Chapter 5

Hydrodynamic Evolution Of The Plasma

In the last chapter we have tried to study the preequilibrium evolution of the plasma in phase space through kinetic description.But the kinetic equations are not always very convenient to the study the collective processes, because of their complicated structure. On the other hand, under appropriate conditions it is possible to study non-equilibrium plasma, with a little lesser difficulty , with the help of hydrodynamic equations. There have been efforts[1] to obtain such a description of pre-equilibrium phase from the Boltzman-Vlasov kinetic equations. It is worth noting here that though the usual hydrodynamic description is applicable when the mean free path is much less than the scale length of the system and as a consequence the system reaches local thermodynamic equilibrium. On the other hand, if one assumes that the plasma is cold[2] i.e $V_{ph} >> V_{thermal}$ (such that one can define a fluid element over which the particles can have a coherent velocity) and particle density inside the Debye sphere is $>> 1$, then one can factorize in the cold collisionless limit the distribution function as $f(x,p,t) = g(x,t)\delta(p - \overline{p})$ and obtain a closed hydrodynamic description even in the collisionless limit.

In view of this, we proceed to formulate a classical hydrodynamic description for gluons in the cold collisionless limit, starting from the gauge covariant kinetic equation given by Elze, Gyulassy and Vasak (EGV[3]) neglecting all the terms of the order of $\hbar$ and higher. This may be justified because the collective effects we consider have length scales much greater than the compton wavelength, so that quantal corrections may not be very important.

We take the moments of the EGV equation, as described in the last chapter to get a most general hydrodynamic description for the gluons. This involves a set of 48 coupled nonlinear partial differential equations and would be difficult to solve even numerically.

The organisation of this chapter is as follows. In section two we start from the gluon kinetic equation of chapter 4 and take the momentum moments of this equation to generate a set of hydrodynamic equations for gluons under certain approximations. In section three we start from the classical kinetic equation for quarks, take the momentum moment of those equations to generate a set of chromo hydrodynamical equations to describe the space-time evolution of pre-equilibrium quark matter. In section four we show the formal similarity between the gluon hydrodynamic equations and the quark hydrodynamic equations.

In section five we study the collective oscillations of the plasma. We show the existence of certain conservation laws following from the hydrodynamic equations and try to solve these equations numerically obeying these conservation laws. The numerical solutions x show the existence of chaotic oscillàtions.

5.1 Towards Hydrodynamics Of Quarks And Gluons

In this section starting from the gluon kinetic equations we derive the gluon hydrodynamic equations. To derive the gluon hydrodynamic equations one essentially starts from the gluon kinetic equations of chapter 4. The kinetic equation can be written in terms of the combinations of different components of the distribution function that get coupled to each other (as a consequence of the nonabelian nature of the fields) to describe the evolution of the plasma.

From equation (4.7) we define diagonal components,

$$\begin{aligned} G^{11} &= 2G^{1} \\ G^{22} &= 2G^{2} \\ G^{33} &= 2G^{3} \end{aligned} \tag{5.1}$$

the symmetric combinations,

$$\begin{aligned} 2S^1 &= G^{23} + G^{32} \\ 2S^2 &= G^{31} + G^{13} \\ 2S^1 &= G^{12} + G^{21} \end{aligned} \tag{5.2}$$

and lastly the antisymmetric combinations,

$$\begin{aligned} 2iQ^1 &= G^{23} - G^{32} \\ 2iQ^2 &= G^{31} - G^{13} \\ 2iQ^3 &= G^{12} - G^{21} \end{aligned} \tag{5.3}$$

Since gluons belong to the adjoint representation of the appropriate unitary group, the distribution function for the gluons, for the SU(2) case can have three distinct irreducible representations i.e scalar, vector and second rank symmetric tensor in color space. It is worth recalling that for the quarks the distribution function can have only the scalar and vector representations. In the equations above S corresponds to the symmetric rank two tensor , Q corresponds to the vector representation and $tr\,(G)$ corresponds to the scalar representation in color space. Hence the hydrodynamic equations for quarks and gluons in general would not be the same. It is difficult to make much progress with the most general distribution, because of lack of knowledge about the 'd' coefficients arising out of anticommutation relations of the generators in the adjoint representations of the unitary group. In order to make some progress, in the next step, we will make an assumption, that all the symmetric combinations of the distribution function are zero. With this assumption one can show that the equations (4.7) reduce to

$$\begin{aligned} p_\mu \partial^\mu Q^1 - gp^\mu \left[A^2_\mu Q_3 - A^3_\mu Q_2\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 \left(G^2 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^2 - gp^\mu \left[A^3_\mu Q_1 - A^1_\mu Q_3\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^2 \left(G^1 + G^3\right)\right] &= 0 \\ p_\mu \partial^\mu Q^3 - gp^\mu \left[A^1_\mu Q_2 - A^2_\mu Q_1\right] + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 \left(G^1 + G^2\right)\right] &= 0 \end{aligned} \tag{5.4}$$

and

$$\begin{aligned} p_\mu \partial^\mu G^1 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \\ p_\mu \partial^\mu G^2 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^3 Q^3 + F_{\mu\nu}{}^1 Q^1\right] &= 0 \\ p_\mu \partial^\mu G^3 + g/2 p_\mu \partial_p{}^\nu \left[2F_{\mu\nu}{}^1 Q^1 + F_{\mu\nu}{}^2 Q^2\right] &= 0 \end{aligned} \tag{5.5}$$

The space time and color evolution of the macroscopic observables will be described by quantities obtained after taking moments of the one particle distribution function. For instance, the 4-color current flow in space will be given by

$$\int p^{\mu} Q_a(x,p)\, d^4p = Q_a{}^{\mu}(x) \tag{5.6}$$

and the general four momentum flow is given by

$$\int p^{\mu} \sum_{a=1}^{3} G_a(x,p)\, d^4p = G^{\mu}(x) \tag{5.7}$$

This flow tensors can further be decomposed (following Stewart[4]) as

$$Q_a{}^{\mu} = n_a U^{\mu} E \qquad \text{and} \qquad G^{\mu} = n U^{\mu} E \tag{5.8}$$

Here n_a is the color triplet number density of gluons, E is their energy and $U^{\mu}(x)$ is the four velocity, with

$$EU^{\mu} = \frac{\int p^{\mu} Q_a(x,p)\, d^4p}{\int Q_a(x,p)\, d^4P} \quad \text{and} \quad U^{\mu} U_{\mu} = 0 \tag{5.9}$$

In fact following equation (5.8) one can decompose any higher order tensor. For example a second rank tensor can be written as,

$$Q_a{}^{\mu\nu} = n_a U^{\mu} U^{\nu} E^2 \tag{5.10}$$

To derive the hydrodynamic equation from the kinetic equations (5.4) and (5.5) we take the zeroth (and the first) momentum moments of those equations. In the next stage we assume $G_1 = G_2 = G_3 = G/3$ and add all the components of equations (5.5) to get

$$\begin{aligned} &\partial_{\mu} \quad G^{\mu} = 0 \\ &\partial_{\mu} \quad Q_a{}^{\mu} - g\epsilon_{abc} A_{\mu}{}^{b} Q^{\mu}{}_{c} = 0 \end{aligned} \tag{5.11}$$

In the following step, as was shown in equation (5.8) we decompose the 4-vectors in equation (5.11) to arrive at

$$\begin{aligned} &\partial_{\mu} \quad [n_a U^{\mu}] - g\epsilon_{abc} A_{\mu}{}^{b} n_c U^{\mu} = 0 \\ &\partial_{\mu} \quad [nU^{\mu}] = 0 \end{aligned} \tag{5.12}$$

From equation (5.12), one can show by defining a quantity called color charge, as $I_a = \frac{n_a}{n}$, that

$$\begin{aligned} &\partial_{\mu} \qquad [nU^{\mu}] = 0 \\ &\textit{and} \\ &U^{\mu}\partial_{\mu} \quad [I_a] - g\epsilon_{abc} A_{\mu}{}^{b} I_c U^{\mu} = 0 \end{aligned} \tag{5.13}$$

Similarly from the first momentum moment equation one can get, the force equation or conservation of energy momentum relation

$$U^{\mu}\partial_{\mu}U^{\nu} = \frac{g}{E}F_{a}{}^{\mu\nu}U_{\mu}I^{a} \tag{5.14}$$

For studying the collective behaviour of the system one needs to solve the equation

$$\begin{aligned} \partial_{\mu}\left[nU^{\mu}\right] &= 0 \\ U^{\mu}\partial_{\mu}U^{\nu} &= \tfrac{g}{E}F_{a}{}^{\mu\nu}U_{\mu}I^{a} \\ \partial_{\mu}\left[n_{a}U^{\mu}\right] - g\epsilon_{abc}A_{\mu}{}^{b}n_{c}U^{\mu} &= 0 \\ \textit{and} \qquad & \\ D_{\mu}F_{a}{}^{\mu\nu} = J_{a}{}^{\mu} &= gnI_{a}U^{\mu} \end{aligned} \tag{5.15}$$

self consistently. The first two equations above are the usual continuity and force balance equations respectively. The third equation, characteristic of non-abelian dynamics, is the color evolution equation. In the next section we will show that the quark hydrodynamic equations also take the same form but without any such approximation.

5.2 Towards Quark Hydrodynamics

In this section we try to arrive at the hydrodynamic equations for quarks as obtained by Kajantie and Montonen[5], starting from the kinetic equations of quarks as gotten by EGV. To derive the equations for classical quark matter, we proceed as follows. We start from the gauge covariant kinetic equations of Elze, Gyulassy and Vasak, setting the terms of the order of $\hbar = 0$.

$$p^{\mu}D_{\mu}W(x,p) + g/2p^{\mu}\partial_{p}{}^{\nu}\left[F_{\mu\nu}, W(x,p)\right]_{+} = 0 \tag{5.16}$$

Here $[,]_{+}$ means anticommutator. The distribution function W(x,p) apart from its spin structure, is a hermitian matrix in color space. It can be written in terms of a color singlet and triplet components for SU(2) as

$$W(x,p) = \frac{1}{2}\langle G\rangle 1 + \frac{1}{2}\sum_{a=1}^{3}\lambda_{a}\langle G^{a}\rangle \tag{5.17}$$

Here $\langle G\rangle = TrW(x,p)$ and $\langle G_{a}\rangle = Tr\left[\lambda_{a}W(x,p)\right]$. Using equation (5.17) one can write equation(5.16) in terms of a set of coupled partial differential equations. The coupling is between the singlet and triplet distribution function for quarks. They are as follows,

$$\begin{aligned} p_{\mu}\partial^{\mu}\langle G\rangle + gp_{\mu}\partial_{p}{}^{\nu}\left[F_{\mu\nu}{}^{a}\langle G^{a}\rangle\right] &= 0 \\ p_{\mu}\partial^{\mu}\langle G^{a}\rangle + g\epsilon_{abc}p^{\mu}A^{b}{}_{\mu}\langle G^{c}\rangle + gp_{\mu}\partial_{p}{}^{\nu}\left[2F_{\mu\nu}{}^{a}\langle G^{a}\rangle\right] &= 0 \end{aligned} \tag{5.18}$$

As it has been shown in the earlier section, one can take the momentum moments of these equations to generate the hydrodynamic equations. Before we go for generating the hydrodynamic equations it is worth noting that since the quarks are massive particles the 4-velocities for quarks obey

$$U^{\mu}U_{\mu} = 1 \qquad \text{and} \qquad mU^{\mu} = \frac{\int p^{\mu}G_a(x,p)\,d^4p}{\int G_a(x,p)\,d^4p} = \frac{\int p^{\mu}G(x,p)\,d^4p}{\int G(x,p)\,d^4p} \tag{5.19}$$

On taking the zeroth moment of equation (5.18) we arrive at

$$\begin{gathered} \partial_{\mu}[G^{\mu}] = 0 \\ \textit{and} \\ \partial_{\mu}[G_a{}^{\mu}] + g\epsilon_{abc}A_{\mu}{}^{b}G_c{}^{\mu} = 0 \end{gathered} \tag{5.20}$$

Decomposing 4-vectors $G^{\mu} = mnU^{\mu}$ and $G_a{}^{\mu} = mn_aU^{\mu}$ we get,

$$\begin{gathered} \partial_{\mu}[nU^{\mu}] = 0 \\ \textit{and} \\ \partial_{\mu}[U^{\mu}n_a] - g\epsilon_{abc}A_{\mu}{}^{b}n_cU^{\mu} = 0 \end{gathered} \tag{5.21}$$

Defining $I^a = \frac{n_a}{n}$ and using the two equations one can obtain the color evolution equation, namely

$$U^{\mu}\partial_{\mu}[I_a] - g\epsilon_{abc}A_{\mu}{}^{b}I_cU^{\mu} = 0 \tag{5.22}$$

Similarly on taking the first momentum moment of equation (5.18) and using similar decomposition as before we arrive at

$$U^{\mu}\partial_{\mu}U^{\nu} = \frac{g}{m}F_a{}^{\mu\nu}U_{\mu}I^a \tag{5.23}$$

Here repeated indices are summed up. If one multiplies equation (5.23) by U^{ν} one can show that the condition $U^{\mu}U_{\mu} = 1$ is satisfied. One can rewrite equation (5.23) as

$$U^{\mu}\partial_{\mu}U^{\nu} = \frac{g}{m}F_a{}^{\mu\nu}J_{\mu}{}^{a} \quad \text{with} \quad J^{\mu}{}_{a} = nU_{\mu}I^a \tag{5.24}$$

One can derive an identical set of hydrodynamic equations as Kajantie and Montonen, provided one considers a multicomponent distribution function, $G_A(x,p)$ where the label A stands for different species of quarks and one decomposes the color singlet and color triplet 4-vectors and Lorentz tensors as

$$\begin{aligned} G_A{}^{\mu} &= m_An_AU_A{}^{\mu} \\ G_{aA}{}^{\mu} &= m_An_{aA}U_A{}^{\mu} \\ G_a{}^{\mu\nu} &= m_An_{aA}U_A{}^{\mu}U_A{}^{\nu} \\ G_{aA}{}^{\mu\nu} &= m_An_{aA}U_A{}^{\mu}U_A{}^{\nu} \end{aligned} \tag{5.25}$$

The repeated indices A are not summed. Starting from gauge covariant kinetic equations of EGV, we have derived at the Kajantie Montonen hydrodynamic equations for both quarks and gluons. In the next section we will study their collective behaviour non perturbatively, by numerically solving the equations.

5.3 Study of Collective Oscillation of the Plasma

In the earlier section we have obtained the quark hydrodynamic equations. These equations for different species of quarks can be written as,

$$\begin{aligned} \frac{\partial n_A}{\partial t} + \nabla (n_a V_A) &= 0 \\ \left(\frac{\partial}{\partial t} + V_A \nabla\right) V_A &= \tfrac{g}{m} I_A{}^a \left[E_A + V_A \times B_A\right] \\ \left[\frac{\partial}{\partial t} + V_A \nabla\right] I_A{}^a &= g\epsilon_{abc} \left[A_b{}^o - V_A A_b\right] I_A c \end{aligned} \tag{5.26}$$

To study the collective oscillations of the system one has to solve these equations along with the Yang-Mills equations self-consistently. In order to extract the essential non-abelian physics, we have simplified the equations by removing most of the non-essential complications by assuming that in the x and y directions the plasma is homogeneous so that the fluid variables have zero gradient in x and y direction, i.e $\partial_x = \partial_y = 0$. Next we will look for special solutions (ref Coleman[6]) where the fluid as well as the field variables are functions of a single variable denoted by

$$\tau = t\beta + z \tag{5.27}$$

In this stationary frame ansatz, β is the frame velocity. We also assume that of the two species of quarks, one is much heavier than the other, such that the second species is relatively at rest compared to the first and it acts as a (neutralising) background. So with these assumptions one can solve the fluid equations analytically to get

$$n_A = \frac{n_o \beta}{V_A + \beta} \tag{5.28}$$

Here $n_{o1} = n_{o2} = n_o$ is the equilibrium density. Note that for $V_2 = 0$ we get $n_2 = n_o$. For the velocities we have assumed V_{1x} and V_{1z} to be nonzero, so that on solving the respective equations we get V_{1x} and V_{1z}

$$V_{1x} = V_x = \frac{(g I_a A^a{}_x)}{m} \tag{5.29}$$

$$V_{1z} = V_z = \frac{1}{2mn_0\beta}\left[(\beta^2-1)\left(\dot{A_a}^{x2} - \dot{A}(0)_a{}^{x2}\right) + \beta^2\left(\dot{A_a}^{z^2} - A(\dot{0})_a{}^{z^2}\right)\right]$$
$$-g(\beta^2-1)\,\epsilon_{abc}\left(A^x{}_aA^z{}_bA^x{}_c\right) - \frac{g^2}{2}\left[(A^x_cA^z_b)^2 - (A^z_bA^z_c)(A^x_cA^x_b)\right]$$
$$-gI_{a0}A^z{}_a n_0\beta \tag{5.30}$$

$$I_a = -\frac{\epsilon_{abc}}{n_o\beta}\left[\left(\beta^2-1\right)(A^x_b)\left(\dot{A}^x{}_c\right) + \beta^2 A^z_b\dot{A}^z{}_c\right] - \frac{gA^x{}_b}{n_0\beta}\left(A^z_aA^x_b - A^x_aA^z_b\right) + I_{a0} \tag{5.31}$$

Thus we have expressed n_A, V_A and I_A in terms of the color potentials and their derivatives. The Yang Mills equations are

$$\left(\beta^2-1\right)\ddot{A_a}^x + g\epsilon_{abc}\left[2A^z{}_b\dot{A}^x{}_c + \dot{A}^x{}_c\dot{A}^z{}_b\right] + g^2A^z{}_b\left(A^x{}_aA^z{}_b - A^z{}_aA^x{}_b\right) = -\left(\frac{gn_o\beta I_aV_x}{V_z+\beta}\right) \tag{5.32}$$

$$\left(\beta^2\right)\ddot{A_a}^z - g\epsilon_{abc}\left[A^x{}_b\dot{A}^x{}_c\right] - g^2A^x{}_b\left(A^x{}_aA^z{}_b - A^z{}_aA^x{}_b\right) = -\left(\frac{gn_o\beta I_aV_z}{V_z+\beta}\right) \tag{5.33}$$

From the geometry that we have chosen, one can see that there is no force in the x direction. However in the x direction there exists a canonical momentum which is obviously conserved from equation (5.29). From these sets of equations (i.e (5.29) to (5.33)) one gets two conserved quantities

$$I_1{}^2 + I_2{}^2 + I_3{}^2 = constant \tag{5.34}$$

$$\begin{aligned}&(\beta^2-1)\left[\dot{A}^{x^2}_a - \dot{A}^{x^2}_a(0)\right]\\ &+\beta^2\left[\dot{A}^{z^2}_a - \dot{A}^{z^2}_a(0)\right]\\ &+g^2A^z{}_bA^x{}_c\left(A^z{}_bA^x{}_c - A^z{}_cA^x{}_b\right)\\ &+n_om\left[V_x{}^2 + V_z{}^2\right] = \varepsilon\end{aligned} \tag{5.35}$$

where ε can be termed as energy. The first equation is the color conservation equation and the second equation is the energy conservation equation. These quantities depend on the initial value of the variables and the given parameters and they remain constant throughout the evolution of the system. We next scale the variables to bring the equations to dimensionless form. We choose

$$\begin{aligned}A_a{}^x &= a_oA_a{}^x\\ I_a{}^x &= I_oI_a{}^x\end{aligned} \tag{5.36}$$

Where, a_o and I_o have dimension of length and charge. Next we redefine our independent variable as

$$\zeta = \omega_p \tau \tag{5.37}$$

where τ is like time and ω_p is abelian plasma frequency parameter.

$${\omega_p}^2 = \frac{\left(g^2 {n_o}^2 {I_o}^2\right)}{m} \tag{5.38}$$

We also introduce two more dimensionless parameter

$$\begin{aligned} t &= \left(\tfrac{g n_o I_o}{m}\right) \\ r &= \left(\tfrac{g a_o}{\omega_p}\right) \end{aligned} \tag{5.39}$$

One can check that $r * t = \epsilon$ is the non abelian parameter defined in (ref:7). The scaled equations then take the form

$$V_x = (I_a {A^a}_x) \tag{5.40}$$

$$\begin{aligned} V_z = \; & \tfrac{t}{2\beta}\left[(\beta^2-1)\left(\dot{A_a}^{x^2} - \dot{A}(0)_a{}^{x2}\right) + \beta^2\left(\dot{A_a}^{z^2} - A(\dot{0})_a{}^{z^2}\right)\right] \\ & -\tfrac{rt}{\beta}(\beta^2-1)\,\epsilon_{abc}\left({A^x}_a {A^z}_b {A^x}_c\right) - \tfrac{r^2 t}{2\beta}\left[(A^x_c A^z_b)^2 - (A^z_b A^z_c)(A^x_c A^x_b)\right] \\ & -{A^z}_a \end{aligned} \tag{5.41}$$

$$I_a = -\frac{rt\epsilon_{abc}}{\beta}\left[\left(\beta^2-1\right)(A^x_b)\left(\dot{A^x}_c\right) + \beta^2 A^z_b \dot{A^z}_c\right] - \frac{g r^2 t {A^x}_b}{\beta}\left(A^z_a A^x_b - A^x_a A^z_b\right) + 1 \tag{5.42}$$

The Yang Mills equations are

$$\left(\beta^2-1\right)\ddot{A_a}^x + r\epsilon_{abc}\left[2{A^z}_b \dot{A^x}_c + \dot{A^x}_c \dot{A^z}_b\right] + r^2 {A^z}_b\left({A^x}_a {A^z}_b - {A^z}_a {A^x}_b\right) = -\left(\frac{\beta I_a V_x}{tV_z+\beta}\right) \tag{5.43}$$

$$\left(\beta^2\right)\ddot{A_a}^z - r\epsilon_{abc}\left[{A^x}_b \dot{A^x}_c\right] - r^2 {A^x}_b\left({A^x}_a {A^z}_b - {A^z}_a {A^x}_b\right) = -\left(\frac{\beta I_a V_z}{tV_z+\beta}\right) \tag{5.44}$$

We will now solve these equation numerically with the help of 4th order Runge Kutta iteration scheme, as the analytical solution is beyond our reach. Before describing the numerical results we would like to show that in the absence of any nonlinearity parameter the equations for the potentials take the uncoupled form

$$\begin{aligned}(\beta^2 - 1)\,\ddot{A^x}_a + \textstyle\sum_{a=1}^{3} A_a{}^x &= 0\\ (\beta^2)\,\ddot{A^z}_a + \textstyle\sum_{a=1}^{3} A_a{}^x &= 0\end{aligned} \tag{5.45}$$

which indicates that in absence of all the nonlinearity the modes execute a simple harmonic oscillation with frequency $\frac{\sqrt{3}}{\beta}$ when β is large.

Next we choose the initial conditions as in reference (7), and Fig(5.1) shows the same earphone like oscillation obtained in ref:7 fig(2). Here the $A^x{}_a$ and $\dot{A}^x{}_a$ are all zero throughout the course of integration.

In our plots we show the momentum profiles. To see the effect of a small transverse perturbation on longitudinal oscillation we keep all $A^x = 0$, and $\dot{A}^x \simeq 10^{-5}$. One can see from the figures(5.2) and (5.3) that till t little beyond $600\omega_p$ the velocity profiles execute the same mode, when there is a catastrophic jump in the velocity profile in x direction and correspondingly the coherent oscillation in z direction breaks up into a chaotic one.

One interesting thing has been observed in all the oscillation that they try to execute a coherent mode for couple of periods, even if the profile is globally chaotic. We have carried out an extensive numerical analysis of the above hydrodynamic equations[7], namely taking different initial conditions for both the field variables and their first derivatives, varying the parameters of the equations such as non-abelian nonlinearity parameter, plasma nonlinearity parameter and the frame velocity β. The central observation of all the numerical experiment is that, except for a very few points in the parameter space, for most of the other points the system tends to go to a state of chaotic oscillation. On carrying out the FFT (fast fourier transform) analysis of the numerical solutions we have seen that the most dominant frequency for both V_x and V_z components are the same, implying energy equilibration.

5.4 Conclusion

The fundamental conclusion we reach, in addition to the conclusion of Bhatt et. al, is that, other than the non linearity parameters, even the presence of a small

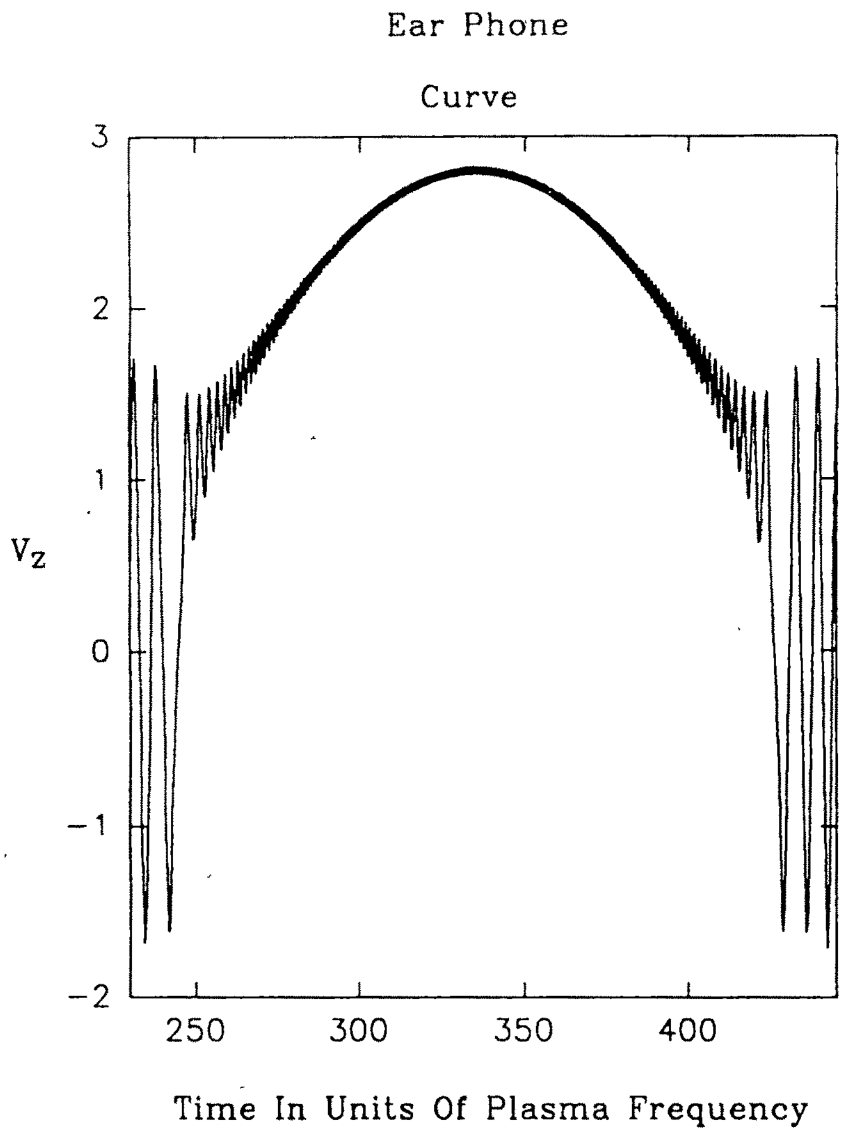

Figure 5.1: The Profile for V_z Oscillation (with out the transverse field).

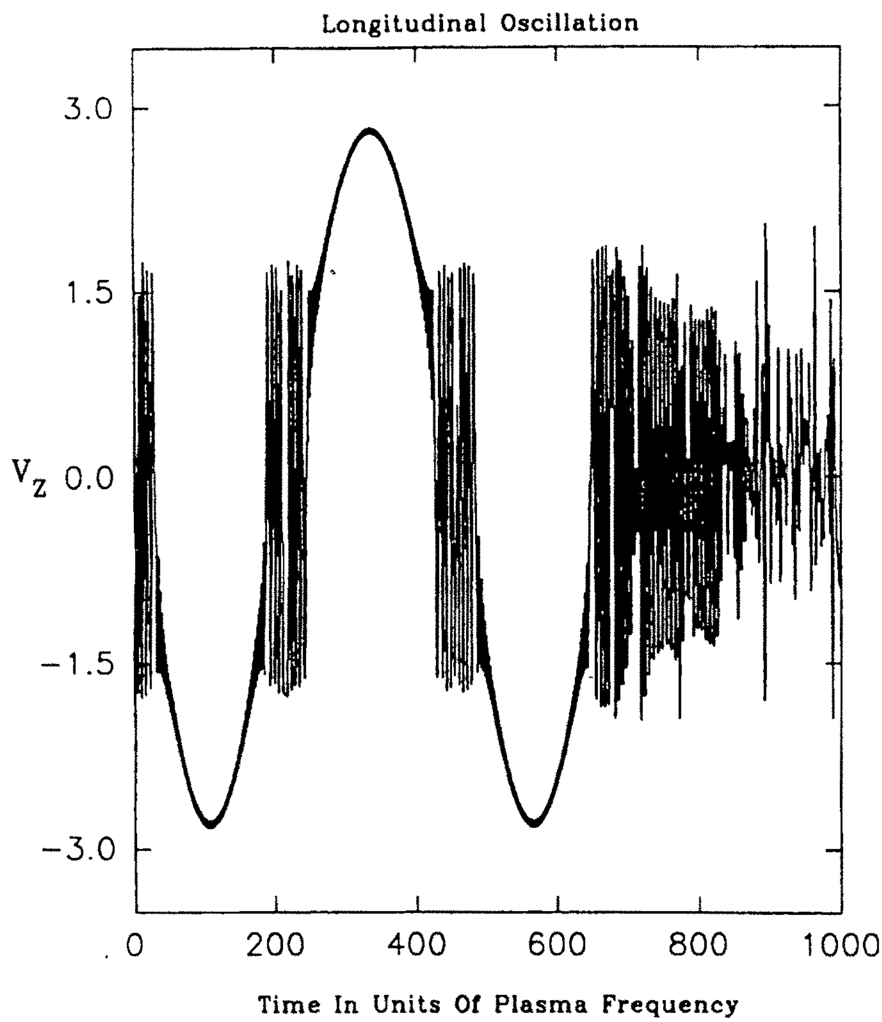

Figure 5.2: The Profile for V_z Oscillation (with the transverse field).

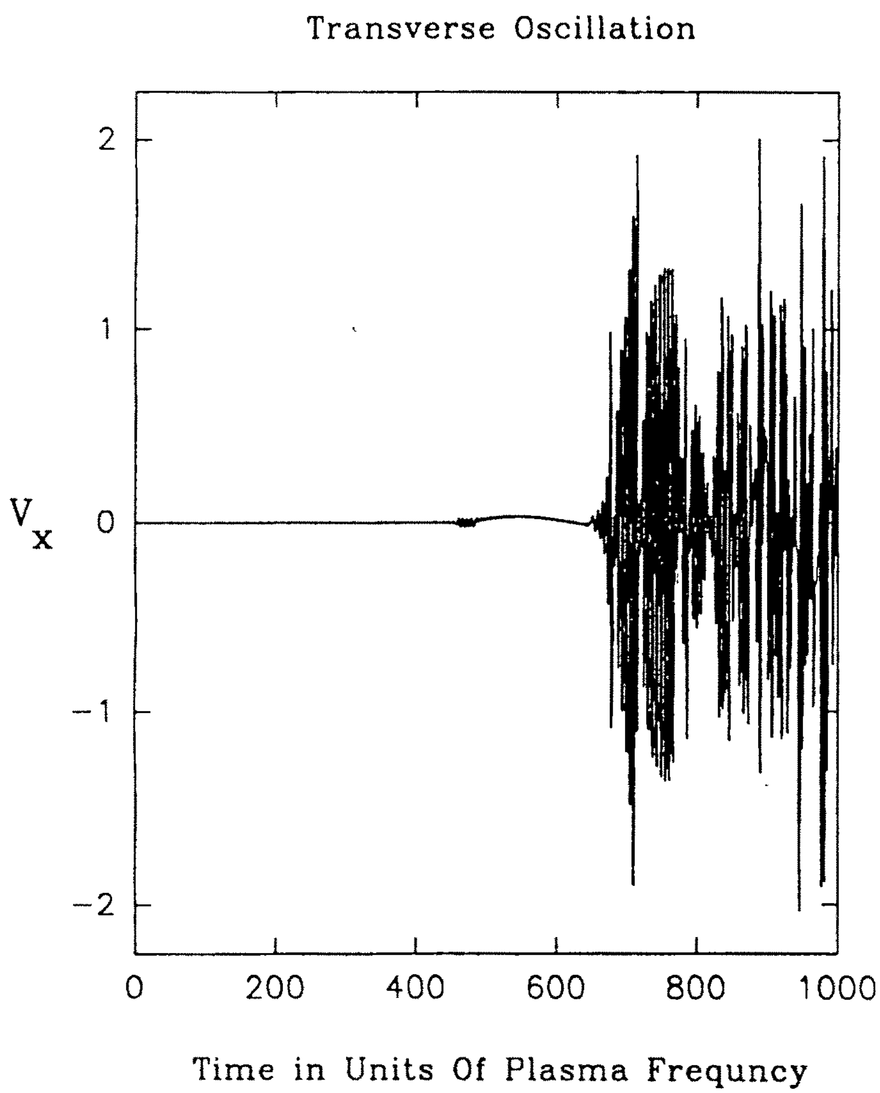

Figure 5.3: The Profile for V_x Oscillation

transverse field can also produce chaos in the otherwise regular longitudinal oscillation. Even though these transverse fields are orders of magnitude less,to begin with, but given sufficient enough time they become comparable to that of longitudinal oscillations. This onset of chaos is related to energy being equilibrated between the two components A_x and A_z . From the autocorrelation function one can find out the time taken by the system to reach a chaotic state of oscillation or the state of energy equilibrium. Secondly there exists some kind of a memory in the system that drives it to behave in the same way after a regular interval of time. Thirdly we have seen that the nonabelian nonlinearity is capable of setting the system into chaos. The factor on which the growth rate of oscillation in the x direction of the velocity depends, seems to be the quantity β, i.e the frame velocity. Totally chaotic regime shows that the energy transfer takes place around the most dominant frequency of oscillation and V_z component drives the V_x oscillation. Though We have not exhausted the space of all possible parameters and initial values for these system of equations, but we think we have been successful in showing a class of solutions where all the specialities as mentioned above, are all present.

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