APPENDIX

In this appendix, several results are tabulated in theory section. Mainly the dp , dp_z integrals appearing in the second Born term of the HHOB theory are evaluated. All the integrals are evaluated using standard integration techniques (Gradshiteyn and Ryzhik, 1965). The evaluation of these integral are as follows :

A .1 Evaluation of integral $I_1(.......)$:

$$I_1(q^2, u^2, v^2) = \int_0^\infty \int_0^{2\pi} \frac{p \, dp \, d\phi}{(p^2 + u^2)\,(|q - p^2| + v^2)}$$

$$= \int_0^\infty \frac{p \, dp \, d\phi}{(p^2 + u^2)\,[\,(p^2 + q^2 + v^2)^2 - 4 q^2 p^2\,]^{1/2}}$$

$$= \frac{\pi}{E} \, \mathrm{Log}\left[\frac{(q^2 + v^2)\,(q^2 + v^2 + E) - u^2 (v^2 - q^2)}{u\,(E + v^2 - u^2 - q^2)}\right]$$

$$\{ \text{ where } E^2 = u^4 + (q^2 + v^2)^2 - 2 u^2 (v^2 - q^2) \}$$

$$= \frac{\pi}{u^2 - v^2} \, \mathrm{Log}\left(\frac{u^2}{v^2}\right) \qquad \text{when } q \longrightarrow 0$$

$$= \frac{\beta i^2 + y^2}{\beta i^2} \text{ when } \quad u^2 = \beta_i^{\,2} + y^2 \; ; \; v^2 = \beta_i^{\,2}$$

These integral results are used to obtain the closed form of $I_1(......)$ and $I_1'(...)$, which are occured in the imaginary part of the second Born approximation.

A.2 Evaluation of integral $I_2(.......)$:-

$$I_2(q^2, \beta_1^2, y^2) = \mathcal{P} \int d\underline{p} \; {}_{-\infty}\int^{+\infty} \frac{dP_z}{(p_z - \beta_1)\,(|\underline{q} - \underline{p}|^2 + p_z^2)\,(p^2 + p_z^2)}$$

$$= \int \frac{d\underline{p}}{(p^2 - |\underline{q} - \underline{p}|^2 + y^2)} \mathcal{P} \; {}_{-\infty}\int^{+\infty} \frac{dP_z}{(p_z - \beta_1)} \left[\frac{1}{p_z^2 - |\underline{q} - \underline{p}|^2} - \frac{1}{p_z^2 + p^2 + y^2} \right]$$

$$= -\Pi \beta_1 \left[\int \frac{d\underline{p}}{(|\underline{q} - \underline{p}|^2 - p^2 + y^2)\; p\; (p^2 + \beta_1^2)} - \int \frac{d\underline{p}}{(p^2 - |\underline{q} - \underline{p}|^2 + y^2)\,(p^2 + \beta_1^2 + y^2)\,(p^2 + y^2)^{1/2}} \right]$$

$$= -\Pi \beta_1 [\; I_2' - I_2'' \;]$$

where,

$$I_2' = {}_{0}\int^{\infty} \frac{p\,dp}{p(p^2 + \beta_i^2)} \; {}_{0}\int^{2\pi} \frac{d\phi}{q^2 + y^2 - 2qp\cos\phi}$$

$$= 2\Pi \; {}_{0}\int^{A} \frac{dp}{(p^2 + \beta_1^2)\,((q^2 + y^2)^2 - 4q^2p^2)^{1/2}} \qquad A = \frac{q^2 + p^2}{2q}$$

$$= \frac{2\Pi}{2q} \; {}_{0}\int^{A} \frac{dp}{(p^2 + \beta_1^2)\,(A^2 - p^2)} = \frac{\pi^2}{\beta_1((q^2 + y^2)^2 + 4q^2\beta_1^2)^{1/2}}$$

and,

$$I_2'' = {}_{0}\int^{\infty} \frac{p\,dp}{(p^2 + y^2)^{1/2}\,(p^2 + \beta_1^2 + y^2)} \quad I_\phi$$

$$I_\phi = {}_0\int^{2\pi} \frac{d\phi}{(y^2 - q^2 + 2qp\cos\phi)} = \frac{2\pi}{((y^2 - q^2) - 4q^2p^2)^{1/2}} ;$$

$$\left(\left|\frac{y^2 - q^2}{2q}\right| > p\ ,\ y^2 > q^2\right) = - {}_0\int^{2\pi} \frac{d\phi}{q^2 - y^2 - 2qp\cos\phi}$$

$$= \frac{-2\Pi}{((y^2 - q^2) - 4q^2p^2)^{1/2}} ; \quad \left|\frac{y^2 - q^2}{2q}\right| > p, \quad q^2 > y^2$$

$$= \frac{2\Pi\ \mathrm{sgn}\ (y^2 - q^2)}{((y^2 - q^2)^2 - 4q^2p^2)^{1/2}}$$

$$\therefore I_{2}'' = \frac{\pi\ \mathrm{sgn}\ (y^2 - q^2)}{2q}\ {}_0\int^{A^2} \frac{dx}{(x + p^2 + y^2)\ (x + y^2)^{1/2}\ (A^2 - x)^{1/2}}$$

$$\left(A^2 = \frac{(y^2 - q^2)^2}{4q^2}\right)$$

$$= \frac{\Pi\ \mathrm{sgn}\ (y^2 - q^2)}{\beta_1[(y^2 + q^2)^2 + 4q^2\beta_1^2]^{1/2}}\left[\frac{\Pi}{2} - \sin^{-1} A_1\right]$$

Now combining the integral term I_2' and I_2'', one can write the integral I_2 as follows :

$$I_2(q^2, \beta_1^2, y^2) = \frac{-\Pi^3}{((q^2 + y^2)^2 + 4q^2\beta_1^2)^{1/2}}\left[1 - \mathrm{sgn}(y^2 - q^2)\left\{\frac{1}{2} - \frac{\sin^{-1} A1}{\Pi}\right\}\right]$$

$$= \frac{-\Pi^3}{y^2}\left[\frac{1}{2} + \frac{\sin^{-1} A1}{\pi}\right] \quad \text{when } q \longrightarrow 0$$

where,

$$A_i = 1 - \frac{2\beta i^2 (y^2 - q^2)^2}{(y^2 + q^2)^2 (y^2 + \beta i^2)} ; \; A_1' = 1 - \frac{2\beta i^2}{y\,(y^2 + \beta^2)}$$

A.3 Evaluation of integral $I_3(.....)$: -

$$I_3(\beta_i^2 , y) = P \int \int_{-\infty}^{+\infty} \frac{dp \;\; dPz}{(p_z - \beta_i)\,(p^2 + p_z^2 + y^2)}$$

$$= 2\pi \int_0^{\infty} p\, dp \;\; P \int_{-\infty}^{+\infty} \frac{dPz}{(p_z - \beta_i)\,(p^2 + p_z^2 + y^2)}$$

$$= 2\pi \int_0^{\infty} p\, dp \;\; \frac{-\pi\beta i}{(p^2 + \beta_i^2 + y^2)\,(p^2 + y^2)^{1/2}}$$

$$= - 2\pi^2 \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{y}{\beta i}\right) \right]$$

A. 4 . Evaluation of the integral $I_4(.....)$: -

$$I_4(q^2, \beta_i^2, y_1^2, y_2^2) = P \int dp \int_{-\infty}^{+\infty} \frac{dPz\,(|q - p|^2 + y2^2 + Pz^2)^{-1}}{(p_z - \beta_i)\,(p^2 + p_z^2 + y_1^2)}$$

$$= \int dp\, P \int_{-\infty}^{+\infty} \frac{dPz}{Pz - \beta i} \left[\frac{1}{(p^2 - |q - p|^2 + y_1^2 - y_2^2)} \right.$$

$$\left. \left\{ \frac{1}{|q - p|^2 + p_z^2 + y_2^2} - \frac{1}{p^2 + p_z^2 + y_1^2} \right\} \right]$$

$$= \int dp\, P \int_{-\infty}^{+\infty} \frac{dPz}{(p_z - \beta_i)\,(p^2 - |q - p|^2 + Y)}$$

$$\left[\frac{1}{|q - p|^2 + Pz^2 + Y2^2} - \frac{1}{p^2 + Pz^2 + Y1^2} \right]$$

Evaluation of the dp and dp_z integrals is similar to that of I_2' (A . 2) evaluation. Using those earliar results, the present $I_4(....)$ can be written as follows :

$$I_4(q^2, \beta_1^2, y_1^2, y_2^2) = -\pi^2 [\text{sgn}(y^2 - q^2) \{ \frac{1}{2E1} - \frac{\sin^{-1} A1}{\pi E1} \}$$

$$- \text{sgn}(y^2 - q^2) \{ \frac{1}{2E1} - \frac{\sin^{-1} A2}{\pi E2} \}]$$

where,

$$E_1^2 = (Y + q^2)^2 + 4 q^2 (\beta_1^2 + y_1^2)$$

$$E_2^2 = (Y - q^2)^2 + 4 q^2 (\beta_1^2 + y_2^2)$$

$$A_1 = 1 - \frac{2\beta_1^2 (Y + q^2)^2}{[(Y + q^2)^2 + 4 q^2 y_1^2] (\beta_1^2 + y_1^2)} ; Y = y_1^2 - y_2^2$$

$$A_2 = 1 - \frac{2\beta_1^2 (Y - q^2)^2}{[(Y - q^2)^2 + 4 q^2 y_2^2] (\beta_1^2 + y_2^2)}$$

In the forward direction q ---> Ø , this $I_4(.....)$ tends to a finite form like $I_2(.....)$.

These typical results of the form (A . 2 , A . 3 , A . 4) are used to obtain the close form of $I_2(....)$, $I_2'(....)$ and $I_3(....)$, I_3' (...) and $I_4(...)$ in the real part of the second Born approximation.

It is also shown that all the integrals tend to a finite value in the forward direction i.e. when q ---> Ø.