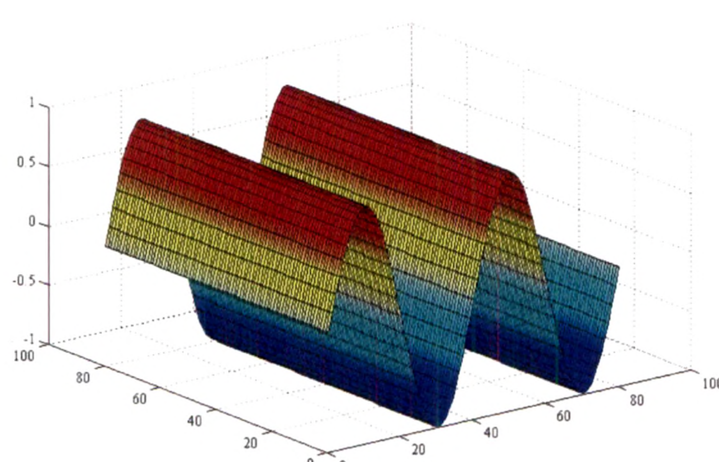
Chapter 3

PREDICTION OF WEEKLY SOIL TEMPERATURES BY ARTIFICIAL NEURAL NETWORKS AND HARMONIC ANALYSIS



3.1 INTRODUCTION

To predict the soil temperatures (ST) at different depths, it is required to study its annual cycle by computing their mean values for each month or a season. Plotting of the weather parameters, here soil temperatures (ST) against time point can explain its periodical variations (Fig. 3.1 and Fig. 3.2). Here time point is the standard week (SW) numbers from 1 to 52 (Ghadeker [39]). List of these weeks according to dates are given in the Table A-3.1 in the Appendix. Figure 3.1 and 3.2 show the ST at morning (MN) and afternoon (AN), respectively for the selected three depths namely, 5 cm, 10 cm and 20 cm.

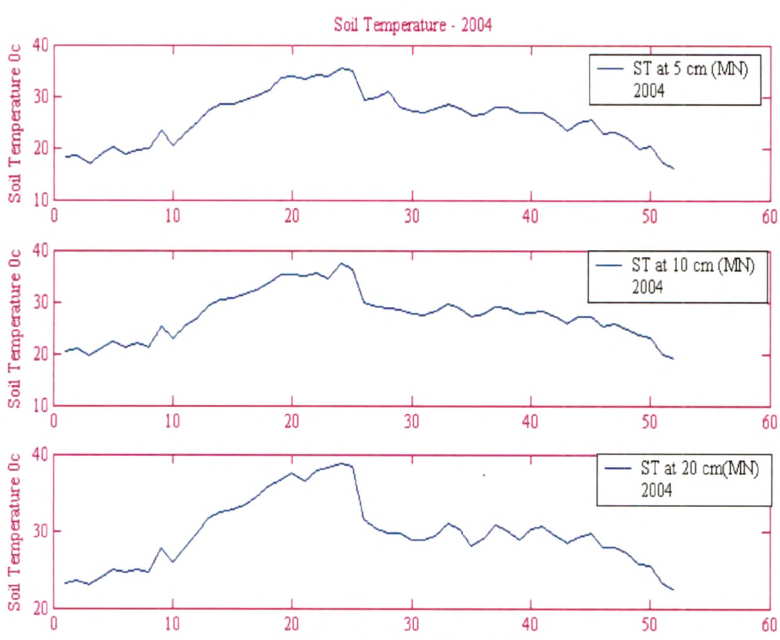


Fig: 3.1

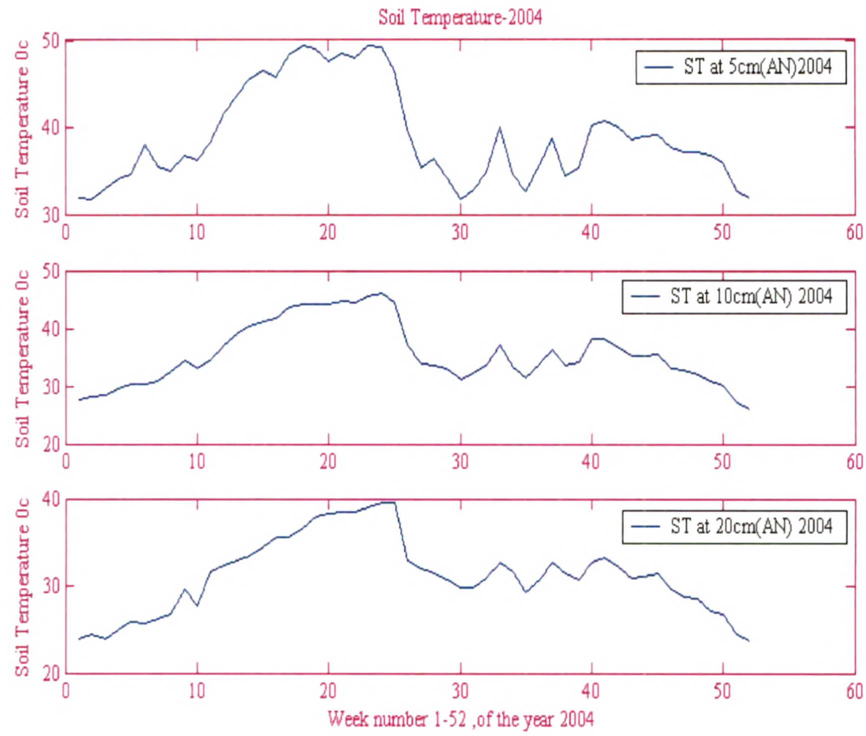


Fig: 3.2

Annual range of the soil temperature at the depth 5 cm is the result of long-term thermal equilibrium between the soil and the atmosphere. During the warm season, soil temperature decreases with depth and the associated downward heat flux builds up the soil's heat store. In the season of winter the gradient is reversed and the heat store is gradually depleted. The spring and autumn are transitional periods when the ST gradients reverse the sign.

These reversals are important biological triggers to soil pathogens, soil born insects and many other chemical activities. This shows the importance of soil temperature and so its estimation in Agriculture. Bocock *et al.* [16] have estimated the soil temperatures from air temperatures and other climatic variables.

3.2 PROBLEM FORMULATION

In this chapter we predict the soil temperature at three depths namely, 5 cm, 10 cm and 20 cm for morning and afternoon hours by using Artificial Neural Network algorithm and harmonic analysis techniques. Anand station is selected for a case study. In ANN we prove a convergence theorem for McChulloch type network. The proof is based on fixed-point theorem approach from functional analysis.

If we consider random values of weather parameters like soil temperature at fixed time interval like an hour, a week or a year then that parameter can be considered as a discrete random variable. Soil temperature can be considered as time dependent random variable But soil temperature is also highly correlated to Air Temperature (AT), Wind Speed (WS), Relative Humidity (RH), Rainfall (RF) etc.

In this chapter prediction of soil temperature is carried out by two methods. These are

(I) Prediction of soil temperature by Artificial Neural Network with

(a) Three inputs,

(b) Two inputs,

(c) One input .

and

(II) Prediction of soil temperature by Harmonic Analysis (HA)

3.3 (I) (a) PREDICTION OF SOIL TEMPRATURE BY ANN USING 3-INPUTS AND 3-OUTPUTS.

Meteorological parameters are always interlinked or highly affected by other parameters like relative humidity, wind speed air temperature etc. We consider a general set up where we have m weather parameters, $y_1, y_2, y_3 \ldots y_m$ and each one is affected by n factors $x_1, x_2 \ldots x_n$. That is, we have the following functional relationship between x_i's and y_i's:

$$\left.\begin{array}{l} y_1 = f_1(x_1, x_2, x_3 \ldots x_n) \\ y_2 = f_2(x_1, x_2, x_3 \ldots x_n) \\ y_3 = f_3(x_1, x_2, x_3 \ldots x_n) \\ \ldots \ldots\ldots \ldots \\ y_m = f_m(x_1, x_2, x_3 \ldots x_n) \end{array}\right\} \quad (3.1)$$

where, the functions f_i's are unknown Now the prediction is possible only if we identify these functions There are various techniques to approximate these functions in which we assume some forms to the functions. Commonly used method is Multiple Regression technique (Walker [164]; Murphy [110]).

We consider a single layer neural network having McCulloch- Pitts Type neurons (McCulloch *et al.* [105]) and use the generalized Widrow-Hoff (Widrow *et al.* [169]) algorithm to train the network. We give conditions on the learning rate and the transfer functions, which will guarantee the convergence of the generalized Widrow-Hoff algorithm To prove the convergence we make use of

Fixed- point theorem. Our convergence theorem generalizes an earlier convergence theorem proved by Hui *et al.* [67].

3.3.1 METHOD: (I)

MULTIPLE REGRESSION TECHNIQUE

Let us assume a linear relationship

$$f_i(x_1, x_2, x_3) = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3.$$

where a_{ij}'s are unknown constants to be determined. For each equation if we have k-samples then (3.1) becomes

$$\left.\begin{array}{l} y_1^{\,j} = a_{11}x_1^{\,j} + a_{12}x_2^{\,j} + . \quad + a_{1n}x_n^{\,j} \\ y_2^{\,j} = a_{21}x_1^{\,j} + a_{22}x_2^{\,j} + \ldots \ + a_{2n}x_n^{\,j} \\ y_3^{\,j} = a_{31}x_1^{\,j} + a_{32}x_2^{\,j} + \ldots.. + a_{3n}x_n^{\,j} \\ \quad \ldots \qquad .. \qquad \ldots \\ y_m^{\,j} = a_{m1}x_1^{\,j} + a_{m2}x_2^{\,j} + . \ . + a_{mn}x_n^{\,j} \\ \\ where\ , \quad j = 1,2,3. \quad k \end{array}\right\} \tag{3.2}$$

where x_1^j's are assumed to be 1 for j = 1,2, ...k. These are mk equations in mn unknowns a_{ij}, $1 \le i \le m, 1 \le j \le n$. Let us define the following matrices and vectors.

$$X = \begin{bmatrix} 1 & x_2^1 & \ldots & x_n^1 \\ 1 & x_2^2 & . & x_n^2 \\ \ldots & \ldots & .. & .. \\ 1 & x_2^k & \ldots & x_n^k \end{bmatrix}; \quad y_i = \begin{bmatrix} y_i^{\,1} \\ y_i^{\,2} \\ . \\ . \\ y_i^{\,k} \end{bmatrix}; \quad A = \begin{bmatrix} a_{11} & a_{21} & . & a_{m1} \\ a_{12} & a_{22} & . & a_{m2} \\ . & . & . & . \\ a_{1n} & a_{2n} & . & a_{mn} \end{bmatrix}; \quad Y = [y_1, y_2, .., y_m]$$

The mk equations in (3.2) can be represented in the following matrix equation

$$XA = Y \tag{3 3}$$

We want to find the matrix A satisfying the above matrix equation. In general X is a singular rectangular matrix. Therefore, we find a least square solution as described below:

To find A which minimizes the error

$$E = \frac{1}{2} || XA-Y ||^2$$

We write E in the inner-product form

$$E=\frac{1}{2}\langle XA-Y, XA-Y\rangle.$$

The necessary condition for minimum is

$$\frac{\partial E}{\partial A} = \frac{1}{2}(\langle XA-Y, X\rangle + \langle X, XA-Y\rangle) = 0.$$

Since, the matrices and vectors are real we have,

$$< XA-Y, X >= 0$$

This implies that $\langle X^*(XA-Y), I\rangle = 0$

$$(X^*X)A - X^*Y = 0$$

$$A = (X^*X)^{-1}X^*Y$$

That is, $A = X^+Y$

where, $X^+ = (X^*X)^{-1}X^*$, the pseudo - inverse of X.

For any given $x = [x_1, x_2, \ldots\ldots x_n]$ the prediction is done by the equation

$$y = Ax \quad (3\ 4)$$

3.3.2 DATA

Here, three inputs Relative Humidity (RH), Wind Speed (WS) and Air Temperature (AT) are considered to predict the soil temperature (ST) at surface of the earth. These data for Anand station are obtained from the Land Surface Process Experiment held for Sabarmati River Basin in the year 1997 (LASPEX-1997).

3.3.3 RESULTS AND DISCUSSION

Figure 3.3 is the plotting of standard week number and predicted soil temperature by Multiple Regression technique and actual soil temperature. Actual soil temperature and predicted soil temperature have non-significant difference. During the rainy season, that is in the standard week of 30th to 40th (in the month of August) this difference is large but it is non significant.

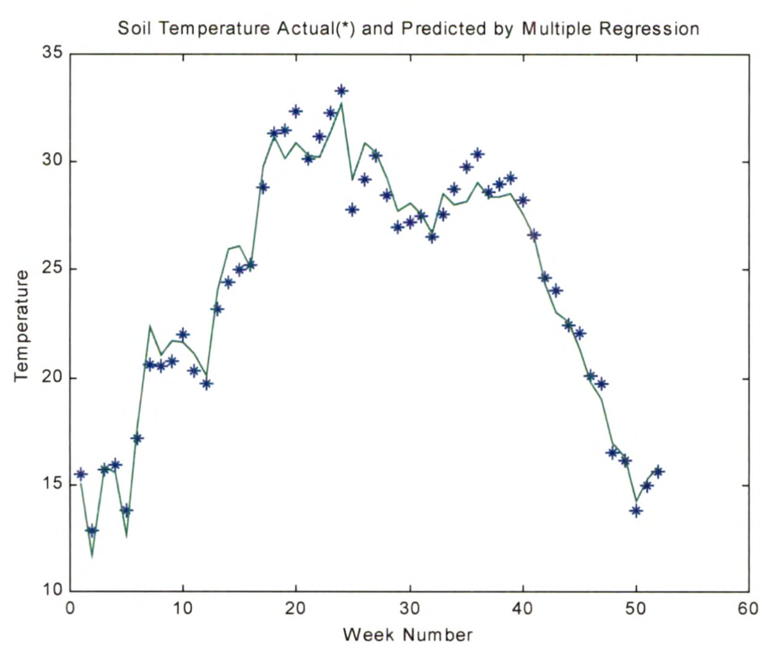


Fig: 3.3

3.4 METHOD II
NEURAL NETWORK APPROACH: (MCCULLOCH TYPE NEURONS)

It is observed that if the number of parameters and number of observations are large, the regression method will be very difficult due to involvement of inverse of large matrices, whereas for ANN, increasing of parameters may not make the method impossible, though it may take long time to train a network.

For practical problems, X is a very large matrix and the Least Square Technique to obtained the coefficients is tedious due to the requirement of inversion (pseudo-inverse is defined in chapter 2) of large rectangular matrix. To overcome this difficulty we now consider neural network algorithm to get the functional relationship between the set of input data and output data.

We consider a neural network having McCulloch Pitts type neurons, each having 3 inputs value namely, Relative Humidity (RH), Wind Speed (WS) and Air Temperature (AT). Architecture of the ANN is shown in the Figure 3.4.

- **STRUCTURE OF THE NETWORKS**

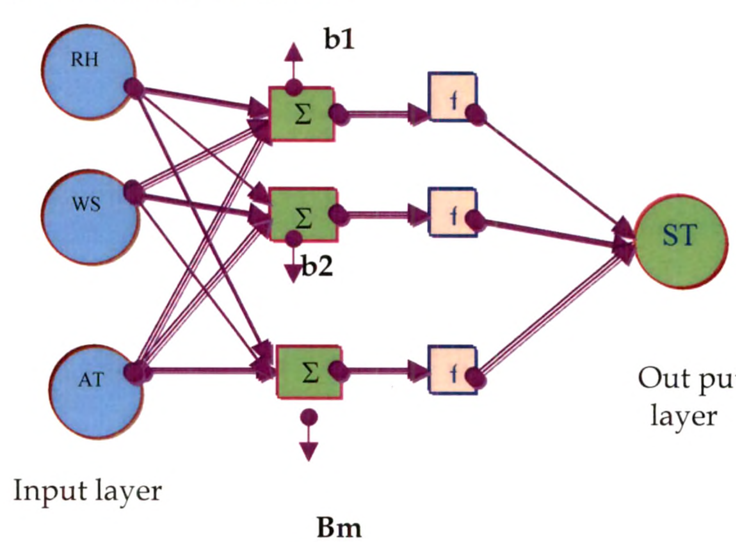


Fig: 3.4

Each element of the input vector x is connected to each neuron input through the weight matrix W=(w_{ij}) where w_{ij} denotes strength of the connection from the j[th] input to the ith neuron. The i[th] neuron has a summer that gathers its weighted input $w_{ij}x_j$ and the bias b_i to form its net input net_i given by

$$net_i = \sum_j w_{ij}x_j + b_i$$

Finally, this scalar input net_i is passed through the transfer function f_i of the i[th] neuron to obtain its output

$$y_i = f_i(net_i)$$

For notational convention, we define the following vectors and matrices:

$$W = \begin{bmatrix} w_{11} & w_{12} & . & w_{1n} \\ w_{21} & w_{22} & . \; . & w_{2n} \\ . & . & . \; . & . \\ w_{(m-1)1} & w_{(m-1)2} & . \; . & w_{(m-1)n} \\ w_{m1} & w_{m2} & . & w_{mn} \end{bmatrix}; \quad y = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_m \end{pmatrix}; \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_m \end{pmatrix}; x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{pmatrix}$$

Let us define a nonlinear function $F = \Re^m \to \Re^m$ by

$$F(net_1, net_2, net_3, ..., net_m) = \begin{pmatrix} f_1(net_1) \\ f_2(net_2) \\ . \\ \\ f_m(net_m) \end{pmatrix}$$

Using these notations the output vector y of the network can be written as

$$y = F(Wx + b)$$

3.4.1 LEARNING ALGORITHMS

The main task in designing a neural network for a given set of input and out put data is determination of a suitable weight matrix W. We start with a random weight matrix W. For each input vector we can calculate the network's output y using this random weight matrix W. The difference between the calculated output and the actual output (target vector) is known as the error. Training a network means finding out suitable values for the network weights w_{ij} and biases b_i such that sum of the squares of the errors is minimized

We note that if $f_i(net_i) = net_i$, the network becomes a linear network known as ADALINE (Widrow,[170]). In this case this network is similar to the regression equation (3.4) where $w_{ij} = a_{ij}$, $b_i = a_{i1}$. To train a linear network, Widrow and Hoff gave an algorithm known as Widrow- Hoff Algorithm. In that, we minimize the sum – squared error (sse) given by

$$sse = \tfrac{1}{2}\sum_k e(k)^2 = \tfrac{1}{2}\sum_k \| d(k) - y(k) \|^2,$$

where d_k's are the desired output and y_k's are the calculated output. The Widrow-Hoff calculates small changes for neuron's weights and biases in the direction that decreases the neuron's error. This direction is found by taking the derivative of the sum-squared error with respect to the parameters W and b.

Therefore,

$$\frac{\partial sse}{\partial w_{ij}} = \tfrac{1}{2}\frac{\partial}{\partial w_{ij}} \| d(i) - \sum_{j=1}^{N} w_{ij}x_j - b_i \|^2$$

Widrow -Hoff rule is implemented by making changes to the weights in the opposite direction from the direction that error is increasing. Therefore, the weight change is given by $\Delta W_{ij} = \alpha\, e_i x_j$, where α is constant known as learning rate. In matrix form,

$$\Delta W = \alpha < e, x > .$$

Similarly, the change in bias is given by $\Delta b = \alpha\, e$.

3.4.2 CONVERGENCE THEOREM

In this section we prove a convergence result for the generalized Widrow-Hoff algorithm applied to the network. We assume that the activation functions f_i are Lipschitz continuous and strictly monotone (Sec 2.5.5; Chap. 2) We give condition on the learning parameter α under which the generalized Widrow-Hoff algorithm converges. To prove the result we make use Fixed-point theorem. Our result generalizes the result obtained by Hui and Zak [67].

We make use of the following notations.

$$w_{i1} = b_i, \quad \mathrm{x} = (x_1, \quad x_2, \quad . \quad . \quad x_n)^T,$$
$$W_1 = (w_{11}, \quad w_{12}, \quad . \quad . \quad w_{1n})^T,$$
$$W_2 = (w_{21}, \quad w_{22}, \qquad w_{2n})^T$$
$$.$$
$$W_m = (w_{m1}, \quad w_{m2}, \quad . \quad . \quad w_{mn})^T$$

Let $W = \begin{bmatrix} w_{11} & w_{12} & . & . & w_{1n} \\ w_{21} & w_{22} & . & . & w_{2n} \\ . & . & . & . & . \\ w_{(m-1)1} & w_{(m-1)2} & . & . & w_{(m-1)n} \\ w_{m1} & w_{m2} & . & . & w_{mn} \end{bmatrix}$, $u_i = x^T W_i$

Therefore, we have,

$$Wx = \begin{pmatrix} x^T W_1 \\ x^T W_2 \\ . \\ . \\ x^T W_m \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ . \\ . \\ u_m \end{pmatrix},$$

and the error vector is given by

$$e = \begin{pmatrix} e_1 \\ e_2 \\ . \\ . \\ e_m \end{pmatrix} = \begin{pmatrix} yd_1 - f_1(u_1) \\ yd_2 - f_2(u_2) \\ . \\ . \\ yd_m - f_m(u_m) \end{pmatrix},$$

where yd_i's are the desired (actual)output and $f_i(u_i)$'s are the computed output. If α is the learning rate then the generalized Widrow – Hoff Algorithm is given by

$$W^{k+1} = W^k + \frac{\alpha\, ex^T}{x^T x}, \tag{3.6}$$

where W^{k+1}, W^k and ex^T are $m \times n$ matrices.

Multiplying (3.6) by x we get

$$W^{k+1}x = W^k x + \frac{\alpha e x^T x}{x^T x} \tag{3.7}$$

$$u^{k+1} = u^k + \alpha\, e$$

$$= u^k + \alpha \begin{pmatrix} yd_1 - f_1(u_1) \\ yd_2 - f_2(u_2) \\ . \\ . \\ yd_m - f_m(u_m) \end{pmatrix}$$

Define a nonlinear function $F : \Re^m \to \Re^m$ by

$$F(u) = u + \alpha\,[\mathrm{yd} - \mathrm{f(u)}]$$

$$\text{where, } \mathrm{yd} = \begin{pmatrix} yd_1 \\ yd_2 \\ . \\ . \\ yd_m \end{pmatrix},$$

$$f(u) = \begin{pmatrix} f_1(u_1) \\ f_2(u_2) \\ . \\ . \\ f_m(u_m) \end{pmatrix}$$

3.4.3 LEMMA 3.1. If f_i is strictly monotone for each i =1,2…m, then for each yd $\in$ Range(f) there exists a u* in $\Re^m$ such that

(a) (u*)=yd

(b) The operator F has a fixed point.

Proof:

Let $yd = [yd_1, yd_2,..., yd_m]^T \in$ Range (*f*). Since f_i is strictly monotone for each i , there exist $u_i^* = f_i^{-1}(yd_i)$ in $\Re$. Obviously $u^* = [u_1^*, u_2^*,..., u_m^*]^T$ is the required u* in (a). Since $F(u) = u + \alpha\,[\text{yd - f(u)}]$, part (b) follows immediately from the fact that u* is a fixed point of F.

Assumptions:

$$\| \text{u - v - } \alpha(\text{f(u) - f(v)}) \| \leq \| \text{u - v} \| \tag{3.8}$$

Claim:

Under the above assumption (3.8), the fixed point u* of F is unique

Proof: Let u* and v* be two fixed points of F

i.e. u*=F(u*),

v*=F(v*)

$$u^* - v^* = F(u^*) - F(v^*) = u^* - v^* - \alpha(f(u^*) - f(v^*)$$

$$\| u^* - v^* \| = \| F(u^*) - F(v^*) \| = \| u^* - v^* - \alpha(f(u^*) - f(v^*) \| \leq \| u^* - v^* \|$$
$$\Rightarrow \| u^* - v^* \| = 0$$
$$\Rightarrow u^* = v^*$$

- **ANOTHER CONDITION**

1) $< f(u) - f(v), u - v > \geq c \, \| u - v \|^2$

2) $\| f(u) - f(v) \| \leq \beta \, \| u - v \|$

3) $\alpha \, \beta^2 \leq 2c$

Then F has a unique fixed point.

Proof:

We show that above conditions imply Assumption (3.8).

To prove that $\| u - v - \alpha\,(f(u) - f(v)) \|^2 \le \| u - v \|^2$

$$\begin{aligned}
&\textit{Consider} < u - v - \alpha\,(f(u) - f(v)), u - v - \alpha\,(f(u) - f(v)) > \\
&= \| u - v \|^2 \; -2\alpha < f(u) - f(v), u - v > + \alpha^2 \; \| f(u) - f(v) \|^2 \\
&\le \| u - v \|^2 \; -2\alpha c \| u - v \|^2 \; + \alpha^2 \beta^2 \; \| u - v \|^2 \\
&= (1 - 2c\alpha + \alpha^2 \beta^2) \| u - v \|^2 \le \| u - v \|^2 \qquad \text{by (3)}
\end{aligned}$$

- **Claim**· If $u^{k+1} = F(u^k)$, then $\{u^{k+1}\}$ converges to the unique fixed point of F.

Proof: Let u* be the fixed point of F

Then $0 \le \| u^{k+1} - u^* \| - \| F(u^k) - F(u^*) \| \le \| u^k - u^* \|$

The sequence $\| u^1 - u^* \|$, $\| u^2 - u^* \|$,.. .. ...is decreasing and bounded below

By monotone convergence theorem, this sequence converges

i.e. $\lim_{k \to \infty} \| u^k - u^* \| = L < \infty$

Let $\lim_{k \to \infty} (u^{k+1} - u^k) = y$

i.e.

$\lim_{k \to \infty} (u^{k+1}) = u^* + y$

$$F\begin{pmatrix} \lim(u^k) \\ j \to \infty \end{pmatrix} = \begin{pmatrix} \lim F(u^k) \\ j \to \infty \end{pmatrix} = \begin{pmatrix} \lim u^{k+1} \\ k \to \infty \end{pmatrix} = y + u^*$$

Thus, y+u* is a fixed point of F. But F has a unique fixed point and hence.

. y=0,

$\therefore \lim_{k \to \infty} u^{k+1} = u^*$

This completes the proof.

The above discussion leads to the following convergence theorem.

Suppose that the transfer function f1, f_2, f_3.......f_n satisfy the following conditions :

(1) $|f_i(u) - f_i(v)| \le \beta_i |u-v|$; for all u, v $\in$ R, i = 1,2,.. n

(2) $(f_i(u) - f_i(v))\,(u-v) \ge c_i |u-v|^2$; for all u, v $\in$ R, i = 1, 2,.. n

where,

$f : R^n \to R^n$ is defined by

$$f(u_1, u_2, \ldots u_n) = \begin{pmatrix} f_1(u_1) \\ f_2(u_2) \\ ---- \\ f_n(u_n) \end{pmatrix}$$

Let $\beta = \max_i (\beta_i)$ $c = \min_i (c_i)$

The generalize Widrow Hofff learning algorithm converges for the McCulloch Pitt type neural network if the learning rate α is satisfy

$$\alpha \le \frac{2c}{\beta^2}.$$

3.4.4 RESULTS AND DISCUSSION (McCulloch type ANN)

Here, three inputs, Relative Humidity (RH), Wind Speed (WS) and Air Temperature (AT) are considered to predict the soil temperature (ST) at surface of the earth. Figure 3.5 shows the predicted soil temperature standard week wise. It is significant with actual soil temperature. Here we use f(x) = tanhx as the transfer function. This function is both Lipschitz continuous and monotone and the above theorem applies here. The McCulloch type network with generalized Widrow Hoff (3 6) gives better result than multiple linear regression method Note that when the transfer functions are identify mappings, the McCulloch type network produces to multiple regression model.

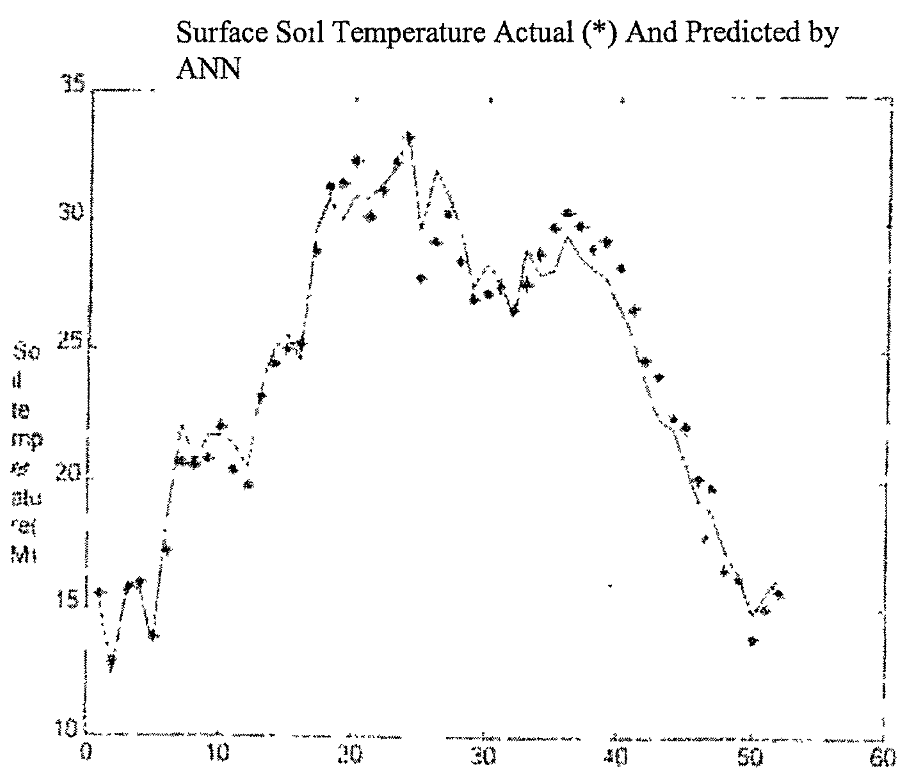


Fig: 3.5

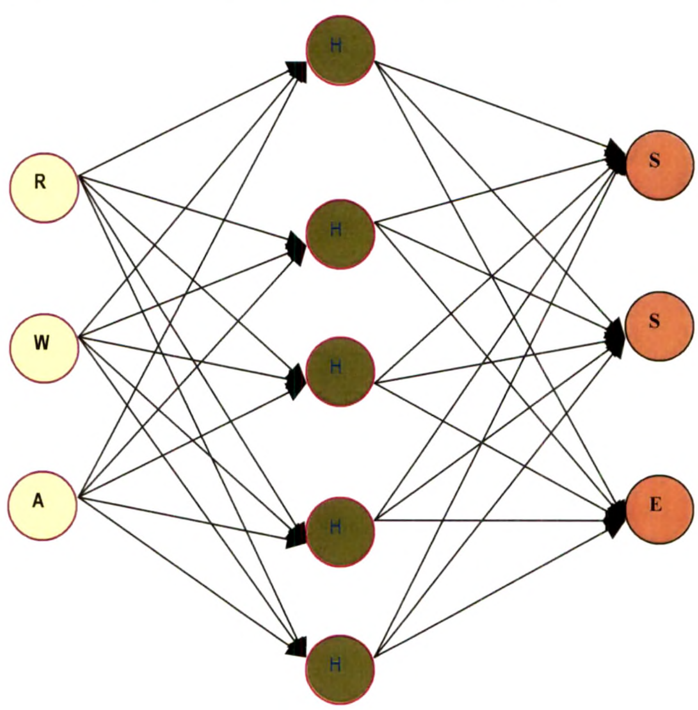
3.5 METHOD III

PREDICTION OF ST BY THREE INPUTS USING MULTILAYERED ANN

Here we consider a multilayer network to predict the soil temperature. The input variables are Relative Humidity (RH%), Wind speed (WS) and Air Temperature (AT) and outputs are ST (Morning), ST (Afternoon) and Evaporation (EP).

All the standard weekly data are taken from the Land Surface Process Experiment (LASPEX) -1997 over Sabarmati River Basin site of Anand.

❖ **STRUCTURE OF THE NEURAL NETWORKS**



Input layer with 3-iputs RH, WS and AT

Hidden layer

Output layer with 3-outputs ST (MN), ST (AN) and EP.

Fig: 3.6

3.5.1 RESULT AND DISCUSSION (Multilayered ANN)

Figure 3.7 shows the predicted soil temperature by three inputs employing ANN Transfer function used is $f_i(x) = \tanh(x)$ for i= 1, 2, 3.

Actual soil temperature and predicted soil temperature have non-significant difference. There is some large deviation but non significant, found in the rainy season that is in the month of August and standard Week (SW) of 30 to 40.

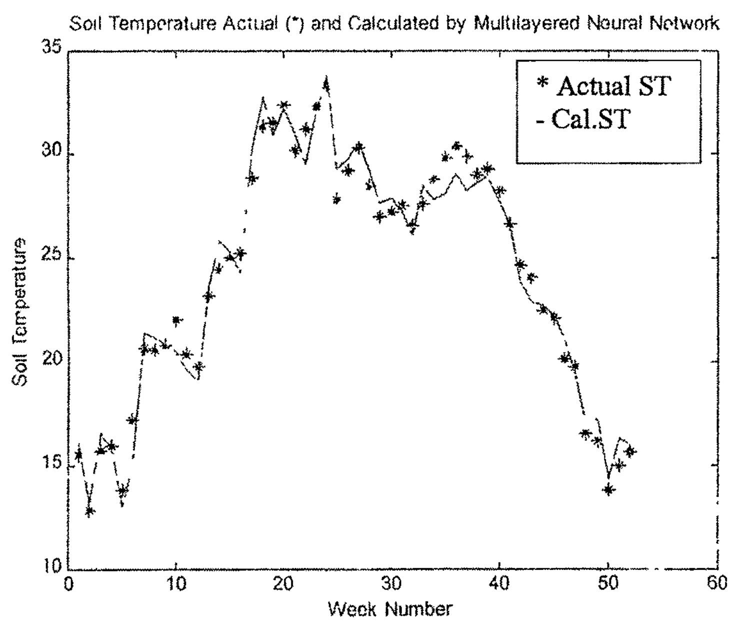


Fig: 3.7

3.6 (I) Case (b) PREDICTION OF ST BY TWO INPUTS USING MULTILAYERED ANN

Prediction of soil temperature is done by ANN method. For case (b) Artificial Neural Network (ANN) is applied for prediction of soil temperature by two inputs, the air temperature and time

Two sets of Data Series (DS) namely, Air Temperature at 1-mt height and time in minute. Minute DS is converted in to hourly DS from 10.00 hr to 20.00 hr for the month of December 1997.

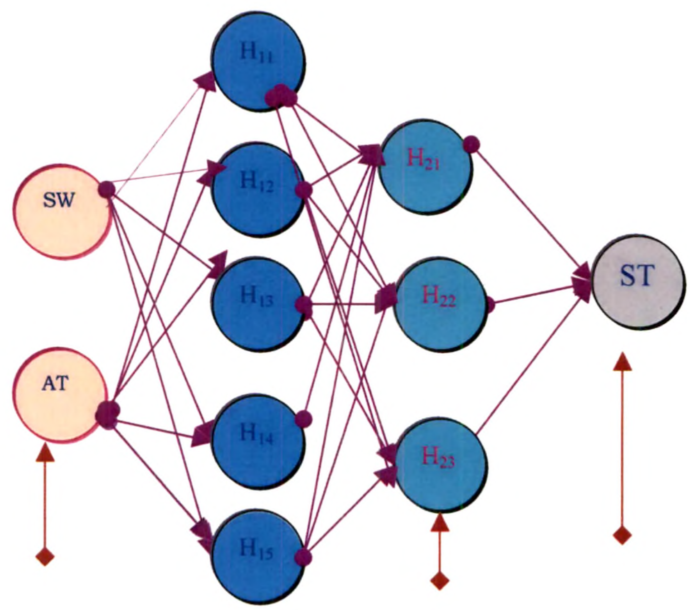
First a set of input and actual output data has been used for the training purpose of the networks. This trained network is applied for prediction of soil temperature. Network is shown in the Figure 3.8

In the shown network first hidden layer has five neurons and second hidden layer has three During the learning of the network, parameter values used are momentum (0.5) and learning rate (0.0005). Hyperbolic tangent sigmoidal function (Cybenko, [25]) and linear function are used as transfer functions. The hidden layer is the one in which the nodes are not connected directly to the input or output of the system.

3.6.1 RESULTS AND DISCUSSION (Multilayer ANN)

Figure 3.9 shows the predicted soil temperature by two inputs air temperature and time in hour using ANN with two hidden layers. This figure shows the result for 10.00 hr to 20.00 hr in the month of December 1997. Predicted soil temperature is smoother than actual observed. Actual soil temperature are highly affected by unexpected disturbances on the surface temperature like cloud cover, cold wind, greenery vegetation etc. while soil temperature obtained by ANN is not accounting such variations.

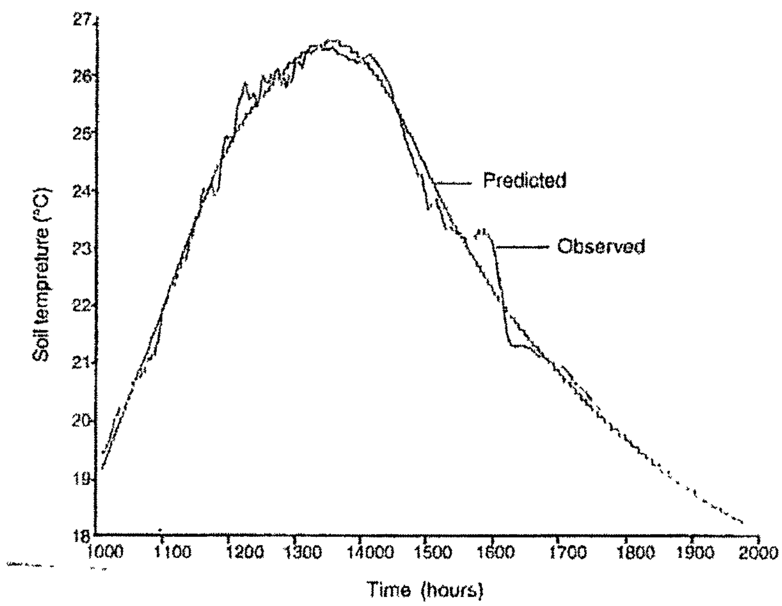
❖ STRUCTURE OF THE NN



Input layer With 2-nodes	Hidden layer-1 with 5 nodes	Hidden layer-2 with 3-nodes	Output layer with one node

Fig: 3.8

It is evident from Figure 3.9 that result obtained by ANN has non-significant difference with actual soil temperature.



Observed and predicted soil temperature at 0-5 cm depth

Fig: 3.9

3.7 (I) case (c) PREDICTION OF ST BY ONE VARIABLE, (TIME) USING (ANN) WITH BACKPROPAGATION ALGORITHM

Here we predict the soil temperature (ST) with one input variable, namely the time point (Standard Week (SW))

Data series 1982 to 2003 of soil temperature (ST) at the depths of 5 cm, 10 cm and 20 cm for morning and afternoon is used to train the ANN. These trained network is used to predict the soil temperature (MN and AN) of the year 2004 for all the depths

Similarly, NN is trained by the DS 1982-2004 of soil temperature (MN and AN) for all the depths. Then soil temperature of the year 2005 is predicted.

3.7.1 METHOD

We employ ANN with back propagation algorithm for training the network consists of one input (time point (SW 1^{st} to 52^{nd})) and one out put (ST). For the depths 5 cm, 10 cm and 20 cm at morning (MN) and afternoon (AN) time period, different networks are trained.

- **STRUCTURE OF THE NETWORKS:**

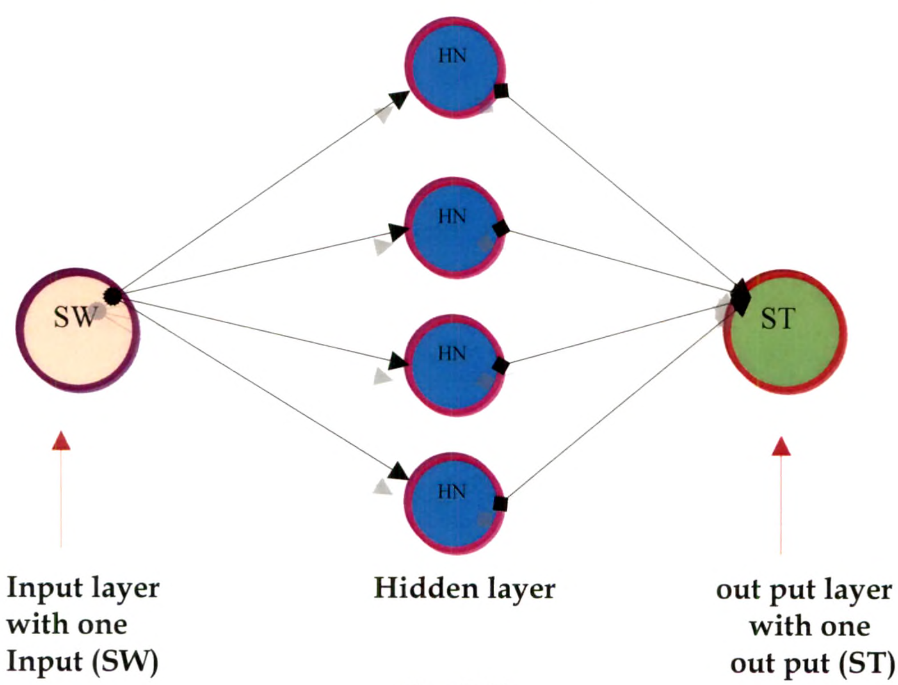


Fig: 3.10

Here, ANN is trained by soil temperature data series (DS) of the year 1982 to 2003 for the depth 5 cm (MN) to predict the ST of the year 2004 with using parameter values as per the Table 3.1.

During the training, error that is difference between calculated output by ANN and actual is minimized by Back propagation method. For each depth and at each time period morning (MN) and afternoon (AN) for the year of 2004 and

2005, different networks have been trained by selecting parameters as showed in the Table: 3.5.

Programmes are developed in MATLAB and given in the Appendix.

3.7.2 RESULT AND DISCUSSION

Prediction of soil temperature for all the depths for morning and afternoon are depicted in the Figure 3.11 to 3.22. Values of the Absolute Maximum Difference (AMD), Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) are given in the Table 3.1.

Prediction for 5 cm (MN) is shown in the Figure 3.11. Difference between predicted and actual is plotted in the same figures as subplots. High difference between actual and predicted soil temperature is found during the standard week of 5^{th} -10^{th} and 30^{th} -40^{th}.

The absolute maximum difference (AMD) for soil temperature (5 cm (MN)) of 2004 is 4.67 ^{0}c. Root Mean Square Error (RMSE) 1.86 ^{0}c and Percentage of Average Error (PAE) 7.19% is obtained during the analysis. With the help of student t -test predicted soil temperatures (STs) have non-significant difference with actual one

Predicted soil temperature at the depth of 5 cm (AN) for the year of 2005, depicted in the Figure 3.14. The soil temperature differences are also shown. AMD is 9 50 ^{0}c with RMSE 3.81 ^{0}c and PAE 8 98%. Here RMSE is higher than the prediction of the year of 2004 for the same depth.

To predict the soil temperature at the depth 10 cm (MN) of the year 2004, DS of the year 1982-2003 has been used to train the ANN.

Difference between actual and predicted soil temperature is increasing in the period of rainy days that is 23rd to 42nd standard weeks and prediction by ANN is under estimating soil temperature (10 cm (MN, 2004)) (Figure 3.14).

Similarly, soil temperatures (ST) of the year 2004 and 2005 at depth 10 cm (AN), 20 cm (MN) and 20 cm (AN) are found by ANN and shown in the Figure 3.16 to 3.22. These all the predicted temperatures are significant to actual temperatures by student t-test for two tails.

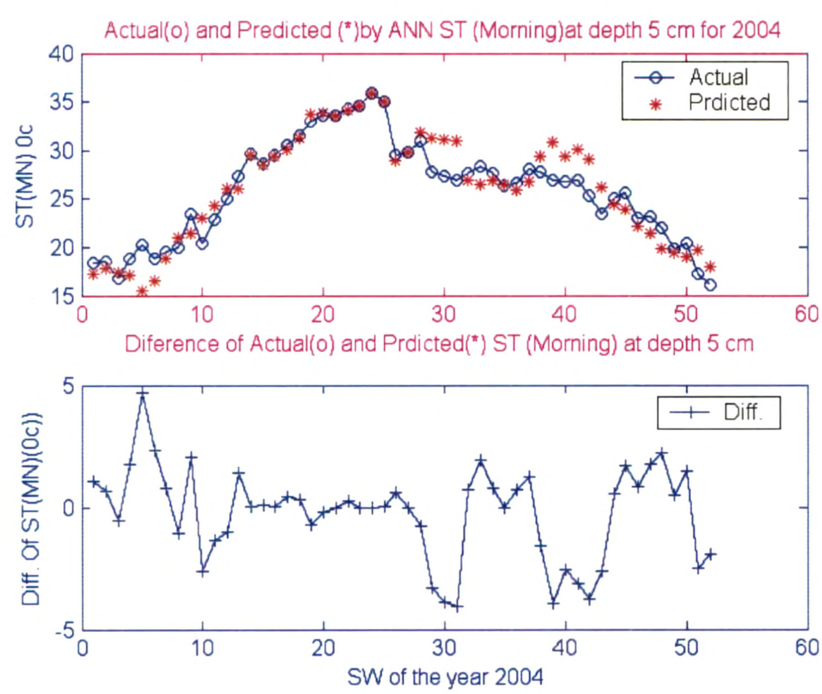
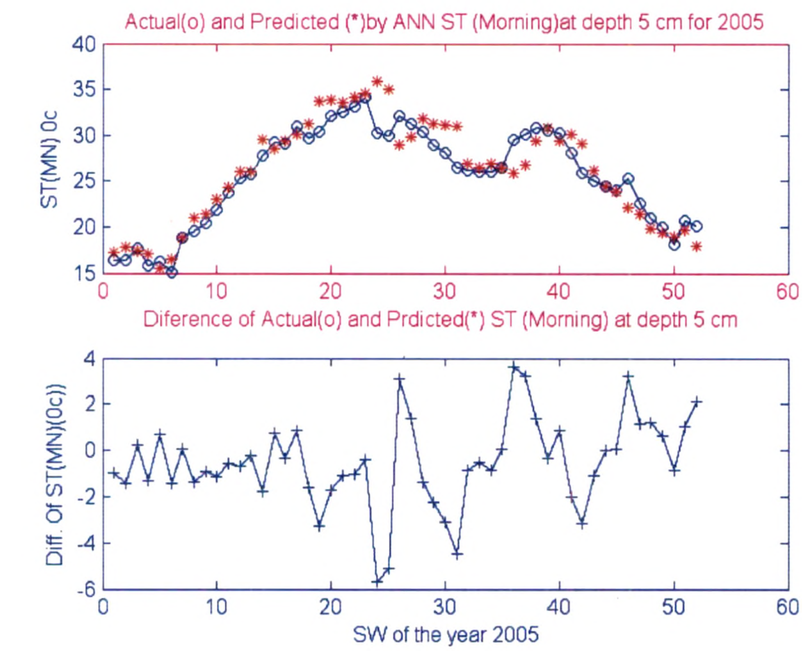


Fig: 3.11



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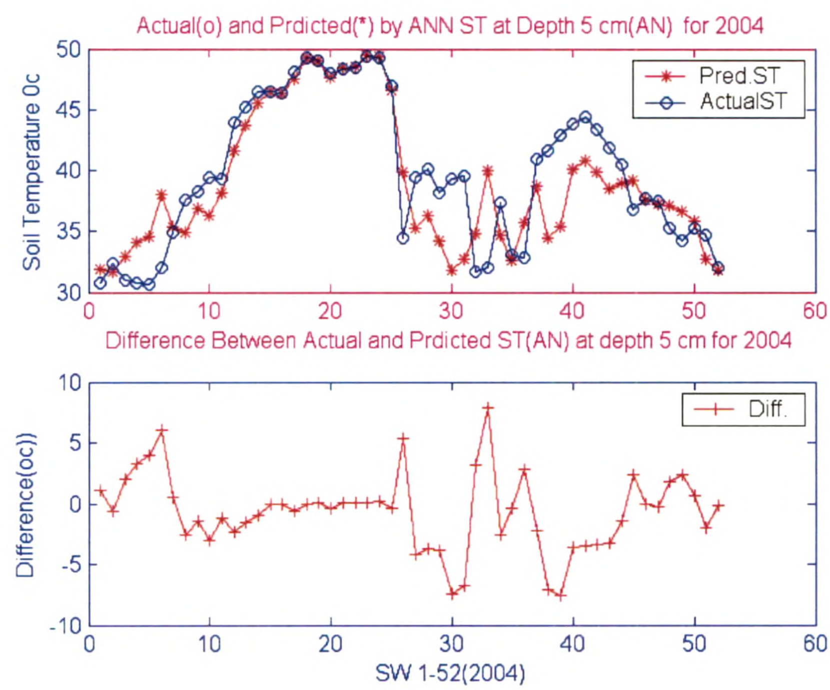

Fig: 3.13

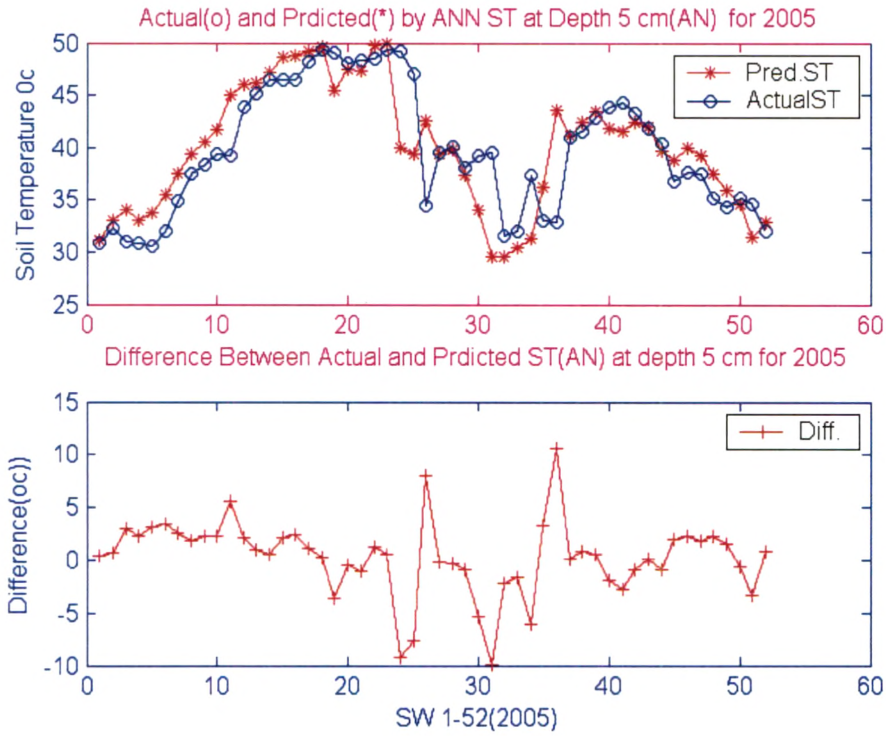

Fig: 3.14

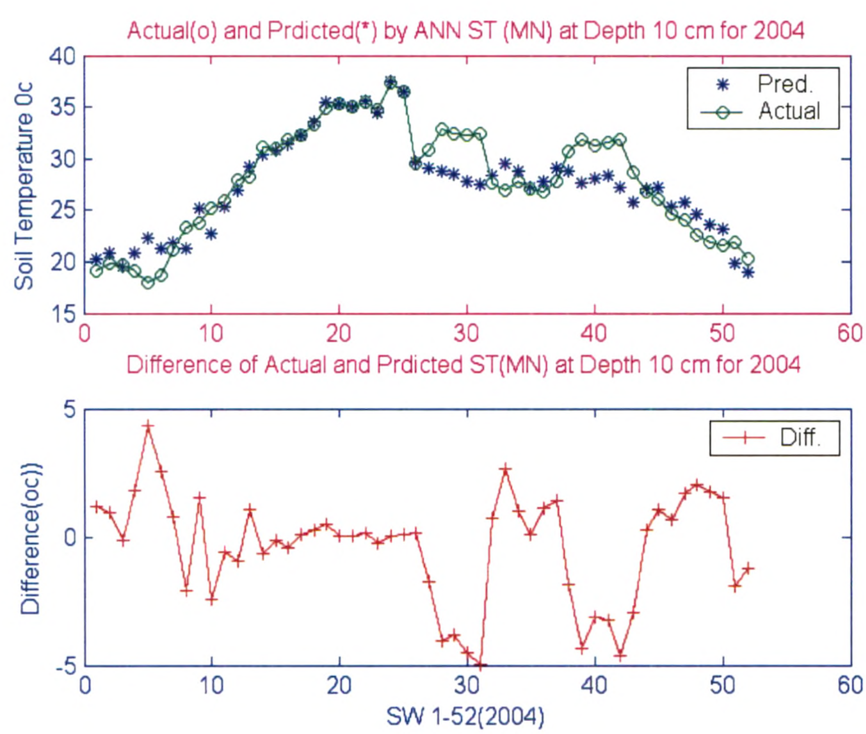

Fig: 3.15

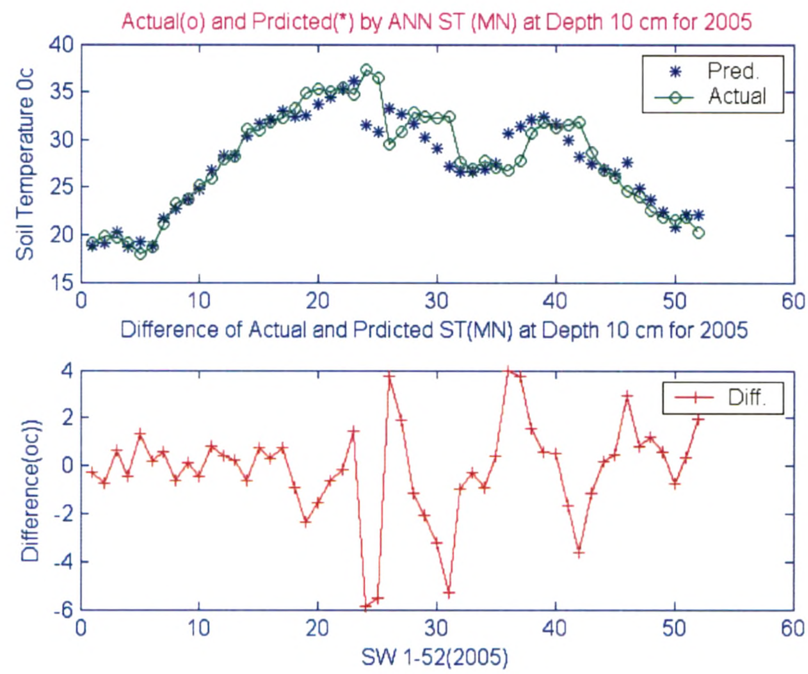

Fig: 3.16

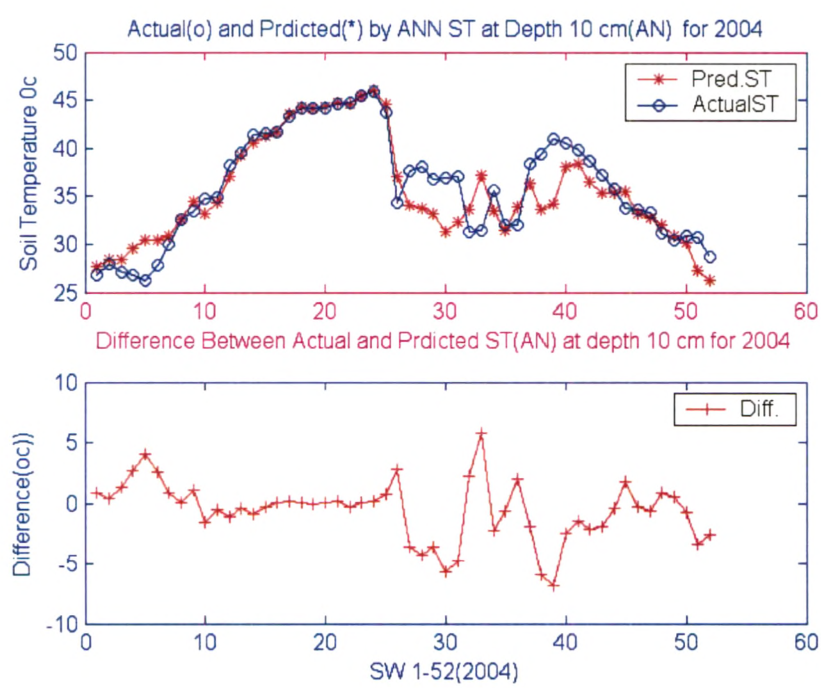

Fig: 3.17

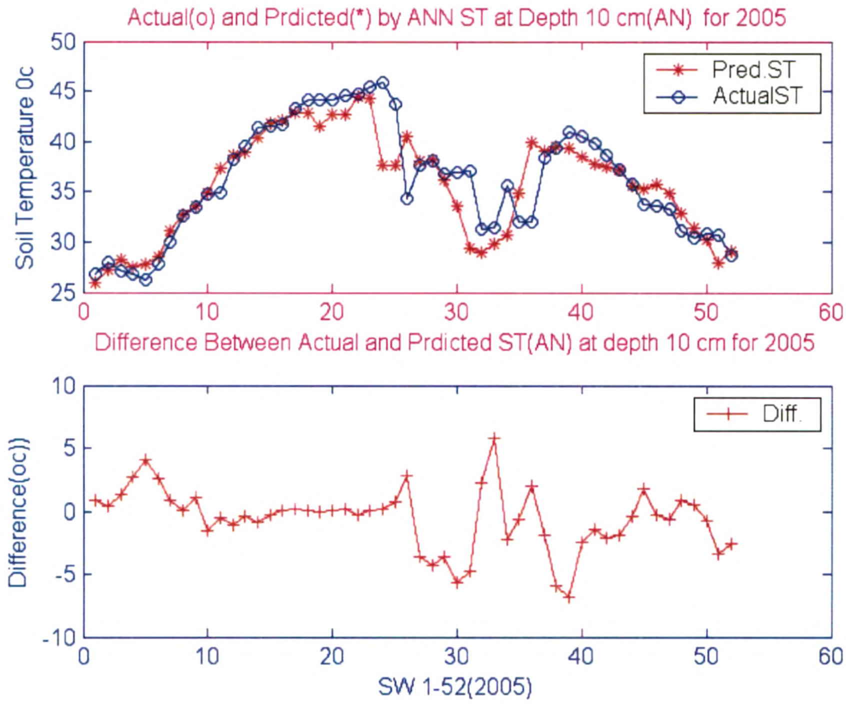

Fig: 3.18

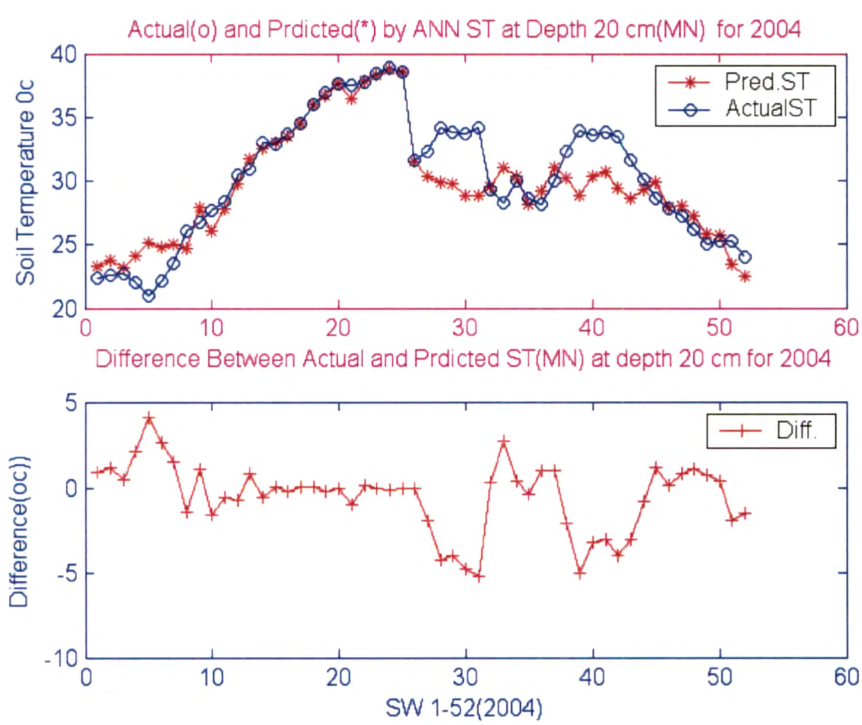

Fig: 3.19

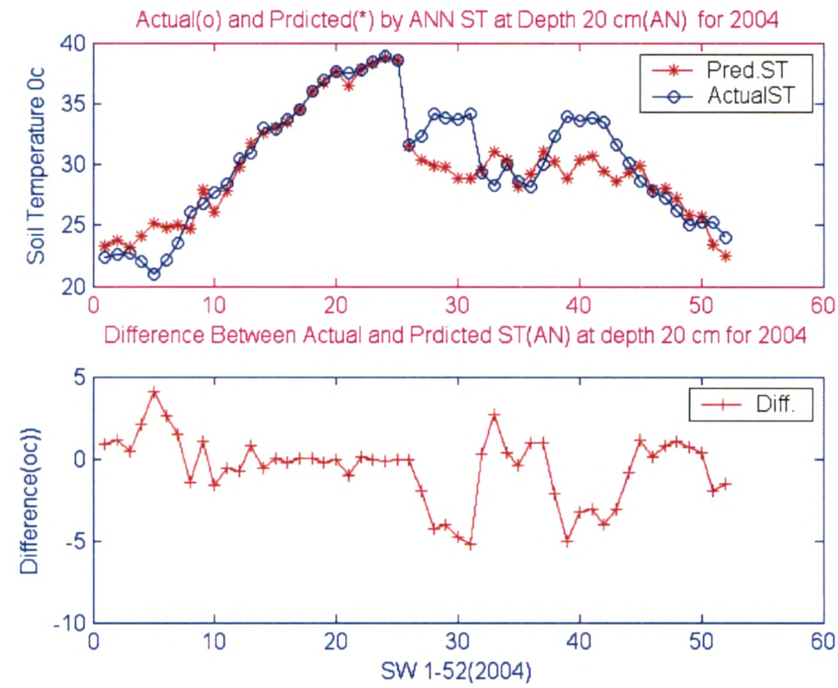

Fig: 3.20

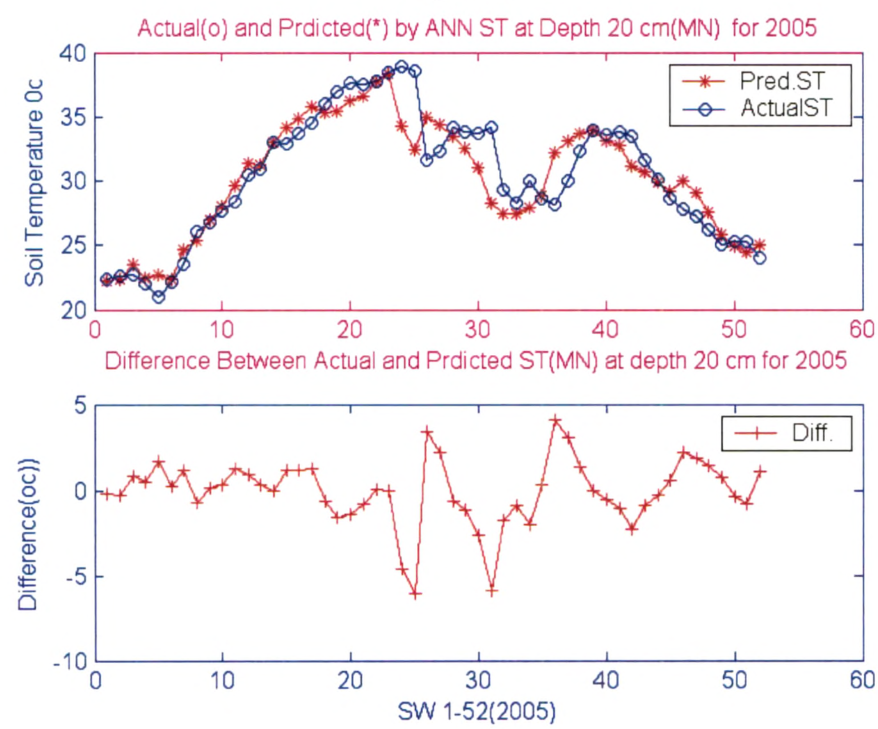

Fig: 3.21

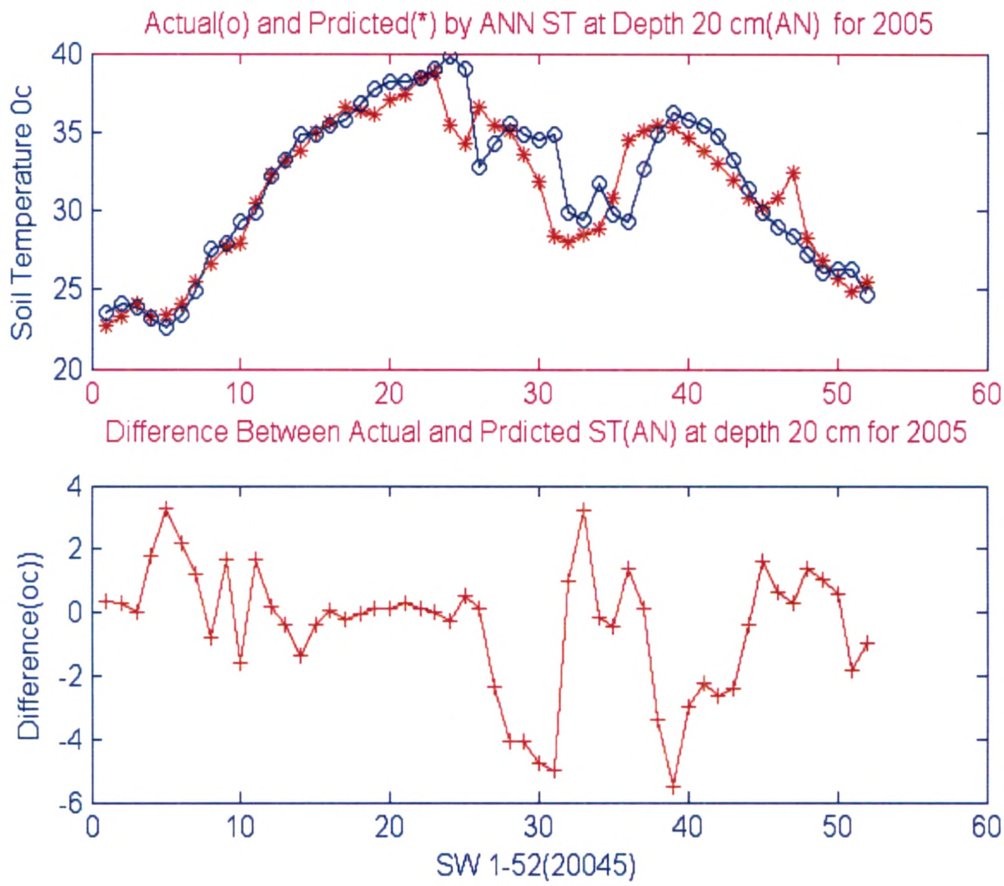
Actual(o) and Prdicted(*) by ANN ST at Depth 20 cm(AN) for 2005
Soil Temperature 0c
Difference Between Actual and Prdicted ST(AN) at depth 20 cm for 2005
Difference(oc))
SW 1-52(20045)

Fig: 3.22

TABLE 3.1

DETAILS OF THE PARAMETER VALUES USED IN ANN TRAINING AND RESULTS

Sr. No.	Predicting Year	ST at Depth	Number of Epochs used	Learning Rate 'r'	Momentum 'm'	No. of Neurons	Error Goal	Absolute max. Diff. (AMD) (^0C)	RMSE (^0C)	Percentage Of Average Error (PAE)
	2004	MN.	13755			140	10^{-19}	4.67	1.86	7 19
1	2005		7044	0.001	0.5	140	10^{-15}	5.7	1.99	7.75
		5 cm								
		AN								
	2004		14866	0.001	0.5	140	10^{-22}	7.93	3.23	8.29
	2005		12763			140	10^{-23}	10.68	3.59	8.98
	2004	MN.	19252	0.001	0.5	140	0.001	4.93	2.11	7.68
2	2005		4104			140	10^{-5}	5.93	2.23	8.06
		10 cm								
	2004	AN	13148	0.001	0.3	140	10^{-22}	6.80	2.47	6.95
	2005		12225			140	10^{-19}	6.79	2.48	6.95
	2004	MN.	13675		0.5	140	10^{-22}	5.25	2.08	6 99
3	2005		12269	0.001	0.3	140	10^{-22}	6 1	1.93	6.39
		20 cm								
		AN.								
	2004		12883	0.001	0.3	140	10^{-21}	5.54	2.02	6.49
	2005		15541			140	10^{-22}	5.54	2 02	6 46

3.7.3 CONCLUSION

Here, prediction of soil temperatures (ST) is made by ANN with different training methods. Different networks are used for each depths of 5 cm, 10 cm and 20 cm for mornıng (MN) and afternoon (AN) periods for the year 2004 and 2005. During this analysis the following conclusıon has been made.

i) Obtained minimum Absolute Maximum Difference (AMD) & Root Mean Square Error (RMSE) and Percentage of error (PAE) are 4.67 ^{0}C & 1.86 ^{0}C, respectively in the year of 2004, at the depth 5 cm (MN) (Table: 3.1)

ii) Fluctuation and variance (36.68 ^{0}C) in the soil temperatures (AN) at the surface (5 cm) are high (Fig: 3.2 and Table· 3.1), therefore, during the prediction of soil temperatures for the year of 2005, computed Absolute Maximum Difference (AMD), Root Mean Square Error (RMSE) and PAE that are 10 68 ^{0}C, 3 59 ^{0}C and 8.98% , respectively are highest.

ııi) Due to disturbance of the nature, like highest rainfall in the year 2005, in each case of each depth for morning (MN) and afternoon periods (AN), 2005 year has large Absolute Maximum Difference (AMD) & Root Mean Square Error (RMSE) in comparison to the other years.

iv) Each and every case of prediction by ANN with actual soil temperatures (ST) has large variations in the period of rainy days that is standard week from 22^{nd} to 42^{nd}

v) All the predictions by ANN are significant to actual soil temperatures (ST).

3.8 SOIL TEMPERATURE PREDICTION BY HARMONIC ANALYSIS

3.8.1 DESCRIPTION OF HA

The soil temperature is useful to estimate in and out diurnal heat flux of soil, which control the atmospheric air temperature and determine microbiological activities in the soil. The physical condition of soil is the key factor in determining the magnitude of daily soil temperature. Soil temperatures are measured at a specific observatory and the data is not available for all other locations. In the absence of measured soil temperature observation one can estimate soil temperature as a dependent variable of heat flux, type of the soil, and air temperature by different statistical/mathematical techniques.

Bocock *et al.* [16] reviewed different methods for estimation of soil temperature. These methods involved (i) Linear Regression of soil temperature on air temperature (ii) Multiple regression of soil temperature on other climatic variables and (iii) Harmonic analysis using average sine curves for soil and air temperature and derivation from these for particular years. They observed that linear regression gave the least precise estimates, with less difference between other methods Outside the period, soil temperature was over estimated slightly by other methods. Harmonic analysis giving the most precise and accurate estimation.

The observed standard weekly soil temperature at 5cm depth for three consecutive years (2002, 2003 and 2004) is depicted in Figure 3.23. Pattern indicates that the soil temperature process repeats as periodic phenomena with period of 52 standard weeks. Also, the soil temperature process is a continuous function of time, without very rapid oscillations Therefore, we can employ the tools of harmonic analysis to study the process.

The Fourier theorem ensures that a periodic function satisfying some smooth properties (Dirichlet's conditions) can be represented by an infinite series of sine and cosine function. A brief discussion of Fourier series is given in section 2.6 of chapter 2.

Carson [17], Krishnan *et al.* [89] and Bocock [15] have used the method of harmonic analysis to predict the weather parameters like soil temperature and air temperature.

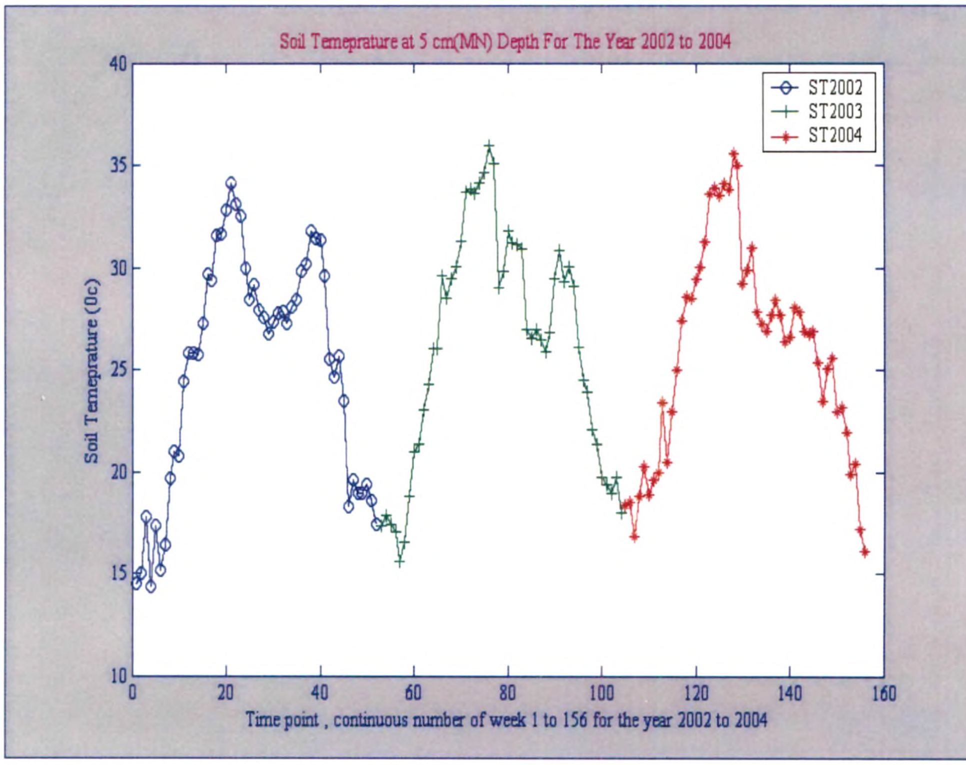

Fig: 3.23

3.8.2 HA IN PROBLEM SOLVING

The standard weekly (SW) mean soil temperature at various depths follows the pattern of a periodic function (Fig: 3.23). Harmonic analysis employing Fourier series can be used to accurately describe the thermal cycle at particular depth as follows:

$$T(t) = T_0 + \sum_{k=1}^{N}\left(a_k * \cos\left(\frac{2\pi}{P}kt\right) + b_k * \sin\left(\frac{2\pi}{P}kt\right)\right), \tag{3.7}$$

t = 0, 1, 2…,51

where T(t) is the soil temperature at time point t, p = 52, N is the total number of Harmonics used, T_0 is the Mean of soil temperature of the data series, C_k is the Amplitude of the k^{th} Harmonic and t is time, t = 0, 1, 2, 3…51; fundamental period P = 52.

From this equation, $T(t) = T_0 + \sum_{k=1}^{N}\left(C_k \ * \sin\left(\frac{2\pi}{P}kt + A_k \right)\right)$ (3.8)

where, C_k is the Amplitude and A_k is the Phase Angle given by

$$C_k = \sqrt{a_k^{\ 2} + b_k^{\ 2}} \ ; \quad A_k = \tan^{-1}(\frac{a_k}{b_k}) \tag{3.9}$$

where a_k and b_k are the Fourier Coefficients, estimated by formula

$$b_k = \frac{2}{no}\sum[T(t) \ \sin(\frac{2\pi}{P}k\,t)] \ ; \tag{3.10}$$

$$a_k = \frac{2}{no}\sum[T(t) \ \cos(\frac{2\pi}{P}k\,t)] \ ; \tag{3.11}$$

where, k=1,2,…N=no/2.

3.8.3 "HARMONIC" IN DETAIL

The first harmonic has the fundamental period of P. The second harmonic has a period of half of the fundamental period, that is, P/2 Similarly, third harmonic has period of one third of the fundamental period, P/3, up to the number of harmonics N equals half of the number of observations. This last harmonic N has period of P/no, where no=52, is the total number of observations. The different harmonics are treated as an independent entity; each harmonic may have a different physical cause.

Complete cycle of soil temperature (ST) is fairly well described by the first harmonic in most of the cases (Pearce [122]). It is always not possible to account the complete variations in ST at once but the individual harmonics can be explained variance in parts.

Some times periodical function may not have sinusoidal character, then at this stage harmonic is simply a mathematical representation equivalent to a periodical function and harmonic does not have any physical meaning.

Thus first problem covers, soil temperature prediction by, Harmonic analysis and their amplitudes (C_k) are found for all the used number of harmonics, N=26. These amplitudes are used to find out the total variations in the data accounted by total number of harmonics in percentages That is $\frac{C_k^{\ 2}}{2s^2} \times 100$; where, s^2 is the variance in observed data. (Kulshrestha *et al.* [92]).

3.8.4 DATA

In this analysis standard weekly data series of ST is used from 1982 to 2005, for the depths of 5 cm, 10 cm and 20 cm of 'Morning (MN)(7.38 hr)' and 'Afternoon (AN)(14 38 hr)' from the observatory, Dept. of Agril Meteorology, B A College of Agriculture, Anand Agricultural University, Anand.

In Harmonic analysis, data series from 1982 to 2003 is averaged (Table A- 3.2) and used to find out a_k and b_k ; k=1,2,3....N=26, Fourier Coefficients (FC) from formula (3.10) and (3.11) are found. These are used to predict the STs of the year 2004 and 2005. Their Mean (T_0) and variances (s^2) are given in the Table 3.2.

TABLE 3.2
STATISTICAL MEASURES OF DS OF THE PREDICTING YEARS.

Sr. No	Predicting Year	ST at depth	Mean ^0C	Variance ^0C
1	2004	MN.	25.85	26.88
	2005		25.69	28.04
	2004		38.97	31.06
	2005	AN.	39.96	34.68
2	2004	MN.	26.65	12.67
	2005		27.68	23.73
		10 cm		
	2004.	AN.	35.53	28.93
	2005		35.70	26.88
3	2004	MN.	29.76	19.05
	2005		30.20	20 12
		20 cm		
	2004	AN.	31.15	19.15
	2005		31.27	21.20

Table 3.2 shows that in the year 2005, ST (5 cm (AN)) has highest variance (34.68 ^0C). At the depth of 10 cm (AN) in the year 2004, highest

variance (28.93 ^{0}C) is observed At the depth 20 cm (AN) in the year 2005, observed variance is highest (21.20 ^{0}C).

3.8.5 COMPUTATION

In our case T (t) is the soil temperature, at time t=0, 1, 2...51, that is to be predicted from formula (3.7). In this formula,

T_0 = mean soil temperature (Appendix, Table: A-3.1),

P=52, k= 1, 2, 3. .26.

There, are $\frac{N}{2}$-1 =25 sine and $\frac{N}{2}$=26 cosine terms in the equation (3.7), Fourier coefficient (FC), b_{26} is always zero. The first two terms of the series (3.7) is completing the complete one cycle in the one fundamental period that is 52 weeks. The third and fourth terms varies rapidly twice, that means these two terms completing the one cycle in half of the fundamental period, 26 weeks (Figure · 3.31, 3.37 and 3 41). The last two terms have very less period to complete the cycle. That is (2P/no) =2 weeks. The period of k^{th} harmonic is inversely proportional to k.

3.8.6 COMPUTATION OF FOURIER COEFFICIIENTS

Harmonic analysis starts with finding the values of FCs of series (3.8) from historical data. During the computation of Fourier coefficicients, T(t) are the recorded historical values (Table : A-3.1) of soil temperature Generally, maximum number of harmonics is no/2 (Guenni [51])

Computations of the Fourier coefficients are carried out as per the following steps:

Step I

All the sines and cosines are computed for the value of $\frac{2\pi}{P}kt$ for k =1,2,...26. Table A-3.3 in Appendix shows these sines and cosines values for k =1,2,3. Then each element in the columns are multiplied by the corresponding soil temperature observed at time point, t=0,1,2,...51.

Step II

This is repeated in the columns corresponding to all 26 harmonics.

Step III

These columns sums for every column is found out and multiplied by the 1/26.

Denote these cosines and sines sums after multiplying with 1/26 by a_1, a_2 a_3 a_k and b_1, b_2, b_3 b_k , respectively. These are Fourier coefficients (FCs).

After having computed the FCs for the 26 number of harmonics, formula 3.7 is used for soil temperatures are prediction

3.8.7 COMPUTATION OF HARMONIC

In general, kth harmonic is ($b_k \sin(\frac{2\pi}{P}kt) + a_k \cos(\frac{2\pi}{P}kt)$)

After having computed the FCs for the number of harmonics then adds them for each value of t and adds it to its mean. If no mistake has been made, the sum of the mean and harmonics would add up to the original time series ST.

It is required to find out total variance of soil temperature which is accounted by the harmonics. The equation for the variance accounted for are given by

first harmonic is $C_1^2/(2^*s^2)$,

second harmonic is $C_2^2/(2^*s^2)$,

third harmonic is $C_3^2/(2^*s^2)$ and similarly for

kth harmonic is $C_k^2/(2^*s^2)$

Here s^2 is the variance in the data series of soil temperature. Since, all harmonics are uncorrelated and therefore no two harmonics can explain the same part of the s^2, the variances explained by the different harmonics can be added.

Thus, total variance explained by total number of k harmonics is given by,

$$C_1^2/(2^*s^2) + C_2^2/(2^*s^2) + C_3^2/(2^*s^2)\ldots\ldots\ldots + C_k^2/(2^*s^2)$$

Required programmes are developed in MATLAB.

3.8.8 RESULTS AND DISCUSSION

Table 3.2 shows the statistical measures during the Harmonic analysis at all the depths for morning (MN) and afternoon periods. Soil temperature at 5 cm depth is also known as surface temperature Always, surface temperature has high variations in the afternoon (AN) hour due to change in solar radiations. Table 3.2 shows that soil temperature at 5 cm depth in the afternoon (AN) period, has large mean (39.96 ^{0}C) and variance (36.68 ^{0}C) in comparison to other soil temperature (ST) at other depths.

In the case of at 20 cm depth, in the year of 2004 variance is 19.05 ^{0}C which can be covered fully by first two harmonic and ıt approximately 100%. (Table: 3.4). Predicted soil temperature (ST) for 5 cm depth in the period of afternoon are significant covering 99.45 % (year-2004) and 88.32 % (year-2005) Highest difference found between Actual and Predicted soil temperature is 6.24 ^{0}C (Table: 3.4, Fig: 3.38).

The computed 26 Amplitudes are plotted in the Figure 3.32, 3.38 and 3.42. Five phase angles out of 26 are showed in the Table 3.3.

Fig. 3.14 and Fig. 3.15 show that predicted soil temperature are significant to actual soil temperature. Accuracy of the predicted soil temperature are tested by student's t-test and found non-significant difference with actual soil temperature.

At 10 cm depth for mornıng (MN) and afternoon (AN) the accounted percentage of variance are 96.54% and 91.5%, 84.33% and 90.83% for the year of 2004 and 2005 respectively (Table: 3.4). Predictıon of soil temperature is shown in the figures 3.15 to 3.18. The highest difference found is 4.74 ^{0}C between actual and predicted soil temperatures (10 cm (AN)).

Figure 3.37 shows the plots of the harmonics (NH) used during the prediction of soil temperature. First harmonic has biggest amplitude, its period is one fundamental period of 52 weeks, and during this time period second harmonic oscillates two times and so on for other harmonics. Graphs of harmonics in the Figure 3.31 3.37 and 3.41 show that higher harmonics contribute les in covering to variance.

Same way prediction of soil temperatures by HA at depth 20 cm for morning and afternoon are plotted in the Figure 3.36, 3.39 and 3.40.

Root Mean Square (RMSE) and Percentage of Error (PAE) are found and provided in the Table 3.5. Due to the high fluctuation in the surface temperature, highest RMSE (1.74 ^{0}C) and PAE (6.74%) are found at the depth 5 cm (MN). While, lowest RMSE (1.39) and PAE (4.44%) are found at the depth 20 cm (AN) for the year 2005. This is due to the fact that at the depth 20 cm, variation in the soil temperature is less in comparison with that at 5 cm and 10 cm depth.

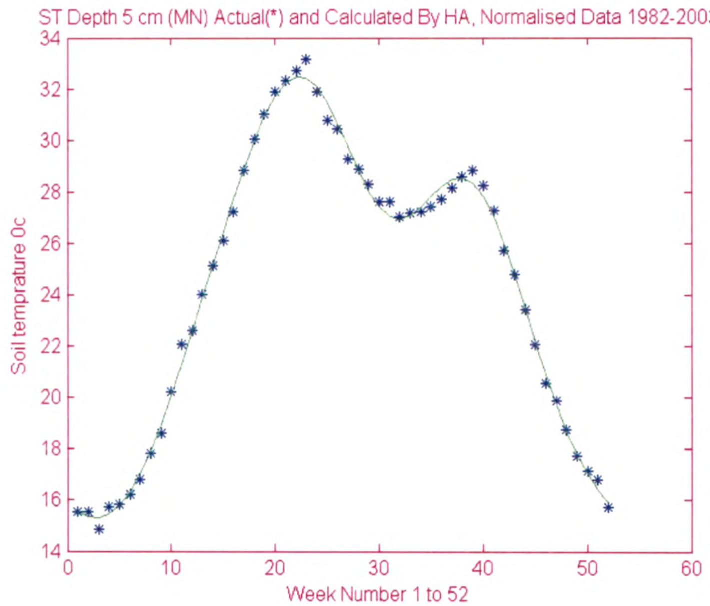

Fig: 3.24

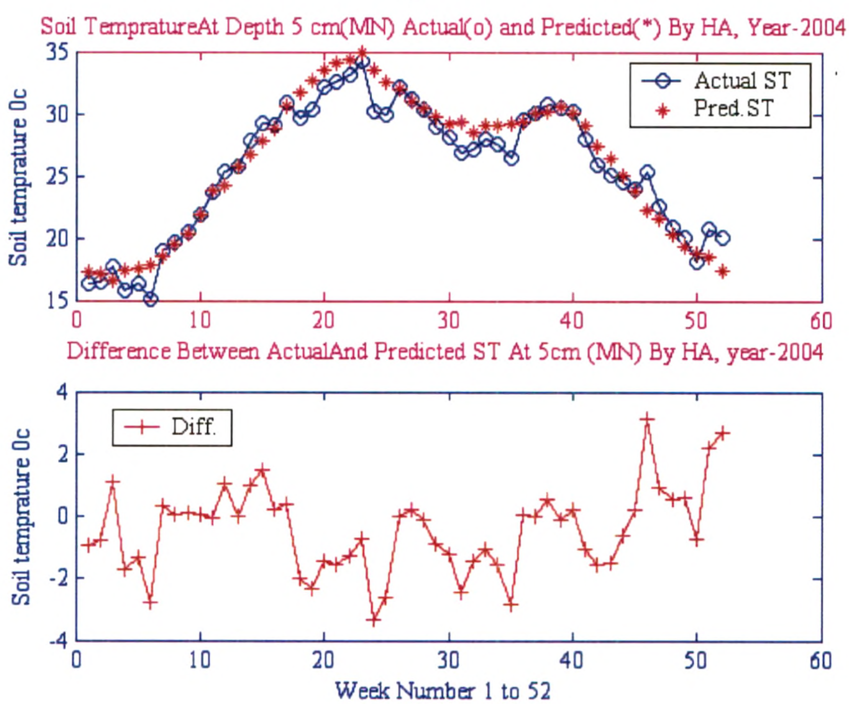

Fig: 3.25

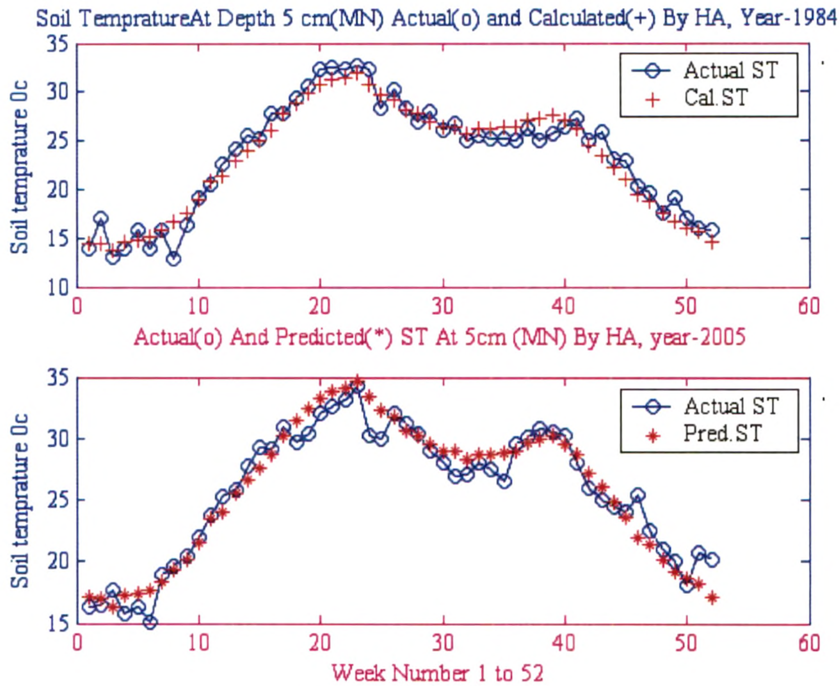

Fig: 3.26

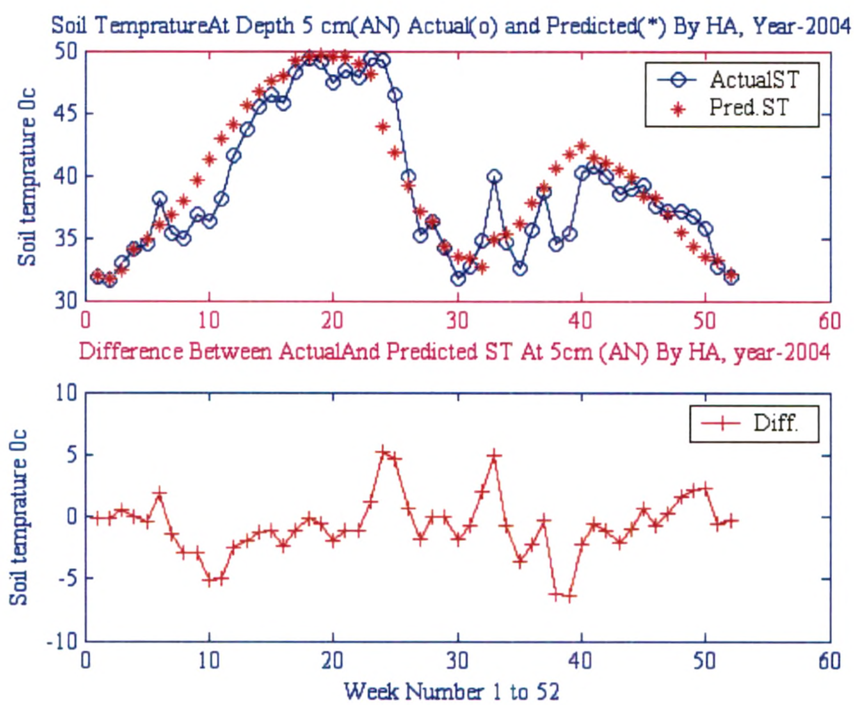

Fig: 3.27

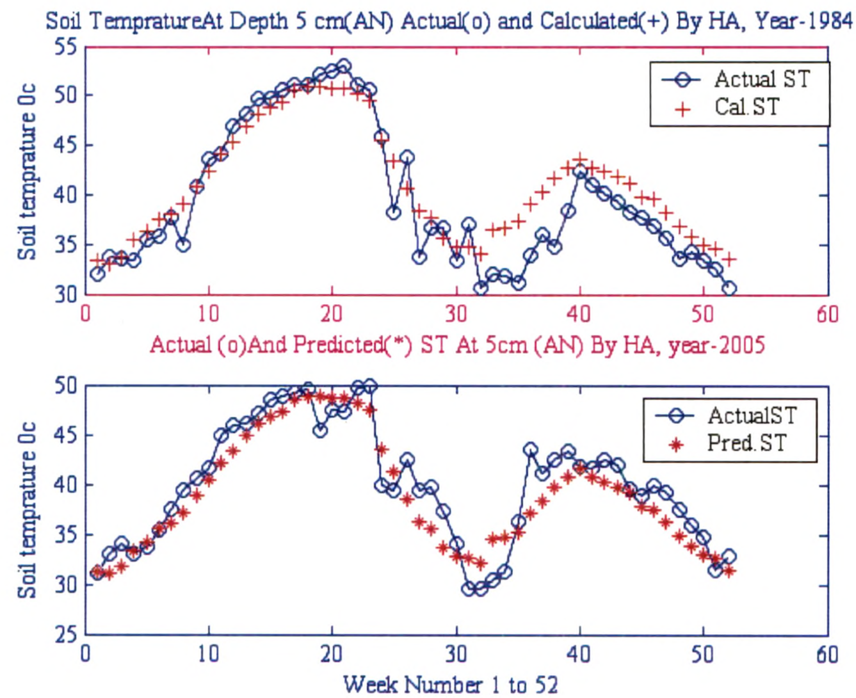

Fig: 3.28

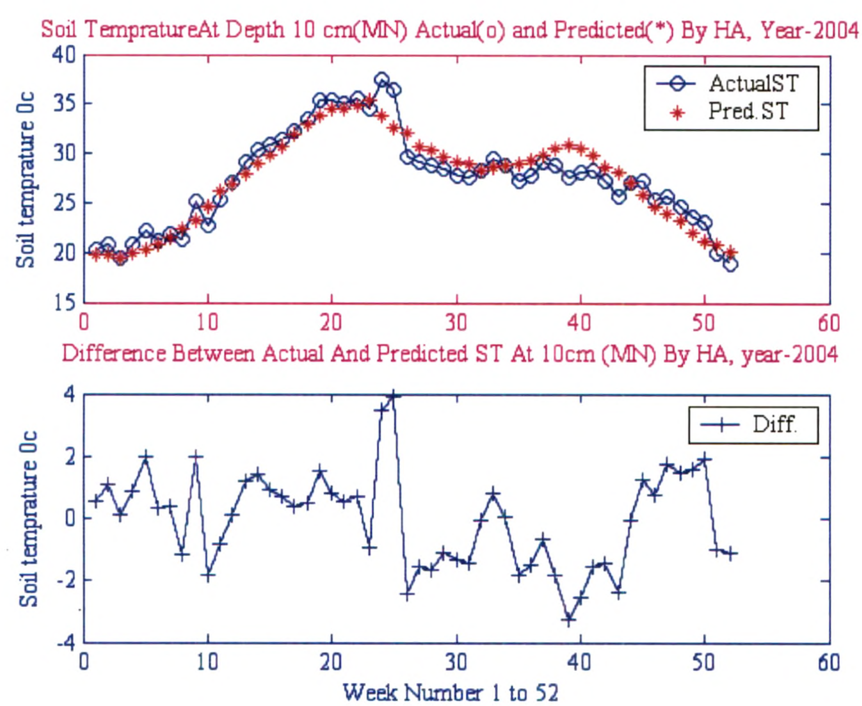

Fig: 3.29

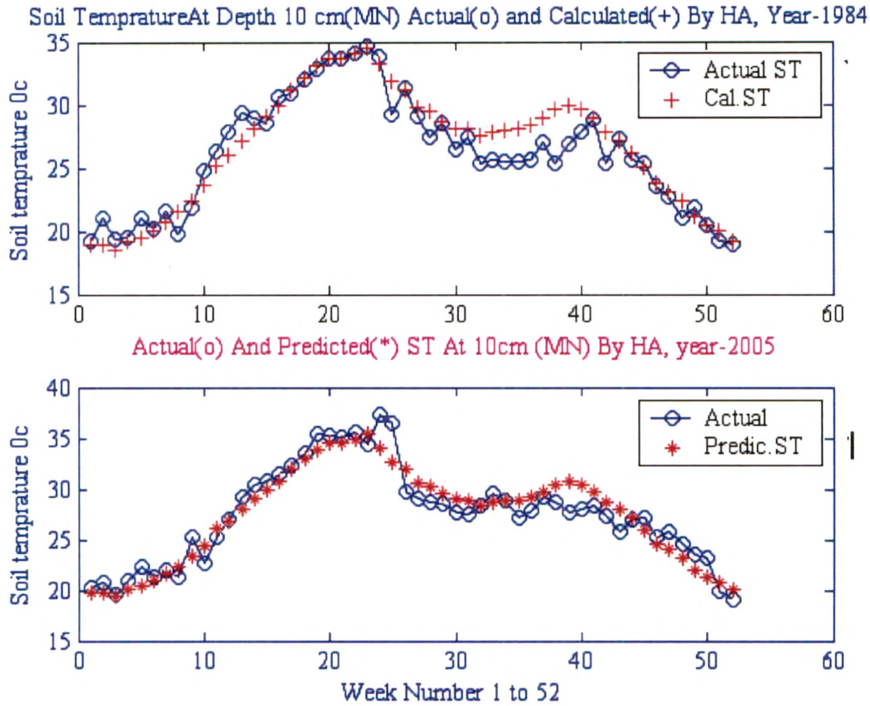

Fig: 3.30

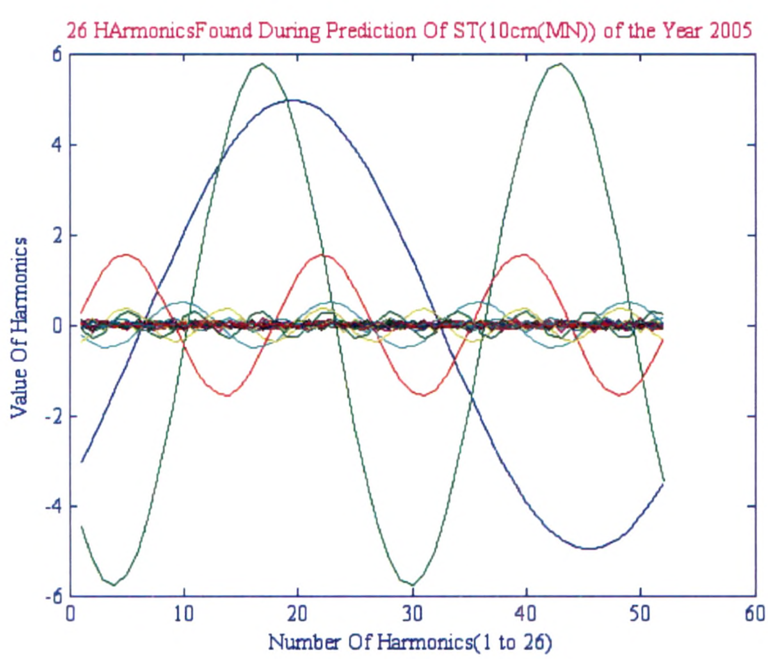

Fig: 3.31

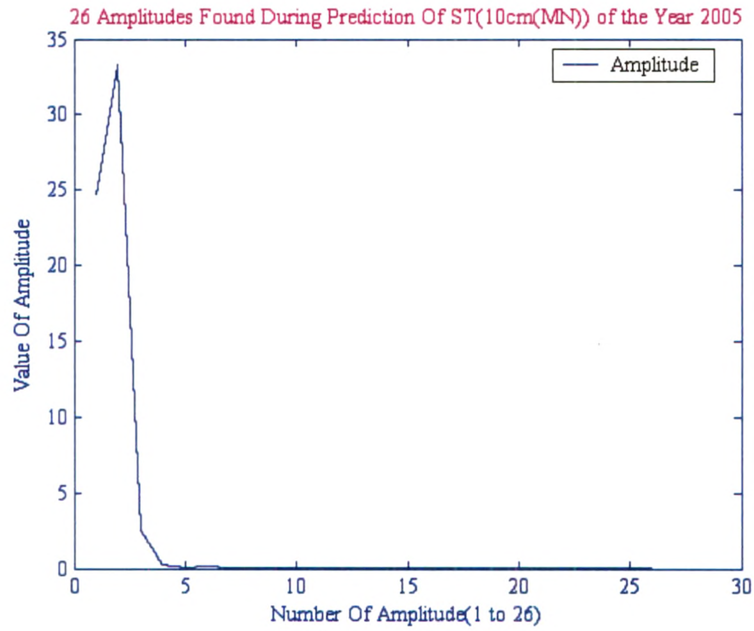

Fig: 3.32

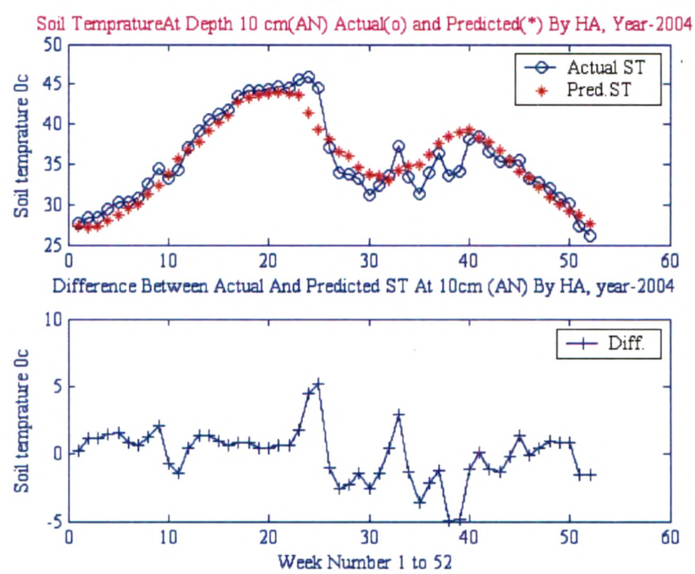

Fig: 3.33

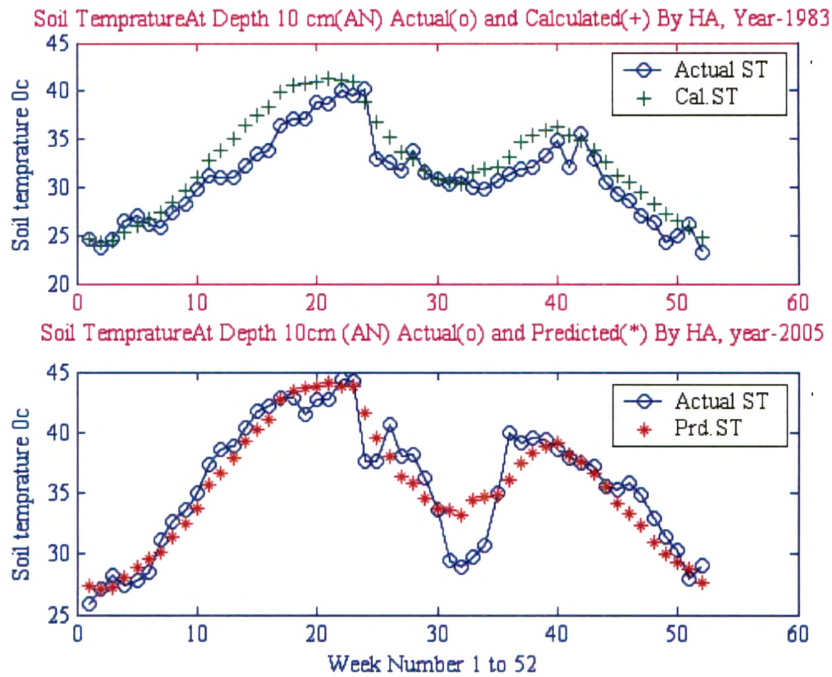

Fig: 3.34

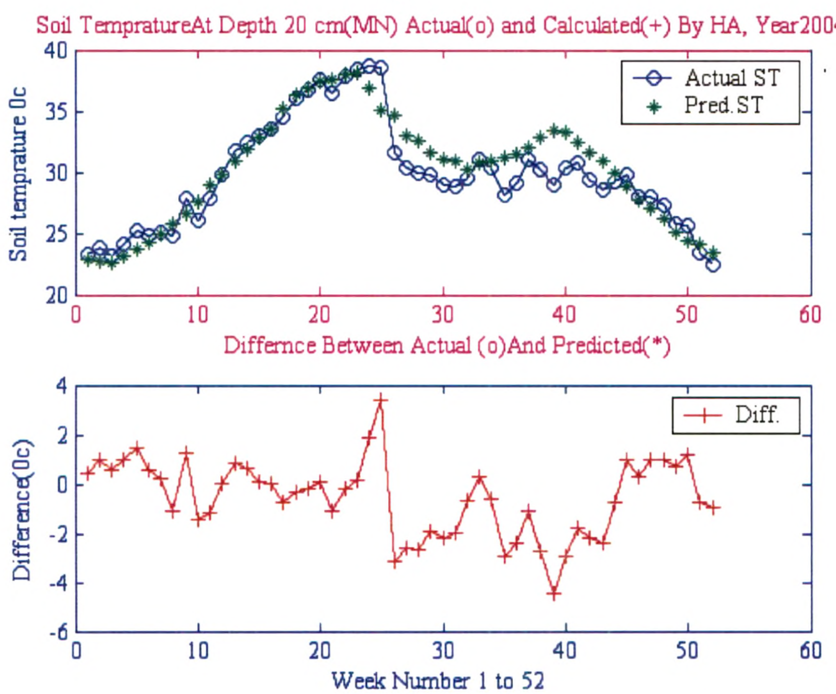

Fig: 3.35

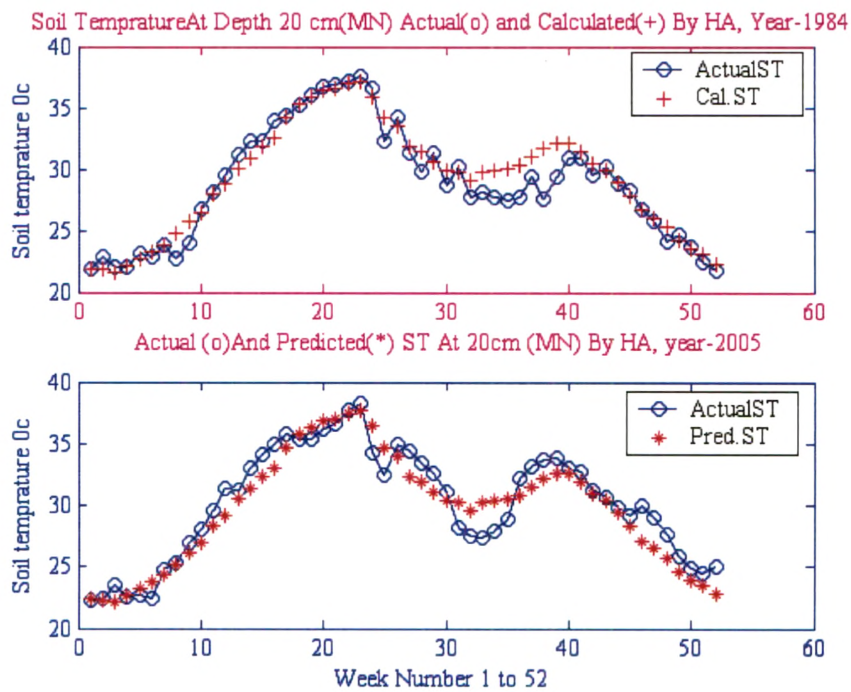

Fig: 3.36

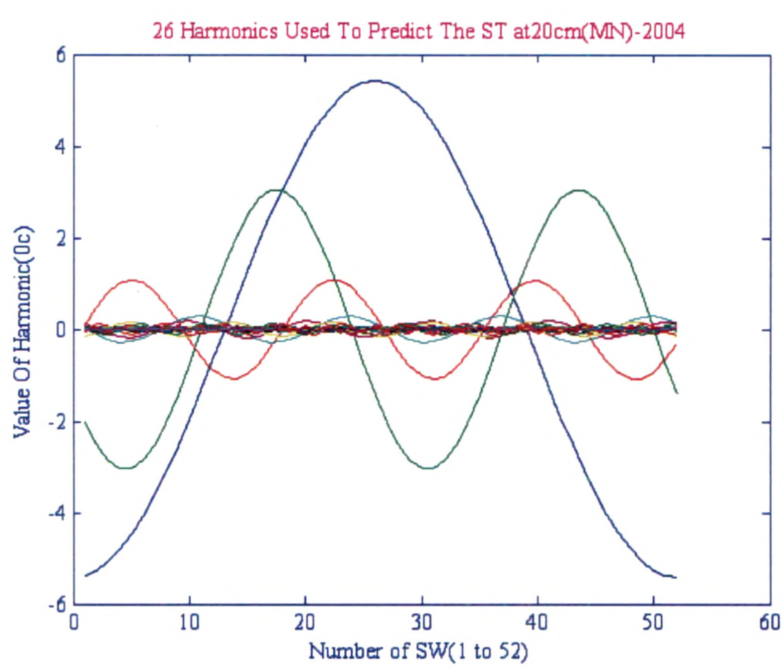

Fig: 3.37

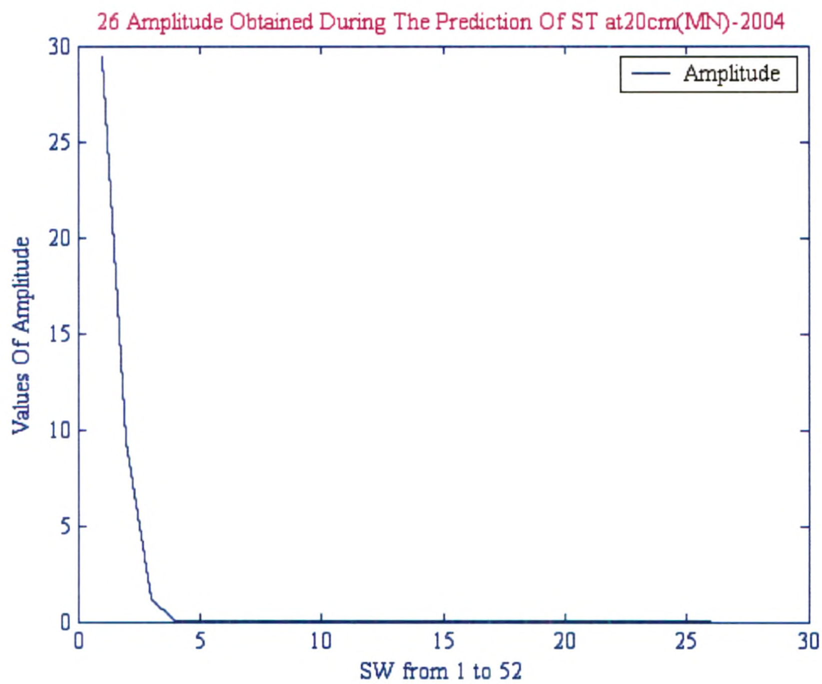

Fig: 3.38

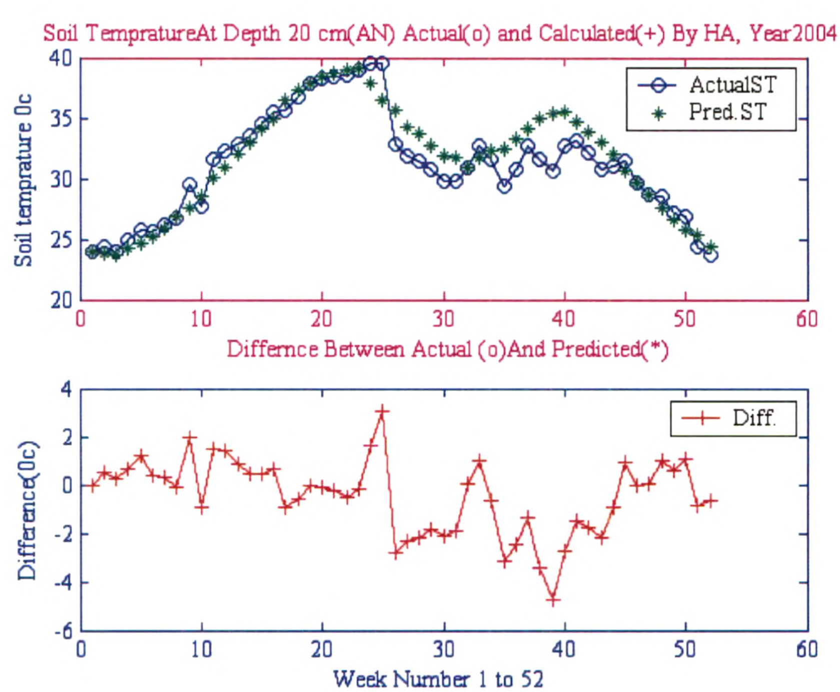

Fig: 3.39

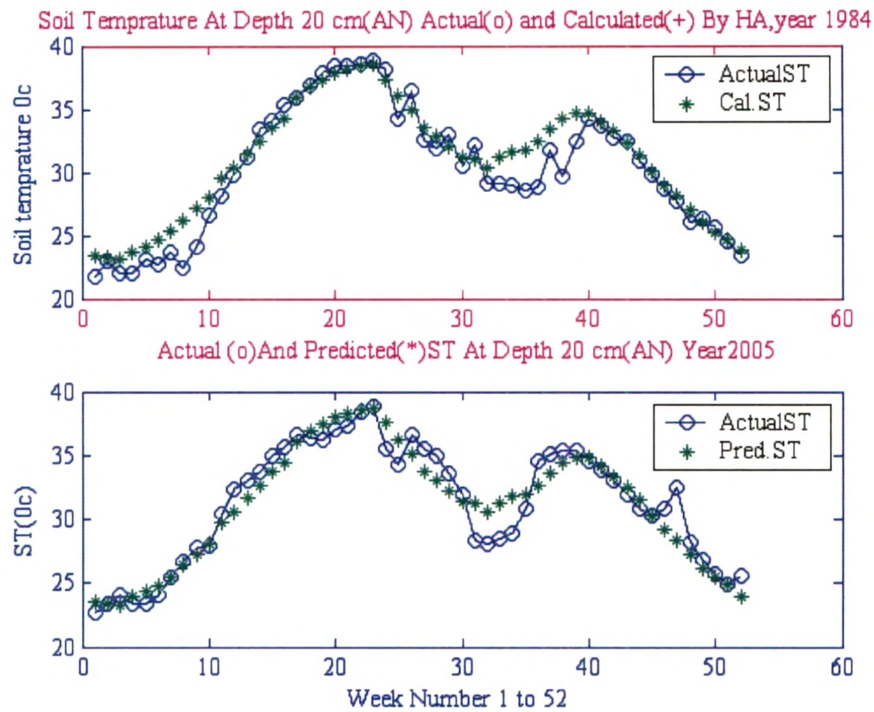

Fig: 3.40

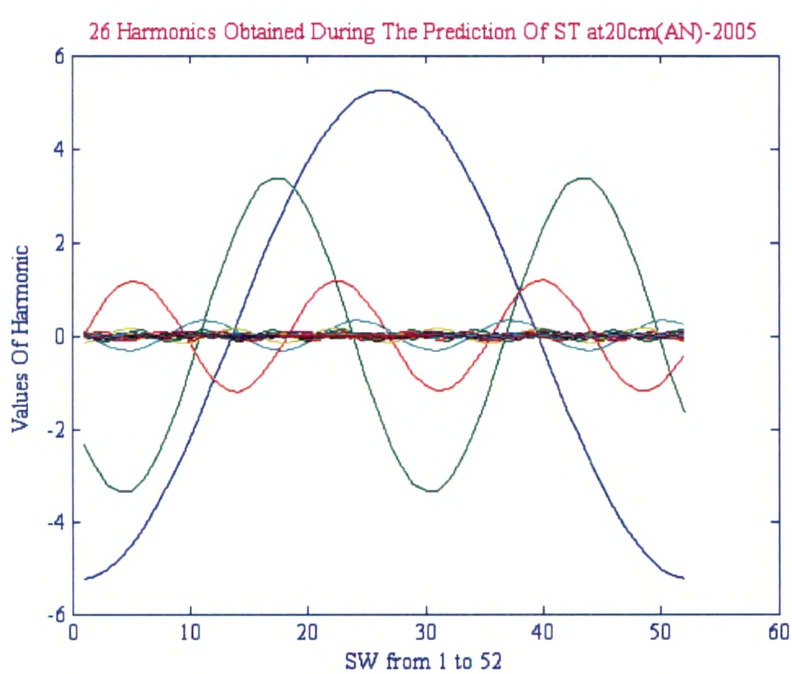

Fig: 3.41

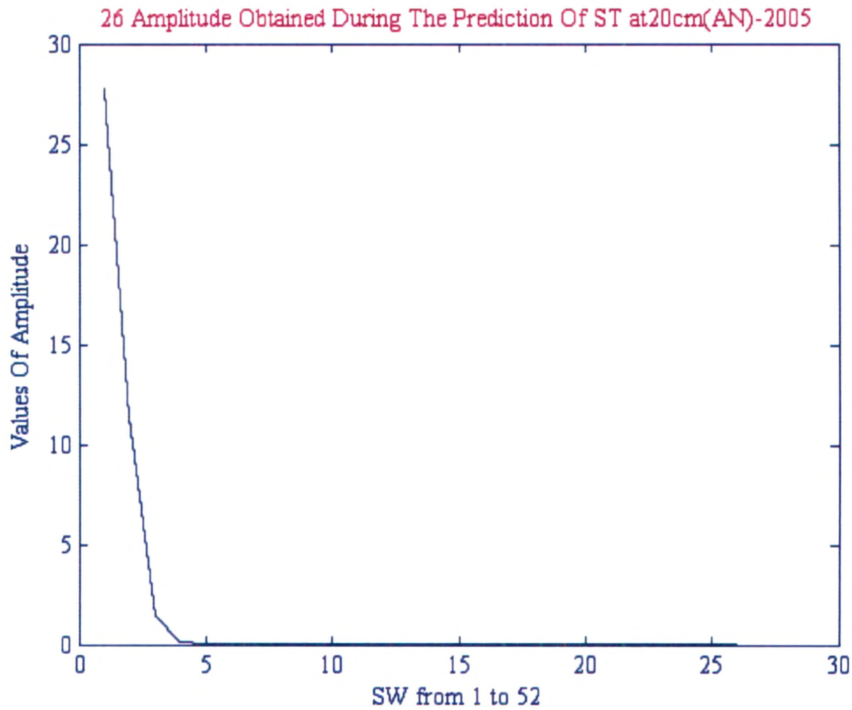

Fig: 3.42

TABLE 3.3
FIRST FIVE HARMONICS' PHASE ANGLE

Sr. No	Depth (cm)	First Five Phase Angle (Radian)	Sr. No	Depth (cm)	First Five Phase Angle (Radian)
1	5(MN)	-1.4017	16	10(AN)	-1.3939
2		-0.5553	17		-0.6075
3		-0.1064	18		-0.1577
4		0.9512	19		0.6143
5		1.4249	20		0.9584
6	5(AN)	-0.7803	21	20(MN)	-1.5645
7		-0.6460	22		-0.4818
8		-0.1949	23		-0.2729
9		0.0291	24		0.4953
10		0.3902	25		1.2292
11	10(MN)	-1.5045	26	20(AN)	-1.4958
12		-0.5330	27		-0.5215
13		-0.2712	28		-0.3306
14		0.6942	29		0.8093
15		1.4919	30		0.8129

TABLE 3.4
VARIATIONS ACCOUNTED FROM THE DATA BY INDIVIDUAL HARMONICS.

Sr. No	Predicting year	ST at Depth	No. of Harmonics m	Variations accounted by first five harmonics (%)	Total of variations accounted by 26 Harmonics (%)	Sr. No	Predicting year	ST at Depth	No. of Harmonics m	Variations accounted by first five harmonics (%)	Total of variations accounted by 26 Harmonics (%)
1	2004	5cm(MN)	26	100 54	112 17	8	2004	10cm(AN)	26	46 88	
				13.82						33.89	84 33
				2 08						3	
				0 23						0 22	
				0.097						0 039	
2	2005	5cm(MN)	26	95 24		9	2005	10cm(AN)	26	50 5	
				13.21	110.88					36.49	90 83
				1.94						-	
				0 21						-	
				0 0697						-	
3	2004	5cm(AN)	26	39.82		10	2004	20cm(MN)	26	77 4	
				54 5	99.45					24 73	≈100 00
				4.04						3 09	
				0.44						0 23	
				0.013						0 09	

Cont..

TABLE 3.4 (Cont.)

Sr. No	Predicting year	ST at Depth	No. of Harmonics	Variations accounted by first five harmonics (%)	Total of variations accounted by 26 Harmonics (%)	Sr. No	Predicting year	ST at Depth	No. of Harmonics	Variations accounted by first five harmonics (%)	Total of variations accounted by 26 Harmonics (%)
4	2005	5cm(AN)	26	35.7		11	2005	20cm(MN)	26	72.5	
				48.09	88 32					23 42	99.4
				3 55						2.88	
				0.39						0.22	
				0.017						0.059	
5	2004	10cm(MN)	26	71.61		12	2004	20cm(AN)	26	73.25	
				17.6	96.54					29.81	≈ 100.00
				1.81						-	
				0.17						-	
				0.06						-	
7	2005	10cm(MN)	26	71.61		13	2005	20cm(AN)	26	65.46	
				17.6	91.5					26.9	96.21
				1.81						3.38	
				0.17						0.26	
				0.057						0.018	

TABLE 3.5
RMSEs DURING HA AT DIFFERENT DEPTHS

Sr.No	Predicting year	ST at Depth (cm)	Absolute Maximum diff. (AMD) (^0C)	RMSE (^0C)	Percentage of Average Error (%)
1	2004	5 (MN)	3.33	1.74	6.74
2	2005	5 (MN)	3.52	1.36	5.32
3	2004	5 (AN)	6.40	2.44	6.26
4	2005	5 (AN)	6.37	2.37	5.93
5	2004	10 (MN)	3.63	1.58	5.76
7	2005	10 (MN)	2.75	1.31	4.73
8	2004	10 (AN)	4.74	1.89	5.30
9	2005	10 (AN)	4.67	1.90	5 32
10	2004	20 (MN)	3.88	1.62	5.46
11	2005	20 (MN)	3.26	1.42	4.73
12	2004	20 (AN)	4.15	1.58	5.09
13	2005	20 (AN)	4.17	1 39	4.44

3.8.9 CONCLUSION

Soil Temperatures (ST) is an important agro-meteorological element which influences seed germination, soil microbiological activity and chemical activity in the soil. Measurement of soil temperature at a large number of places in a region is difficult; this can be overcome by estimating soil temperature by Mathematical techniques. The following conclusions have been made while Harmonic analysis is employed.

i) The total variation accounted by the 26 of harmonics ranges between 84.33 to 112.17 %

ii) The testing the results of the Harmonic analysis on an individual year 2004 and 2005 for the depths 5, 10, and 20 for both the time period morning and afternoon resulted in a good agreement between the observed and estimated soil temperature except for the monsoon period of standard week from 23 to 35.

    Monsoon activity does not allow a simple sine curve to explain the complete annual cycle. The soil temperature profiles during monsoon are different from those of the other seasons due to infiltration of rainwater and their retention capillaries.

iii) Soil temperature at different depths could be predicted with fairly good accuracy by Harmonic analysis.

3.8.10 COMPARISON BETWEEN TWO METHODS

I) (a) ANN with Back Propagation algorithm and
(b) McCulloch Type with

II) Harmonic Analysis

TABLE 3.6
LIST OF AMDS AND RMSEs OBTAINED BY BOTH THE METHODS

Sr. No	Depth	AMD By HA (°C)	AMD By ANN (°C)	RMSE By HA (°C)	RMSE By ANN (°C)	PAE By HA (%)	PAE By ANN (%)
1	Depth 5 cm 2004 Morning	3.22	4.67	1.74	1.86	6.74	7.19
	2005	3.42	5.70	1.36	1 99	5.32	7.75
2	Depth 5 cm 2004 After noon	6.37	7.93	2.44	3.23	6.26	8.29
	2005	5.27	**9.50 largest**	2 37	**3.59 largest**	5.93	8.98
3	Depth 10 cm 2004 Morning	3.53	4.93	1.58	2.11	5 76	7.68
	2005	**2.63 smallest**	5.91	**1.31 smallest**	2 23	4.73	8.06
4	Depth 10 cm 2004 After noon	4.83	6.79	1.90	2.47	5.30	6.95
	2005	4.80	8.31	1.88	2.48	5.32	6.95
5	Depth 20 cm 2004 Morning	3 70	5.25	1.62	2.08	5.46	6.99
	2005	3.08	6.07	1.42	1.93	4.73	6.39
6	Depth 20 cm 2004 After noon	4 14	5.54	1.58	2.02	5.09	6.49
	2005	4.20	5.31	1.39	2 02	4.44	6.46

3.8.11 ADVANTAGES OF HA

1. Soil Temperature analysis by HA gives complete study of waves of solar energy by deriving amplitudes and phase angels. Also, it accounts percentages of variances by different number of harmonics (NH).

2. There is only one parameter, that needs to change to obtained a required accuracy in the model.
3. Obtained Root Mean Square Error, Percentage of Error and Absolute Mean Difference are very less in comparison to ANN method.

3.8.12 ADVATAGES OF ANN

1. ANN can be applied to any continuous process. ANN is a model free technique to establish a relationship between two variables. This model can be used for more than one input and output.
2. ANN can be improved by further training of the network with a suitable algorithm. Root Mean Square Error (RMSE) and Absolute Maximum Difference (AMD) obtained by both the methods are listed in the Table 3.6

3.9 CONCLUSION

In this chapter the soil temperature at 3 depth, 5 cm, 10 cm and 20 cm are predicted by using techniques like multiple regression, McCulloch Pitt, Multi layer and Harmonic analysis techniques. We have also proved a convergence theorem for McCulloch Pitt type network by invoking fixed-point theorem.

Though the harmonic analysis found to have less Root Mean Square Error (RMSE) and Percentage of Average Error (PAE) in comparison with ANN method, the difference is not significant. Here, RMSE and PAE are ranges by ANN are from 1.86 ^{0}C to 3.59 ^{0}C and 6.39% to 8.98%, respectively. During the application of harmonic analysis found RMSE and PAE are ranges from 1.31 0C to 2.44 0C and 4.44% to 6 74%.