Chapter 2

$Q\bar{Q}$Production In Presence Of Oscillating External Field

2.1 Introduction

In this chapter, we will discuss the process of formation of Quark Gluon Plasma in Relativistic Heavy Ion Collision (**RHIC**). In particular we will concentrate on the mechanism by which the initial beam energy in RHIC gets deposited in a small volume in the speculated form of quark gluon plasma through the production of quark anti-quark pairs. The process of quark anti-quark production in RHIC has attracted the attention of many workers for over a decade.

This complex process of pair production, inspite of being visited many a time by many workers taking into account different physical conditions, till today, stands as one of the most elegant model whose potential is far from being exhausted. The production of $q\bar{q}$ pairs from vacuum in the flux tube model [1], basically owes its existence to the classic paper of Schwinger[2], where in the context of Quantum Electro Dynamics (QED), it was shown that in the presence of very strong external electric field, QED vacuum becomes unstable and it starts emitting e^+e^- pairs at the expense of the electric field till the field strength falls bellow a critical value comparable to the square of the mass of the produced particles.

Along the same line, the $q\bar{q}$ pair production in RHIC is also assumed to take place by the decay of the flux tubes formed between the two receding nuclei due to the multiple exchange of soft gluons. This process continues till the energy stored in the chromo-electric field/unit length becomes less than the mass of the produced $q\bar{q}$ pairs. In addition to extending Schwinger's QED calculation to the QCD case, efforts have also been made to include effects such as, the screening of the external electric field[3], finite size of the nuclei[4], moving boundary conditions[5], radial confinement[6] etc. It is worth noting that in all these works the external chromo-electric field has always been considered to be constant in both space and time.

In this chapter we will contest the validity of this assumption and in fact argue that the basic nature of QCD lagrangian demands the electric field to be time dependent. The actual evaluation of $q\bar{q}$ pair production rate by us however has been carried out for an external field which is homogeneous in space but oscillating sinusoidally in time.

The organisation of this chapter is as follows. In section two we will review briefly the Schwinger mechanism followed by the physical picture of flux tube formation in relativistic heavy ion collisions. In section three we justify, from exact solutions of the classical SU(2) Yang-Mills equations, why the external chromo-electric field has to be time dependent rather than constant. This is followed by section four where we will try to give an order of magnitude estimate of the field strength and the frequency of oscillation attainable in relativistic heavy ion collision. In section five we compute the pair production rate in a time varying field with different values of field strength and frequency of oscillation. Lastly we conclude by stating the scope of further improvement of our results.

2.2 Schwinger Mechanism: A Brief Outline

The production of particle antiparticle pairs by a classical external field via Schwinger mechanism is a general phenomenon that reflects a much broader physical reality, i.e instability of vacuum under external perturbations. This idea has been used in a variety of theories in different contexts, ranging from QED, QCD, Transport theory[7], Relativistic Heavy Ion Collision, Gravitation[8], Early Universe[9] and even in String theory[10].In the following passage, we will elaborate on the physics[11] of this process for the simple case of QED.

Let us consider a system to consist of vacuum (including virtual particle antiparticle pairs) subjected to an external electric field. In order to create an on-shell particle antiparticle pair from the vacuum, the virtual particle antiparticle have to be moved away from each other over a distance d $\geq$ the compton wave length of the particles, with a corresponding energy loss (of the system) $\sim$ 2m. Now in the presence of an external electric field (with assumed strength $E \geq E_c \sim m^2$), because of vacuum polarisation,if the virtual particle-antiparticle are moved apart by a distance d the energy gained by the system, at the expense of the external field, will be gEd. If the distance d $\geq$ Compton wave length of the particles, the energy gained by the system in putting the pairs on shell becomes more than 2m. Since it is energetically always favourable for a system (i.e vacuum) to go to its lowest energy state, pairs will be emitted from vacuum till the field strength falls below the critical field strength E_c.

The production of $q\bar{q}$ pairs in RHIC has also been explained by the same principle via the flux tube model. This model was introduced independently by Low[1] and by Nussinov[1] to account for the observed scaling behaviour of scattering cross-sections in hadron hadron collisions.In nucleus nucleus collisions, it assumes that at high energy, when the two highly Lorentz contracted nuclei pass through each other, the partons of one nucleus interact with the partons of the other nucleus by the exchange of soft (color octet) gluons. If the fly by time of the nuclei is less than the time scale of interaction of the partons, the receding nuclei get randomly color charged by exchange of soft gluons. Since a colored object cannot exist free in nature the color octet partons in the receding nuclei get connected to each other by means of color flux tubes with color electric fields inside them. This color flux tube decays producing $q\bar{q}$ pairs in the same way as described previously for the QED case of Schwinger.

With this picture in mind, the dynamical evolution of the plasma produced in RHIC, including $q\bar{q}$ pair creation, has been studied[2-7] by many others. We however will be content to examine the effect of oscillating external chromo-electric field on the pair production rate, since this has not been investigated before.

2.3 Some Exact Solutions of Yang Mills Equations

In this section we will establish that, because of the presence of the nonlinear terms in the Lagrangian, the gluons produced in RHIC, polarise the medium between the two receding nuclei, generating an electric field that undergoes, characteristic non-linear, non-abelian oscillations in time. For this purpose we will make certain assumptions based on the geometry of the problem. These assumptions however do not change the qualitative nature of our observation.

The first assumption is that each of the color charged nucleus has a uniform distribution of color charge in the plane transverse to the direction of motion so that there exists no gradient of the fields in this direction. Our second assumption is that these color charges produce a chromo-electric field such that A_0 and A_z are the only nonzero potentials. Although in principle a magnetic field can also be present, we will not consider it here since it cannot transfer energy to the system to create pairs. Our third assumption is that the region between the two nuclei can be treated as vaçuum and we will neglect the curvature effects near the boundaries. With these simplifying assumptions, the dynamics of the gluon fields can essentially be described in (1+1) dimensions rather than (3+1) dimensions. Therefore in order to get information about the nature of the classical gluon fields one needs to solve the classical Yang-Mills field equations in (1+1) dimensions.

2.3.1 Solution of Yang Mills equations in (1+1) dimensions.

We next show that in (1+1) dimensions the Yang-Mills equations have a solution with a sinusoidally time varying component whose frequency depends on the amplitude. We first write the sourceless Yang Mills equation in (1+1) dimensions

$$D_\mu F^{\mu\nu} = 0 \tag{2.1}$$

where the Greek indices μ and ν take values 0 and 1 only. The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig\,[A_\mu,] \tag{2.2}$$

with g as the coupling constant and $g[A_\mu,]$ as the commutator bracket. Since we are working with an SU(2) color symmetry, A_μ is defined as $A_\mu = {A_\mu}^a \tau_a$ where τ_a are the generators obeying the commutation rules

$$\left[\tau^a, \tau^b\right] = i\varepsilon_{abc}\tau^c \tag{2.3}$$

The indices a,b,c takes values from one to three. Further, the only non zero components of the vector field in this case are A_0 and A_z, and we have chosen the axial gauge $A_z = 0$. With this choice of the gauge we get from equation (2.1) for $\nu = 0$,

$$\partial^2{}_z A_a{}^0 = 0 \qquad (2.4)$$

whose solution is

$$A_a{}^0(t) = \alpha_a(t) z + \beta_a \qquad (2.5)$$

Here α_a and β_a are arbitrary integration constants. In order to find out an exact solution of this equation we take α_a to depend on time and β_a to be a constant. Equation (2.1) for $\nu = 1$ gives

$$\partial_0 \partial_z A_a{}^0 + g\epsilon_{abc} A_b{}^0 \partial_z A_c{}^0 = 0 \qquad (2.6)$$

Substituting the solution (2.5) in equation (2.6) we arrive at

$$\dot{\alpha}_a(t) + g\epsilon_{abc}\alpha_c\beta_b = 0 \qquad (2.7)$$

One can derive a conservation law from this equation namely

$$\alpha_a(t)\alpha_a(t) = constant \qquad (2.8)$$

A summation over repeated indices is implied.

We solve this set of coupled first order linear differential equations by Euler's method; i.e we choose a solution of the form

$$\alpha_a(t) = a_a e^{pt} \qquad (2.9)$$

Substituting equation (2.9) in equation (2.7), we obtain a set of coupled algebraic equations whose solution is of the form

$$\alpha_1 = \beta_1 + \beta_1\beta_3 [\, e^{i\omega t} + e^{-i\omega t}] - i\omega\beta_2 [\, e^{i\omega t} - e^{-i\omega t}] \qquad (2.10)$$

$$\alpha_2 = \beta_2 + \beta_2\beta_3 [\, e^{i\omega t} + e^{-i\omega t}] + i\omega\beta_1 [\, e^{i\omega t} - e^{-i\omega t}] \qquad (2.11)$$

$$\alpha_3 = \beta_3 + \beta_3{}^2 [\, e^{i\omega t} + e^{-i\omega t}] - \omega^2 [\, e^{i\omega t} + e^{-i\omega t}] \qquad (2.12)$$

Here $\omega = [(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2]^{\frac{1}{2}}$.

Once the α's are known, one gets the solution for the A_0's by substituting equations (2.10), (2.11) and (2.12) in equation (2.5). Without giving the unnecessary mathematical details, the final expression is

$$A_0^1 = [\beta_1 + \beta_1\beta_3 [\, e^{i\omega t} + e^{-i\omega t}] - i\omega\beta_2 [\, e^{i\omega t} - e^{-i\omega t}]]\, z + \beta_1 \tag{2.13}$$

$$A_0^2 = [\beta_2 + \beta_2\beta_3 [\, e^{i\omega t} + e^{-i\omega t}] + i\omega\beta_1 [\, e^{i\omega t} - e^{-i\omega t}]]\, z + \beta_2 \tag{2.14}$$

$$A_0^3 = \left[\beta_3 + {\beta_3}^2 [\, e^{i\omega t} + e^{-i\omega t}] - \omega^2 [\, e^{i\omega t} + e^{-i\omega t}]\right] z + \beta_3 \ . \tag{2.15}$$

Thus from the solution it is clear that the electric field inside a chromo-electric flux tube oscillates with frequency $\omega = [(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2]^{\frac{1}{2}}$, which depends on the amplitude of oscillation.

It may be pointed out that there also exists an exact time dependent vacuum solution of (SU(2)) Yang-Mills equations of the type[12]

$$A_\mu^\alpha = (0, H\delta_1^\alpha, H\delta_2^\alpha, H\delta_3^\alpha) \tag{2.16}$$

where

$$H = \frac{B}{\sqrt{g}} cn \left[\sqrt{2g} B \,(t - t_o)\right] \tag{2.17}$$

In eq.(2.16), μ (=0,1,2,3) is the Lorentz index, α (=1,2,3) is the color index and δ_i^α is the Kronecker delta. In eq. (2.17) cn represents the Jacobi elliptic function and B is a constant determining the amplitude of the oscillating field. Physically, this solution represents a non-linear collective oscillation of gluons with a characteristic amplitude dependent time period $\sim \left(\sqrt{2g}B\right)^{-1}$, which is a manifestation of the intrinsic nonlinearity present in the system.

As we will see this time varying nature of the field changes the pair production rate quite significantly over that due to the constant field i.e the Schwinger estimate.

2.4 Estimating The Parameters

Having established the fact, that the chromo-electric field inside the flux tube (because of the self-interaction of the fields) should be oscillating in time, we next explore its consequences on the rate of spontaneous pair production from vacuum.

For this purpose, one can in principle take either of the exact solutions and compute the rate of pair production. But considering the computational difficulties associated in working with such exact solutions, we will content ourselves with a spatially homogeneous chromo-electric field that oscillates sinusoidally in time. We take the vector potential to be

$$A_\mu^a(x) \;=\; (0,0,0,\ A_a(t))\ and\ A_a(t) \;=\; -\frac{E_a \cos\omega_o t}{\omega_o} \qquad (2.18)$$

and for of SU(3) color symmetry a goes over (=1, 2,....8)

Here E_a's are constants and ω_0 is the characteristic collective frequency for the gauge fields. Now to determine the pair production rate one has to give an estimate of the frequency and amplitude of the external chromo-electric field produced in RHIC. For this purpose, we make use of the solution of Yang-Mills field equations given by equations(2.16)- (2.17).From this solution taking each $A^a \equiv A$ one can write the r.m.s chromo-electric field strength as

$$E \equiv \left(\sqrt{8}\right)\left(\sqrt{2}gB\right)\left(\tfrac{B}{\sqrt{g}}\right) = 4B^2 \qquad (2.19)$$

apart from an uninteresting constant.
Since $\omega_0 = \left(\sqrt{2}gB\right)$, and replacing B in terms of E from the equations above, we get the expression for frequency

$$\omega_0 = \sqrt{\tfrac{gE}{2}} \qquad (2.20)$$

in terms of a gauge invariant chromo-electric field defined as

$$E = \left[\textstyle\sum_{a=1}{}^{8} {E_a}^2\right]^{\frac{1}{2}} \qquad (2.21)$$

Once ω_0 is known in terms of the chromo-electric field strength, one is left with the determination of the strength of the external field attainable in RHIC. In order to estimate it, one has to first make an estimate of the color charge deposited on each of the receding nuclei after the collision. Following Kerman, Matsui and Svetitsky[13], it is usually assumed that at very high energy in a nucleon nucleus interaction multiple gluons are exchanged. In each interaction, with the exchange of each gluon there is an exchange of color charge t_a (where t_a is the matrix in the adjoint representation of the symmetry group). Thus after ν such exchanges of gluons, the total color charge that gets accumulated on the target nucleus is

$$\vec{T} = \sum_{j=1}^{\nu} \vec{t}_j \tag{2.22}$$

If the color orientations amongst these exchanged gluons are uncorrelated, one can assume, after ν such interactions, that the r.m.s color charge deposited on the target nucleus is

$$\langle T^2 \rangle^{\frac{1}{2}} = \sqrt{\nu} \langle t^2 \rangle \tag{2.23}$$

From this relation one can say that, after ν interactions, the amount of color charge deposited on the target nucleus is proportional to the square root of the number of interactions i.e

$$Q \propto \sqrt{\nu} \tag{2.24}$$

One can relate (see ref.13) the number of pairs produced to the number of interactions or the total color charge as

$$\frac{dN_{pair}}{dy} \propto \sqrt{\nu} \tag{2.25}$$

Here N_{pair} is the number of pairs produced and y is the rapidity. Moreover if one assumes the number of hadrons produced to be proportional to the number of pairs produced then

$$\left(\frac{dN}{dy}\right)_{pA} = \left(\frac{dN}{dy}\right)_{pp} \sqrt{\nu} \tag{2.26}$$

i.e the multiplicity for proton nucleus collision scales as the square root of the number of interactions times the multiplicity in proton proton collisions. So, from the multiplicities of the produced particles one can compute the number of collision that each nucleon undergoes in a p-A collision.

If σ_{p-p} and σ_{p-A} be the cross sections for proton proton and proton nucleus collisions then one can write phenomenologically that

$$\nu = A\frac{\sigma_{pp}}{\sigma_{pA}} \tag{2.27}$$

(where A is the mass number of the target nucleus).From simple geometrical considerations one can show that $\frac{\sigma_{pp}}{\sigma_{pA}}$ scales as $A^{-\frac{2}{3}}$ and hence the number of collisions from p-p to p-A should scale as $A^{\frac{1}{3}}$. This implies that the amount of color charge deposited in p-A collision on the target nucleus scales as $A^{\frac{1}{6}}$ times that in the p-p collision.

In high energy central collision of two heavy nuclei, the individual constituent nucleons of each nuclei can be thought of scattering through the other nuclei. So in the light of the foregoing discussion, total number of interactions, compared to p-p collision, should scale as $A_T^{\frac{1}{3}}\ A_P^{\frac{1}{3}}$, where A_T and A_P are the target and projectile mass numbers respectively. This implies, that the amount of color charge deposited should scale as $A_T^{\frac{1}{6}} A_P^{\frac{1}{6}}$ from p-p to A-A collision. The earlier relation implies that the chromo-electric field strength, should scale from p-p to A-A collision as $A_T^{\frac{1}{6}}\ A_P^{\frac{1}{6}}$.

After establishing the scaling behaviour of the chromo-electric field from p-p to A-A collision, the only task one is left with is to evaluate the strength of the chromo-electric field produced in p-p collisions. If the flux tube produced in p-p collision generates a string tension σ then the field energy stored per unit length of the tube is

$$E^2 = \frac{2\sigma}{area} \tag{2.28}$$

From Gauss law one can write[14]

$$E \text{ area} = g, \qquad \text{where g is the coupling constant.} \tag{2.29}$$

On using the equations (2.28) and (2.29) one can derive that

$$gE = 2\sigma \tag{2.30}$$

The quantity σ is usually evaluated from the Regge slope parameter and its value has been estimated to be around $0.2GeV^2$[14]. Because of the final state interactions[15] (basically screening effect), the effective field strength generated initially in p-p collision, gets reduced to around .2 GeV^2. Once we know that the field strength produced in p-p collision is 0.2 GeV^2 one can compute the value of the field produced in A-A collision, from the scaling law

$$E_{A_T-A_P} \sim A_T^{\frac{1}{6}} A_P^{\frac{1}{6}} E_{p-p} \tag{2.31}$$

Following Pavel and Brink[6] the magnitude of the field strength produced in the collision of S^{32} on S^{32} has been estimated to be, $gE \leq 0.6GeV^2$ and for U - U collisions it is $gE \leq 1.2GeV^2$. These values of gE imply a variation of ω_0 between 0.32 GeV to 0.87 GeV,a number obviously not close to zero. This nonzero value of ω_0 certainly implies that caution should be exercised before estimating, the number of particles produced in RHIC, using Schwinger's expression[2].

2.5 Estimation Of Pair Production Rate

Having obtained estimates of the field strengths and the frequencies of oscillation of the fields produced in RHIC, we will concentrate next on the computation of the pair production rate of spin zero bosons with SU(2) color symmetry, in the presence of a sinusoidally oscillating background chromo-electric field. For fermions the final result will get modified by numerical factors only. In the discussion of our calculation we will not provide derivation of the standard field theory results, instead we will refer to the sources where they could be found.
The probability that the vacuum remains vacuum, in the presence of an external field, can be written in terms of the S matrix as

$$|\langle 0 \mid S \mid 0\rangle|^2 \equiv |S_o(A)|^2 = exp\left[-\int d^4x W(x)\right] \tag{2.32}$$

where $\langle 0 \mid S \mid 0\rangle$ is the vacuum expectation value of S-matrix in the presence of the color potential A^a_μ and $W(x)$ is the pair creation probability per unit volume per unit time. The quantity S_0 can be shown [16] to be equal to

$$S_0 = Det\left(G^{-1}G_0\right) = expTr\left[ln\left(G^{-1}G_0\right)\right] \tag{2.33}$$

where G_0 and G are the free propagator and the propagator in presence of the external field respectively, defined as

$$G_0 = \frac{1}{P^2 - m^2 + i\varepsilon} \quad \text{and} \quad G = \frac{1}{(P - gA)^2 - m^2 + i\varepsilon} \tag{2.34}$$

The trace in equation (2.33) is defined over spinor, color and coordinate spaces. In terms of scattering operators T and $\bar{T}$ defined as

$$T = V + V\frac{1}{P^2 - m^2 + i\varepsilon}T \quad and \quad \bar{T} = V + V\frac{1}{P^2 - m^2 - i\varepsilon}\bar{T} \tag{2.35}$$

(2.36)

with $\bar{T} = \gamma^0 T^\dagger \gamma^0$ and $V = G_o{}^{-1} - G^{-1}$, one can show that

$$|S_o(A)|^2 = exp\left[Trln\left(1 - T\rho_+ T^+ \rho_-\right)\right] \tag{2.37}$$

$$W(x) = -tr\langle x \mid ln\left(1 - T\rho_+ T^+ \rho_-\right) \mid x\rangle \tag{2.38}$$

Here $\rho_\pm$ are the projectors over positive and negative energy states defined as

$$\rho_\pm = 2\pi\theta_\pm\left(p^2\right)\delta\left(p^2 - m^2\right)$$

It should be noted that the operators $T\,(T^+)$ as well as $\rho_\pm$ are matrices in color, spinor and coordinate space. In equation (2.37)the symbol Tr stands for integration over the continuous variables and trace over the color and spinor indices, whereas in equation (2.38) tr stands for trace over color and spinor indices only.
On expanding the logarithm in equation (2.38) and retaining the first term (i.e neglecting the production probability of 2 ,3 or more pairs) one gets

$$W = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} dt . \frac{1}{2\pi} \int \frac{d^3p}{(2\omega)^2} \mid \langle -\omega \mid T \mid \omega \rangle \mid^2 \tag{2.39}$$

Here $\omega = (p^2 + m^2)^{1/2}$ and m is the mass of the spin zero colored particle. The backward "scattering" amplitude $\langle -\omega \mid T \mid \omega \rangle$ is then evaluated by solving the color coupled Klein Gordon equations in external color potential. For the color SU(2) group the equations to be solved are (τ_α , $\alpha = 1,2,3$ are Pauli matrices).

$$\left[\left(\partial_o^2 - \nabla^2\right) + 2igA_\alpha \tau_\alpha \partial_3 + g\,(A_\alpha)^2 + m^2\right] \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = 0 \tag{2.40}$$

with appropriate asymptotic conditions in time.

More precisely, we look for solutions of eq. (2.40) having the form[17]

$$\begin{array}{llcl} t \to -\infty & \varphi_+(t) & = & e^{-i\omega t} + b_+ e^{i\omega t} \\ & \varphi_-(t) & = & e^{-i\omega t} + b_- e^{i\omega t} \\ t \to +\infty & \varphi_+(t) & = & a_+ e^{-i\omega t} \\ & \varphi_-(t) & = & a_- e^{-i\omega t} \end{array} \tag{2.41}$$

Since a negative energy particle at $t \to -\infty$ is equivalent to a positive energy antiparticle at $t \to +\infty$, the backward "scattering" amplitude $\langle -\omega \mid T \mid \omega \rangle$ and hence the pair creation probability, can be determined from the coefficients b_+ and b_-. Actually one has

$$W \propto \frac{\mid b_+ \mid^2 + \mid b_- \mid^2}{2} \tag{2.42}$$

In order to proceed with the solution of equation (2.40), it is easy to show that, the above equations can be decoupled by a unitary transformation in color space defined by,

$$U^+ = \begin{pmatrix} (E_3 + E)/N_1, & (E_1 - iE_2)/N_1 \\ (E_3 - E)/N_2, & (E_1 - iE_2)/N_2 \end{pmatrix} \tag{2.43}$$

where $E^2 = E_1^2 + E_2^2 + E_3^2$, $N_1^2 = 2E^2 + 2E_3E, N_2^2 = 2E^2 - 2E_3E$. The (column vector) wave function in turn transforms into

$$U^+ \begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} \tag{2.44}$$

For the spatially homogeneous system that we are considering (note that this ignores the confinement effect discussed by some earlier workers[6]), the decoupled equations are,

$$\left[\partial_o^2 + m^2 + p^2 \mp 2gp_3\left(\frac{E}{\omega_o}\right)\cos\omega_o t + g^2\left(\frac{E}{\omega_o}\right)^2\cos^2\omega_o t\right]\begin{pmatrix}\Psi_+\\ \Psi_-\end{pmatrix} = 0 \qquad (2.45)$$

with $p^2 \equiv p_1^2 + p_2^2 + p_3^2 \equiv p_\perp^2 + p_3^2$.
Following Brezin and Itzykson[17], these decoupled equations are solved using the boundary conditions

$$\begin{array}{rrcl} t \to -\infty & \Psi_+(t) & = & Ae^{-i\omega t} + Be^{i\omega t} \\ & \Psi_-(t) & = & Ce^{-i\omega t} + De^{i\omega t} \\ t \to +\infty & \Psi_+(t) & = & Ee^{-i\omega t} \\ & \Psi_-(t) & = & Fe^{-i\omega t} \end{array} \qquad (2.46)$$

After finding the coefficients A, B, C, D we finally express them in terms of b_+ and b_- respectively. We have solved for the coefficients A, B, C, D from equation (2.45) by W.K.B method, choosing a solution of the form

$$\begin{array}{rcl} \Psi_+(t) & = & \alpha_a(t)e^{-i\chi_a(t)} + \beta_a(t)e^{i\chi_a(t)} \\ \Psi_-(t) & = & \alpha_b(t)e^{-i\chi_b(t)} + \beta_b(t)e^{i\chi_b(t)} \end{array} \qquad (2.47)$$

where

$$\chi_a(t) = \int_0^t d\bar{t}\omega_a(\bar{t}) \qquad \text{and} \qquad \chi_b(t) = \int_0^t d\bar{t}\omega_b(\bar{t}) \qquad (2.48)$$

and assuming $\frac{gE}{m^2} \ll 1$ along with the conditions $\frac{\dot{\omega}_a(t)}{\omega_a{}^2(t)} \ll 1$ and $\frac{\dot{\omega}_b(t)}{\omega_b{}^2(t)} \ll 1$ where $\omega_a(t) = \left[m^2 + \left(p_3 - \frac{E}{\omega_0}t\right)^2\right]^{\frac{1}{2}}$ and $\omega_b(t) = \left[m^2 + \left(p_3 + \frac{E}{\omega_0}t\right)^2\right]^{\frac{1}{2}}$. One assumes here that the external field is switched on and off adiabatically.
From equation (2.46), we obtain an order of magnitude estimate of pair creation probability, in the case $\omega_o \ll m$,

$$W \simeq \frac{\alpha_s E^2}{2\pi}\frac{1}{g(\gamma) + \frac{1}{2}\gamma g'(\gamma)} exp\left[-\frac{\pi m^2}{gE}g(\gamma)\right] \qquad (2.49)$$

where

$$\begin{array}{rcl} g(z) & = & \frac{4}{\pi}\int_o^1 dy\left[\frac{1-y^2}{1+z^2y^2}\right]^{1/2} \\ and & & \\ \gamma & = & \frac{m\omega_o}{gE}, \quad \alpha_s = \frac{g^2}{4\pi} \end{array} \qquad (2.50)$$

As shown by Brezin and Itzykson[16], one can recover the static Schwinger limit from equations (2.49) and (2.50) by taking $\omega_o \to 0$ independently of gE in such a way that $\gamma = \frac{m\omega_o}{gE} \to 0$. In this case, one obtains the Schwinger result

$$W_s \simeq \frac{\alpha_s E^2}{2\pi} exp\left[-\frac{\pi m^2}{gE}\right] \tag{2.51}$$

To consider the case of oscillating non-abelian fields we must take ω_o to be dependent on E in the manner discussed after equation (2.20) i.e. that $\omega_o = \sqrt{gE/2}$. Equations (2.49) and (2.50) now show that $\gamma = \frac{m\omega_o}{gE} = \frac{m}{\sqrt{2gE}} = \frac{1}{2}\frac{m}{\omega_o} \gg 1$ and that the pair creation probability W takes the form of 'multigluon' production, viz.,

$$W_g \simeq \frac{\alpha_s E^2}{8}\left[\frac{g^2E^2}{4m^2\omega_o^2}\right]^{\frac{2m}{\omega_o}}\left[\omega_o = \sqrt{\frac{gE}{2}} \ll m\right] \tag{2.52}$$

where $\frac{2m}{\omega_o}$ is the minimum number of gluons required to produce a pair. Incidentally following Sakurai[17], one can also compute the pair production rate using ordinary perturbation theory, when $\omega_0 \gg m$. Here the transition amplitude is given by the S matrix element

$$S_{fi} = -g\langle q\bar{q} \mid \int d^4x \overline{\Psi}^{(-)}{}_{\alpha}\,(\gamma)_{\alpha\beta}\,\Psi^{(-)}{}_{\beta}A^a_\mu\tau^a \mid 0\rangle \tag{2.53}$$

Here Ψ are the quark fields operator, ${A_\mu}^a$ are the classical external fields and τ_a are the pauli matrices respectively. The square of this amplitude will give us the probability of transition from vacuum to $q\bar{q}$ pairs. An integration over the available phase space gives the total pair production probability. On taking the external field as sinusoidally oscillating in time and carrying out the integration one arrives at the pair production rate

$$W_p \simeq \frac{\alpha_s E^2}{6}\left(1+\frac{2m^2}{\omega_0^2}\right)\sqrt{\left(1-\frac{4m^2}{\omega_0^2}\right)} \tag{2.54}$$

If one considers the strong field limit i.e $gE \gg m^2$ then one can see that the perturbative formula for pair production reduces to

$$W_p \simeq \alpha_s E^2 \tag{2.55}$$

(ignoring the numerical factors). This result also follows from Schwinger's expression, since in the limit $m^2/gE \ll 1$ we can expand the exponential in powers of $\frac{m^2}{gE}$ and retain only the first term ignoring the others to arrive at the same expression. Next we consider the various limits, by defining $x = \frac{m^2}{gE}$ and $n = \frac{2m}{\omega_0}$. One can then see that the ratio between Schwinger and the multigluon production rate is

$$\frac{W_s}{W_g} \simeq (xn)^{2n} e^{-\pi x} \tag{2.56}$$

Since an exponential dominates over any finite order polynomial this expression shows that for $n \sim x \gg 1$ the multigluon ionisation process of vacuum dominates over the Schwinger process.

Before we obtain the numerical estimate of the pair production rate , we would like to comment on the numerical value of the particle mass to be used in the computation. In the literature the numerical estimate of the pair production rate has been carried out using constituent as well as current quark masses. In our view, since the flux tube model takes into account the localisation of color flux and the effect of confinement, it is more appropriate to consider constituent quark mass for numerical estimation. Moreover as has been discussed earlier, in order to produce an on shell $q\bar{q}$ pair from vacuum, the external field has to move them over a distance, of the order of compton wavelength ($\sim \frac{\hbar}{m_q c}$) of the particles. For current quark mass this distance is around $\sim 20fm$, which appears unreasonable for $A - A$ collisions. We therefore propose that for pair creation via flux tube model $\frac{\hbar}{m_q c} \leq 1fm$ i.e $m_q \geq 200MeV$.

In any case we have numerically evaluated the pair production rate using the expressions in the three limits i.e perturbative, multigluon ionisation and Schwinger, with different values of the chromo-electric field and mass. The results are shown in Table-I. They show the following features:

1.If m = 10 MeV, then for values of gE ranging from 0.05 GeV^2 to 1.5 GeV^2, the pair creation probability $W_s \approx W_g \approx W_p$. For $m_s = 150MeV$, W_p is larger in p-p collisions and W_g is significant in A - A collisions.

2.For the production of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, pairs with constituent quark masses and field strength $gE \leq 0.5GeV^2$ the pair creation probability W_p dominates in p-p and in A-A collisions. For $gE \geq 0.5GeV^2$ the multigluon ionisation of pairs from the vacuum is larger in A-A collisions.

Table Caption

Table 1. Pair creation probability W_s, W_g and W_p (in units $(\mathrm{fm})^{-4}$) for different values of mass m (GeV) and field strength gE ($(\mathrm{GeV})^2$).

Table 1

m	gE	W_s	W_g	W_p
0.01	0.05	0.021	0.015	0.022
	0.1	0.083	0.076	0.087
	0.2	0.334	0.331	0.350
	0.5	2.09	2.12	2.19
	1.0	8.35	8.35	8.75
	1.5	18.8	18.5	19.7
0.15	0.05	0.005	$\sim 2\times10^{-7}$	0.022
	0.1	0.041	$\sim 4\times10^{-4}$	0.087
	0.2	0.235	0.051	0.350
	0.5	1.81	3.03	2.19
	1.0	7.78	24.5	8.75
	1.5	18.0	65.9	19.7
0.300	0.05	7×10^{-5}	$\sim 2\times10^{-14}$	0.022
	0.1	5×10^{-3}	$\sim 6\times10^{-8}$	0.087
	0.2	0.081	7×10^{-4}	0.350
	0.5	1.19	1.06	2.19
	1.0	6.30	28.1	8.75
	1.5	15.6	112.5	19.7
0.500	0.05	3×10^{-9}	3×10^{-25}	0.022
	0.1	3×10^{-5}	6×10^{-14}	0.087
	0.2	7×10^{-3}	5×10^{-7}	0.350
	0.5	0.434	0.103	2.19
	1.0	3.81	17.5	8.75
	1.5	11.1	133.9	19.7

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