Chapter 4

Evolution In Phase Space

4.1 Introduction

In the previous two chapters color flux tube model was studied to understand the process of plasma formation in A-A collision. In this chapter we examine how the plasma will evolve before it reaches color and thermal equilibrium.

The study of this phase is crucial because, it will give information about the dynamic processes that are important for reaching equilibrium and also the time it would take to reach the equilibrium. Furthermore the signals for detecting QGP might get modified depending on the pre-equilibrium evolution of the system. Since we are interested in the pre-equilibrium phase of the plasma , we will study the real time phase space evolution of the plasma through kinetic[1] theory followed by hydrodynamic equations.

As the number of degrees of freedom for gluons are more than the same for quarks and moreover since they are massless, the production rate of the gluons will be more than that for the quarks. This has already been evaluated in reference[2]. One can also get them, (approximately) apart from the numerical constants coming from color and spin degrees of freedom, from the rate expressions obtained by us for quarks by setting the, mass for the quarks equal to zero. Due to the color factors the g-g cross section is larger than gq and qq cross sections and as a result of this the gluons will equilibrate[3] faster than the quarks. So in this chapter we will concentrate on the the pre-equilibrium evolution of the gluons.

In the pre-equilibrium phase, right after the nuclear collision, the quarks and gluons will interact by means of binary (perhaps 3 body, 4 body) collisions and also through collective interactions to bring the system to a state of thermal equilibrium. The pre-equilibrium description of plasma has been studied by many authors[4] using kinetic description, by putting a collision term on the right hand side of Boltzman-Vlasov equation for the plasma.

For quarks a binary collision term is justified to some extent if one assumes the number density of quarks to be very small. For gluons this kind of assumption is not justified because of the presence of 3 body, 4 body interaction term in the Lagrangian. Therefore instead of using the Boltzman-Vlasov equation we will use the Vlasov kinetic equation, with the underlying assumption that collective effects arising out of mean fields are more important than the collision terms. This would be the case when a typical time scale for collective behavior $(1/\omega_p)$ is much shorter than the collision time $\frac{1}{\nu_c}$ i.e. $\omega_p >> \nu_c$. Further more there must be enough number of particles in a Debye sphere, i.e n $\lambda_d{}^3 \gg 1$, so that the collective effects dominate. We follow the phase space evolution of the gluonic plasma, starting from the gauge covariant operator valued quantum kinetic equations of gluons given by Elze,Gyulassy and Vasak and taking its classical limit. The classical description of the gluonic plasma is obtained as we take the ensemble average of this equation and then set terms proportional to $\hbar$ to zero. This is justified for studying those collective effects where the waves with wave length $\lambda > \frac{\hbar}{mc}$. It is also worth recalling that classical approaches reproduce many of the collective phenomena in quantal systems.

The organisation of this chapter is as follows. In section two we start with the gauge covariant distribution function[1] for gluons described by Elze, Gyulassy and Vasak and discuss how to obtain a classical kinetic description for gluons from there. In the following section we study a simple model to examine whether non-abelian color dynamics can provide a new equilibration mechanism. Finally we conclude by discussing the scope of further improvement of our result.

4.2 Kinetic Equations

In RHIC when the plasma is produced, the particles will have a characteristic momentum distribution. For the purpose of separating collective effects from the non-collective ones we assume that there are two types of gluons present in the system. The ones with very high four momentum (i.e short scale lengths) describe particle like properties , whereas those with low four momentum, i.e those generated by the interaction amongst the high frequency gluons, describe the collective i.e wave like properties. Therefore, as a result of this assumption the low four momentum gluons are described by the Yang Mills field equations with a source term (4-current) on the right hand side , generated by the high momenta gluons.

We are going to describe here the dynamics of these high momentum gluon fields which will interact among themselves to bring the system close to color and thermal equilibrium. Presently we take only the interaction of these high momentum gluons among themselves, which will be described by a Boltzman-Vlasov like equation for the gluons.

To describe the dynamics of these gluons, following Elze, Gyalassy and Vasak (EGV) (Ref. EGV[1], Elze[5]) one starts with the gauge covariant distribution function for the gluons defined as

$$G_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-1/2\mathrm{y.D}(x)} \; \vec{F}^{\lambda}_{\mu}(x) \right] \left[e^{1/2\mathrm{y.D}(x)} \; \vec{F}_{\lambda\nu}(x) \right]^{\dagger} \tag{4.1}$$

which is an 3×3 matrix for $SU(2)$ case, expressed as a dyadic product of a 3 component vector (color) and its adjoint. In the component notation it can be written as

$$G^{ab}_{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\mathrm{ip.y}/\hbar} \left[e^{-1/2\mathrm{y.D}(x)} \; \vec{F}^{\lambda}_{\mu}(x) \right]^{a} \left[e^{1/2\mathrm{y.D}(x)} \; \vec{F}_{\lambda\nu}(x) \right]^{b} \tag{4.2}$$

Here $y.D = y^0 D_0 + y^1 D_1 + y^2 D_2 + y^3 D_3$ and

$$D_\mu = \partial_\mu - ig\,[\,A_\mu,] \tag{4.3}$$

$$F_{\mu\nu} = \frac{[\,D_\mu, D_\nu\,]}{\text{-ig}} \tag{4.4}$$

μ and ν go from 0 to 3 and a and b go from 1 to 3.

Now operating with $p^\mu.D_\mu$ term, on the distribution function one arrives at the kinetic equation of gluons (see ref. 1). Since we are interested in the classical description we set terms of the order of $\hbar$ equal to zero as in ref. (5) and from there arrive at the following expression.

$$p^\mu.D_\mu G_{\mu\nu} + g/2P^\sigma \partial_p^\tau \; [\; \mathcal{F}_{\sigma\tau}, G_{\mu\nu} \;]_+ \; = g\,(\,\mathcal{F}_{\mu\sigma} G^\sigma_\nu - \quad G_{\mu\sigma}\mathcal{F}^\sigma_\nu\,) \tag{4.5}$$

Here $[,]_+$ means anti-commutator, and

$$D_\mu = \partial_\mu - ig\,[\,\mathcal{A}_\mu,] \tag{4.6}$$

where

$$\mathcal{A}^{ab}_\mu \equiv -if_{\text{abc}} A^c_\mu$$
$$\mathcal{F}^{ab}_{\mu\nu} \equiv -if_{\text{abc}} F^c_{\mu\nu}$$

f_{abc} is the antisymmetric structure constant for $SU(2)$

and g is the coupling constant.

In general, with regard to Lorentz indices, $G_{\mu\nu}$ has a symmetric part[4-5] as well as an antisymmetric part. We neglect the antisymmetric part by taking a spin equlibration ansatz, i.e.

$$G_{\mu\nu}(x,p) = p_\mu p_\nu G(x,p)$$

where $G(x,p)$ is a Lorentz scalar function.

So with this ansatz the r.h.s of equation(4.5) vanishes and, the gluon kinetic equation in color component notation takes the form

$$p^\mu \partial_\mu G^{mn} + g p^\mu.A^c_\mu\,[\,f\text{cma}G^{an} \quad - \quad G^{ma} f\text{can}\,] + i\frac{g}{2} p^\sigma \partial^\tau_p$$
$$[\; f\text{ema}G^{an} + f\text{ean}G^{ma}]\; F^e_{\sigma\tau} = 0 \tag{4.7}$$

All repeated indices are to be summed over.

The assumption of spin equilibration i.e $G_{\mu\nu}(x,p) = p_\mu p_\nu G(x,p)$ leads to the following expression for the gluon current ref(4,5)

$$J_c^\mu = ig \int G_{ab} f_{abc} p^\mu d^4p \tag{4.8}$$

To study the collective behavior of the system, we solve the YM field equations with the current (equation(4.8)) on the right hand side. The basic idea here, as explained before, is that because of the self interaction the high momentum gluons generate a low momentum long wavelength mean field which in turn acts as a source term for a mean Yang-Mills field equations. For studying the collective properties one has to solve these equations self consistently,i.e

$$D_\mu F^{\mu\nu} = J^\nu \tag{4.9}$$

along with the gluon kinetic equations (equation(4.5)).

4.3 A New Mechanism For Equilibration

As mentioned earlier, in this section, we propose to analyse a simple model which exhibits mechanisms for equilibration arising entirely from the non-abelian nature of the color dynamics. In this model we assume that the equilibrium distribution function has the form

$$G_{ab}^{eq} = \frac{n_{ab}}{(e^{p_o\beta} - 1)} \tag{4.10}$$

Here n_{ab}'s are the elements of a matrix in color space p_o is the zeroth component of the four momentum and β is the temperature of the system and the important point is that, the off-diagonal elements of the distribution function are nonzero. In equilibrium, we have chosen the distribution to have a simple Bose-Einstein form, so as to avoid momentum space contribution to collective effects. The important point, that we would like to bring home, is the hitherto unconsidered role of the color degrees of freedom as a source of free energy. Further we take, the classical fields $F_{\mu\nu}$ and A in the kinetic equations (4.8) - (4.9) to be diagonal in color space(i.e abelian dominance[1,5] approximation) and the zeroth component of the vector field to be finite and other components are zero.

We then carry out a stability analysis of the resulting system of equations about the equilibrium distribution function G_{eq}^{ab}. On linearising the equations about the aforementioned equilibrium distribution, one arrives at

$$k_\mu p^\mu \delta G_{mk} = \frac{g}{2T} p_\mu F_e^{\mu o} \left[f_{ema} n_{ak} - f_{bek} n_{mb} \right] cosh^2 \left(\frac{p_o}{2T} \right) \tag{4.11}$$

Using equations (4.8) and (4.11) we have solved for the current produced by the fluctuations and it is

$$J_o{}^n = \frac{(gT)^2}{6}\left[\frac{\omega}{|k|} ln\left|\frac{\omega+k}{\omega-k}\right| - 2\right] A^n{}_o(k) \tag{4.12}$$

On using relation (4.9) we next get

$$k^2 A_o{}^a(k) = C(\omega, k)\left[2n_{kk}A_o{}^a - A_o{}^b(n_{ab} + n_{ba})\right] \tag{4.13}$$

Here the repeated indices are summed up and

$$C(\omega, k) = \frac{(gT)^2}{6}\left[\frac{\omega}{|k|} ln\left|\frac{\omega+k}{\omega-k}\right| - 2\right] \tag{4.14}$$

From equation (4.13) the matrix dispersion relation comes out to be

$$k^2 - C(\omega,k)\begin{bmatrix} 2(n_{22}+n_{33}) & -(n_{12}+n_{21}) & -(n_{13}+n_{31}) \\ -(n_{12}+n_{21}) & 2(n_{33}+n_{11}) & -(n_{32}+n_{23}) \\ -(n_{13}+n_{31}) & -(n_{32}+n_{23}) & 2(n_{22}+n_{11}) \end{bmatrix} = 0 \tag{4.15}$$

If we set $n_{11} = n_{22} = n_{33} = \frac{n}{2}$ and $n_{12} = n_{21} = n_{23} = n_{32} = n_{31} = n_{13} = s$ then ,in the long wavelength limit one gets the following dispersion relation (for the long wavelength gluons),

$$\omega^2 = \frac{3k^2(n-s)}{1+s-n} \tag{4.16}$$

From equation (4.16) we see that if $s > n$ there will be an instability in the system. Clearly the instability is related to the color degrees of freedom and would then drive the system towards a distribution which is diagonal in color space. This mechanism may provide us with some insight about the manner in which an arbitrary distribution function in color space becomes color diagonal and attains color equilibration.

4.4 Conclusion

In this chapter we have looked for the plasma oscillations in QGP through the semiclassical kinetic equations for gluons, derived by Elze, Gyulassy and Vasak. Though the dispersion relation has been derived under the approximations that the mean fields are basically abelian in nature and of them only A_o is finite, but these simplifying assumptions still carry some nontrivial nonabelian dynamical signatures in it. In particular the existence of the off-diagonal (in color space) components of the distribution function, is the signature of gluon gluon interactions , a purely non-abelian effect and is seen to be responsible for damping or instability. Incidentally on performing the same analysis with an equilibrium distribution function i.e diagonal in color space no such signature of instability or damping is found[5].

Usually, the damping, can originate from three different kinds of sources; for instance, it can be collisional relaxation damping, decay of plasmons into particle antiparticle pairs or gluon gluon pairs. Production of quark antiquark pairs from vacuum is similar to electron positron pair production through plasmon decay as encountered in high T QED plasma. On the the other hand gluon going to two gluons is a typical non-abelian effect, typical of QCD plasma. Since the physical situation we are considering here does not have any collisional relaxation process in it, and neither have we considered the presence of quarks and antiquarks here, so the existence of instability or damping corresponds to the last process. This damping signifies passage of energy from wave mode to particle mode. Conversely an instability would signify the passage of energy from particle mode to wave mode.

In our view, the non-abelian interactions amongst the gluons, try to take the system, with strong initial color fluctuations, to a stable equilibrium. To get a correct picture, of the physics of this process, one ought to solve these coupled partial non linear set of differential equations. Instead we will try to explore some special solutions numerically and some more of their collective properties under different approximation schemes.

References

1. H.T.Elze, M. Gyulassy and D. Vasak, Nucl. Phys. B276, (1986), 706
   H.T.Elze, M. Gyulassy and D. Vasak, Phys. Lett. 177B, (1986), 402
   D. Vasak M. Gyulassy and H.T.Elze, Ann. Phys.(N.Y) , 173, (1987), 462
   H.T.Elze and U.Heinz, in *Quark Gluon Plasma*, Ed. R.Hwa World Scientific, Singapore, 1990)

2. M. Gyulassy and A. Iwajaki, Phys. Lett.165B, (1985),157

3. E.V.Shuryak, Phys. Rev. Lett. 68, (1992), 3270

4. K. Geiger, Phys. Rev.D46, (1992), 4986
   S. Mrowcz'ynskii, in *Quark Gluon Plasma*, Ed. R.Hwa World Scientific, Singapore, 1990); and references therein

5. H.T. Elze, Z. Phys C38, (1988), 211