APPENDIX

Mathematical Treatment of Decay of Isobars Formed in Nuclear Reaction:

Let T be a target containing N_T nuclei per unit area and ϕ be the flux of α-particles incident on it for a period of time t_i. During this period of irradiation, two isobaric nuclei P and D are produced through (α,xn) and (α, pxn) reactions with cross sections σ_p and σ_d respectively. Also the isobaric nuclei are all radioactive with disintegration constants λ_p and λ_d respectively and forming a radioactive decay chain

$$P \xrightarrow{\lambda_p} D \xrightarrow{\lambda_d}$$

here P stands for parent and D stands for daughter nucleus.

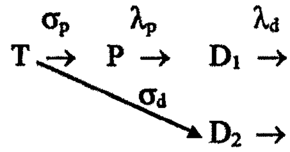
After a period of irradiation t_i, the products are allowed to decay for a waiting time t_w and then the activity of the daughter nucleus D is measured upto the time $t_w + t_c$ starting from t_w .Treating T as a long lived ($\lambda_T \cong 0$) leading member of a radioactive series.

$$T \xrightarrow[\sigma_P]{\lambda_T} P \xrightarrow{\lambda_p} D \xrightarrow{\lambda_d}$$

The reaction yield of the parent P through (α, xn) reaction with cross section σ_P can be simulated by using standard Batemann's equations /1/ and making the substitutions $T\,\lambda_T = \phi\, N_T\, \sigma_P$ and $\lambda_T \cong 0$, similarly the reaction yield of the daughter D through (α,pxn) reaction with cross section σ_d can be simulated by the radioactive series.

$$T \xrightarrow[\sigma_d]{\lambda_T} P \xrightarrow{\lambda_d}$$

With the following substitutions in Batemann's equations $T\,\lambda_T = \phi\, N_T\,\sigma_d$ and $\lambda_T \cong 0$. For simplicity of notation, let the letters T, P and D also stands for the members of radioactive nuclei of that kind, present at the instant of time under consideration.The above described scenario can be schematically represented as follows



Stage 1 : Upto the end of irradiation (t_i), according to Batemann's /1/ equations ,

$$P\,\lambda_p = (T\,\lambda_T)\,\frac{\lambda_p}{\lambda_p - \lambda_T}\,\left(1 - e^{(\lambda p - \lambda T)\,ti}\right)$$

Substituting $\lambda_T = 0$ and $T\,\lambda_T = \phi\,N_T\,\sigma_p$, one has

$$P_X = \frac{\phi\;N_T\;\sigma_p}{\lambda_p}\,\left(1 - e^{-\lambda p\,ti}\right) \tag{2}$$

$$D_1\lambda_d = T\lambda_T\left[\frac{\lambda_p}{\lambda_p - \lambda_T}\,\frac{\lambda_d}{\lambda_d - \lambda_T} + \frac{\lambda_p}{\lambda_T - \lambda_p}\,\frac{\lambda_d}{\lambda_d - \lambda_p}\,\left(e^{-(\lambda p - \lambda T)\,ti}\right) + \frac{\lambda_p}{\lambda_p - \lambda_d}\,\frac{\lambda_d}{\lambda_T - \lambda_d}\,\left(e^{-(\lambda d - \lambda T)\,ti}\right)\right]$$

substituting $\lambda_T = 0$ and $T\,\lambda_T = \phi\,N_T\,\sigma_p$,one has

$$D_1 = \frac{\phi\,N_T\,\sigma_p}{\lambda_d}\left[1 - \frac{\lambda_d}{\lambda_d - \lambda_P}\,e^{-\lambda p\,ti} - \frac{\lambda_p}{\lambda_p - \lambda_d}\,e^{-\lambda d\,ti}\right] \tag{3a}$$

also $$D_2\,\lambda_d = (T\lambda_T)\,\frac{\lambda_d}{\lambda_d - \lambda_T}\,\left(1 - e^{-(\lambda d - \lambda T)\,ti}\right)$$

substituting $\lambda_T = 0$ and $T\,\lambda_T = \phi\,N_T\,\sigma_d$, one has

$$D_2 = \frac{\phi\,N_T\,\sigma_d}{\lambda_d}\,\left(1 - e^{-\lambda d\,ti}\right) \tag{3b}$$

Combining (3a) and (3b) one defines

$D_X = D_1 + D_2$

$$D_X = \frac{\phi\,N_T\,\sigma_p}{\lambda_d}\,\left(1 - \frac{\lambda_d}{\lambda_d - \lambda_p}\,e^{-\lambda p\,ti} - \frac{\lambda_p}{\lambda_p - \lambda_d}\,e^{-\lambda d\,ti}\right) + \frac{\phi\,N_T\,\sigma_d}{\lambda_d}\,\left(1 - e^{-\lambda d\,ti}\right) \tag{3}$$

Stage 2: Irradiation ended, reaction is stopped . T is no longer present.

$$P \xrightarrow{\lambda p} D \xrightarrow{\lambda d}$$

At the start of the series decay i.e. at $t = 0$, $P = P_X$ and $D = D_X$

$P = P_X\, e^{-\lambda p t}$ is parent (4)

To obtain D , the rate equation is $dD/dt = P\lambda_p - D\,\lambda_d$

$\therefore\ dD/dt + D\lambda_d = P\lambda_p$, multiply both side by $e^{\lambda d\, t}$

$e^{\lambda d\, t}\ (dD/dt + D\lambda_d) = P\lambda_p\, e^{\lambda d\, t}$

$d/dt\,(De^{\lambda d\, t}) = P\lambda_p\, e^{\lambda d\, t}$

Integrating both sides, we get $De^{\lambda d\, t} = P_X\lambda_p \int e^{\lambda d\, t}\, dt + K_d$

$$D\,e^{\lambda d\, t} = P_X\,\lambda_p \int e^{-\lambda p t}\ e^{\lambda d\, t}\ dt + K_d$$

$$= \frac{P_X\,\lambda_p}{\lambda_d - \lambda_p}\ e^{(-\lambda p + \lambda d)t} + K_d$$

At $t = 0$, $D = D_X$

$$\therefore D_X = \frac{P_X\,\lambda_p}{\lambda_d - \lambda_p} + K_d$$

$$\therefore K_d = D_X - \frac{P_X\lambda_p}{\lambda_d - \lambda_p}$$

$$\therefore D = \frac{P_X\ \lambda_p}{\lambda_d - \lambda_p}\,(e^{-\lambda p t} - e^{-\lambda d t}) + D_X\ e^{-\lambda d t} \qquad (5)$$

The number of radioactive daughter nuclei that have decayed in the time between t_w and $t_w + t_c$ be designated as

$$[D_d]_{tw}^{tw+tc} = \int_{tw}^{tw+tc} \lambda_d\ D\,dt$$

$$= \int_{tw}^{tw+tc} \lambda_d\ \frac{P_X\,\lambda_p}{\lambda_d - \lambda_p}\,(e^{-\lambda p t} - e^{-\lambda d t}) + \int_{tw}^{tw+tc} D_X\, e^{-\lambda d t}\ \lambda_d$$

$$= \frac{P_X\,\lambda_p\,\lambda_d}{\lambda_d - \lambda_p}\left[\frac{e^{-\lambda p t}}{-\lambda_p} - \frac{e^{-\lambda d t}}{-\lambda_d}\right]_{tw}^{tw+tc} + D_X\,\lambda_d\left[\frac{e^{-\lambda d t}}{-\lambda_d}\right]_{tw}^{tw+tc}$$

$$= P_X \lambda_p \frac{\lambda_d}{\lambda_d - \lambda_p} \left\{ \frac{e^{-\lambda p\, tw} - e^{-\lambda p\,(tw+tc)}}{\lambda_p} - \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$+ D_X \lambda_d \left\{ \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$\therefore [D_d]_{tw}^{tw+tc} = \phi N_T \sigma_p (1 - e^{-\lambda p\, ti}) \frac{\lambda_d}{\lambda_d - \lambda_p} \left\{ \frac{e^{-\lambda p\, tw} - e^{-\lambda p\,(tw+tc)}}{\lambda_p} - \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$+ \phi N_T \sigma_p \left\{ 1 - \frac{\lambda_d}{\lambda_d - \lambda_p} e^{-\lambda p\, ti} - \frac{\lambda_p}{\lambda_p - \lambda_d} e^{-\lambda d\, ti} \right\} \left\{ \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$+ \phi N_T \sigma_d (1 - e^{-\lambda d\, ti}) \left\{ \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\} \qquad (6)$$

These $[D_d]_{tw}^{tw+tc}$ nuclei that had decayed in time interval between t_w and $t_w + t_c$ by emitting characteristic γ - rays of energy E_γ with an abundance of θ_γ per decay, these γ rays are detected by a detector with a photopeak efficiency P_γ corrosponding to the energy E_γ and recorded as a full enrgy peak of area A_γ

$$\therefore \quad A_\gamma = [D_d]_{tw}^{tw+tc} P_\gamma \theta_\gamma$$

$$\therefore \frac{A_\gamma}{P_\gamma \theta_\gamma} = [D_d]_{tw}^{tw+tc} \qquad (7)$$

N_T be the number of target atoms per unit area and is given as $N_T = W_i P_i N_{av} / A_i$ (8)

where W_i is the weight of the target foil per unit area, P_i is the isotopic abundance of the target . A_i is the mass number of the target and N_{av} is the Avagadro's number.Using (6), (7) and (8) one finally gets

$$\frac{A_i A\gamma}{\phi\theta\gamma P\gamma\, W_i P_i\, N_{av}} = \sigma_p (1 - e^{-\lambda p\, ti}) \frac{\lambda_d}{\lambda_d - \lambda_p} \left\{ \frac{e^{-\lambda p\, tw} - e^{-\lambda p\,(tw+tc)}}{\lambda_p} - \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$+ \sigma_p \left\{ 1 - \frac{\lambda_d}{\lambda_d - \lambda_p} e^{-\lambda p\, ti} - \frac{\lambda_p}{\lambda_p - \lambda_d} e^{-\lambda d\, ti} \right\} \left\{ \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\}$$

$$+ \sigma_d (1 - e^{-\lambda d\, ti}) \left\{ \frac{e^{-\lambda d\, tw} - e^{-\lambda d\,(tw+tc)}}{\lambda_d} \right\} \qquad (9)$$

/1/ Batemann, Proc. Cambridge Phil. Soc., **15**,423(1910).