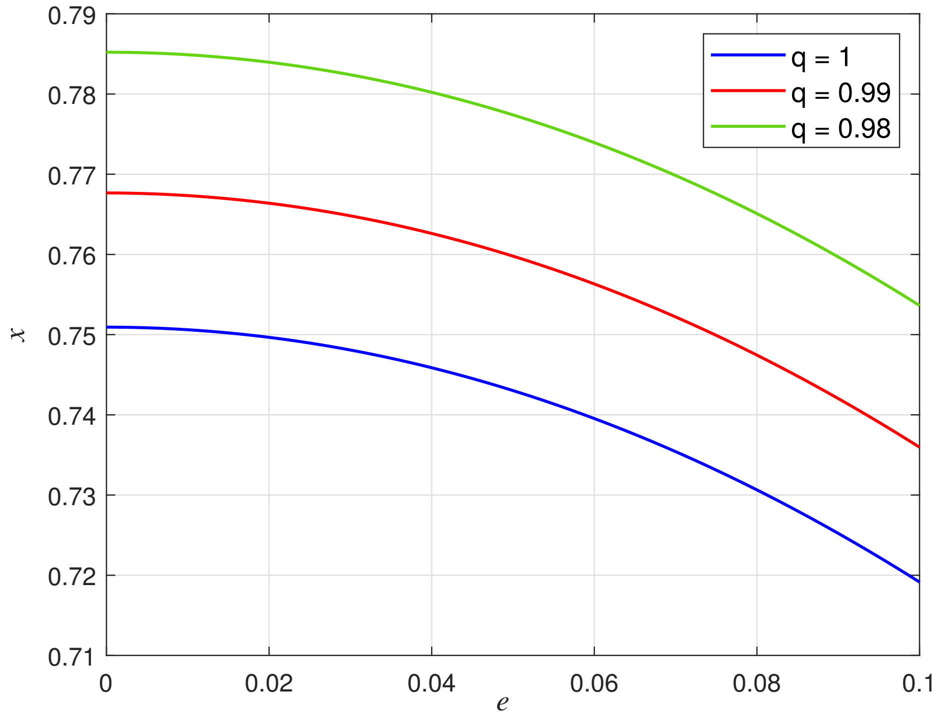


FIGURE 6.3: Variation in location of 1 : 2 resonant orbits against variation in e for $C = 2.77$

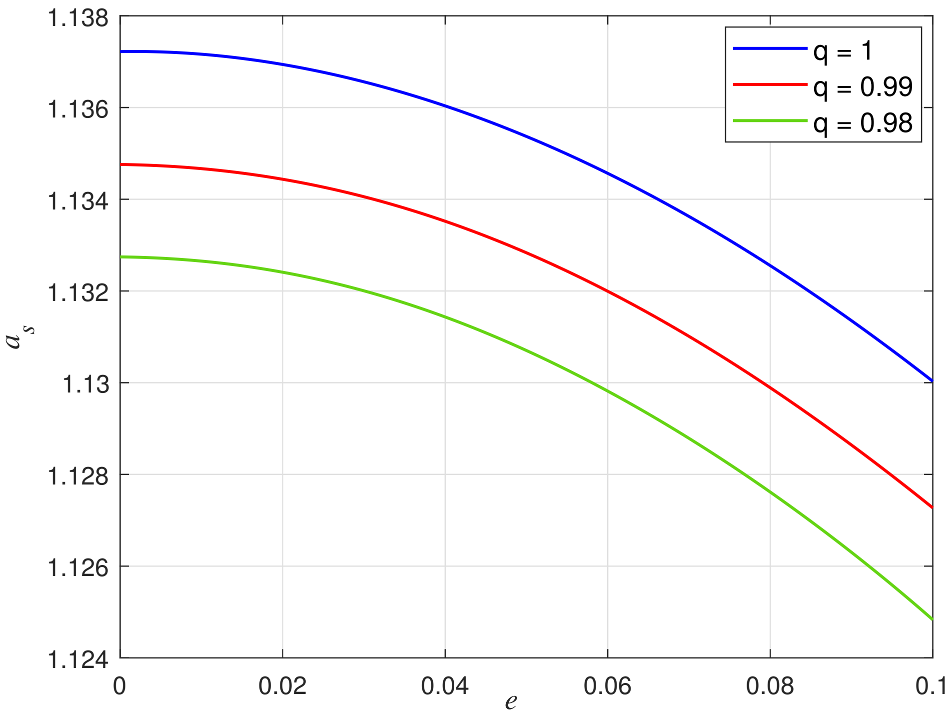
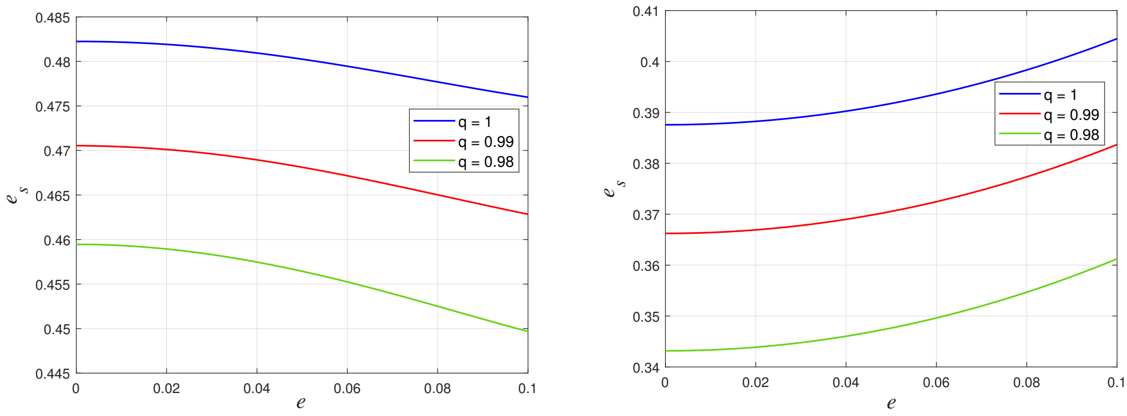


FIGURE 6.4: Variation in semi-major axis of 5 : 6 resonant orbits against variation in e for $C = 2.85$



(A) e_s vs e for $1:2$ resonant orbits (B) e_s vs e for $4:5$ resonant orbits

FIGURE 6.5: Variation in e_s against variation in e for $C = 2.85$

TABLE 6.2: Orbital parameters of spacecraft for $C = 2.77$ and $q = 0.99$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p + s$
	1	0.767668	12.5410	1.6017	0.5205	0.4989	1:2
	2	0.698900	18.8290	1.3154	0.4684	0.6704	2:3
0.000	3	0.663105	25.1128	1.2150	0.4540	0.7552	3:4
	4	0.641110	31.3971	1.1631	0.4485	0.8063	4:5
	5	0.625923	37.6832	1.1305	0.4461	0.8414	5:6
	1	0.764813	12.5524	1.5941	0.5200	0.5025	1:2
	2	0.696547	18.8455	1.3131	0.4693	0.6722	2:3
0.030	3	0.660949	25.1343	1.2138	0.4552	0.7563	3:4
	4	0.639070	31.4234	1.1623	0.4499	0.8071	4:5
	5	0.623951	37.7152	1.1300	0.4475	0.8420	5:6
	1	0.759145	12.5836	1.5795	0.5192	0.5095	1:2
	2	0.691850	18.8815	1.3086	0.4711	0.6757	2:3
0.052	3	0.656600	25.1728	1.2113	0.4577	0.7587	3:4
	4	0.634940	31.4700	1.1607	0.4527	0.8088	4:5
	5	0.619933	37.7679	1.1288	0.4505	0.8434	5:6
	1	0.742040	12.6662	1.5384	0.5174	0.5301	1:2
	2	0.677660	18.9706	1.2952	0.4766	0.6861	2:3
0.090	3	0.643584	25.2823	1.2041	0.4652	0.7655	3:4
	4	0.622545	31.5977	1.1559	0.4612	0.8138	4:5
	5	0.607882	37.9157	1.1252	0.4595	0.8474	5:6

6.3.2 Effects of radiation pressure

For studying the effects of radiation pressure on exterior resonant orbits, three different values of q in the interval $[0.98, 1.00]$ are considered. In Fig. 6.3, the curves in blue, red

TABLE 6.3: Orbital parameters of spacecraft for $C = 2.77$ and $q = 0.98$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p + s$
	1	0.785216	12.5333	1.5990	0.5087	0.5002	1:2
	2	0.717610	18.8256	1.3122	0.4529	0.6729	2:3
0.000	3	0.681959	25.1096	1.2117	0.4369	0.7583	3:4
	4	0.659934	31.3949	1.1599	0.4308	0.8097	4:5
	5	0.644740	37.6849	1.1275	0.4279	0.8448	5:6
	1	0.782402	12.5493	1.5915	0.5082	0.5037	1:2
	2	0.715217	18.8418	1.3098	0.4537	0.6747	2:3
0.030	3	0.679744	25.1297	1.2105	0.4382	0.7594	3:4
	4	0.657825	31.4200	1.1590	0.4322	0.8106	4:5
	5	0.642681	37.7126	1.1269	0.4294	0.8455	5:6
	1	0.776760	12.5812	1.5770	0.5072	0.5107	1:2
	2	0.710441	18.8756	1.3053	0.4555	0.6782	2:3
0.052	3	0.675306	25.1685	1.2080	0.4407	0.7618	3:4
	4	0.653592	31.4671	1.1574	0.4350	0.8123	4:5
	5	0.638551	37.7651	1.1256	0.4324	0.8469	5:6
	1	0.75969	12.6625	1.5357	0.5051	0.5315	1:2
	2	0.69607	18.9682	1.2919	0.4610	0.6888	2:3
0.090	3	0.662036	25.2825	1.2007	0.4484	0.7687	3:4
	4	0.640900	31.5979	1.1526	0.4437	0.8174	4:5
	5	0.626215	37.9176	1.1221	0.4416	0.8510	5:6

and green colours show the variation in location of $1 : 2$ resonant orbits corresponding to $q = 1.00, 0.99$ and 0.98, respectively. Due to increase in the solar radiation pressure, the first order resonant orbits move closer to the Saturn (Fig. 6.6 and Tables 6.1-6.9).

The changes in the values of a_s for $q = 1.00, 0.99$ and 0.98 are shown graphically in Fig. 6.4 with blue, red and green curves. Here, $C = 2.85$ is considered. It can be observed from Fig. 6.4 that a_S decreases due to increase in the solar radiation pressure. From Tables 6.1–6.9, similar conclusions can be derived for the variation in a_s for other first order resonant orbits.

Due to increase in solar radiation pressure, periods of $p : p + 1, p \in \{1, 2, , 3, 4, 5\}$ resonant orbits decrease for all values of e and C. Fig. 6.5 shows that as the value of q decreases or solar radiation pressure increases, the eccentricity e_s decreases for $1 : 2$ and $4 : 5$ resonant orbits. Further, from Tables 6.1-6.9, it can be observed that the value of e_s decreases due to increase in solar radiation pressure for $1 : 2, 2 : 3, 3 : 4, 4 : 5$ and $5 : 6$ resonant orbits.

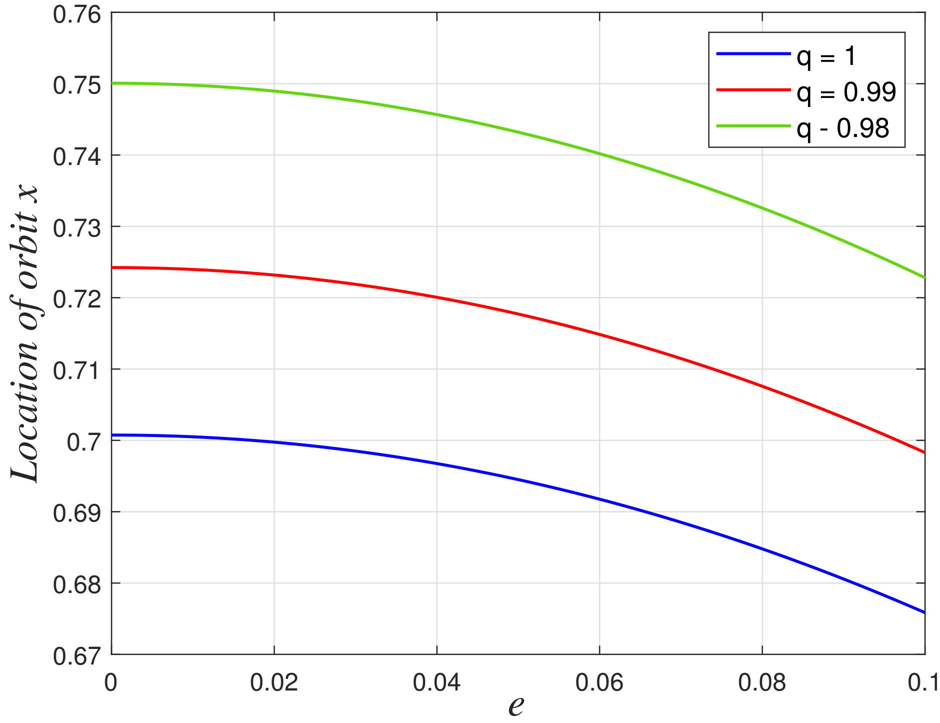


FIGURE 6.6: Variation in location of 5 : 6 resonant orbits against variation in e for $C = 2.85$

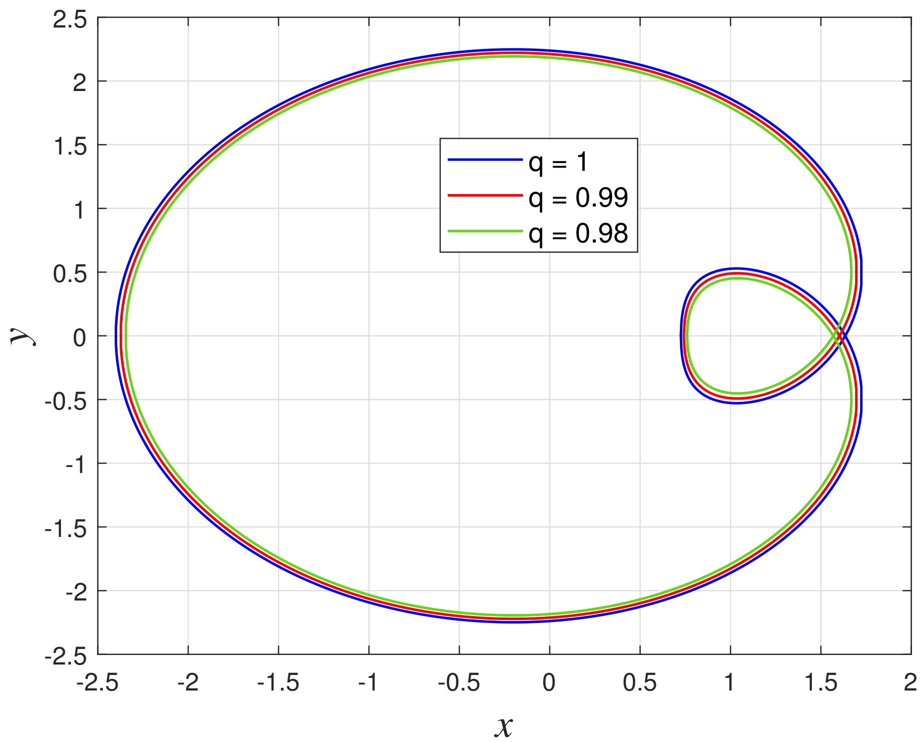
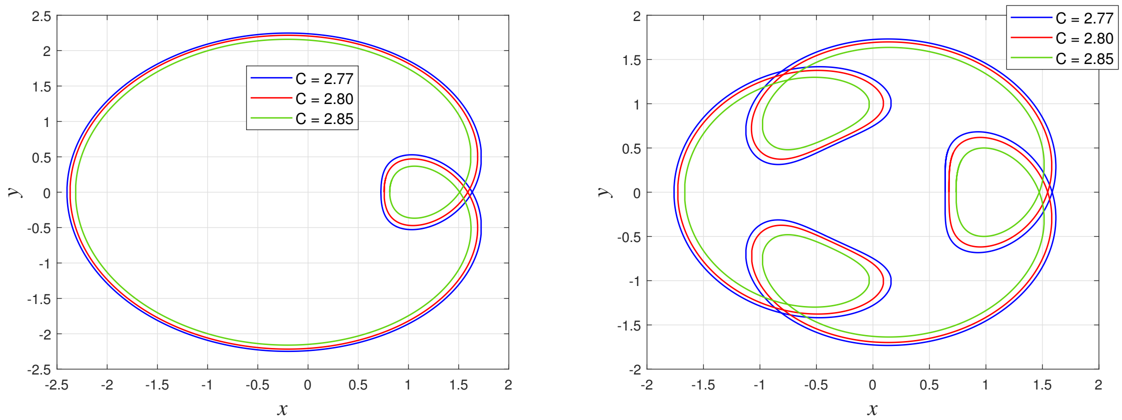


FIGURE 6.7: Variation in size of 1 : 2 resonant orbits due to variation in q for $C = 2.77$



(A) $1:2$ resonant orbits for $q = 1$ (B) $3:4$ resonant orbits for $q = 0.99$

FIGURE 6.8: Variation in size of orbits against variation in Jacobi constant

TABLE 6.4: Orbital parameters of spacecraft for $C = 2.8$ and $q = 1$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p:p+s$
0.000	1	0.782343	12.5404	1.6088	0.5135	0.4957	1:2
	2	0.713850	18.8270	1.3205	0.4591	0.6666	2:3
	3	0.677945	25.1119	1.2195	0.4438	0.7510	3:4
	4	0.655830	31.3994	1.1674	0.4379	0.8019	4:5
	5	0.6405591	37.6869	1.1348	0.4353	0.8367	5:6
0.030	1	0.779491	12.5535	1.6011	0.5130	0.4992	1:2
	2	0.711468	18.8442	1.3181	0.4600	0.6683	2:3
	3	0.675750	25.1336	1.2183	0.4451	0.7521	3:4
	4	0.653720	31.4222	1.1666	0.4393	0.8027	4:5
	5	0.638506	37.7133	1.1342	0.4368	0.8374	5:6
0.052	1	0.773789	12.5822	1.5864	0.5120	0.5062	1:2
	2	0.706692	18.8777	1.3136	0.4618	0.6718	2:3
	3	0.671320	25.1712	1.2158	0.4476	0.7545	3:4
	4	0.649503	31.4683	1.1649	0.4422	0.8044	4:5
	5	0.634419	37.7685	1.1330	0.4398	0.8387	5:6
0.090	1	0.756622	12.6652	1.5450	0.5101	0.5267	1:2
	2	0.692338	18.9712	1.3002	0.4673	0.6822	2:3
	3	0.658078	25.2824	1.2085	0.4552	0.7613	3:4
	4	0.636882	31.5997	1.1601	0.4507	0.8094	4:5
	5	0.622121	37.9165	1.1294	0.4489	0.8427	5:6

The radiation pressure also affects the size of periodic orbits. In Fig. 6.7, $1:2$ resonant orbits for $C = 2.77$ are plotted in blue, red and green colours corresponding to $q = 1.00, 0.99$ and 0.98, respectively. The orbits shrink due to increase in solar radiation

TABLE 6.5: Orbital parameters of spacecraft for $C = 2.8$ and $q = 0.99$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p + s$
	1	0.800241	12.5335	1.6066	0.5017	0.4967	1:2
	2	0.733060	18.8233	1.3174	0.4433	0.6689	2:3
0.000	3	0.697350	25.1068	1.2164	0.4264	0.7539	3:4
	4	0.675223	31.3951	1.1643	0.4198	0.8051	4:5
	5	0.659930	37.6830	1.1318	0.4166	0.8400	5:6
	1	0.797426	12.5503	0.5011	1.5991	0.5002	1:2
	2	0.73063	18.8389	1.3151	0.4442	0.6707	2:3
0.030	3	0.695100	25.1283	1.2151	0.4277	0.7551	3:4
	4	0.673067	31.4202	1.1634	0.4212	0.8060	4:5
	5	0.657800	37.7070	1.1311	0.4182	0.8408	5:6
	1	0.791753	12.5800	1.5843	0.5001	0.5072	1:2
	2	0.725782	18.8725	1.3105	0.4459	0.6742	2:3
0.052	3	0.690574	25.1671	1.2126	0.4302	0.7575	3:4
	4	0.668722	31.4646	1.1617	0.4241	0.8078	4:5
	5	0.653602	37.7639	1.1299	0.4213	0.8421	5:6
	1	0.774646	12.6645	1.5428	0.4977	0.5278	1:2
	2	0.711228	18.9687	1.2970	0.4514	0.6847	2:3
0.090	3	0.677041	25.2802	1.2052	0.4380	0.7644	3:4
	4	0.655764	31.5967	1.1568	0.4328	0.8129	4:5
	5	0.640965	37.9135	1.1262	0.4306	0.8463	5:6

pressure. Also, the loop of these orbits shrink. The orbits in Fig. 6.7 are plotted by considering $e = 0.09$. In Pathak et al. (2016), the authors have analyzed the effects of solar radiation pressure on exterior resonant periodic orbits in the photogravitational Sun-Earth and Sun-Mars systems. Above results agree with their conclusions. So, the effect of solar radiation pressure on the parameters of resonant orbits are similar in CRTBP and ERTBP.

6.3.3 Effects of Jacobi constant

The value of Jacobi constant affect all parameters of resonant orbits. The data of parameters of periodic orbits for $q = 1$ corresponding to $C = 2.77, 2.80$ and 2.85 are given in Tables 6.1, 6.4 and 6.7, respectively. These values of Jacobi constant are selected randomly satisfying the condition $C \leqslant C_M$. In Tables 6.2, 6.5 and 6.8, the values of parameters of resonant orbits corresponding to $C = 2.77, 2.80$ and 2.85 are given for $q = 0.99$, respectively, and in Tables 6.3, 6.6 and 6.9, the parameters for

TABLE 6.6: Orbital parameters of spacecraft for $C = 2.8$ and $q = 0.98$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p+s$
	1	0.819109	12.5264	1.6056	0.4896	0.4972	1:2
	2	0.753560	18.8167	1.3147	0.4266	0.6710	2:3
0.000	3	0.718193	25.1022	1.2135	0.4079	0.7567	3:4
	4	0.696103	31.3911	1.1613	0.4003	0.8082	4:5
	5	0.680785	37.6757	1.1289	0.3967	0.8433	5:6
	1	0.816301	12.5431	1.5979	0.48899	0.5007	1:2
	2	0.751101	18.8345	1.3124	0.4275	0.6728	2:3
0.030	3	0.715877	25.1232	1.2122	0.4092	0.7579	3:4
	4	0.693858	31.4133	1.1605	0.4018	0.8092	4:5
	5	0.678622	37.7065	1.1283	0.3983	0.8439	5:6
	1	0.810662	12.5735	1.5831	0.4877	0.5078	1:2
	2	0.746173	18.8689	1.3077	0.4292	0.6763	2:3
0.052	3	0.711251	25.1654	1.2096	0.4118	0.7603	3:4
	4	0.689390	31.4602	1.1587	0.4048	0.8109	4:5
	5	0.674259	37.7603	1.1270	0.4014	0.8454	5:6
	1	0.793610	12.6577	1.5412	0.4848	0.5286	1:2
	2	0.731362	18.9646	1.2942	0.4346	0.6870	2:3
0.090	3	0.697361	25.2787	1.2022	0.4196	0.7674	3:4
	4	0.676054	31.5962	1.1537	0.4137	0.8162	4:5
	5	0.661221	37.9126	1.1232	0.4110	0.8497	5:6

$q = 0.98$ corresponding to $C = 2.77, 2.80$ and 2.85 are given.

It can be observed from Tables 6.1, 6.4 and 6.7 that as the Jacobi constant increases, resonant orbits shift towards the second primary, the Saturn. Similar change in locations of orbits is observed for $q = 0.99$ and 0.98. The period and eccentricity (e_s) decrease with the increase in Jacobi constant for all values of q and e whereas the semi-major axis increases.

For $C = 2.77, 2.80$ and 2.85, the periodic orbits are plotted in Fig 6.8 with blue, red and green colours, respectively. In Fig. 6.8(A), $1 : 2$ resonant orbits having one loop are plotted for $q = 1$ and in Fig. 6.8(B), $3 : 4$ resonant orbits having three loops are plotted for $q = 0.99$. From Fig. 6.8(A) and Fig. 6.8(B), it can be observed that the orbits as well as loops contract due to increase in the value of Jacobi constant. Further, these results agree with the results of Pathak et al. (2016) which shows that the effects

TABLE 6.7: Orbital parameters of spacecraft for $C = 2.85$ and $q = 1$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p+s$
	1	0.838670	12.5215	1.6203	0.4822	0.4904	1:2
	2	0.773783	18.8140	1.3251	0.4158	0.6632	2:3
0.000	3	0.738392	25.0991	1.2226	0.3958	0.7483	3:4
	4	0.716156	31.3858	1.1698	0.3876	0.7995	4:5
	5	0.700742	37.6732	1.1372	0.3836	0.8341	5:6
	1	0.83586	12.5398	1.6126	0.4814	0.4939	1:2
	2	0.771282	18.8333	1.3227	0.4166	0.6649	2:3
0.030	3	0.736000	25.1183	1.2212	0.3971	0.7495	3:4
	4	0.713852	31.4100	1.1689	0.3891	0.8003	4:5
	5	0.698499	37.7028	1.1365	0.3852	0.8348	5:6
	1	0.830191	12.5718	1.5974	0.4801	0.5010	1:2
	2	0.766230	18.8647	1.3179	0.4184	0.6686	2:3
0.052	3	0.731260	25.1630	1.2187	0.3997	0.7519	3:4
	4	0.709246	31.4572	1.1672	0.3921	0.8022	4:5
	5	0.693988	37.7560	1.1352	0.3884	0.8363	5:6
	1	0.813015	12.6530	1.5545	0.4768	0.5219	1:2
	2	0.751130	18.9632	1.3041	0.4238	0.6792	2:3
0.090	3	0.717001	25.2778	1.2110	0.4077	0.7590	3:4
	4	0.695501	31.5931	1.1620	0.4012	0.8075	4:5
	5	0.680562	37.9131	1.1314	0.3982	0.8406	5:6

of Jacobi constant on parameters of exterior resonant orbits are invariant under the eccentricity of the orbit of the primaries (e) and mass factor (μ) of the system.

6.4 Analysis of resonance

The analysis of different families of first order exterior resonant orbits in the Sun-Saturn system shows that all these orbits possess inner loops. In dimensionless system, the semi-major axis of the Saturn is $a = 1.007684$. Using equations (6.2), the semi-major axis of orbit, a_s, is obtained. From equation (6.3), the approximate ratio $p : p+s$ is calculated using the values of a and a_s.

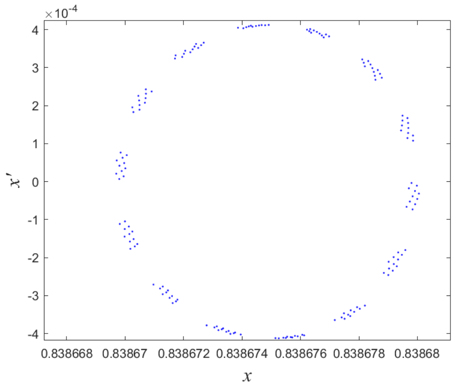
In Tables 6.1-6.9, the column $(a/a_s)^{3/2}$ contains the approximate value of the ration $p : p+s$ for various values of e, q and C. The ratio $p : p+s$ was obtained for 63 different families of periodic orbits. Analysis of each family shows that the value of

TABLE 6.8: Orbital parameters of spacecraft for $C = 2.85$ and $q = 0.99$

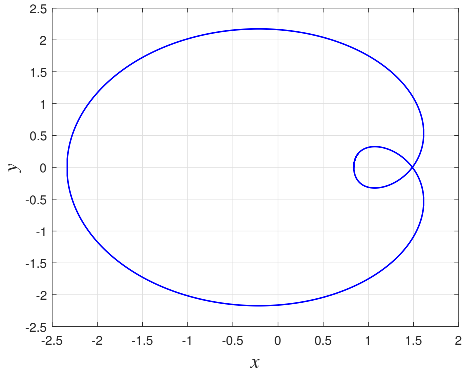
e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p + s$
	1	0.858968	12.5140	1.6229	0.4705	0.4893	1:2
	2	0.796407	18.8040	1.3236	0.3981	0.6643	2:3
0.000	3	0.761650	25.0897	1.2204	0.3757	0.7503	3:4
	4	0.739581	31.3763	1.1674	0.3662	0.8020	4:5
	5	0.724224	37.6652	1.1348	0.3615	0.8368	5:6
	1	0.856152	12.5283	1.6147	0.4696	0.4930	1:2
	2	0.793830	18.8201	1.3211	0.3989	0.6662	2:3
0.030	3	0.759201	25.1137	1.2190	0.3770	0.7515	3:4
	4	0.737194	31.4030	1.1665	0.3678	0.8029	4:5
	5	0.721870	37.6934	1.1340	0.3632	0.8376	5:6
	1	0.850517	12.5612	1.5991	0.4679	0.5002	1:2
	2	0.788712	18.8591	1.3163	0.4006	0.6698	2:3
0.052	3	0.754256	25.1519	1.2163	0.3796	0.7541	3:4
	4	0.732400	31.4505	1.1647	0.3709	0.8048	4:5
	5	0.717181	37.7507	1.1327	0.3666	0.8391	5:6
	1	0.833433	12.6472	1.5552	0.4639	0.5216	1:2
	2	0.773278	18.9570	1.3022	0.4060	0.6807	2:3
0.090	3	0.739548	25.2714	1.2085	0.3878	0.7614	3:4
	4	0.718130	31.5895	1.1594	0.3803	0.8103	4:5
	5	0.703185	37.9082	1.1286	0.3767	0.8436	5:6

ratio $p : p + s$ is of the form $p : p + 1$, $p \in \{1, 2, 3, 4, 5\}$, which shows the first order resonance. Also, the existence of internal loops shows the exterior resonance. Further, the PSS corresponding to each orbit contains only one island which confirms the first order resonance.

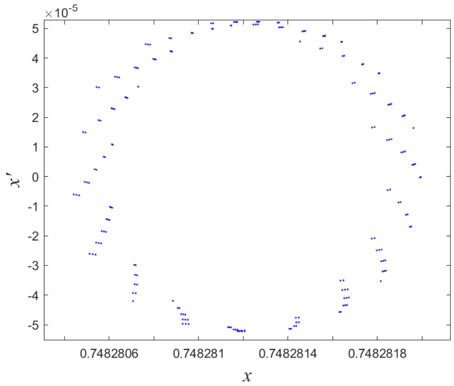
In Figs. 6.9, the PSS and their corresponding orbits for $C = 2.85$ are given. In Fig. 6.9(A), the PSS at $x_0 = 0.83867$ for $q = 1$ and $e = 0$ for $1 : 2$ resonant orbit is given and the corresponding orbit is given in Fig. 6.9(B). In Fig. 6.9(C), the PSS at $x_0 = 0.748282$ for $q = 0.99$ and $e = 0.07$ is given and the corresponding $3 : 4$ resonant orbit is given in Fig. 6.9(D). In Fig. 6.9(E) and Fig. 6.9(F), the PSS and corresponding orbit at $x_0 = 0.791623$ for $q = 0.98$ and $e = 0.1$ are given, respectively.



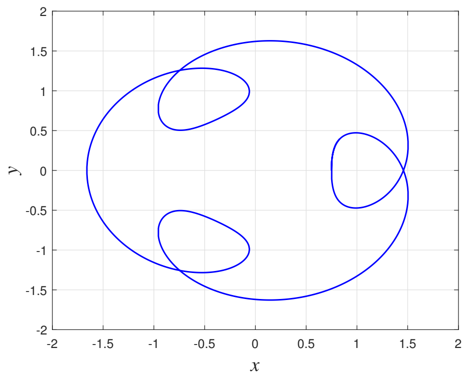
(A) PSS at $x_0 = 0.83867$ for $1:2$ resonant orbit with $q = 1, e = 0$



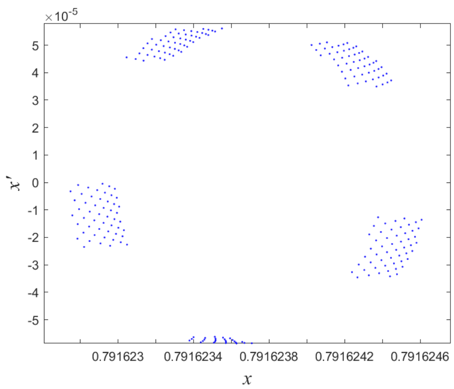
(B) Orbit related to PSS in Fig. 6.9(A)



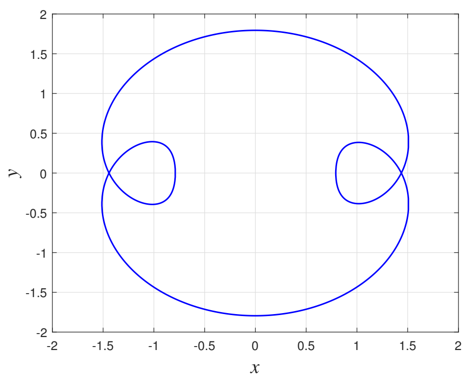
(C) PSS at $x_0 = 0.748282$ for $3:4$ resonant orbit with $q = 0.99, e = 0.07$



(D) Orbit related to PSS in Fig. 6.9(C)



(E) PSS at $x_0 = 0.791623$ for $2:3$ resonant orbit with $q = 0.98, e = 0.1$



(F) Orbit related to PSS in Fig. 6.9(E)

FIGURE 6.9: Exterior first order resonant PSS and orbits for $C = 2.85$

TABLE 6.9: Orbital parameters of spacecraft for $C = 2.85$ and $q = 0.98$

e_p	No. of loops	x	T	a_s	e_s	$(a/a_s)^{3/2}$	$p : p+s$
	1	0.880481	12.4963	1.6294	0.4595	0.4863	1:2
	2	0.820918	18.7912	1.3235	0.3795	0.6643	2:3
0.000	3	0.787087	25.0786	1.2190	0.3541	0.7516	3:4
	4	0.765323	31.3657	1.1656	0.3432	0.8038	4:5
	5	0.750065	37.6515	1.1327	0.3376	0.8391	5:6
	1	0.877691	12.5128	1.6208	0.4583	0.4902	1:2
	2	0.818280	18.8091	1.3209	0.3803	0.6663	2:3
0.030	3	0.784517	25.0999	1.2176	0.3554	0.7529	3:4
	4	0.762810	31.3919	1.1646	0.3448	0.8048	4:5
	5	0.747589	37.6817	1.1320	0.3393	0.8399	5:6
	1	0.87211	12.5515	1.6043	0.4562	0.4978	1:2
	2	0.813000	18.8440	1.3158	0.3819	0.6702	2:3
0.052	3	0.779400	25.1447	1.2148	0.3582	0.7555	3:4
	4	0.757787	31.4418	1.1627	0.3480	0.8068	4:5
	5	0.74264	37.7387	1.1305	0.3429	0.8415	5:6
	1	0.855092	12.6360	1.5583	0.4511	0.5200	1:2
	2	0.797210	18.9506	1.3012	0.3871	0.6815	2:3
0.090	3	0.764100	25.2649	1.2066	0.3665	0.7632	3:4
	4	0.742840	31.5822	1.1571	0.3578	0.8127	4:5
	5	0.727950	37.9025	1.1263	0.3534	0.8463	5:6

6.5 Conclusions

In this chapter, the first order exterior resonant orbits in the photogravitational Sun-Saturn ERTBP are computed using the technique of PSS and the effects of eccentricity of orbit of the primaries, solar radiation pressure and Jacobi constant on parameters of these orbits are analyzed.

The analysis of PSS shows that the first order resonant orbits exist with resonance ratio $p : p + 1,\ p \in \{1, 2, 3, 4, 5\}$ for different values of eccentricity of orbit of the primaries, solar radiation pressure and Jacobi constant. It is observed that the orbits shift towards the Sun and the semi-major axis of these orbits decrease as the eccentricity of the orbit of the primaries increase. Further, the eccentricity of the $1 : 2$ resonant orbits decrease while the reverse effect is observed for the remaining $p : p+1 (p \in \{2, 3, 4, 5\})$ resonant orbits due to increase in the value of eccentricity of primaries' orbit. Also, the periods of these orbits increase with the increase in primaries' eccentricity.

Due to solar radiation pressure, orbits proceed towards the Saturn and shrink in size. This perturbation force decreases periods, semi–major axes and eccentricities of resonant orbits. An increase in the value of Jacobi constant also shift orbits towards the Saturn and they contract in size. Further, period and eccentricity increase while semi–major axis decreases for all resonant orbits.